International Journal of Innovative Computing, Information and Control
Volume **9**, Number **1**, January **2013** ISSN 1349-4198
pp. 153–164

DESIRED COMPENSATION ADAPTIVE ROBUST CONTROL OF MOBILE SATELLITE COMMUNICATION SYSTEM WITH DISTURBANCE AND MODEL UNCERTAINTIES

Jun Jiang[1], Qingwei Chen[1], Bin Yao[2] and Jian Guo[1]

[1]School of Automation
Nanjing University of Science and Technology
No. 200, Xiaolingwei, Nanjing 210094, P. R. China
jiangjun1985@gmail.com

[2]School of Mechanical Engineering
Purdue University
West Lafayette, IN 47907, USA

Received October 2011; revised March 2012

Abstract. *The tracking and stabilizing control of a typical ship-mounted mobile satellite communication system (MSCS) is studied in this paper. Severe parametric uncertainties and uncertain nonlinearities exist in the dynamic model of the MSCS. Therefore, a discontinuous projection-based desired compensation adaptive robust control (DCARC) strategy is adopted to compensate for both the parametric uncertainties and the uncertain nonlinearities to achieve precise posture trajectory tracking control. The resulting controller accurately tracks the desired posture and effectively handles the effects of various parameter variations and the hard-to-model nonlinearities. The designed DCARC controller and other three controllers are tested in the comparative simulations under three different situations. The simulation results show the tracking and stabilizing validity of the proposed controller.*
Keywords: Ship-mounted, Mobile satellite communication system, Desired compensation, Adaptive control, Nonlinear robust control, Uncertainties

1. **Introduction.** A mobile satellite communication system (MSCS) is a device installed on moving carriers such as ships and cars. It can globally communicate with satellites when the carrier is moving. It adjusts its posture automatically to eliminate the disturbance from the motion of the carrier and to maintain the pointing direction of the antenna. Then continuous satellite communication can be obtained.

The MSCS has many advantages, such as high speed, broadband, good adaptability, global communication. Therefore, it can be applied in many areas and is becoming a hotspot of research. Tseng and Teo [1] analyzed the kinematics of a two-axis ship-mounted tracking antenna and designed a fuzzy controller based on a driving motor model without modeling the whole system. Chang and Lin [2] compared the performances of the PI controller and the fuzzy controller based on a simple second-order linear model of a mobile tracking satellite antenna system. Ming et al. [3] used the H_∞ theory to design a controller for a simplified linear model of a ship-mounted tracking antenna system. Soltani et al. [4,5] designed a fault tolerant controller based on a kinematic model of a two-axis ship-mounted satellite tracking antenna. Liu et al. [6] designed a robust adaptive controller with disturbance observer for vehicular radar servo system, which combines the merits of disturbance observer, adaptive backstepping method and sliding mode control.

As reviewed above, most of these studies are based on simplified single-axis models, in which many kinematic and dynamic characteristics of the MSCS are neglected. The

MSCS is a typical multiple-input and multiple-output (MIMO) nonlinear system with multiple degrees of freedom (DOF). The subsystems of the MSCS are not independent with each other but are affected by the cross coupling effects. Moreover, the MSCS is installed on a moving carrier which is a non-inertial frame. The attitude of the whole system is affected by the pose and the motion of the moving base. The motion of carrier will exert some inertial forces on the MSCS. Therefore, if high precision tracking is needed, we need to design a feasible controller based on a more precise model of the MSCS.

In practice, the system not only has all the control difficulties associated with the coupled MIMO complex dynamics, but also the added difficulties with the large extent of model uncertainties existed in the system. In other words, there exist severe parametric uncertainties and uncertain nonlinearities, which come from uncompensated model terms due to unknown model errors, sensor errors and other disturbance. All these problems mentioned above are hard to solve by traditional linear control theories. Hence, advanced control theories are needed to effectively handle the parametric uncertainties and the uncertain nonlinearities in the system. Control of uncertain nonlinear dynamics has been one of the mainstream areas of focus in control community during the past twenty years. Some nonlinear control methods have been popular: adaptive control (AC) [7,8], robust adaptive control (RAC) [9,10], deterministic robust control (DRC) [11,12] and nonlinear robust control (NRC) [13,14]. However, those methods have been presented as competing control design approaches, with each having its own benefits and limitations.

The recently proposed adaptive robust control (ARC) has been shown to be a very effective control strategy for systems with both parametric uncertainties and uncertain nonlinearities [15-18]. By integrating the fundamentally different working mechanisms of the AC and the NRC, the developed ARC theory is able to preserve the theoretical performance results of both design approaches while overcoming their well-known practical performance limitations. The ARC approach effectively integrates adaptive control with robust control through utilizing on-line parameter adaptation to reduce the extent of parametric uncertainties and employing certain robust control laws to attenuate the effects of various uncertainties. In ARC, a projection-type parameter estimation algorithm is used to solve the design conflict between adaptive control and robust control. Thus, high final tracking accuracy is achieved while guaranteeing excellent transient performance. Desired compensation adaptive robust control (DCARC), which was proposed by B. Yao [19-21], is an improved ARC strategy based on a desired compensation adaptation law. The DCARC controller has the following desirable features: (a) the regressor can be calculated offline and thus on-line computation time can be reduced, and (b) the effect of measurement noise is minimized since the regressor does not depend on actual measurements. Consequently, a faster adaptation rate can be chosen in implementation to speed up the transient response and to improve overall tracking performance.

High tracking precision and good efficiency are two key performance requirements of the MSCS. Many sensors are installed on the MSCS to feedback the system's posture information. Under complicated working situations, there is unavoidable measurement noise in the sensors' signals. However, the features of the DCARC are very suitable for the MSCS. The regressor used in the DCARC does not depend on actual measurements, so the effect of measurement noise can be minimized. Furthermore, the regressor can be designed offline with a faster adaptation rate, so the transient response can be speeded up and the tracking performance can be improved. Then, high tracking precision and good efficiency of the MSCS can be achieved.

In this paper, the tracking and stabilizing control of the MSCS shown in Figure 1 is considered. Unlike the former studies based on single-axis models, a MIMO tri-axis nonlinear model, which includes the kinematic and dynamic features of the MSCS, is used

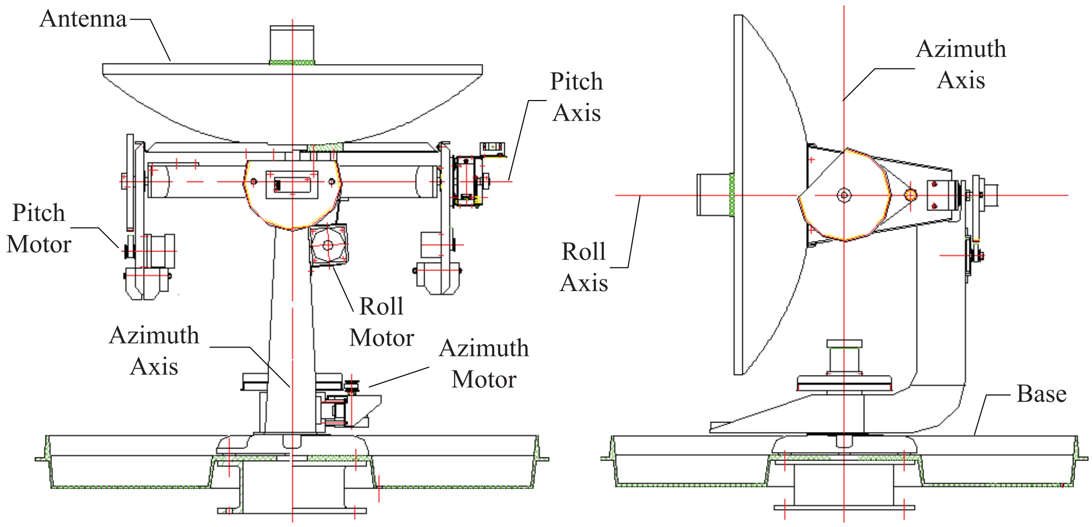

FIGURE 1. System composition

as the control object. Based on the tri-axis nonlinear model, the desired compensation adaptive robust control (DCARC) strategy is applied to solve the tracking and stabilizing control problem of the MSCS. With the designed DCARC controller, the parametric uncertainties and the lumped uncertain nonlinearities of the system can be reduced and the system can accurately track an aimed satellite with a satisfying transient response. Furthermore, a theoretical performance result is given and proved. At last, the designed DCARC controller and other three controllers are tested under three different cases in the comparative simulations. Simulation results show the tracking and stabilizing validity of the proposed DCARC controller.

2. **Problem Formulation.** A typical ship-mounted tri-axis MSCS is studied in this paper as shown in Figure 1. It mainly consists of three subsystems. From the outside to the inside, they are the azimuth subsystem, the roll subsystem and the pitch subsystem, respectively. The antenna for receiving and transmitting satellite signals is installed on the pitch subsystem. A bracket block with attitude sensors is installed coaxially with the pitch axis. The azimuth subsystem and the pitch subsystem are used for tracking the aimed satellite by adjusting the azimuth angle (yaw angle) and the pitch angle of the antenna. In the meantime, the rotations of the three subsystems are used to eliminate the disturbance in the motion of the carrier and keep the pointing direction of the antenna.

2.1. **Dynamic model.** The MSCS shown in Figure 1 is a typical MIMO nonlinear system with multiple DOF. The dynamic model of the MSCS based on a ship at sea can be described by

$$\bar{M}(x)\ddot{x} + \bar{C}(x, \dot{x}, \omega)\dot{x} + \bar{N}(x, \dot{x}, \omega) + \tilde{D} = \bar{\tau} \tag{1}$$

where $\bar{M}$ is the inertial matrix, $\bar{C}$ is called the Coriolis martrix, and $\bar{C}(x, \dot{x}, \omega)\dot{x}$ gives the Coriolis and centrifugal force terms in the equations of motion. $\bar{N}(x, \dot{x}, \omega)$ is the inertial force terms exerted by the motion of the ship. $\tilde{D}$ is the lumped uncertain nonlinearities, which come from uncompensated model terms due to unknown model errors, sensor errors and other disturbance. Note that $\bar{C}$ and $\bar{N}$ are not only dependent on x and $\dot{x}$ but also the motion of the ship ω_b and $\dot{\omega}_b$, which can be obtained from the sensors. For simplicity, we use ω to represent the disturbance ω_b and $\dot{\omega}_b$ from the ship.

The dynamic Equation (1) satisfies the following properties [21]:

Property 2.1. *$\bar{M}(x)$ is symmetric and positive definite, and there exist $\bar{k}_m > 0$ and $\bar{k}_M > 0$ such that $\bar{k}_m I_{3\times3} \le \bar{M}(x) \le \bar{k}_M I_{3\times3}$.*

Property 2.2. *$\dot{\bar{M}} - 2\bar{C} \in \mathbb{R}^{3\times3}$ is a skew-symmetric matrix. For any 3×1 vector x we have*

$$x^T(\dot{\bar{M}} - 2\bar{C})x = 0 \tag{2}$$

Property 2.3. *$\bar{M}(x)$, $\bar{C}(x, \dot{x}, \omega)$ and $\bar{N}(x, \dot{x}, \omega)$ can be linearly parametrized in term of the moments of inertia β and written as*

$$\bar{M}(x)\ddot{x}_r + \bar{C}(x, \dot{x}, \omega)\dot{x}_r + \bar{N}(x, \dot{x}, \omega) = Y(x, \dot{x}, \dot{x}_r, \ddot{x}_r, \omega)\beta \tag{3}$$

where $Y(x, \dot{x}, \dot{x}_r, \ddot{x}_r, \omega)$ is the regressor, $\beta = [I_{1x}, I_{1y}, I_{1z}, I_{2x}, I_{2y}, I_{2z}, I_{3x}, I_{3y}, I_{3z}]^T$ is the moment of inertia vector, $\dot{x}_r$ and $\ddot{x}_r$ are any reference vectors.

2.2. **Control aim.** The control aim of the MSCS is to make the antenna point to the satellite accurately and keep this pointing direction when the carrier is moving. Essentially, that is to track a desired space posture, which can be described by the yaw, pitch, roll angles of the antenna, and adjust the pose of the system by the rotations of the three subsystems to eliminate the disturbance from the motion of the carrier. So, the ultimate goal is to track the desired yaw, pitch, roll angles of the antenna and maintain them unchanged.

2.3. **Trajectory planning.** Suppose x_r is the reference command in the workspace. In practical application, not only the final tracking but also a quick and good transient process is desired. In most time, the initial posture of the antenna is not the same as the reference one ($x(0) \neq x_r(0)$). If the tracking error is too large, the transient of tracking cannot be guaranteed due to the limitations of the real system.

Trajectory planning can be used to solve this problem. Through a proper trajectory generation algorithm with the initial value $x_d(0)$ chosen the same as $x(0)$, one can always generate a feasible desired trajectory $x_d(t)$ that converges to the reference command $x_r(t)$ with a prescribed transient. $x_d(t)$ can be designed offline by the performance specifications according to the structure of the system.

3. Desired Compensation Adaptive Robust Control.

3.1. **Design issues to be addressed.** Generally the system is subjected to parametric uncertainties due to the moments of inertia β, and uncertain nonlinearities represented by $\tilde{D}$ that come from uncompensated model terms due to unknown model errors, sensor errors and other disturbance.

In order to use parameter adaptation to reduce the parametric uncertainties for an improved performance, it is necessary to linearly parameterize the model of the MSCS. Noting the property (3), the dynamic model (1) can be rewritten as

$$\bar{M}(x)\ddot{x} + \bar{C}(x, \dot{x}, \omega)\dot{x} + \bar{N}(x, \dot{x}, \omega) + \tilde{D} = Y(x, \dot{x}, \dot{x}, \ddot{x}, \omega)\beta + \tilde{D} = \bar{\tau} \tag{4}$$

Although there exist parametric uncertainties and uncertain nonlinearities, the boundaries of the uncertainties may be known by estimations or measurements in the practical application of the MSCS. Therefore, the following practical assumption is normally made:

Assumption: The extent of the parametric uncertainties and uncertain nonlinearities are known, i.e.,

$$\beta \in \Omega_\beta \triangleq \{\beta : \beta_{\min} < \beta < \beta_{\max}\} \tag{5}$$

$$\tilde{D} \in \Omega_{\tilde{D}} \triangleq \left\{\tilde{D} : \left\|\tilde{D}\right\| \le \delta_{\tilde{D}}\right\} \tag{6}$$

where $\beta_{\min} = [I_{1x\min}, \cdots, I_{3z\min}]^T$ and $\beta_{\max} = [I_{1x\max}, \cdots, I_{3z\max}]^T$ are the known lower and upper bounds of the unknown parameter vector β, respectively. $\delta_{\tilde{D}}(t)$ is a known bounded function, which is the bound of the uncertain nonlinearities $\tilde{D}$. In (5), the operation $<$ for two vectors is performed in terms of the corresponding elements of the vectors.

3.2. **Projection mapping.** Let $\hat{\beta}$ denote the estimates of β and $\tilde{\beta} = \hat{\beta} - \beta$ the estimation errors. In view of (5), the following projection-type parameter adaptation law is used to guarantee that the parameter estimates remain in the known bounded region all the time.

$$\dot{\hat{\beta}} = Proj_{\hat{\beta}}(\Gamma\sigma) \tag{7}$$

$$Proj_{\hat{\beta}}(\bullet_i) = \begin{cases} 0 & \text{if } \hat{\beta}_i = \beta_{i\max} \text{ and } \bullet_i > 0 \\ 0 & \text{if } \hat{\beta}_i = \beta_{i\min} \text{ and } \bullet_i < 0 \\ \bullet_i & \text{otherwise} \end{cases} \tag{8}$$

where Γ is a positive definite diagonal matrix of adaptation rates and σ is a parameter adaptation function to be synthesized later. It can be shown [16] that for any adaptation function σ, the projection mapping used in (7) guarantees

$$\begin{aligned} &\mathbf{P1} \quad \hat{\beta} \in \Omega_\beta \triangleq \left\{ \hat{\beta} : \beta_{\min} \le \hat{\beta} \le \beta_{\max} \right\} \\ &\mathbf{P2} \quad \tilde{\beta}^T \left(\Gamma^{-1} Proj_{\hat{\beta}}(\Gamma\sigma) - \sigma \right) \le 0 \quad \forall \sigma \end{aligned} \tag{9}$$

3.3. **Desired compensation adaptive robust controller design.** Define a switching-function-like quantity as

$$p = \dot{e} + K_1 e = \dot{x} - \dot{x}_{eq} \qquad \dot{x}_{eq} \triangleq \dot{x}_d - K_1 e \tag{10}$$

where $e = x - x_d$ is the trajectory tracking error vector, x_d is the desired trajectory to be tracked by x, and K_1 is a positive definite diagonal feedback matrix. If p converges to a small value or zero, then e will converge to a small value or zero since the transfer function from p to e is stable. In this sense, controlling the output tracking error e is the same as regulating p.

The state realization of (10) is

$$\dot{e} = Ae + Bp \tag{11}$$

where $A = -K_1$ and $B = I_{3\times3}$. Because (11) is stable, there exists an s.p.d. solution P for any s.p.d. matrix Q for the following Lyapunov equation,

$$A^T P + PA = -Q \tag{12}$$

and the exponentially converging rate $\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}$ can be any value by assigning the poles of A to the far left plane.

Noting (10) and the property (3),

$$\bar{M}(x)\ddot{x}_{eq} + \bar{C}(x, \dot{x}, \omega)\dot{x}_{eq} + \bar{N}(x, \dot{x}, \omega) = Y(x, \dot{x}, \dot{x}_{eq}, \ddot{x}_{eq}, \omega)\beta \tag{13}$$

$$\bar{M}(x_d)\ddot{x}_d + \bar{C}(x_d, \dot{x}_d, \omega)\dot{x}_d + \bar{N}(x_d, \dot{x}_d, \omega) = Y(x_d, \dot{x}_d, \dot{x}_d, \ddot{x}_d, \omega)\beta \tag{14}$$

Combining (4) and (13), the control input and the model uncertainties are related to p through a first-order dynamics given by

$$\bar{M}(x)\dot{p} + \bar{C}(x, \dot{x}, \omega)p = \bar{\tau} - Y(x, \dot{x}, \dot{x}_{eq}, \ddot{x}_{eq}, \omega)\beta - \tilde{D} \tag{15}$$

Let $h(x, \dot{x}, \dot{x}_{eq}, \ddot{x}_{eq}, \omega)$ be a bounding function satisfying

$$\left\| Y(x_d, \dot{x}_d, \dot{x}_d, \ddot{x}_d, \omega)\tilde{\beta} - \tilde{D} \right\| \le h(x_d, \dot{x}_d, \dot{x}_d, \ddot{x}_d, \omega) \tag{16}$$

For example, choose

$$h(x_d, \dot{x}_d, \dot{x}_d, \ddot{x}_d, \omega) = \|Y(x_d, \dot{x}_d, \dot{x}_d, \ddot{x}_d, \omega)\| \, \beta_M + \delta_{\tilde{D}} \tag{17}$$

where $\beta_M = \|\beta_{\max} - \beta_{\min}\|$.

According to [21], it can be proved that there are known non-negative bounded scalars $\gamma_1(t)$, $\gamma_2(t)$, $\gamma_3(t)$ and $\gamma_4(t)$, which depend on the desired trajectory and A only, such that the following inequality is satisfied

$$\|Y(x_d, \dot{x}_d, \dot{x}_d, \ddot{x}_d, \omega)\beta - Y(x, \dot{x}, \dot{x}_{eq}, \ddot{x}_{eq}, \omega)\beta\| \leq \gamma_1 \|e\| + \gamma_2 \|p\| + \gamma_3 \|p\| \, \|e\| + \gamma_4 \|e\|^2 \tag{18}$$

Noting the structure of (15), a desired compensation adaptive robust control law shown in Figure 2 is designed as follows:

$$\bar{\tau} = \bar{\tau}_a + \bar{\tau}_s, \quad \bar{\tau}_a \triangleq Y(x_d, \dot{x}_d, \dot{x}_d, \ddot{x}_d, \omega)\hat{\beta} \tag{19}$$

where $\bar{\tau}_a$ is the adjustable model compensation needed for achieving perfect tracking, and $\bar{\tau}_s$ is a robust control function having the form of

$$\bar{\tau}_s = \bar{\tau}_{s1} + \bar{\tau}_{s2}, \quad \bar{\tau}_{s1} = -K_{s1}p - K_e e - \gamma_{ep} \|e\|^2 p \tag{20}$$

where $\bar{\tau}_{s1}$ is used to stabilize the nominal system, p is defined in (10), K_{s1} is any symmetric positive definite matrix, $K_e = P$ is defined in (12) and γ_{ep} is a positive scalar. K_{s1}, K_e and γ_{ep} are large enough such that

$$\begin{aligned} &\lambda_{\min}(K_{s1}) \geq \varepsilon_0 + \gamma_2 + \gamma_5 + \gamma_7 \\ &\lambda_{\min}(Q) \geq 2(\varepsilon_0 + \gamma_6 + \gamma_{10}) \\ &\gamma_{ep} \geq \gamma_8 + \gamma_9 \end{aligned} \tag{21}$$

where ε_0 is any positive scalar, and γ_5, γ_6, γ_7, γ_8, γ_9 and γ_{10} are any positive scalars satisfying

$$\begin{aligned} \gamma_5\gamma_6 &= \gamma_1^2 \\ \gamma_7\gamma_8 &= \gamma_3^2 \\ \gamma_9\gamma_{10} &= \gamma_4^2 \end{aligned} \tag{22}$$

$\bar{\tau}_{s2}$ is a robust feedback term used to attenuate the effect of model uncertainties. $\bar{\tau}_{s2}$ is synthesized to satisfy the following robust performance conditions

$$\begin{aligned} &i) \quad p^T \left\{ \bar{\tau}_{s2} + Y(x_d, \dot{x}_d, \dot{x}_d, \ddot{x}_d, \omega)\tilde{\beta} - \tilde{D} \right\} \leq \varepsilon \\ &ii) \quad p^T \bar{\tau}_{s2} \leq 0 \end{aligned} \tag{23}$$

where ε is a design parameter which can be arbitrarily small. Essentially, $i)$ of (23) shows that $\bar{\tau}_{s2}$ is synthesized to dominate the model uncertainties coming from both parametric

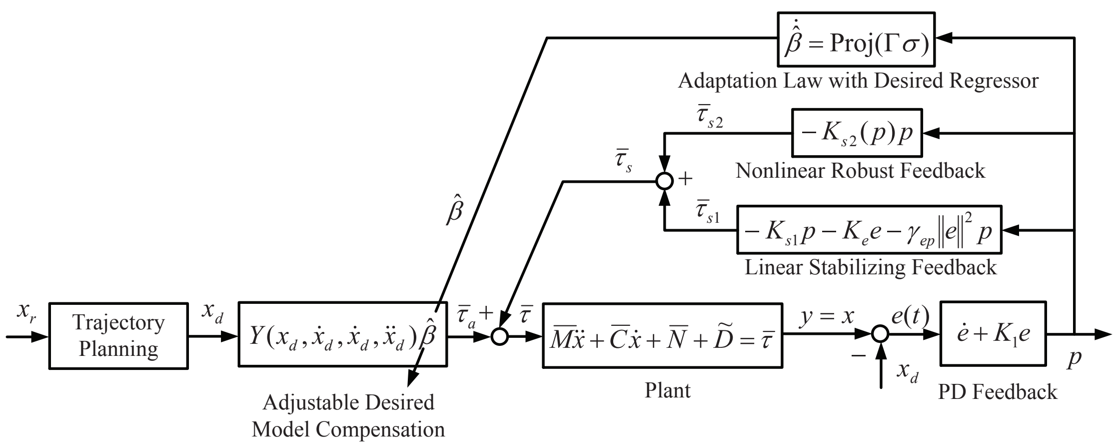

FIGURE 2. Desired compensation adaptive robust controller

uncertainties $\tilde{\beta}$ and uncertain nonlinearities $\tilde{D}$, and ii) of (23) is to make sure that $\bar{\tau}_{s2}$ is dissipating in nature so that it does not interfere with the functionality of the adaptive control part $\bar{\tau}_a$. How to choose $\bar{\tau}_{s2}$ to satisfy constraints like (23) can be found in [17].

With this DCARC control law, the tracking error dynamics shown in (15) can be rewritten as

$$\begin{aligned}\bar{M}(x)\dot{p}+\bar{C}(x,\dot{x},\omega)p =& -K_{s1}p-K_e e-\gamma_{ep}\|e\|^2 p+Y(x_d,\dot{x}_d,\dot{x}_d,\ddot{x}_d,\omega)\beta\\ &-Y(x,\dot{x},\dot{x}_{eq},\ddot{x}_{eq},\omega)\beta+\bar{\tau}_{s2}+Y(x_d,\dot{x}_d,\dot{x}_d,\ddot{x}_d,\omega)\tilde{\beta}-\tilde{D}\end{aligned} \tag{24}$$

and the following theoretical performance holds:

Theorem 3.1. *With the projection type adaption law (7) and an adaptation function of $\sigma=-Y(x_d,\dot{x}_d,\dot{x}_d,\ddot{x}_d,\omega)^T p$, the DCARC law guarantees that:*

A. In general, all signals are bounded. Furthermore, the positive definite function V_s defined by

$$V_s=\frac{1}{2}p^T\bar{M}(x)p+\frac{1}{2}e^T P e \tag{25}$$

is bounded above by

$$V_s\le \exp(-\lambda t)V_s(0)+\frac{\varepsilon}{\lambda}[1-\exp(-\lambda t)] \tag{26}$$

where $\lambda=\frac{2\varepsilon_0}{\max\{\bar{k}_M,\lambda_{\max}(P)\}}$ and $\bar{k}_M I_{3\times 3}$ is the upper bound of $\bar{M}(x)$.

B. If after a finite time t_0 there exist parametric uncertainties only (i.e., $\tilde{D}=0$, $\forall t\ge t_0$), then, in addition to results in A, zero final tracking error is also achieved, i.e., $e\to 0$ and $p\to 0$ as $t\to\infty$.

Proof: Noting Property 2.2 and (12), the time derivative of V_s given by (25) is

$$\begin{aligned}\dot{V}_s &= p^T\bar{M}(x)\dot{p}+\frac{1}{2}p^T\dot{\bar{M}}(x)p+\frac{1}{2}\dot{e}^T P e+\frac{1}{2}e^T P\dot{e}\\ &= p^T\left[\bar{M}(x)\dot{p}+\bar{C}(x,\dot{x},\omega)p\right]+\frac{1}{2}p^T\left[\dot{\bar{M}}(x)-2\bar{C}(x,\dot{x},\omega)\right]p\\ &\quad+\frac{1}{2}(p-K_1 e)^T P e+\frac{1}{2}e^T P(p-K_1 e)\\ &= p^T\left[\bar{M}(x)\dot{p}+\bar{C}(x,\dot{x},\omega)p\right]+\frac{1}{2}e^T(-K_1^T P-PK_1)e+p^T P e\\ &= p^T\left[\bar{M}(x)\dot{p}+\bar{C}(x,\dot{x},\omega)p\right]-\frac{1}{2}e^T Q e+p^T P e\end{aligned} \tag{27}$$

Noting (18), (23) and (24)

$$\begin{aligned}\dot{V}_s &= p^T\Big[-K_{s1}p-K_e e-\gamma_{ep}\|e\|^2 p+Y(x_d,\dot{x}_d,\dot{x}_d,\ddot{x}_d,\omega)\beta-Y(x,\dot{x},\dot{x}_{eq},\ddot{x}_{eq},\omega)\beta\\ &\quad+\bar{\tau}_{s2}+Y(x_d,\dot{x}_d,\dot{x}_d,\ddot{x}_d,\omega)\tilde{\beta}-\tilde{D}\Big]-\frac{1}{2}e^T Q e+p^T P e\\ &= -p^T K_{s1}p-\frac{1}{2}e^T Q e-\gamma_{ep}\|e\|^2\|p\|^2+p^T\left[\bar{\tau}_{s2}+Y(x_d,\dot{x}_d,\dot{x}_d,\ddot{x}_d,\omega)\tilde{\beta}-\tilde{D}\right]\\ &\quad+p^T[Y(x_d,\dot{x}_d,\dot{x}_d,\ddot{x}_d,\omega)-Y(x,\dot{x},\dot{x}_{eq},\ddot{x}_{eq},\omega)]\beta\\ &\le -p^T K_{s1}p-\frac{1}{2}e^T Q e-\gamma_{ep}\|e\|^2\|p\|^2+\gamma_1\|p\|\|e\|+\gamma_2\|p\|^2+\gamma_3\|p\|^2\|e\|\\ &\quad+\gamma_4\|p\|\|e\|^2+\varepsilon\end{aligned} \tag{28}$$

Applying the inequality

$$b_1|y_1||y_2|\le b_2 y_1^2+b_3 y_2^2 \quad \forall y_1,y_2\in R\ b_1,b_2,b_3 \tag{29}$$

where $4b_2b_3 = b_1^2$ to (28), we have

$$\begin{aligned}\dot{V}_s &\leq -p^T K_{s1} p - \frac{1}{2} e^T Q e - \gamma_{ep} \|e\|^2 \|p\|^2 + \gamma_5 \|p\|^2 + \gamma_6 \|e\|^2 + \gamma_2 \|p\|^2 + \gamma_7 \|p\|^2 \\ &\quad + \gamma_8 \|p\|^2 \|e\|^2 + \gamma_9 \|p\|^2 \|e\|^2 + \gamma_{10} \|e\|^2 + \varepsilon \\ &= -p^T \left[K_{s1} - I_{3\times3}(\gamma_2 + \gamma_5 + \gamma_7)\right] p - e^T \left[\frac{1}{2} Q - I_{3\times3}(\gamma_6 + \gamma_{10})\right] e \\ &\quad - (\gamma_{ep} - \gamma_8 - \gamma_9) \|p\|^2 \|e\|^2 + \varepsilon \end{aligned} \tag{30}$$

If (21) is satisfied, (30) becomes

$$\dot{V}_s \leq -\varepsilon_0(\|p\|^2 + \|e\|^2) + \varepsilon \leq -\lambda V_s + \varepsilon \tag{31}$$

Integrating (31) leads to (26). So, A of Theorem 3.1 is proved.

Now consider the situation in B of Theorem 3.1, i.e., $\tilde{D} = 0$, $\forall t \geq t_0$. Choose a positive definite function V_a as

$$V_a = V_s + \frac{1}{2} \tilde{\beta}^T \Gamma^{-1} \tilde{\beta} \tag{32}$$

Noting (28), condition *ii*) of (23), and **P2** of (9), the derivative of V_a satisfies

$$\begin{aligned}\dot{V}_a &= \dot{V}_s + \tilde{\beta}^T \Gamma^{-1} \dot{\tilde{\beta}} \\ &\leq -\varepsilon_0(\|p\|^2 + \|e\|^2) + p^T \bar{\tau}_{s2} + p^T Y(x_d, \dot{x}_d, \dot{x}_d, \ddot{x}_d, \omega) \tilde{\beta} + \tilde{\beta}^T \Gamma^{-1} \dot{\tilde{\beta}} \\ &\leq -\varepsilon_0(\|p\|^2 + \|e\|^2) + \tilde{\beta}^T \Gamma^{-1} \left(\dot{\hat{\beta}} + \Gamma Y(x_d, \dot{x}_d, \dot{x}_d, \ddot{x}_d, \omega)^T p\right) \\ &= -\varepsilon_0(\|p\|^2 + \|e\|^2) + \tilde{\beta}^T \Gamma^{-1} (\dot{\hat{\beta}} - \Gamma \sigma) \\ &\leq -\varepsilon_0(\|p\|^2 + \|e\|^2) \end{aligned} \tag{33}$$

in which the fact that the unknown parameter vector is assumed to be constant has been used in obtaining $\dot{\tilde{\beta}} = \dot{\hat{\beta}}$. Thus, $\forall t$, $V_a(t) \leq V_a(0)$, which leads to $p \in \mathcal{L}_\infty$ and $\tilde{\beta} \in \mathcal{L}_\infty$. Furthermore, $\forall t$, integrating (33),

$$\int_0^t \varepsilon_0(\|p(v)\|^2 + \|e(v)\|^2) dv \leq -(V_a(t) - V_a(0)) \leq V(0) \tag{34}$$

which implies $p \in \mathcal{L}_2$. Since all signals are bounded, noting (24), thus $\dot{p} \in \mathcal{L}_\infty$. So, p is uniformly continuous. By Barbalat's lemma, $p \to 0$ as $t \to \infty$.

4. **Comparative Simulation Results.** In this section, the effectiveness of the proposed method is demonstrated by simulations. Here, four control methods, an adaptive controller (AC), a deterministic robust controller (DRC), an adaptive robust controller (ARC) and the proposed desired compensation adaptive robust controller (DCARC), are compared in three cases: 1) tracking and stabilizing without disturbance; 2) tracking and stabilizing with non-persistent disturbance; 3) tracking and stabilizing with persistent disturbance.

Suppose the orientation of the aimed satellite is $x_r = [0°, 45°, 60°]$, and the initial posture of the MSCS is $x(0) = [-30°, 0°, 0°]$. The ship is hit by the wave on three different orientations. The yaw, pitch and roll orientations are all affected by the angular disturbance $15\sin(0.25\pi t)°$, which are known from the sensors. The system parameters in simulation are given in Table 1. I_1, I_2 and I_3 are the moments of inertia of the three subsystems, respectively. $\hat{\beta}(0)$ is the initial value of $\hat{\beta}$. $\hat{\beta}_{\max}$ and $\hat{\beta}_{\min}$ are the upper and lower bounds of $\hat{\beta}$, respectively.

TABLE 1. Parameters in simulations

Parameters	Value
I_1	$diag[0.788, 0.716, 0.852]$
I_2	$diag[0.465, 0.432, 0.448]$
I_3	$diag[0.147, 0.153, 0.135]$
$\hat{\beta}(0)$	$[0.8; 0.8; 0.8; 0.45; 0.45; 0.45; 0.15; 0.15; 0.15]$
$\hat{\beta}_{\max}$	$[0.9, 0.9, 0.9, 0.5, 0.5, 0.5, 0.2, 0.2, 0.2]$
$\hat{\beta}_{\min}$	$[0.65; 0.65; 0.65; 0.4; 0.4; 0.4; 0.1; 0.1; 0.1]$
K_1	$diag[12, 12, 12]$
K_{s1}	$diag[4.6, 4.6, 4.6]$
Γ	$0.1I_{9\times 9}$

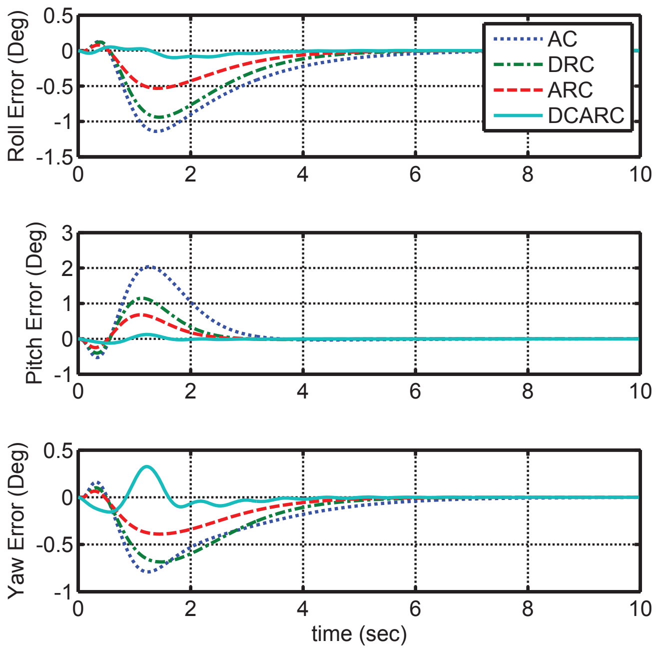

FIGURE 3. Tracking and stabilizing without disturbance

4.1. **Tracking and stabilizing without disturbance.** In the first situation, the carrier is stationary ($\omega_b = 0$ and $\dot{\omega}_b = 0$). Therefore, $\bar{N}(x, \dot{x}, \omega) = 0$, which means that there is no inertial force from the ship. The tracking results are shown in Figure 3.

As shown, when there is no disturbance, the four controllers can track the desired trajectories accurately with good transient responses. The trajectory tracking errors $e \to 0$ as $t \to \infty$, which verifies the excellent tracking capability of the proposed algorithms. Furthermore, the tracking errors of the ARC and the DCARC converge to zero more quickly than the AC and the DRC. The DCARC also has the smallest transient tracking error and the quickest tracking speed.

4.2. **Tracking and stabilizing with non-persistent disturbance.** In the second situation, the ship is hit by the wave before $t = 4s$. After $t = 4s$, the disturbance disappears. Therefore, $\bar{N}(x, \dot{x}, \omega) \neq 0$ before $t = 4s$, which means that there are inertial forces from

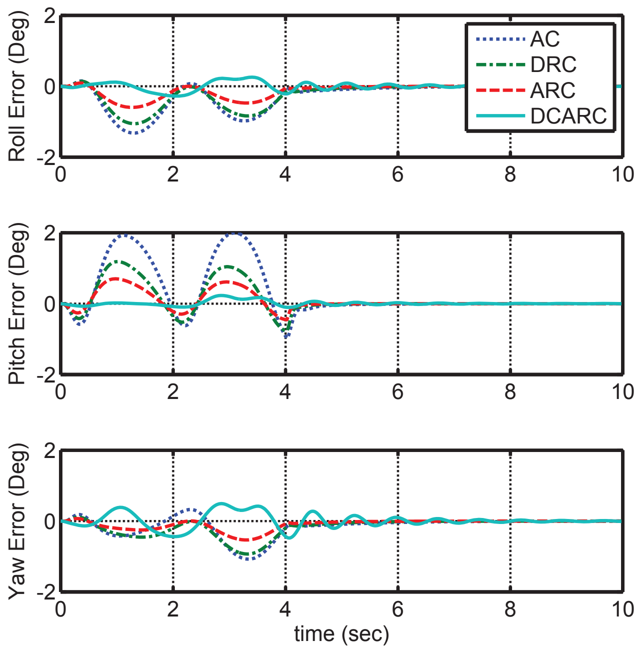

FIGURE 4. Tracking and stabilizing with non-persistent disturbance

the ship before $t = 4s$. The purpose of this case is to test the performances of the four controllers when the environment changes. The tracking results are shown in Figure 4.

When the disturbance exists, the four controllers can track the desired trajectories quickly and accurately. The tracking errors e are small. The DCARC perform much better than the other three controllers. When the disturbance is gone, $e \to 0$ as $t \to \infty$. the tracking errors of the ARC and the DCARC converge to zero more quickly than the AC and the DRC. The DCARC performs better than the ARC on roll angle and pitch angle tracking, but a little worse on the yaw angle tracking. Because, at $t = 4s$, the yaw angle tracking error of the DCARC is coincidently large, but the yaw angle tracking error of the ARC is small. So, it seems that the yaw angle tracking error of the DCARC converge to zero a little bit slower than the ARC. On the whole, the DCARC performs best in the four.

4.3. **Tracking and stabilizing with persistent disturbance.** In the third situation, the ship is hit by the wave all the time. Disturbance exists all the time even in the initial tracking period. It requires that the MSCS can track the aimed satellite and eliminate the disturbance from the carrier in the same time.

As shown in Figure 5, when the disturbance exists, the four controllers can track the desired trajectories quickly and accurately. The tracking trajectory errors e alway remain in small regions. Furthermore, the DCARC has the smallest transient tracking errors in the four controllers.

The results in three cases all indicate that the DCARC and the ARC performs much better than the AC and the DRC. Due to unchanged parameter estimations, there are unmatched model errors in the DRC all the time. So, the large tracking errors come into existence. Fortunately, the robust part of the DRC works. So, the DRC can track the desired trajectory eventually. Although the AC updates the parameter estimations all the time, the robustness of the AC is not very good. When there exists disturbance, the

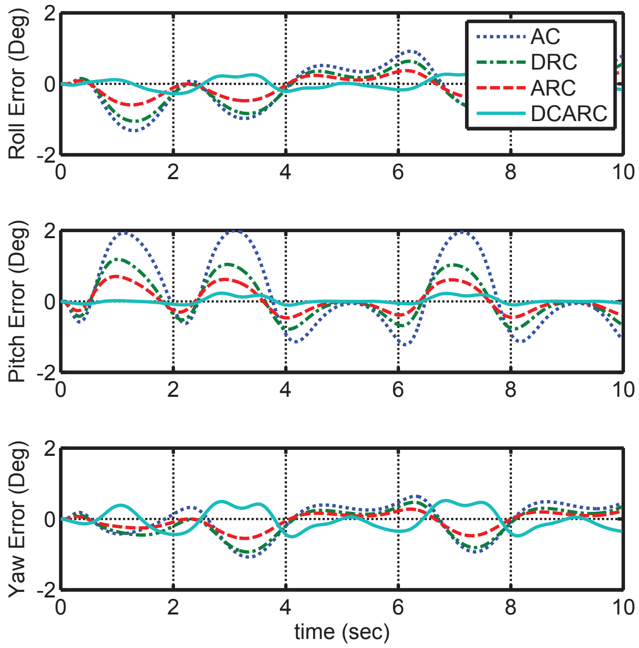

Figure 5. Tracking and stabilizing with persistent disturbance

tracking errors of the AC are even worse than the DRC. The ARC theory effectively integrates adaptive control with robust control through utilizing online parameter adaptation to reduce the extent of parametric uncertainties and employing certain robust control laws to attenuate the effects of various uncertainties. In ARC, a projection-type parameter estimation algorithm is used to solve the design conflict between adaptive control and robust control. Thus, high final tracking accuracy is achieved while guaranteeing excellent transient performance. The DCARC performs a little better than the ARC. Because the regressor in the DCARC only depends on the states of the desired trajectory, which can reduce the effect of measurement noise. Moreover, a faster adaptation rate is chosen in the DCARC to speed up the transient response and to improve overall tracking performance.

5. **Conclusion.** The tracking and stabilizing control of a typical ship-mounted mobile satellite communication system is studied in this paper. Unlike the former studies on MSCSs, a MIMO tri-axis nonlinear model, which includes the kinematic and dynamic features of the MSCS, is used as the control object. In the tri-axis model, there are not only inertial forces and coupling nonlinear terms but also parametric uncertainties and uncertain nonlinearities. Hence, the desired compensation adaptive robust control theory is applied to deal with large parametric uncertainties and uncertain nonlinearities of the system. Based on the dynamic model of the system, a DCARC controller with trajectory planning is designed to make the system to track an aimed satellite with a satisfying transient response. The DCARC controller and other three controllers are tested under three different cases in the comparative simulations. The simulation results show the tracking and stabilizing effectiveness of the proposed DCARC controller.

Acknowledgment. This work is supported in parts by the National Natural Science Foundation of China (Grant No. 61074023 and 60975075), the Natural Science Foundation of Jiangsu Province of China (BK2008404), the Science and Technology Pillar Program of Jiangsu Province of China (Grant No. BE2009160), and the Innovation

Project of Graduate Students of Jiangsu Province of China (CXZZ_0254). The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

REFERENCES

[1] H. C. Tseng and D. W. Teo, Ship-mounted satellite tracking antenna with fuzzy logic control, *IEEE Trans. on Aerospace and Electronic Systems*, vol.34, no.2, pp.639-645, 1998.

[2] P. K. Chang and J. M. Lin, Mobile satellite antenna tracking system design with intelligent controller, *WSEAS Trans. on Systems and Control*, vol.3, no.5, pp.435-446, 2008.

[3] A. Ming, T. Yamaoka, T. Kida and M. Satoh, Accuracy improvement of ship mounted tracking antenna for satellite communications, *Proc. of IEEE Conf. Mechatronics & Automation*, Niagara Falls, Ont., Canada, pp.1369-1374, 2005.

[4] M. N. Soltani, R. Izadi-Zamanabadi and R. Wisniewski, Reliable control of ship-mounted satellite tracking antenna, *IEEE Trans. on Control Systems Technology*, vol.19, no.1, pp.221-228, 2011.

[5] M. N. Soltani, R. Izadi-Zamanabadi and R. Wisniewski, Robust FDI for a ship-mounted satellite tracking antenna: A nonlinear approach, *Proc. of IEEE Conf. Control Applications*, San Antonio, TX, USA, pp.757-762, 2008.

[6] X. Liu, Q. Huang and Y. Chen, Robust adaptive controller with disturbance observer for vehicular radar servo system, *International Journal of Control, Automation and Systems*, vol.9, no.1, pp.169-175, 2011.

[7] M. Krstic, I. Kanellakopoulos and P. V. Kokotovic, *Nonlinear and Adaptive Control Design*, Wiley, New York, 1995.

[8] R. A. Freeman, M. Krstic and P. V. Kokotovic, Robustness of adaptive nonlinear control to bounded uncertainties, *Proc. of IFAC World Congress*, vol.F, pp.329-334, 1996.

[9] J. S. Reed and P. A. Ioannou, Instability analysis and robust adaptive control of robotic manipulators, *IEEE Trans. on Robotics and Automation*, vol.5, no.3, 1989.

[10] M. M. Polycarpou and P. A. Ioannou, A robust adaptive nonlinear control design, *Proc. of American Control Conference*, pp.1365-1369, 1993.

[11] V. I. Utkin, *Sliding Modes in Control Optimization*, Springer Verlag, 1992.

[12] A. S. I. Zinober, *Deterministic Control of Uncertain Control System*, Peter Peregrinus Ltd., London, United Kingdom, 1990.

[13] R. A. Freeman and P. V. Kokotovic, Design of softer robust nonlinear control laws, *Automatica*, vol.29, no.6, pp.1425-1437, 1993.

[14] Z. Qu, *Robust Control of Nonlinear Uncertain Systems*, John Wiley and Sons, Inc., 1998.

[15] B. Yao and L. Xu, Adaptive robust control of linear motors for precision manufacturing, *International J. of Mechatronics*, vol.12, no.4, pp.595-616, 2002.

[16] B. Yao, F. Bu, J. T. Reedy and G. C. T. Chiu, Adaptive robust control of single-rod hydraulic actuators: Theory and Experiments, *IEEE/ASME Transactions on Mechatronics*. vol.5, no.1, pp.79-91, 2000.

[17] B. Yao and M. Tomizuka, Adaptive robust control of SISO nonlinear systems in a semi-strict feedback form, *Automatica*, vol.33, no.5, pp.893-900, 1997.

[18] B. Yao and M. Tomizuka, Adaptive robust control of MIMO nonlinear systems in semi-strict-feedback forms, *Automatica*, vol.37, no.9, pp.1305-1321, 2001.

[19] B. Yao, Desired compensation adaptive robust control, *ASME Journal of Dynamic Systems, Measurement, and Control*, vol.131, no.6, pp.1-7, 2009.

[20] B. Yao, Advanced motion control: From classical PID to nonlinear adaptive robust control, *Proc. of IEEE International Workshop on Advanced Motion Control*, Nagaoka, Japan, pp.815-829, 2010.

[21] B. Yao, Adaptive robust control of robot manipulators: Theory and comparative experiments, *Proc. of the 2nd CWC on ICIA*, pp.442-447, 1997.