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IMPROVED DELAY-DEPENDENT ROBUST EXPONENTIAL STABILIZATION CRITERIA FOR UNCERTAIN TIME-VARYING DELAY SINGULAR SYSTEMS

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Abstract. *This paper is concerned with the stability and stabilization for singular systems with time-varying delay. Firstly, by defining a novel Lyapunov function, a delay-dependent stability criterion, which ensures that the nominal unforced singular time-varying delay system is regular, impulse free and asymptotically stable, is established in terms of integral inequality approach (IIA) and linear matrix inequalities (LMIs). Then based on the obtained criteria, the exponential robust stability and stabilization problems are solved and the explicit expressions of the desired state feedback control laws are also given. Numerical examples are given to demonstrate the effectiveness and the benefits of the obtained results.*
Keywords: Exponential stability, Time delay singular systems, Linear matrix inequality (LMI), Integral inequality approach (IIA)

1. **Introduction.** Singular systems, which are also known as descriptor systems, semi-state-space systems and generalized state-space systems are dynamic systems whose behaviors are described by both differential equations (or difference equations) and algebraic equations. These systems can preserve the structure of practical systems and have extensive applications in power systems, robotic systems and networks [2,3,5,6]. Since singular time-delay systems are matrix delay differential equations coupled with matrix difference equations, the study for such systems is much more complicated than that for standard state-space time-delay systems. For singular systems with delays, several kinds of simple Lyapunov-Krasovskii functionals, i.e., functionals parameterized with constant matrices, have been proposed, which lead to different levels of conservatism due to the different model transformations and the bounding techniques for some cross-terms [1-3,14,15,17,18]. Recently, there has been a growing interest in the study of such more general class of delay systems; see [1-3,5-7,13-15,17,18], and the references therein.

On the other hand, many uncertain factors exist in practical systems. Uncertainty in a control system may be attributed to modeling errors, measurement errors, parameter variations and a linearization approximation [9]. Therefore, study on the robust stability of uncertain singular systems with time-varying delay becomes significantly important. The results obtained will be very useful to further research for roust stability and control design of uncertain singular control systems with time-varying delay. To the best of our knowledge, there is few contribution dealing with the problems of robust stability singular systems with delay [6,7,13-15,17,18].

Formally speaking, these conditions provide only the asymptotic stability of singular time-delay systems. In [12], the global exponential stability for a class of singular systems with multiple constant time delays is investigated and an estimate of the convergence rate of such systems is presented. One may ask if there exists a possibility to use the linear matrix inequality (LMI) approach for deriving exponential estimates for solutions of singular time-varying delay systems. In [16], exponential stability conditions in terms of LMIs are given but no estimate of the convergence rate is presented. In [15], robust exponential stability of uncertain singular Markovian jump time-delay systems was also investigated. To the best of our knowledge, the robust exponential stabilization problem for singular time-varying delay systems has not yet been fully investigated.

In this paper, the problem of delay-dependent exponential robust stabilization for uncertain singular systems with time-varying state delay is investigated. First, an improved delay-dependent asymptotic stability criterion for normal singular time-varying delay systems is given, which is less conservative. This robust stabilization criterion, which is obtained without using model transformation and bounding technique for cross-product terms, is much less conservative and includes some existing results as its special case. Using this result, the problem of state feedback robust exponential stabilization for uncertain singular time-varying delay system is discussed. Finally, numerical examples are given to illustrate the effectiveness of the result. All results are derived in the LMI framework and the solutions are obtained by using LMI toolbox of Matlab.

2. **Stability Analysis.**

Consider the following uncertain singular system with time-varying delay in the state

$$E\dot{x}(t) = (A_0 + \Delta A_0(t))x(t) + (A_1 + \Delta A_1(t))x(t - h(t)) + (B + \Delta B(t))u(t), \quad t > 0 \quad (1a)$$

with the initial condition

$$x(\theta) = \phi(\theta), \quad t \in [-h, 0] \quad (1b)$$

where $x(t) \in R^n$ is the state vector of the system; A_0, A_1 and B are known real matrices with appropriate dimensions. The matrix $E \in R^{n\times n}$ maybe singular, without loss generality, we suppose rank $E = r \leq n$; $h > 0$ denotes time delay. $\phi(\theta)$ denotes the initial function. $h(t)$ is a time varying delay in the state, and h is an upper bound on the delay $h(t)$ and $h(t)$ is a time-varying delay satisfying

$$0 \leq h(t) \leq h, \quad \dot{h}(t) \leq h_d, \quad (2)$$

where h and h_d are positive constants.

The time-varying parameter uncertainties $\Delta A_0(t)$, $\Delta A_1(t)$ and $\Delta B(t)$ are assumed to be in the form of

$$\begin{bmatrix} \Delta A_0(t) & \Delta A_1(t) & \Delta B(t) \end{bmatrix} = MF(t) \begin{bmatrix} N_0 & N_1 & N_b \end{bmatrix} \quad (3)$$

where M, N_0, N_1 and N_b are known real constant matrices with appropriate dimensions, and $F(t)$ is an unknown, real, and possibly time-varying matrix with Lebesgue-measurable elements satisfying

$$F^T(t)F(t) \leq I, \quad \forall t. \quad (4)$$

Now, the objective is to design a stabilizing state-feedback controller in the form of

$$u(t) = Kx(t) \quad (5)$$

where $K \in R^{m\times n}$ is a gain matrix to be determined. Our aim is to develop a delay-dependent robust stabilization method which provides a controller gain K as well as an upper bound h of the delay such that the closed-loop system is stable.

To facilitate the following discussion, we will introduce some definitions and lemmas, which are essential for the development of our main results.

Definition 2.1. [5] *The pair* (E, A_0) *is said to regular if* $\det(sE - A_0)$ *is not identically zero.*

Definition 2.2. [5] *The pair* (E, A_0) *is said to be impulse free if* $\deg(\det(sE - A_0)) = \operatorname{rank} E$.

Definition 2.3. *For a given scalar* $h > 0$, *the nominal unforced singular time-varying delay system (1) is said to be regular and impulse free for any constant time delay* h *satisfying* $0 \le h(t) \le h$, *it the pairs* (E, A_0) *and* $(E, A_0 + A_1)$ *are regular and impulse free.*

Lemma 2.1. [8] *The linear singular system*

$$E\dot{x}(t) = Ax(t) \tag{6}$$

is regular, impulse free, and stable, if and only if there exists a matrix P *such that* $P^T E = E^T P \ge 0$ *and* $A^T P + PA < 0$.

Lemma 2.2. [10, 11] *For any positive semi-definite matrices*

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & X_{33} \end{bmatrix} \ge 0 \tag{7a}$$

the following integral inequality matrix holds

$$\begin{aligned} &-\int_{t-h(t)}^{t} \dot{x}^T(s)\, X_{33}\, \dot{x}(s) ds \\ &\le \int_{t-h(t)}^{t} \begin{bmatrix} x^T(t) & x^T(t-h(t)) & \dot{x}^T(s) \end{bmatrix} \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h(t)) \\ \dot{x}(s) \end{bmatrix} ds \end{aligned} \tag{7b}$$

Lemma 2.3. [4] *The following matrix inequality*

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} < 0 \tag{8a}$$

where $Q(x) = Q^T(x)$, $R(x) = R^T(x)$ *and* $S(x)$ *depend affine on* x, *is equivalent to*

$$R(x) < 0 \tag{8b}$$

$$Q(x) < 0 \tag{8c}$$

and

$$Q(x) - S(x)R^{-1}(x)S^T(x) < 0 \tag{8d}$$

Lemma 2.4. [4] *Given matrices* $Q = Q^T$, M *and* N *with appropriate dimensions,*

$$Q + MF(t)N + N^T F^T(t) M^T < 0 \tag{9a}$$

for all F *satisfying* $F^T(t)F(t) \le I$, *if and only if there exists some* $\varepsilon > 0$ *such that*

$$Q + \varepsilon M M^T + \varepsilon^{-1} N^T N < 0 \tag{9b}$$

Firstly, we consider the nominal system (1) $(\Delta A_0(t) = \Delta A_1(t) = \Delta B(t) = 0)$ *as follows:*

$$E\dot{x}(t) = (A_0 + BK)x(t) + A_1 x(t - h(t)) \tag{10}$$

For the singular time-varying delay nominal system (10), we will give a stability condition by using an integral inequality approach (IIA) as follows.

Theorem 2.1. *For given positive scalars h and h_d, the singular time-varying delay nominal system (10) is asymptotically stable if there exist symmetric positive-definite matrices $W = W^T > 0$, $U = U^T > 0$, $Z = Z^T > 0$ and a positive semi-definite matrix $T = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{12}^T & T_{22} & T_{23} \\ T_{13}^T & T_{23}^T & T_{33} \end{bmatrix} \geq 0$ and a matrix Y with appropriate dimension such that the following LMIs holds:*

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} \\ \Omega_{12}^T & \Omega_{22} & \Omega_{23} \\ \Omega_{13}^T & \Omega_{23}^T & \Omega_{33} \end{bmatrix} < 0 \tag{11a}$$

$$E^T(W - T_{33})E \geq 0 \tag{11b}$$

and

$$W^T E = E^T W \geq 0 \tag{11c}$$

where

$$\begin{aligned}
&\Omega_{11} = WA_0^T + A_0W + BY + Y^TB^T + U + E^T(hT_{11} + T_{13} + T_{13}^T)E, \\
&\Omega_{12} = A_1W + E^T(hTX_{12} - T_{13} + T_{23}^T)E, \quad \Omega_{13} = h(WA_0^T + Y^TB^T), \\
&\Omega_{22} = -(1 - h_d)U + E^T(hT_{22} - T_{23} - T_{23}^T)E, \quad \Omega_{23} = hWA_1^T, \quad \Omega_{33} = -hZ.
\end{aligned}$$

Further, the control gain may be obtained as $K = YW^{-1}$.

Proof: Choose a Lyapunov-Krasovskii functional candidate as follows:

$$V(x_t) = x^T(t)PEx(t) + \int_{t-h(t)}^{t} x^T(s)Qx(s)ds + \int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)\,E^T\,RE\dot{x}(s)dsd\theta \tag{12}$$

Calculating the time derivative of (12) along the trajectory of (10) yields

$$\begin{aligned}
\dot{V}(x_t) &= x^T(t)[P(A_0 + BK) + (A_0 + BK)^T\,P]x(t) + x^T(t)P\,A_1\,x(t - h(t)) \\
&\quad + x^T(t - h(t))\,A_1^T\,Px(t) + x^T(t)Qx(t) - x^T(t - h(t))Qx(t - h(t)) \\
&\quad + \dot{x}^T(t)h\,E^T\,RE\dot{x}(t) - \int_{t-h}^{t} \dot{x}^T(s)\,E^T\,RE\dot{x}(s)ds \\
&= x^T(t)[P(A_0 + BK) + (A_0 + BK)^T\,P + Q]x(t) + x^T(t)P\,A_1\,x(t - h(t)) \\
&\quad + x^T(t - h(t))\,A_1^T\,Px(t) - x^T(t - h(t))(1 - \dot{h}(t))Qx(t - h(t)) + \dot{x}^T(t)hR\dot{x}(t) \\
&\quad - \int_{t-h}^{t} \dot{x}^T(s)\,E^T(R - X_{33})E\dot{x}(s)ds - \int_{t-h}^{t} \dot{x}^T(s)\,E^T\,X_{33}\,E\dot{x}(s)ds \\
&\leq x^T(t)[P(A_0 + BK) + (A_0 + BK)^T\,P + Q]x(t) + x^T(t)P\,A_1\,x(t - h(t)) \\
&\quad + x^T(t - h(t))\,A_1^T\,Px(t) - x^T(t - h(t))(1 - h_d)Qx(t - h(t)) + \dot{x}^T(t)hR\dot{x}(t) \\
&\quad - \int_{t-h(t)}^{t} \dot{x}^T(s)\,E^T(R - X_{33})E\dot{x}(s)ds - \int_{t-h(t)}^{t} \dot{x}^T(s)\,E^T\,X_{33}\,E\dot{x}(s)ds
\end{aligned} \tag{13}$$

According to Lemma 2.2 and Leibniz-Newton formula $x(t)-x(t-h(t)) = \int_{t-h(t)}^{t} \dot{x}(s)ds$, it is clear that the following inequality is true:

$$
\begin{aligned}
&-\int_{t-h}^{t} \dot{x}^T(s)\, X_{33}\, \dot{x}(s)ds \\
&\le \int_{t-h(t)}^{t} \left[\begin{array}{ccc} x^T(t) & x^T(t-h(t)) & \dot{x}^T(s) \end{array} \right] \left[\begin{array}{ccc} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & 0 \end{array} \right] \left[\begin{array}{c} x(t) \\ x(t-h(t)) \\ \dot{x}(s) \end{array} \right] ds \\
&\le x^T(t)h\, X_{11}\, x(t) + x^T(t)h\, X_{12}\, x(t-h(t)) + x^T(t)\, X_{13} \int_{t-h(t)}^{t} \dot{x}(s)ds \\
&\quad + x^T(t-h(t))\, hX_{12}^T\, x(t) + x^T(t-h(t))h\, X_{22}\, x(t-h(t)) \qquad (14)\\
&\quad + x^T(t-h(t))\, X_{23} \int_{t-h(t)}^{t} \dot{x}(s)ds + \int_{t-h(t)}^{t} \dot{x}^T(s)ds\, X_{13}^T\, x(t) \\
&\quad + \int_{t-h(t)}^{t} \dot{x}^T(s)ds\, X_{23}^T\, x(t-h(t)) \\
&= x^T(t)[h\, X_{11} + X_{13}^T + X_{13}]x(t) + x^T(t)[h\, X_{12} - X_{13} + X_{23}^T]x(t-h(t)) + x^T(t \\
&\quad - h(t))[hX_{12}^T - X_{13}^T + X_{23}]x(t) + x^T(t-h(t))[h\, X_{22} - X_{23} - X_{23}^T]x(t-h(t))
\end{aligned}
$$

Substituting (14) into (13) yields

$$
\dot{V}(x_t) \le \xi^T(t)\Xi\xi(t) - \int_{t-h(t)}^{t} \dot{x}^T(s)\, E^T(R - X_{33})E\dot{x}(s)ds \qquad (15)
$$

where $\xi^T(t) = \left[\begin{array}{cc} x^T(t) & x^T(t-h) \end{array} \right]$ and $\Xi = \left[\begin{array}{cc} \Xi_{11} & \Xi_{12} \\ \Xi_{12}^T & \Xi_{22} \end{array} \right]$, with

$$
\begin{aligned}
\Xi_{11} &= P(A_0 + BK) + (A_0 + BK)^T P + Q + E^T(h\, X_{11} + X_{13} + X_{13}^T)E \\
&\quad + h\, (A_0 + BK)^T R(A_0 + BK), \\
\Xi_{12} &= P\, A_1 + E^T(h\, X_{12} - X_{13} + X_{23}^T)E + h\, (A_0 + BK)^T\, R\, A_1, \\
\Xi_{22} &= -(1 - h_d)Q + E^T(h\, X_{22} - X_{23} - X_{23}^T)E + h\, A_1^T\, R\, A_1\, .
\end{aligned}
$$

Since $R - X_{33} \ge 0$, then the last one part in (15) is less than or equal to 0. Therefore, using the Schur complement of Lemma 2.3, with some effort we can show that (15) guarantees of $\dot{V}(x_t) < 0$, if $\dot{V}(x_t) < 0$, then

$$
\Psi = \left[\begin{array}{ccc} \Psi_{11} & \Psi_{12} & \Psi_{13} \\ \Psi_{12}^T & \Psi_{22} & \Psi_{23} \\ \Psi_{13}^T & \Psi_{23}^T & \Psi_{33} \end{array} \right] < 0 \qquad (16a)
$$

and

$$
E^T(R - X_{33})E \ge 0 \qquad (16b)
$$

where

$$
\begin{aligned}
&\Psi_{11} = P(A_0 + BK) + (A_0 + BK)^T P + Q + E^T(h\, X_{11} + X_{13} + X_{13}^T)E, \\
&\Psi_{12} = P\, A_1 + E^T(h\, X_{12} - X_{13} + X_{23}^T)E, \quad \Psi_{13} = h\, (A_0 + BK)^T R, \\
&\Psi_{22} = -(1 - h_d)Q + E^T(h\, X_{22} - X_{23} - X_{23}^T)E, \quad \Psi_{23} = h\, A_1^T\, R, \quad \Psi_{23} = -hR.
\end{aligned}
$$

By pre- and post-multiplying $diag\{P^{-1}, P^{-1}, R^{-1}\}$ to (16a) and apply the change of variables such that $W = P^{-1}$, $Y = KW$, $P^{-1}QP^{-1} = U$, $P^{-1}X_{ij}P^{-1} = T_{ij}$ $(i, j = 1, 2, 3)$,

$Z = R^{-1}$, then we obtain (11a). Applying $\begin{bmatrix} R^{-1} & P^{-1} \end{bmatrix} \begin{bmatrix} R \\ -X_{33} \end{bmatrix} P^{-1} = W - T_{33}$ to (16b) to yields to (11b). This ends the proof.

Now, extending Theorem 2.1 to uncertain time-varying delay singular system (1) yields the following Theorem 2.2.

Theorem 2.2. *For given positive scalars h and h_d, the time-delay singular system (1) is asymptotically stable if there exist symmetric positive-definite matrices $W = W^T > 0$, $U = U^T > 0$, $Z = Z^T > 0$, $\varepsilon > 0$, and a positive semi-definite matrix $T = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{12}^T & T_{22} & T_{23} \\ T_{13}^T & T_{23}^T & T_{33} \end{bmatrix} \geq 0$ and a matrix Y with appropriate dimension such that the following LMIs holds:*

$$\overline{\Omega} = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \varepsilon M & WN_0^T + Y^T N_b^T \\ \Omega_{12}^T & \Omega_{22} & \Omega_{23} & 0 & WN_1^T \\ \Omega_{13}^T & \Omega_{23}^T & \Omega_{33} & h\varepsilon M & 0 \\ \varepsilon M^T & 0 & h\varepsilon M^T & -\varepsilon I & 0 \\ N_0 W + N_b Y & N_1 W & 0 & 0 & -\varepsilon I \end{bmatrix} < 0 \tag{17a}$$

$$E^T(W - T_{33})E \geq 0 \tag{17b}$$

and

$$EW = WE^T \geq 0 \tag{17c}$$

where Ω_{ij}, $(i, j = 1, 2, 3; i < j \leq 3)$ are defined in (11a). Then controller (5) with $K = YW^{-1}$ stabilizes system (1).

Proof: Replacing $A_0 + BK$ and A_1 in (15) with $A_0 + BK + MF(t)(N_0 + N_2 K)$ and $A_1 + MF(t)N_1$, respectively, we apply Lemma 2.4 [4] for system (1) is equivalent to the following condition:

$$\Omega + \Gamma_d F(t) \Gamma_e + \Gamma_e^T F(t) \Gamma_d^T < 0 \tag{18}$$

where $\Gamma_d = \begin{bmatrix} PM & 0 & hRM \end{bmatrix}^T$ and $\Gamma_e = \begin{bmatrix} N_0 + N_b K & N_1 & 0 \end{bmatrix}$.

By Lemma 2.4 [4], a sufficient condition guaranteeing (11) for system (1) is that there exists a positive number $\varepsilon > 0$ such that

$$\Omega + \varepsilon^{-1} \Gamma_d^T \Gamma_d + \varepsilon \Gamma_e^T \Gamma_e < 0 \tag{19}$$

Applying the Schur complement shows that (19) is equivalent to

$$\overline{\Psi} = \begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} & \varepsilon PM & N_0^T + K^T N_b^T \\ \Psi_{12}^T & \Psi_{22} & \Psi_{23} & 0 & N_1^T \\ \Psi_{13}^T & \Psi_{23}^T & \Psi_{33} & h\varepsilon RM & 0 \\ \varepsilon M^T P & 0 & h\varepsilon M^T R & -\varepsilon I & 0 \\ N_0 + N_b K & N_1 & 0 & 0 & -\varepsilon I \end{bmatrix} < 0 \tag{20a}$$

and

$$E^T(R - X_{33})E \geq 0 \tag{20b}$$

where Ψ_{ij}, $(i, j = 1, 2, 3; i < j \leq 3)$ are defined in (16).

By pre- and post-multiplying $diag\{P^{-1}, P^{-1}, R^{-1}, I, I\}$ to (20a) and apply the change of variables such that $W = P^{-1}$, $Y = KW$, $P^{-1}QP^{-1} = U$, $P^{-1}X_{ij}P^{-1} = T_{ij}$, $(i, j = 1, 2, 3)$, $Z = R^{-1}$, then we obtain (17a). Applying $\begin{bmatrix} R^{-1} & P^{-1} \end{bmatrix} \begin{bmatrix} R \\ -X_{33} \end{bmatrix} P^{-1} = W - T_{33}$ to (20b) to yields to (17b). This ends the proof.

3. **Extension to Exponential Stabilization for Time-Delay Systems.** We now present a delay-dependent criterion for exponential stabilization of the singular time-varying delay nominal system (10).

Theorem 3.1. *For any given positive scalars h, α and h_d, the singular time-varying delay nominal system (10) is exponential stabilization with decay rate α if there exist symmetry positive-definite matrices $W = W > 0$, $U = U^T > 0$, $Z = Z^T > 0$, and positive semi-definite matrix $T = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{12}^T & T_{22} & T_{23} \\ T_{13}^T & T_{23}^T & T_{33} \end{bmatrix} \geq 0$ and a matrix Y with appropriate dimension such that the following LMIs holds:*

$$\Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} \\ \Pi_{12}^T & \Pi_{22} & \Pi_{23} \\ \Pi_{13}^T & \Pi_{23}^T & \Pi_{33} \end{bmatrix} < 0 \tag{21a}$$

and

$$E^T(W - T_{33})E \geq 0 \tag{21b}$$

$$EW = WE^T \geq 0 \tag{21c}$$

where

$$\begin{aligned}
\Pi_{11} &= W(A_0 + 0.5\alpha E)^T + (A_0 + 0.5\alpha E)W + BY + Y^T B^T + U \\
&\quad + e^{-\alpha h} E^T(h T_{11} + T_{13} + T_{13}^T)E, \\
\Pi_{12} &= A_1 W + e^{-\alpha h} E^T(hT X_{12} - T_{13} + T_{23}^T)E, \quad \Pi_{13} = h(WA_0^T + Y^T B^T), \\
\Pi_{22} &= e^{-\alpha h}[E^T(hT_{22} - T_{23} - T_{23}^T)E - (1 - h_d)U], \\
\Pi_{23} &= hWA_1^T, \quad \Pi_{33} = -hZ.
\end{aligned}$$

Then controller (5) with $K = YW^{-1}$ stabilizes singular time-varying delay nominal system (10).

Proof: Consider the singular time-varying delay nominal unforced system (10), using the Lyapunov-Krasovskii functional candidate in the following form, we can rewrite

$$V(x_t) = e^{\alpha t} x^T(t)PEx(t) + \int_{t-h}^{t} e^{\alpha s} x^T(s)Qx(s)ds + \int_{-h}^{0}\int_{t+\theta}^{t} e^{\alpha s} \dot{x}^T(s)E^T RE\dot{x}(s)dsd\theta \tag{22}$$

The time derivative of (22) along the trajectory of (10) is given by

$$\begin{aligned}
\dot{V}(x_t) &= e^{\alpha t}\Big\{ x^T(t)\alpha EPx(t) + \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) + x^T(t)Qx(t) \\
&\quad - x^T(t-h(t))\, e^{-\alpha h}(1 - \dot{h}(t))Qx(t-h(t)) + \dot{x}^T(t)h\, E^T RE\dot{x}(t) \\
&\quad - \int_{t-h}^{t} e^{\alpha(s-t)}\, \dot{x}^T(s)\, E^T RE\dot{x}(s)ds \Big\} \\
&\leq e^{\alpha t}\Big\{ x^T(t)[P(A_0 + BK + 0.5\alpha E) + (A_0 + BK + 0.5\alpha E)^T P]x(t) \\
&\quad + x^T(t)P\, A_1\, x(t-h(t)) + x^T(t-h(t))\, A_1^T\, Px(t) \\
&\quad + x^T(t)Qx(t) - x^T(t-h(t))\, e^{-\alpha h}(1 - h_d)Qx(t-h(t)) + \dot{x}^T(t)h\, E^T RE\dot{x}(t) \\
&\quad - \int_{t-h(t)}^{t} e^{\alpha(s-t)}\, \dot{x}^T(s)\, E^T RE\dot{x}(s)ds - \int_{t-h(t)}^{t} e^{\alpha(s-t)}\, \dot{x}^T(s)\, E^T X_{33}\, E\dot{x}(s)ds \Big\}
\end{aligned} \tag{23}$$

Obviously, for any a scalar $s \in [t-h, t]$, we have $e^{-\alpha h} \leq e^{\alpha(s-t)} \leq 1$, and

$$-\int_{t-h(t)}^{t} e^{\alpha(s-t)}\, \dot{x}^T(s)\, E^T\, X_{33}\, E\dot{x}(s)ds \leq -\, e^{-\alpha h} \int_{t-h(t)}^{t} \dot{x}^T(s)\, E^T\, X_{33}\, E\dot{x}(s)ds \tag{24}$$

Applying the proof of Theorem 2.1, we obtain

$$\dot{V}(x_t) \leq e^{\alpha t} \left\{ \xi^T(t)\Xi\xi(t) - \int_{t-h(t)}^{t} \dot{x}^T(s)\, e^{-\alpha h}\, E^T(R - X_{33})E\dot{x}(s)ds \right\} \tag{25}$$

where $\xi^T(t) = \begin{bmatrix} x^T(t) & x^T(t-h) \end{bmatrix}$ and $\Upsilon = \begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} \\ \Upsilon_{12}^T & \Upsilon_{22} \end{bmatrix}$ with

$$\begin{aligned}
\Upsilon_{11} &= (A_0 + BK + 0.5\alpha E)^T P + P(A_0 + BK + 0.5\alpha E) + Q \\
&\quad + e^{-\alpha h}\, E^T(h\, X_{11} + X_{13} + X_{13}^T)E + h\,(A_0 + BK)^T\, R(A_0 + BK), \\
\Upsilon_{12} &= P\, A_1 + e^{-\alpha h}\, E^T(h\, X_{12} - X_{13} + X_{23}^T)E + h\,(A_0 + BK)^T\, R\, A_1, \\
\Upsilon_{22} &= e^{-\alpha h}[E^T(h\, X_{22} - X_{23} - X_{23}^T)E - (1 - h_d)Q] + h\, A_1^T\, R\, A_1\,.
\end{aligned}$$

Just as Theorem 2.1, $R - X_{33} \geq 0$, then the last one part in (25) is less than or equal to 0. Furthermore, using the Schur complement, with some effort we can show that (25) guarantees of $\dot{V}(x_t) < 0$, if $\dot{V}(x_t) < 0$, then

$$\Phi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} \\ \Phi_{12}^T & \Phi_{22} & \Phi_{23} \\ \Phi_{13}^T & \Phi_{23}^T & \Phi_{33} \end{bmatrix} < 0 \tag{26a}$$

and

$$E^T(R - X_{33})E \geq 0 \tag{26b}$$

where

$$\begin{aligned}
\Phi_{11} &= (A_0 + BK + 0.5\alpha E)^T P + P(A_0 + BK + 0.5\alpha E) + Q \\
&\quad + e^{-\alpha h}\, E^T(h\, X_{11} + X_{13} + X_{13}^T)E, \\
\Phi_{12} &= P\, A_1 + e^{-\alpha h}\, E^T(h\, X_{12} - X_{13} + X_{23}^T)E, \quad \Phi_{13} = h\,(A_0 + BK)^T\, R, \\
\Phi_{22} &= e^{-\alpha h}[E^T(h\, X_{22} - X_{23} - X_{23}^T)E - (1 - h_d)Q], \quad \Phi_{23} = h\, A_1^T\, R, \quad \Phi_{33} = -hR.
\end{aligned}$$

By pre- and post-multiplying $diag\{P^{-1}, P^{-1}, R^{-1}\}$ to (26a) and apply the change of variables such that $W = P^{-1}$, $Y = KW$, $P^{-1}QP^{-1} = U$, $P^{-1}X_{ij}P^{-1} = T_{ij}$, $(i, j = 1, 2, 3)$, $Z = R^{-1}$, then we obtain (21a). Applying $\begin{bmatrix} R^{-1} & P^{-1} \end{bmatrix} \begin{bmatrix} R \\ -X_{33} \end{bmatrix} P^{-1} = W - T_{33}$ to (26b) to yields to (21b). This ends the proof.

Now, extending Theorem 3.1 to singular uncertain system with time-varying delay (1) yields the following Theorem 3.2.

Theorem 3.2. *For any given positive scalars h, α and h_d, the uncertain time-varying delay singular system (1) is exponential stabilization if there exist symmetric positive-definite matrices $W = W^T > 0$, $U = U^T > 0$, $Z = Z^T > 0$, $\varepsilon > 0$, and a positive semi-definite matrix $T = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{12}^T & T_{22} & T_{23} \\ T_{13}^T & T_{23}^T & T_{33} \end{bmatrix} \geq 0$ and a matrix Y with appropriate dimension* *such that the following LMIs holds:*

$$\overline{\Pi} = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \varepsilon M & W N_0^T + Y^T N_b^T \\ \Pi_{12}^T & \Pi_{22} & \Pi_{23} & 0 & W N_1^T \\ \Pi_{13}^T & \Pi_{23}^T & \Pi_{33} & h\varepsilon M & 0 \\ \varepsilon M^T & 0 & h\,\varepsilon M^T & -\varepsilon I & 0 \\ N_0 W + N_b Y & N_1 W & 0 & 0 & -\varepsilon I \end{bmatrix} < 0 \tag{27a}$$

$$E^T(W - T_{33})E \geq 0 \tag{27b}$$

and

$$EW = WE^T \geq 0 \tag{27c}$$

where Π_{ij}*,* $(i, j = 1, 2, 3; i < j \leq 3)$ *are defined in (21). Then controller (5) with* $K = YW^{-1}$ *stabilizes system (1). It is, incidentally, worth noting that the uncertain time delay singular system (1) is exponential asymptotically stable, that is, the uncertain parts of the nominal system can be tolerated within allowable time delay* h*.*

When $u(t) = 0$, our method for determining the exponential stability of time-varying delay nominal unforced system (1) in the following Corollary 3.1.

Corollary 3.1. *For any given positive scalars* h *and* α*, the time delay nominal unforced system (1) is exponential stable with decay rate* α *if there exist symmetry positive-definite matrices* $P = P^T > 0$*,* $Q = Q^T > 0$*,* $R = R^T > 0$*,* $\varepsilon > 0$*, and positive semi-definite matrix* $X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & X_{33} \end{bmatrix} \geq 0$ *which satisfy the following inequalities:*

$$\overline{\Phi} = \begin{bmatrix} \overline{\Phi}_{11} & \overline{\Phi}_{12} & \overline{\Phi}_{13} & \varepsilon PM & N_0^T \\ \overline{\Phi}_{12}^T & \overline{\Phi}_{22} & \overline{\Phi}_{23} & 0 & N_1^T \\ \overline{\Phi}_{13}^T & \overline{\Phi}_{23}^T & \overline{\Phi}_{33} & h\varepsilon RM & 0 \\ \varepsilon M^T P & 0 & h\,\varepsilon M^T R & -\varepsilon I & 0 \\ N_0 & N_1 & 0 & 0 & -\varepsilon I \end{bmatrix} < 0 \tag{28a}$$

$$E^T(R - X_{33})E \geq 0 \tag{28b}$$

and

$$P^T E = E^T P \geq 0 \tag{28c}$$

where

$$\overline{\Phi}_{11} = (A_0 + 0.5\alpha E)^T P + P(A_0 + 0.5\alpha E) + Q + e^{-\alpha h} E^T(h X_{11} + X_{13} + X_{13}^T)E,$$
$$\overline{\Phi}_{12} = P A_1 + e^{-\alpha h} E^T(h X_{12} - X_{13} + X_{23}^T)E, \quad \overline{\Phi}_{13} = h A_0^T R,$$
$$\overline{\Phi}_{22} = e^{-\alpha h}[E^T(h X_{22} - X_{23} - X_{23}^T)E - (1 - h_d)Q], \quad \overline{\Phi}_{23} = h A_1^T R, \quad \overline{\Phi}_{33} = -hR.$$

Based on that, a convex optimization problem is formulated to find the bound on the maximum allowable delay bound (MADB) $\overline{h}$ and delay decay rate α which maintains the delay-dependent stability of the uncertain time-varying delay singular system (1).

Remark 3.1. *Theorems in this paper can be used for practical systems, which can be modeled as singular systems with time-varying delays.*

Remark 3.2. *Let* $E = I$*, theorems in this paper can be regarded as an extension of standard state-space time-varying delay systems to singular time-varying delay systems.*

Remark 3.3. *Theorem 3.2 provides delay-dependent asymptotic stability criteria for the uncertain time-varying delay singular system (1) in terms of solvability of LMIs [4]. Based on them, we can obtain the maximum allowable delay bound (MADB) $\overline{h}$ such that (1) is stable by solving the following convex optimization problem*

$$\begin{cases} \text{Maximize} & \overline{h} \\ \text{Subject to} & (27) \text{ and } W > 0, U > 0, Z > 0, \varepsilon > 0, \alpha > 0. \end{cases} \tag{29}$$

Inequality (29) is a convex optimization problem and can be obtained efficiently using the MATLAB LMI Toolbox.

Remark 3.4. *It is interesting to note that h appears linearly in (11), (17), (21), (27) and (28). Thus a generalized eigenvalue problem as defined in [4] can be formulated to solve the minimum acceptable $1/h$ and therefore the maximum $\overline{h}$ to maintain robust stability as judged by these conditions.*

4. **Illustrative Examples.** To demonstrate the effectiveness of our method, we briefly consider the following numerical examples.

Example 4.1. *Consider the following time-varying delay singular systems*

$$E\dot{x}(t) = A_0 x(t) + A_1 x(t - h(t)) + Bu(t) \tag{30}$$

where $E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $A_0 = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$, $A_1 = \begin{bmatrix} \frac{1}{3} & \frac{1}{6} \\ -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$.

It is found that the nominal system (30) is unstable and it is intended to stabilize the controlled system and find the maximum allowable delay bound (MADB) $\overline{h}$ by using memoryless state feedback controller to guarantee that the system (30) is asymptotically stable.

Solution: By taking the parameters $h_d = 0$ and using the LMI Toolbox in MATLAB (with accuracy 0.01), solving the quasi-convex optimization problem (29), the maximum allowable delay bound (MADB), $h \leq \overline{h} = 3.6896$, for which the system is stabilized by the corresponding state feedback gains $K = YW^{-1} = [-15.8926 - 20.3146]$.

In case of $\overline{h} = 3.6896$, solving Theorem 2.1 yields the following set of feasible solutions:

$$W = \begin{bmatrix} 5.3625 & -3.4773 \\ -3.4773 & 2.8432 \end{bmatrix}, \ U = \begin{bmatrix} 11.3677 & -4.7771 \\ -4.7771 & 5.3846 \end{bmatrix}, \ Z = \begin{bmatrix} 66.7305 & -5.9509 \\ -5.9509 & 49.6538 \end{bmatrix}$$
$$T_{11} = \begin{bmatrix} 0.8725 & 0 \\ 0 & 21.7871 \end{bmatrix}, \ T_{12} = \begin{bmatrix} -0.3720 & 0 \\ 0 & 0 \end{bmatrix}, \ T_{13} = \begin{bmatrix} -0.8392 & 0 \\ 0 & 0 \end{bmatrix}$$
$$T_{22} = \begin{bmatrix} 1.0764 & 0 \\ 0 & 21.7871 \end{bmatrix}, \ T_{23} = \begin{bmatrix} 0.8378 & 0 \\ 0 & 0 \end{bmatrix}, \ T_{33} = \begin{bmatrix} 3.1088 & 0 \\ 0 & 21.787 \end{bmatrix}$$

and $Y = [-14.5834 \ - 2.4935]$.

Figures 1 and 2 show that the unstable plant is stabilized by the feedback control. Figure 1 shows the open-loop response, and it can be clearly seen that, in the absence of any control, the system is unstable. Figure 2 shows the closed-loop system response and clearly demonstrates that, under the influence of the proposed controller (5), the system (30) is asymptotically stable for $h \leq 3.68$.

Example 4.2. *Consider the following singular uncertain system with time-varying delay in the state*

$$E\dot{x}(t) = (A_0 + \Delta A_0(t))x(t) + (A_1 + \Delta A_1(t))x(t - h(t)) + (B + \Delta B(t))u(t) \tag{31}$$

where $E = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$, $A_0 = \begin{bmatrix} 1.5 & 0.5 & 1 \\ -1 & 0 & 1 \\ 0.5 & 0 & 1 \end{bmatrix}$, $A_1 = \begin{bmatrix} -1 & 0 & -1 \\ 1 & -1 & 0.5 \\ 0.3 & 0.5 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ *and* $\Delta A_0(t)$, $\Delta A_1(t)$ *and* $\Delta B(t)$ *are of the form of (3) with* $M = \begin{bmatrix} 0.5 \\ 0.2 \\ 0.1 \end{bmatrix}$, $N_0 = \begin{bmatrix} 0.2 & 0.1 & 0.3 \end{bmatrix}$, $N_1 = \begin{bmatrix} 0.1 & 0.2 & 0.5 \end{bmatrix}$, $N_b = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}$.

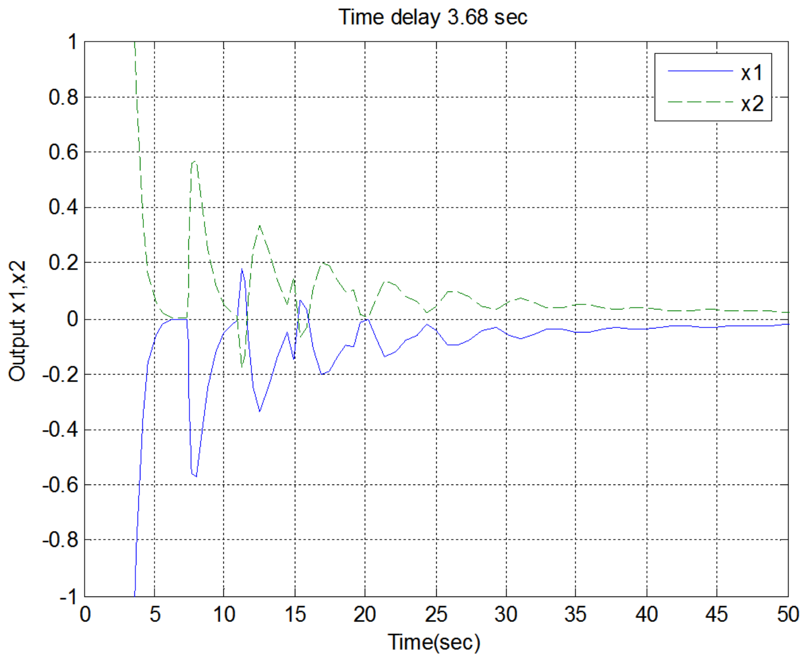

FIGURE 1. Open-loop system response

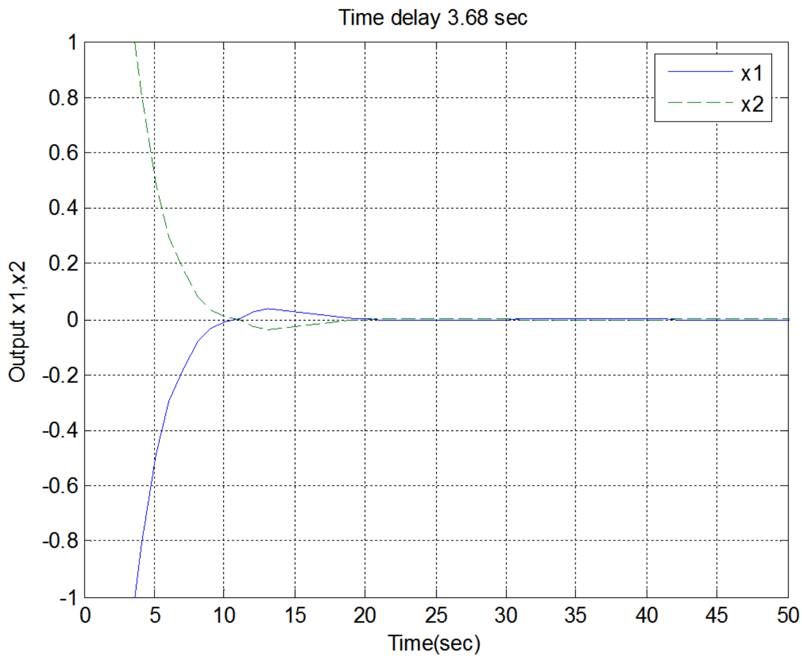

FIGURE 2. Closed-loop system response

Solution: By taking the parameters $h_d = 0$ with these sets of data, for $h = 9.6896$, using Matlab LMI Control Toolbox to solve the LMIs (17a)-(17c), we obtain the feasible solution:

$$W = \begin{bmatrix} 0.6115 & 0.1524 & 0.5709 \\ 0.1524 & 1.3686 & -0.6231 \\ 0.5709 & -0.6231 & 0.9910 \end{bmatrix}, \quad U = \begin{bmatrix} 3.0370 & 2.0391 & 1.6670 \\ 2.0391 & 4.6329 & -1.1387 \\ 1.6670 & -1.1387 & 2.6294 \end{bmatrix},$$

$$Z = \begin{bmatrix} 48.3884 & 14.0950 & 8.6580 \\ 14.0950 & 57.6352 & -9.2450 \\ 8.6580 & -9.2450 & 45.5836 \end{bmatrix}, \quad T_{11} = \begin{bmatrix} 7.2546 & 7.2147 & -7.2147 \\ 7.2147 & 7.2420 & -7.2273 \\ -7.2147 & -7.2273 & 7.2420 \end{bmatrix},$$

$$T_{12} = \begin{bmatrix} -0.0116 & 0.0024 & -0.0092 \\ -0.0072 & 0.0049 & -0.0024 \\ -0.0188 & 0.0073 & -0.0115 \end{bmatrix}, \quad T_{13} = \begin{bmatrix} -0.0882 & 0.0262 & -0.0630 \\ 0.0265 & -0.0320 & -0.0071 \\ -0.0618 & -0.0057 & -0.0701 \end{bmatrix},$$

$$T_{22} = \begin{bmatrix} 7.2875 & 7.1900 & -7.2000 \\ 7.1900 & 7.2530 & -7.2347 \\ -7.2000 & -7.2347 & 7.2429 \end{bmatrix}, \quad T_{23} = \begin{bmatrix} 0.0881 & -0.0262 & 0.0629 \\ -0.0264 & 0.0319 & 0.0071 \\ 0.0618 & 0.0057 & 0.0701 \end{bmatrix},$$

$$T_{33} = \begin{bmatrix} 8.0044 & 6.8931 & -6.5505 \\ 6.8931 & 7.4570 & -7.0922 \\ -6.5505 & -7.0922 & 7.8407 \end{bmatrix}, \quad \varepsilon = 4.0097,$$

$$Y = \begin{bmatrix} -2.8092 & -4.2875 & 1.3681 \\ -2.7746 & 1.9751 & -4.4659 \end{bmatrix}$$

and the stabilizing memoryless state feedback control law can be obtained as

$$u(t) = \begin{bmatrix} -120.3095 & 59.4744 & 108.0832 \\ 15.6177 & -9.0289 & -19.1805 \end{bmatrix} x(t).$$

Hence, according to Theorem 2.2, controller (5) with gain K given in the preceding text stabilizes system (31).

Example 4.3. *Consider the uncertain unforced part of system (1) with* $E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $A_0 = \begin{bmatrix} -0.9 & 0 \\ 0.5 & -2 \end{bmatrix}$, $A_1 = \begin{bmatrix} -1 & 0 \\ -1 & -0.9 \end{bmatrix}$, *and* $\Delta A_0(t)$ *and* $\Delta A_1(t)$ *are of the form of (3) with* $M = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$, $N_0 = \begin{bmatrix} 1 & 1 \end{bmatrix}$, $N_1 = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}$.

Solution: Table 1 provides the comparison of maximum allowable delay bound (MADB) $\overline{h}$ for various h_d by different methods. It is easy to get that, for this example, the delay-dependent stability condition in this paper gives better results than those in [15,17].

Furthermore, by taking the decay rateα, and from Corollary 3.1, we obtain the maximum allowable delay bound (MADB) $\overline{h}$ as shown in Table 2 below. Note that as α increases, the maximum allowable h decreases.

Example 4.4. *Consider the following uncertain singular time delay system*

$$E\dot{x}(t) = (A_0 + \Delta A_0(t))x(t) + (A_1 + \Delta A_1(t))x(t - h(t)) \tag{32}$$

where $E = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$, $A_0 = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix}$, $A_1 = \begin{bmatrix} -2.4 & 2 \\ 0 & 1 \end{bmatrix}$, *the uncertain matrices* $\Delta A_0(t)$ *and* $\Delta A_1(t)$ *are of the form of (3) with* $M = \lambda I$, $N_a = N_b = 0.5I$. *Now, our problem is to estimate the maximum allowable delay bound (MADB)* $\overline{h}$ *to keep the stability of system (32).*

TABLE 1. Comparison of delay-dependent result of Example 4.3

Methods \ h_d	0.2	0.3	0.4	0.5	0.6
Yue and Han [17]	1.5836	1.4897	1.3942	1.2946	1.1843
Wu et al. [15]	1.5895	1.5007	1.4131	1.3251	1.2315
Corollary 3.1	4.9809	4.6067	4.2453	3.8901	3.5244

TABLE 2. Maximum allowable delay bound (MADB) $\overline{h}$ for various stability degree α with $h_d = 0.5$

α	0.1	0.3	0.5	0.7	0.9
Corollary 3.1	2.3391	1.9990	1.5456	1.2345	0.9959

TABLE 3. Comparison of delay-dependent stability condition of Example 4.4

Method \ λ	0.30	0.35	0.40	0.45	0.50
Zhong and Zhi [18]	0.3939	0.3637	0.3279	0.2817	0.2106
Gao et al. [6]	0.7942	0.7689	0.7262	0.6521	0.5054
Wu and Zhou [14]	0.8249	0.7924	0.7438	0.6641	0.5110
Jiang et al. [7]	0.8249	0.7984	0.7678	0.7184	0.6019
Corollary 3.1 ($\alpha = 0$)	0.8456	0.8229	0.7997	0.7750	0.7462

Solution: Fixing $h_d = 0$ and $\lambda = 0.1$, by using the LMI Toolbox in MATLAB (with accuracy 0.01), this above uncertain singular time delay system (32) is asymptotically stable for delay time satisfying $h \leq 0.9387$. The maximum allowable delay bound (MADB) $\overline{h}$ form Corollary 3.1 is shown in Table 3. For comparison, the table also lists the upper bounds obtained from the criteria in [6,7,14,18]. It can be seen that our methods are less conservative.

5. **Conclusion.** This paper discusses the problems of the delay-dependent robust stability and stabilization of singular uncertain systems with time-varying delays. Delay-dependent stability criteria are derived by taking the relationships between the terms in the Leibniz-Newton formula into account. Integral inequality approach (IIA) is employed to express these relationships, and they are easy to obtain because the new criteria are based on linear matrix inequalities (LMIs). Moreover, the stability criteria are extended to the design of a stabilizing state feedback controller. Numerical examples demonstrate that these criteria are effective and are an improvement on previous ones.

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