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IMPROVED ADAPTIVE FUZZY PUNCTUAL KRIGING FILTER FOR IMAGE RESTORATION

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Abstract. *In this paper, we present an intelligent image restoration approach by combining the geostatistical interpolation technique of punctual kriging, fuzzy logic with type-II and fuzzy smoothing based approaches. Images degraded with Gaussian white noise are restored by first utilizing fuzzy logic for selecting pixels that needs kriging. Fuzzy type-II inference system is employed. Type-II fuzzy set has been used to generate fuzzy map for detection of noisy pixels. This fuzzy map of noisy pixels enhances performance of the proposed technique in terms of image restoration and as well as in terms of computational cost. Local neighborhood information is used to ensure noise free pixels in* 3×3 *window to estimate the noisy data. The concept of punctual kriging is then used to estimate the intensity of a noisy pixel. Image restoration performance based comparison has been made against adaptive Weiner filter and existing fuzzy kriging approaches. Experimental results using 450 images and different image quality measures show effectiveness of the proposed approach with fuzzy type-II for detecting noisy pixels and utilization of local information in conjunction with kriging based estimation.*

Keywords: Image restoration, Fuzzy logic, Punctual kriging, Adaptive spatial filtering

1. **Introduction.** Image restoration has become a widely investigated field of image processing, dealing with the reconstruction of images by removing noise and blur from degraded images and making them suitable for human perception. In spite of the advances made by recent methods, it is still a challenging task as these methods have yet to achieve a desirable level of applicability in many realistic scenarios. Moreover, with the ever increasing production of digital contents such as images and videos taken with low resolution cameras and in poor conditions, the importance of image restoration has significantly increased. Images can become corrupted during any of the acquisition, preprocessing, compression, transmission, storage and/or reproduction phases of processing [1]. One of the main aspects in devising intelligent image restoration techniques is noise suppression with keeping edges intact. Further, noise smoothing and edge enhancement are generally considered as contradictory attributes. Since smoothing a region might destroy an edge while sharpening edges might lead to amplification of unnecessary noise [2]; therefore, we present a new spatial filtering technique; an intelligent approach based on punctual kriging and fuzzy logic control, to consider this conflict and to remove noise while efficiently preserving the image details and edge information.

Punctual kriging, named after its developer, D. G. Krige [3] is heavily used in mining and geostatistics based applications. It is an interpolation technique that gives an optimal linear estimate of an unknown parameter at a sampling point in terms of its known values at the surrounding sampling points [4]. The estimation involves calculation of the semi-variances and modeling of semi-variograms from the sampled data. Besides this, kriging has been applied in many other fields as well.

Fuzzy filters have been extensively applied in image processing over the last decade. Choi and Krishnapuram [5] devised fuzzy rule based multiple filters, derived from the method of weighted least squares, for noise removal. Some researchers have also investigated the use of fuzzy clustering for the removal of impulsive noise [6]. In [7], Farbiz and Menhaj have introduced an approach of image filtering based on fuzzy logic control. They have shown how to remove impulsive noise and smooth out Gaussian noise while, simultaneously, preserving image details and edges efficiently. Liang and Looney [8] have proposed a competition fuzzy edge detector to distinguish the noisy pixels from the edge pixels. Further, Khriji and Gabbouj [9] have recently proposed a fuzzy transformation based approach for multichannel image processing. Although fuzzy spatial filters have been widely used, with the increase of local information, the number of fuzzy rules also increases accordingly. To reduce the requirement of such complicated rules, fuzzy control is used as a complementary tool along with the existing techniques to develop better and accurate methods. This is one of the major aims of the investigations presented in this paper.

Besides Pham and Wagner [10,11] have used punctual kriging along with fuzzy sets to enhance images corrupted by Gaussian white noise. They model soft-thresholding by fuzzy sets. In their approach, the pixel intensity in the processed image is a weighted sum of the original (noisy) and the estimated value through kriging. They have evaluated their results qualitatively in comparison with adaptive Wiener filter (AWF). However, their study does not provide any quantitative performance analysis of their proposed technique [12,13]. Further, they apply kriging to all pixels in the degraded image. Considering 3×3 neighborhood, inverse of a kriging matrix of size 9×9 is required, which can make the filtering process computationally expensive. Furthermore, due to zero diagonal, matrix inverse may not always be possible. In addition to this, filter weights also suffer from occurrence of negative values, which leads to an overall poor performance of the filter. Mirza et al. [12] have highlighted these issues and proposed a spatially adaptive fuzzy kriging (SAFK) approach based on punctual kriging, fuzzy logic and fuzzy smoothing. In their scheme, first they employ fuzzy logic based on homogeneity and DAMdistance to generate a fuzzy decision map for the fate of a pixel whether it needs to be kriged or not. In the next step, they apply punctual kriging to estimate the selected noisy pixels. To tackle the matrix inversion failure problem, they have used fuzzy smoothing value as an estimate for the pixel under consideration. In their scheme, they renormalize the positive weights on occurrence of negative weights in punctual kriging.

In most of the image restoration methods, all pixels in local neighborhood are considered to estimate a pixel under consideration without observing whether all of the pixels other than central pixel are noise free or not. Technically speaking, noisy pixels used to estimate a corrupted pixel in local window cannot offer optimal/near optimal estimate. This is because noisy pixel has intensity level away from its actual intensity value depending upon the strength of noise in the degraded image. Even AWF, Pham & Wagner fuzzy kriging (PWFK) [10,11] and SAFK [12] approaches employ the same scenario to estimate the pixel under consideration without taking into account this important fact. So, this certainty needs special attention how to replace intensity values at noisy locations before its use in estimation. In this current work, we propose an intelligent technique based on fuzzy inference system considering fuzzy type-II and utilization of noise free pixels in local neighborhood window in conjunction with punctual kriging for image restoration. This paper makes the following contributions:

a. We introduce an effective fuzzy based kriging methodology for image denoising by exploiting local information in the degraded image.
b. Type-II fuzzy sets with improved fuzzy rules for decision making whether a pixel needs to be estimated or not.
c. We exploit the existence of noise free pixels in local neighborhood to estimate the noisy pixel under consideration. This factor not only improves the results but also gives optimal/near optimal estimates and avoids devastating edges.

Fuzzy type-II with improved member functions, fuzzy rules to generate fuzzy decision map, and taking into account vital information about the pixels (noisy or noise free) to estimate the pixel under consideration in local neighborhood window differentiate this current research from the work presented in [12].

Since separating noise and original signal from a single input image is under constrained, in theory it is very difficult to recover the original signal [14]. Hence in this work, we perform image denoising with following objectives:

a. Edges in an image should be preserved during the denoising process.
b. No artifacts should be visible in the de-noised image.
c. To exploit available local information in the neighborhood window to estimate a noisy pixel.
d. We believe noise free pixels significantly contribute to estimating the noisy pixel whereas noisy pixels other than the pixel under consideration may enhance the noise effect because of their corruptness.
e. An improved, intelligent fuzzy kriging approach based on use of fuzzy decider type-II, and local neighborhood information, punctual kriging and fuzzy averaging.

Although some of these objectives seem contradicting, our proposed method attempts to mutually de-correlate conflicting objectives as much as possible using punctual kriging and a fuzzy rule based approach.

Rest of the paper is structured as follows. Section 2 introduces punctual kriging and variograms, fuzzy inference system, type-II fuzzy sets. It also presents some of the most commonly used image quality measures along with the variogram based quality measures. Section 3 explains the proposed intelligent technique based on punctual kriging and fuzzy approach of adaptive learning. Experimental results and discussion is presented in Section 4. Our findings including directions for future work are given in Section 5.

2. **Related Theory.**

2.1. **Punctual kriging and variogram.**

In literature, it has been proved that Punctual kriging results best linear unbiased estimate of an unknown point on a surface [15]. This estimate is the weighted sum of the known neighboring values around the unknown point and by minimizing the variance of estimation-error these weights can be determined. Kriging uses a variogram model (a concept from geostatistics) to fit the experimental data. Based on the variogram model chosen, known values are assigned optimal weights to calculate the unknown value. Whereas, in an image denoising scenario pixel intensity is available at all points including corrupted ones. So, from the experimental variogram we can directly findout kriging weights to estimate the noisy pixels without employing any theoretical variogram model. For more details on this topic, we refer the readers to [12,13]. Variogram presents the variation of semivariance with respect to distance from a point. Semivariance provides a measure of spatial dependence between samples. Semivariance

[4] of the samples at lag 'd' can be calculated from Equation (1).

$$\gamma(d) = \frac{1}{2} Var(z_{i+d} - z_i) \tag{1}$$

Different distance metrics can be used to identify a group of neighboring samples having the same lag. In the present investigations, however, we have considered the Euclidean metric as the distance measure. The experimental semivariogram is obtained directly by using the sample values from the experimental data.

For a given lag 'd', it is calculated from the available data as

$$\gamma(d) = \frac{1}{2N(d)} \sum_{i=1}^{N(d)} (z_i - z_{i+d})^2 \tag{2}$$

The above expression for experimental semi-variogram depends upon the spatial configuration of the available image data. One has to consider different cases, as to whether the data is aligned or not and whether it is regularly spaced along the alignments. However, in the present case of digital images, the data is aligned and regularly spaced, which makes the estimation of the semi-variogram easy.

Punctual kriging is a linear combination of the neighboring sample values, as given by Equation (3).

$$\hat{z} = \sum_i w_i z_i \tag{3}$$

where w_i are the weights and z_i are the neighboring values of z. It is an unbiased estimator if the weights add up to Equation (1). This additional constraint on weights is given by

$$\sum_i w_i = 1 \tag{4}$$

Statistical variance is measure of how different the estimated value is from its neighboring sample values. It can be found using Equation (5).

$$Var\,(e) = Var\,(z - \hat{z}) \tag{5}$$

A number of such linear unbiased estimators are available, but we find the *best* one in the sense that it has the smallest estimation variance. Thus, the cost function is defined as

$$\varphi(w_i, \lambda) = Var(e) - 2\lambda \left(\sum_i w_i - 1 \right) \tag{6}$$

where λ is the Lagrange multiplier. Differentiating the cost function $\varphi(w_i, \lambda)$ with respect to w_i and λ and setting the differential equal to zero and rearranging the system of equations, these can be written in matrix form as

$$\begin{pmatrix} \gamma(d_{11}) & \gamma(d_{12}) & \cdots & \gamma(d_{1n}) & 1 \\ \gamma(d_{21}) & \gamma(d_{22}) & \cdots & \gamma(d_{2n}) & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \gamma(d_{n1}) & \gamma(d_{n2}) & \cdots & \gamma(d_{nn}) & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \\ \lambda \end{pmatrix} = \begin{pmatrix} \gamma(d_1) \\ \gamma(d_2) \\ \vdots \\ \gamma(d_n) \\ 1 \end{pmatrix} \tag{7}$$

or in matrix-vector notations

$$Aw = b \tag{8}$$

Matrix A is symmetric and has zero diagonal elements. The elements of the matrix are taken from the semi-variogram (defined in Equation (1)) for the current point. Solving Equation (8) gives us the optimal kriging weights $\{w_1, w_2, \cdots, w_n\}$ for estimating the unknown value $\hat{z}$ using its neighbors. However, if A is a singular matrix then punctual kriging fails to estimate pixel intensity. Further, in image de-noising case, the semi-variance matrix A for each marked noisy pixel has to be calculated using neighboring pixels in local window. This may result incorrect estimation when there exist noisy pixels other than the pixel under consideration in local window. Same could be happen in calculating the semi-variance vector b. Special attention is required to handle these problems to avoid incorrect estimation of noisy pixel under consideration and to enhance the overall performance of the system. One of the main contributions of this paper resolves this issue.

2.2. **Fuzzy inference system and fuzzy smoothing.** There are two types of fuzzy inference systems (FIS), that are commonly used, i.e., *Mamdani* and *Takagi-Sugeno* type [16]. Both types of FIS are similar in many aspects; fuzzifying the inputs and applying the fuzzy operator. The Takagi-Sugeno output membership functions are either linear or constant and this aspect mainly differs from the Mamdani type [17]. Since classification of noisy pixels from an image is considered as a nonlinear process. In the proposed approach, nonlinear fuzzy output membership functions are employed to decide the fate of a pixel. Keeping in view this aspect and considering to decide the fate of a pixel whether it needs to be estimated or not, depending upon the local properties of the neighborhood as a challenging problem [12], we have employed Mamdani type FIS.

Type-II fuzzy sets are considered as generalization of the ordinary fuzzy sets, whereby fuzzy sets are characterized by a fuzzy membership function and consequently, the membership value for each member of the set is itself a fuzzy set between 0 and 1 [18]. The main advantages of using a type-II framework are twofold; by using type-II fuzzy sets one can transform a vague pattern classification problem into a precise, well-defined, optimization problem, and secondly type-II fuzzy sets, unlike ordinary fuzzy sets, retain a controlled degree of uncertainty [19]. In this research work, we have employed type-II fuzzy sets for decision making about the pixel's fate (generation of decision map). Further, we exploit the noise free pixels instead of noisy pixels in the local neighborhood window to estimate the pixel under consideration that improves the results as compared with the existing techniques.

Fuzzy logic based smoothing filters have also been applied by many researchers in signal and image processing based applications. These include fuzzy rank selection filter, fuzzy weighted filter, switching fuzzy filter and fuzzy neural network filter [12].

In the present work, we have used both fuzzy based intelligent decision-making and fuzzy smoothing to improve the performance of the proposed spatial adaptive fuzzy kriging filter. The main use of the fuzzy inference system is to generate a fuzzy map from the degraded image, which is then employed by the fuzzy kriging filter to enhance the degraded image. Further, fuzzy smoothing is used to smooth out the unselected pixels within the proposed filter. The current work differs from our previous research findings in [12] in fuzzy decision map generation, fuzzy type-II FIS, improved fuzzy membership functions and rules, and in noisy pixel estimation procedure. The proposed technique offers the following advantages over the existing fuzzy punctual kriging based image restoration approaches:

a. Intelligently differentiate between noisy, non-noisy, edge pixels and improves image restoration results through keeping image detail intact.
b. Type-II fuzzy sets with improved membership functions and rules for fuzzy map generation.
c. Offers optimal/near optimal estimate by utilizing available information in local neighborhood.

d. Robust against different images corrupted through Gaussian white noise of different variances.
e. Computationally efficient as compared with punctual kriging based image restoration techniques at hand.

2.3. **Image quality measures.** Besides mean square error (MSE), peak signal-to-noise ratio (PSNR), weighted peak signal-to-noise ratio (wPSNR) and structure similarity index measure (SSIM) [20,21], other image quality measures in terms of the experimental variograms of the original and degraded images are also used. Whereby, if $\gamma_o(d)$ and $\gamma(d)$ represent the semi-variances at lag d of the original and degraded image respectively, then variogram based image quality measures called variance mean square error (VMSE) and variance peak signal signal-to-noise ratio (VPSNR) can be calculated as

$$\text{VMSE} = \frac{1}{M_d}\sum_{d=0}^{M_d}\left[\gamma_o(d) - \gamma(d)\right]^2 \tag{9}$$

$$\text{VPSNR} = 10\log_{10}\frac{\left[\max\left\{\gamma_o(d)\right\}\right]^2}{\text{VMSE}} \tag{10}$$

Here M_d is the maximum lag for the images. VMSE and VPSNR are global quality measures; however, these do take into account the structural detail information present in the image. Variogram illustrate the variation of semivariance with respect to distance from a point, and semivariance provides a measure of spatial dependence between pixels. Variogram of two different images is different because both images have different distribution of data that can be verified from Figure 5. Further, variogram of an image corrupted with white Gaussian noise shows the up-lifting of the variogram as the noise variance is increased. Furthermore, the general shape and structure of the variogram remains the same for low noise variances as shown in Figure 6. The hypothesis in developing this image quality measure relies on the idea that if a technique which brings the variogram of the restored image very close to that of the original image, will perform better. It can also be verified from Table 1, Figure 7 and Figure 8. Statistical meaning of VMSE is to measure the mean squared error of the variogram of the estimated and the original images. Similarly, VPSNR measures peak signal-to-noise ratio of the estimated and the original images.

3. **The Proposed Approach.** An experimental semi-variogram is the key point for estimating pixel values in a degraded image using kriging. Semi-variogram estimation inherently introduces an element of uncertainty in the overall processing of the image. Fuzzy logic is efficiently well-suited in dealing with such problems. Several traditional image processing techniques have been extended with fuzzy logic giving much improved results, without adding to the complexity of implementing the overall process. Several traditional image processing techniques have been extended with fuzzy logic to simultaneously take care of the conflicting tasks of smoothing and edge preservation. In this paper, we make use of type-II fuzzy logic with improved membership functions and rules to intelligently classify the noisy pixels. The occurrence of singular matrix in kriging is inherently unpredictable as it depends on the variogram for a pixel in the degraded image. The variogram itself depends on neighboring values of a pixel. Such scenarios should be taken care of separately by replacing the processed pixel with a value given by fuzzy 'averaging' or 'median' filter, which ever makes the error variance 'small'. In this experimental study, we assume that the images are corrupted through Gaussian white noise and if singular matrix occurs, we replace the processed pixel by fuzzy averaging. Further, it is observed that for some of the selected pixels, the punctual kriging procedure

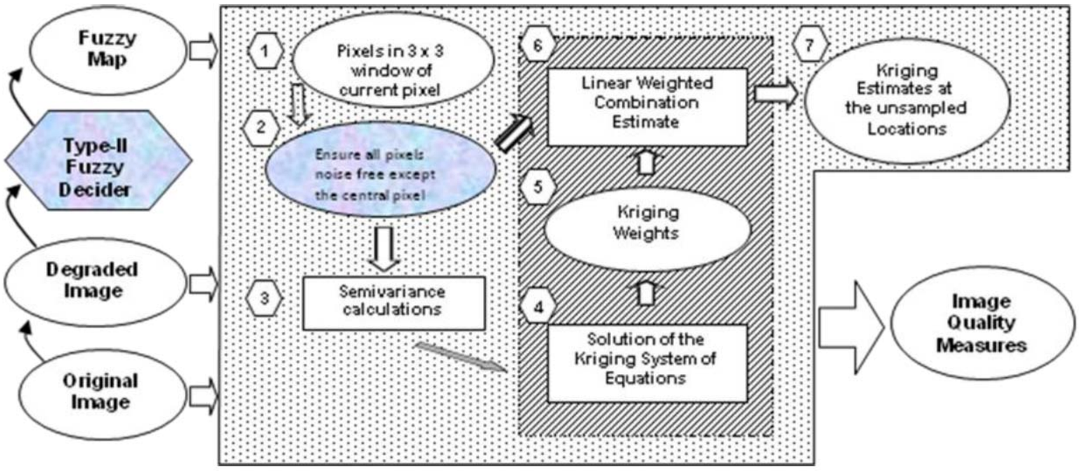


FIGURE 1. Schematic flowchart of the proposed approach

results in negative weights. To handle this problem, we use approximation to reinitialize the weights, i.e., negative weights have been set to zero and positive weights have been renormalized in a similar fashion as described in [12].

In addition to this, kriging procedure is relatively computational expensive as compared with other image smoothing linear filters. This implies that blindly kriging all points in an image would make the estimation procedure infeasible. Further, all pixels in an image may not require estimation as homogenous regions could be enhanced by much less computationally expensive methods like fuzzy averaging. In addition to its computational cost, we need to smooth out noise in certain regions, but still preserve sharpness of the edges. The decision of when to use kriging or not could be related to the smoothness or homogeneity of image structures. Thus, it seems sensible to have a fuzzy logic rule-based system to decide when to perform kriging.

Figure 1 shows the basic architecture of our proposed methodology. Firstly, we generate a map for pixels that needs kriging or not through fuzzy decider. Further, we ensure that there are all noise free pixels around the pixel in question in local window. In case of any noisy pixel found other than the central pixel in local window, we replace it by average of its neighbors lying at one pixel apart. Then selected pixels are estimated by applying punctual kriging. The pixels that are not selected for kriging by the fuzzy decider are processed using the robust fuzzy weighted filter. Lastly, various image quality measures have been employed to analyze the quality of the processed image.

3.1. **Details of different stages of proposed methodology.** In the proposed method, all pixels are not blindly kriged. Rather, based on the homogeneity and deviation of its local neighborhood, a pixel is selected for kriging by a fuzzy logic rule-based system. This fuzzy system is called the *Fuzzy Decider* in our work. The inputs to the Fuzzy Decider are a measure of homogeneity and DAMdistance which is based on the mean and deviation of the 3×3 window around the current pixel. The degree of homogeneity is estimated by Equation (11) as proposed by Tizhoosh [22]. The numerator in Equation (11) is the difference of the maximum and minimum gray values in the region comprising of the 3×3 window around a pixel where, the denominator is the difference of the maximum and minimum gray values in the whole image.

$$\mu_H = \left(\frac{g_{\max}^{local} - g_{\min}^{local}}{g_{\max}^{global} - g_{\min}^{global}} \right) \tag{11}$$

The *DAMdistance* in the rules is simply the difference between the gray value of the current pixel and the mean gray value of its neighbors. The *Fuzzy Decider* is a basic Mamdani type-II fuzzy logic system consisting of the following rules which differs from Mirza et al. [12]. They have employed fuzzy type-I with two rules given below.

If (regionHomogenuity is HomoHigh) or (DAMdistance is acceptable)
then (decision is **KrigingNo**)
If (regionHomogenuity is HomoLow) or (DAMdistance is veryHigh)
then (decision is **KrigingYes**)

3.2. **Type-II fuzzy sets.** The concept of type-II fuzzy logic is actually an extension of type-I fuzzy sets. Type-II fuzzy sets are capable to handle more uncertainties in spite of the fact that they are difficult to use and understand than type-I fuzzy sets. We have used type-II fuzzy set to enhance the fuzzification process for better decision making. In our approach based on type-II fuzzy sets, we have used the following nine rules to decide the pixel's fate.

If (regionHomogenuity is HomoMed) and (DAMdistance is Acceptable)
then (decision is **KrigingNo**)
If (regionHomogenuity is HomoMed) and (DAMdistance is High)
then (decision is **KrigingYes**)
If (regionHomogenuity is HomoMed) and (DAMdistance is VeryHigh)
then (decision is **KrigingYes**)
If (regionHomogenuity is HomoLow) and (DAMdistance is Acceptable)
then (decision is **KrigingNo**)
If (regionHomogenuity is HomoLow) and (DAMdistance is High)
then (decision is **KrigingYes**)
If (regionHomogenuity is HomoLow) and (DAMdistance is VeryHigh)
then (decision is **KrigingYes**)
If (regionHomogenuity is HomoHigh) and (DAMdistance is Acceptable)
then (decision is **KrigingNo**)
If (regionHomogenuity is HomoHigh) and (DAMdistance is High)
then (decision is **KrigingNo**)
If (regionHomogenuity is HomoHigh) and (DAMdistance is VeryHigh)
then (decision is **KrigingYes**)

Figures 2 and 3 show the graphical representations of type-II fuzzy membership functions. Fuzzy decision maps generated by fuzzy type-I and type-II are described below.

3.3. **Generation of fuzzy decision map through fuzzy-II.** In the first stage, the noisy image is presented to the *Fuzzy Decider* that generates a binary image called the *fuzzy decision map*. This decision map is generated through type-II fuzzy set and is provided to the punctual kriging based estimation stage, where the decision of whether to estimate or not is enforced. This helps to reduce the computational time. Effectiveness of fuzzy decision map generated through fuzzy type-I and type-II has been compared for image restoration. Fuzzy decision maps generated through fuzzy type-I and type-II for cameraman image are shown in Figure 4. It can be observed from Figure 4 that fuzzy map generated through type-II avoids to select edge pixels for estimation whereas type-I selects more for estimation including edge pixels results in edge smoothing as compared with type-II fuzzy.

3.4. **Employing punctual kriging for estimation.** In the second stage, an attempt is made to find a *kriging estimate* for pixels selected by the *Fuzzy Decider*. If the attempt fails, the original pixel value is taken as the processed pixel value. The attempt to find

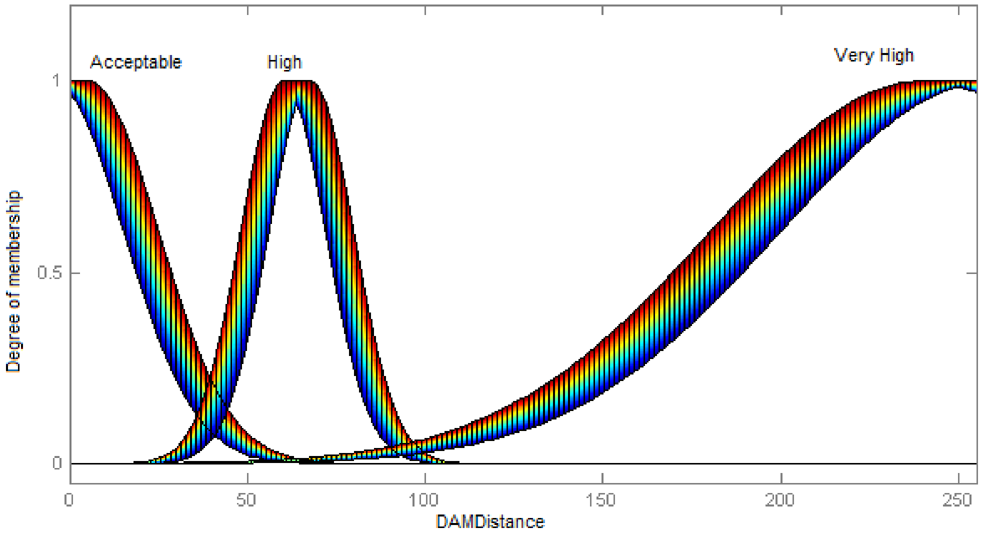


FIGURE 2. Type-II fuzzy membership functions for DAMdistance

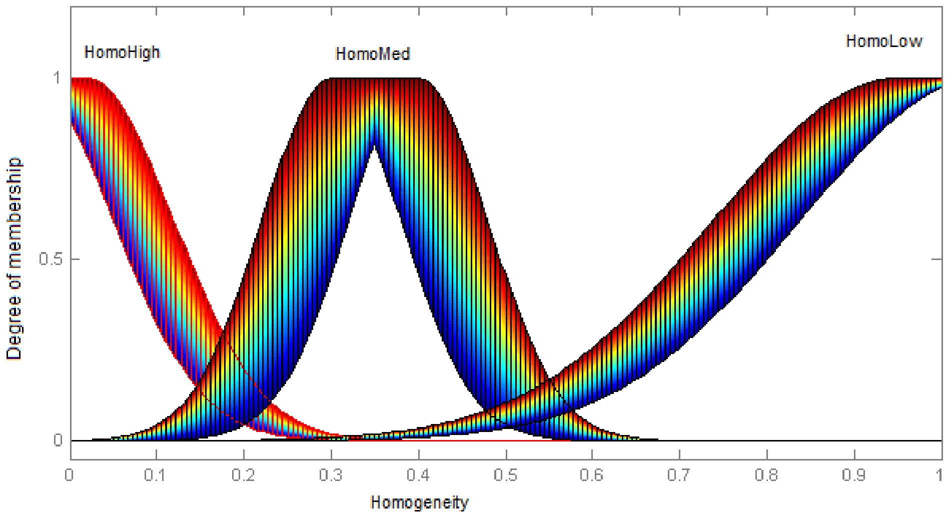
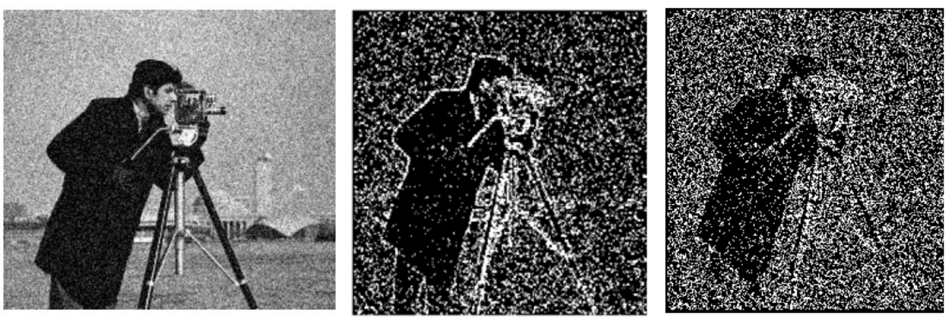


FIGURE 3. Type-II fuzzy membership functions for homogeneity



(a) Noisy cameraman image (b) Fuzzy type-I decision map (c) Fuzzy type-II decision map

FIGURE 4. Result from (a) noisy cameraman image, (b) fuzzy type-I, and (c) fuzzy type-II decision map for cameraman image degraded with variance 0.02

a kriging estimate was found to fail due to two broad reasons: *singular kriging matrix* and *negative-weights*. The pixels that were rejected for kriging by the Fuzzy Decider are processed using the robust fuzzy weighted filter. After this stage, the processed image contains two types of values based on the decision map: kriging estimate and the fuzzy smoothed values. There is a third category of noisy pixels for which kriging could not give optimum weights, i.e., the weights either do not sum up to 1 or the sum of square of weights is not less than or equal to 1. The negative weights in this case are set to zero and the positive weights are normalized to sum to 1.

3.5. **Fuzzy smoothing of pixels not selected for kriging.** In the third stage, the unselected pixels by the Fuzzy Decider are processed using the robust fuzzy weighted filter. After the second stage, the processed image contains two types of values based on the decision map: kriging estimate and original values (unselected pixels). In this stage, a fuzzy smoothing is applied on the unselected pixels.

4. Results and Discussions.

4.1. **Variograms of the original and degraded images.** The experimental semi-variograms of three different types of images (Boat, Blood cells and Lena) have been computed and shown in Figure 5. The shapes of the variograms for all three images near lag zero are continuous. This shows that the pixel values do not change abruptly at lags near zero. However, for Lena and Boat images, fluctuations start appearing for lags greater than 10. This shows that after a lag of 10 pixels, we enter into a new region. Further, in case of Blood cells image, the fluctuations appear after a lag of 20 pixels. The variograms show sharp changes for larger lags.

Figure 6 shows the changes in the experimental variogram when a zero mean Gaussian noise with various variances is added to a particular image. The most interesting aspect to note is the up-lifting of the variogram as the noise variance is increased (see Figure

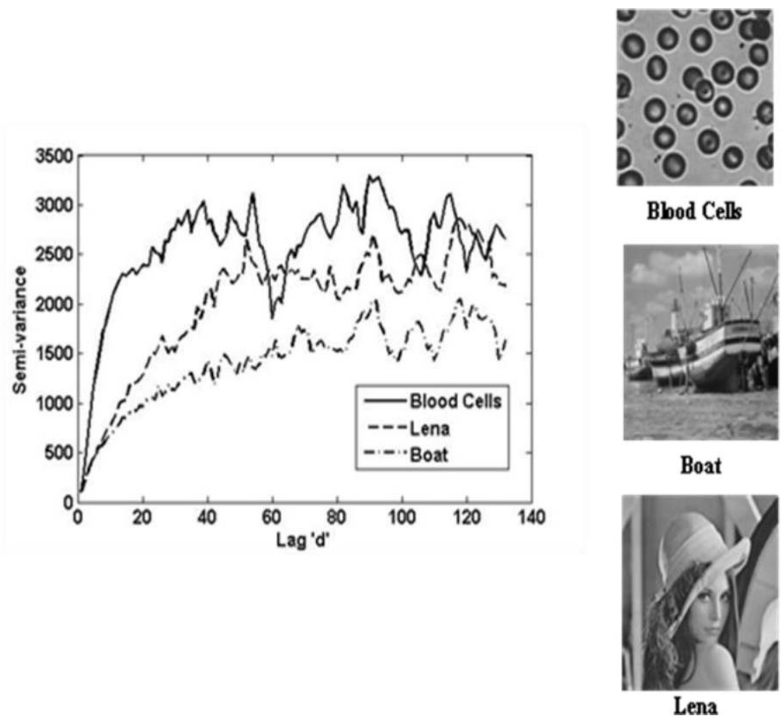


FIGURE 5. Experimental variograms of three different images

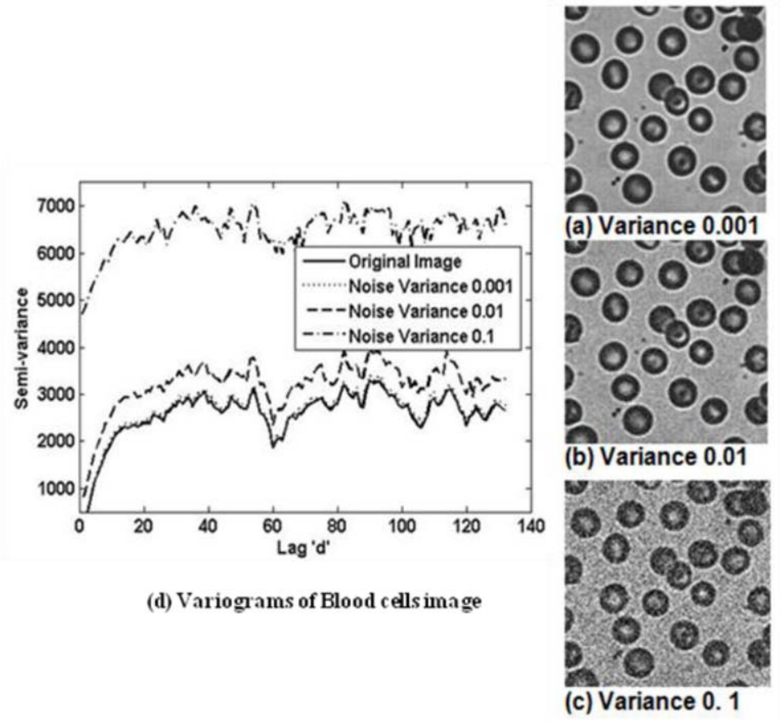


FIGURE 6. (a) – (c) Gaussian noise corrupted images with zero mean and different variances, (d) variograms of Blood cells image with additive Gaussian white noise

6(d)). It is also important to note that the general shape and structure of the variogram stays the same for low noise variances. The abrupt changes in the variogram take place at the same lags. Even for high noise variance, the shape of the variogram remains similar to that of the original image; however, the abrupt changes become more discontinuous. Further, near zero lag, the variogram becomes highly discontinuous as the additive noise variance is increased. These observations have led us to introduce the variogram based image quality measures VMSE and VPSNR, as introduced in Section 2.

Various image quality measures as explained in Section 2 are applied to find out the quality of the processed image as compared to the original image. We have tested the performance of the proposed approach (both using type-II fuzzy sets in SAFK instead of type-I named as spatially adaptive fuzzy-II kriging (SAFK-II) and type-II with local information about the pixels in the neighborhood window around the pixel under consideration) by considering three scenarios.

Firstly, the performance of the proposed method has been tested for additive Gaussian white noise of different variances for a test image. Secondly, the performance is tested for 450 different images corrupted with Gaussian white noise of same variance. This is because the effect of noise may change with the variance of noise as regards the visual distortion for the same image. On the other hand, same noise may affect different images differently as regards the visual distortion. Typical results from the Fuzzy Decider type-I and type-II shown in Figure 4. The white pixels are the ones that need kriging.

4.2. **Scenario 1: performance analysis by varying variance of Gaussian noise.** In the first case, we have considered Boat image as test data. The image is degraded with Gaussian white noise of variance ranging from 0.02 to 0.12. The results obtained from our approach have been compared with that of the AWF, PWFK, SAFK and SAFK-II approaches. The effect of the additive Gaussian noise and its removal by various approaches

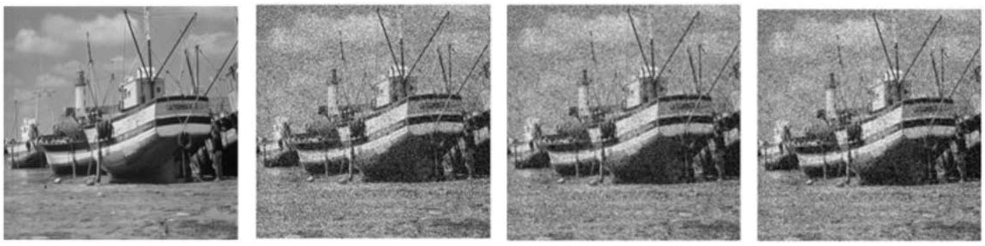
Table 1. Comparison of various approaches against Boat image at different noise variances

Noise Variances	Denoizing Methods	Qualitative Measures MSE	PSNR (db)	SSIM	VMSE	VPSNR (db)
0.12	Noisy Image	5441.96	10.77	0.07	24162143	–8.30
	PWFK	1782.12	15.62	0.15	1671512	2.83
	AWF	1030.22	17.95	0.23	188895	12.30
	SAFK	1001.12	18.25	0.23	170254	12.58
	SAFK-II	987.60	18.36	0.23	162518	12.85
	Proposed Approach	968.61	18.87	0.24	154279	13.18
0.08	Noisy Image	4112.58	11.98	0.10	14130729	–6.43
	PWFK	1297.69	17.00	0.19	850778	5.76
	AWF	793.21	19.13	0.28	132926	13.82
	SAFK	783.70	19.18	0.28	110521	14.63
	SAFK-II	745.04	19.40	0.28	79716	16.05
	Proposed Approach	721.07	19.73	0.29	66047	17.26
0.04	Noisy Image	2336.64	14.44	0.15	4744911	–1.69
	PWFK	738.94	19.44	0.27	230101	11.44
	AWF	471.64	21.19	0.38	53139	17.81
	SAFK	420.29	21.69	0.39	11036	24.63
	SAFK-II	412.22	21.82	0.39	9643	25.22
	Proposed Approach	403.45	21.89	0.40	7255	26.45
0.02	Noisy Image	1241.91	17.18	0.23	1399773	3.60
	PWFK	421.27	21.88	0.37	43712	18.65
	AWF	262.82	23.71	0.51	9538	25.27
	SAFK	229.72	24.29	0.53	1850	32.39
	SAFK-II	225.03	24.36	0.53	1281	33.98
	Proposed Approach	210.59	24.45	0.53	1176	34.25

is shown in Figure 7. Table 1 gives a quantitative comparison among different methods in terms of MSE, PSNR, SSIM, VMSE and VPSNR at test image against various variances. It can be observed from Table 1 that the proposed approach based on fuzzy type-II, and punctual kriging in conjunction with local information offers superior performance at all noise levels compared to the rest of the methods.

The experimental variograms of the original, noisy, and restored images through AWF, PWFK, SAFK, SAFK-II and proposed fuzzy type-II approach are plotted in Figure 8. The image is corrupted with Gaussian noise of variance 0.05. From Figure 8, it is clear that variograms of both the original as well as noisy image retain the structural information about the image and differ only in the semivariance at different lags depending upon the strength of the noise variance. Further, in comparison to the variograms produced by other methods, our approach produces a variogram that is very close to the variogram of the original image. This is also clear from Table 1, where the proposed approach outperforms at all quality measures compared with AWF, PWFK, SAFK and SAFK-II image restoration techniques.

4.3. **Scenario 2: performance analysis using 450 test images.** In the second case, we consider 450 different images as the test data. These images have been corrupted with white Gaussian noise of variance 0.08. Performance analysis of the above-mentioned



(a) Original image (b) Degraded image (c) AWF (d) PWFK

(e) SAFK (f) SAFK-II (g) The proposed approach

Figure 7. Original, noisy and estimated images obtained through AWF, PWFK, SAFK, SAFK-II, and the proposed approach

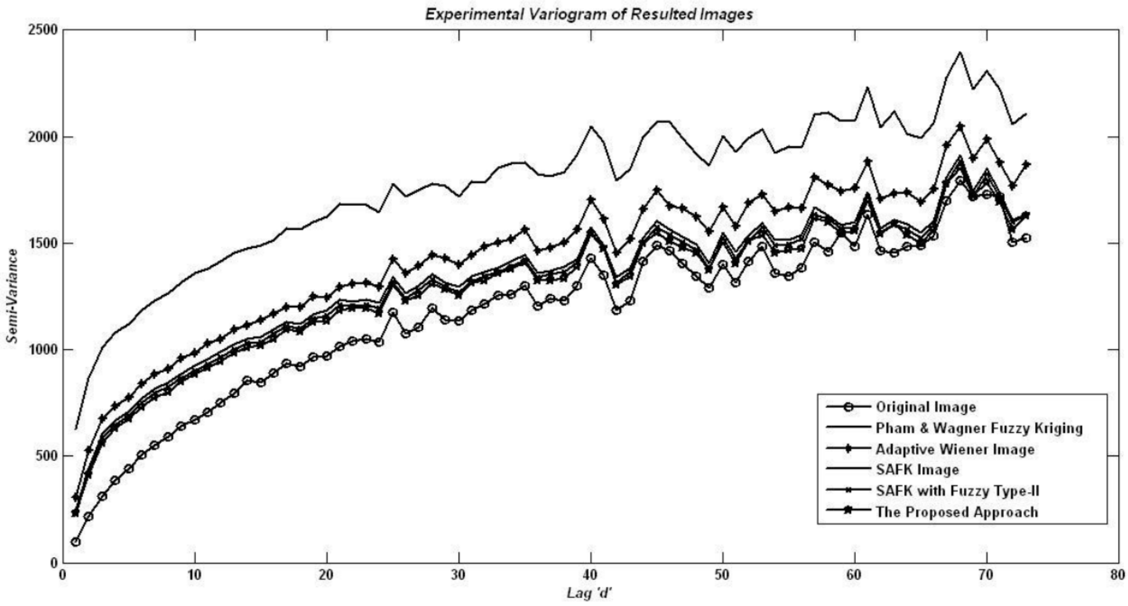


Figure 8. Comparison of the variograms of the original, degraded and processed Boat image

methods is carried out in terms of average values of MSE, PSNR, SSIM, VMSE and VPSNR across 450 test images as shown in Table 2. The graphical representation of various performance measures is shown in Figure 9. Results obtained through the proposed approach have been compared with the existing techniques. It can be observed from results presented in Table 2 that the performance of our proposed approach based on fuzzy type-II, local neighborhood information of the pixels around pixel under estimation and punctual kriging is better as compared with PWFK, SAFK, SAFK-II and AWF in terms of the image quality measures.

TABLE 2. Comparison of various approaches against 450 image database at 0.08 noise variance

Denoizing Methods	Qualitative Measures MSE	PSNR (db)	SSIM	VMSE	VPSNR (db)
Noisy Image	*3826.47*	*12.31*	*0.14*	*11989834*	*−6.454*
PWFK	*1315.80*	*16.94*	*0.23*	*707424*	*6.29*
AWF	*850.20*	*18.74*	*0.33*	*129207*	*14.53*
SAFK	*801.34*	*19.01*	*0.34*	*111842*	*15.35*
SAFK-II	*776.86*	*19.23*	*0.34*	*98308*	*15.89*
The Proposed Approach	*754.27*	*19.46*	*0.35*	*93940*	*16.10*

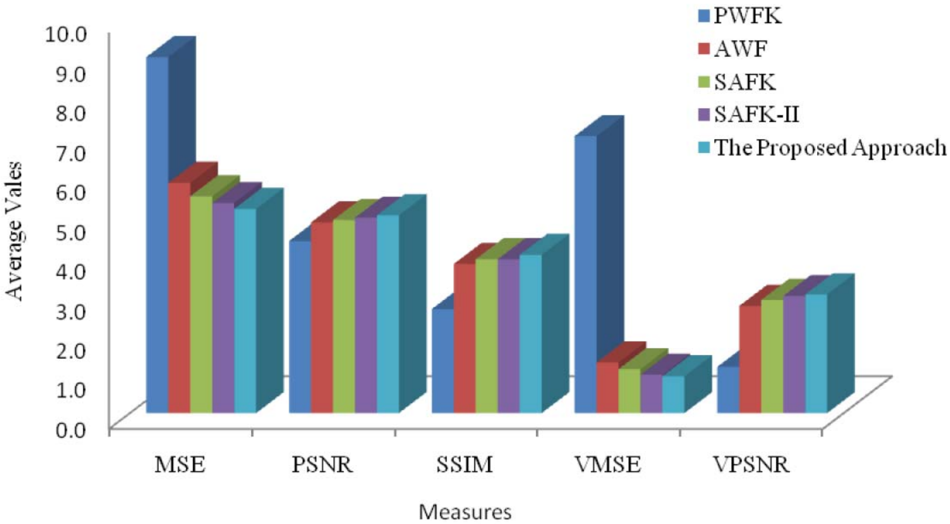


FIGURE 9. Comparison of 450 images test data with different methods at 0.08 noise variance. Average values of various qualitative measures (rescaled for elaboration purpose).

4.4. **Performance comparison using type-I and type-II fuzzy sets.** The results of fuzzy type-I and type-II based punctual kriging approaches are compared using various image quality measures. Number of pixels selected for kriging by the fuzzy decider using fuzzy type-I and type-II are shown in Table 3. From Table 3, it is clear that fuzzy decider using fuzzy type-II selects less number of noisy pixels for high variance compared with that of type-I fuzzy set. The reason for the selection of less number of noisy pixels by type-II fuzzy set is that it avoids over smoothing and preserves edges more efficiently for a noise variance range [0.04 to 0.1]. Whereas for low level of noise, it selects more pixels as compared with type-I fuzzy set. This is because it selects pixels for noise removal that are corrupted but close to homogenous or textured gray level. The performance of the proposed approach is better than AWF, PWFK, SAFK, SAFK-II at all noise levels in terms of MSE, PSNR, SSIM, VMSE and VPSNR. In Figure 8, variograms of the proposed approach based on fuzzy type-II, and utilization of local information in conjunction with punctual kriging based estimation is compared with existing approaches using boat image. It can be observed that the variogram curve of the proposed method is very close to the original image and thus shows the advantage over the existing approaches.

Table 3. Statistics of the pixels selected for kriging by the fuzzy type-I and type-II. The Boat image degraded with white Gaussian noise of different variances.

Statistics of the data	*Gaussian white noise of different variance*					
	0.1	*0.08*	*0.06*	*0.04*	*0.02*	*0.01*
No. of pixels advised for kriging by type-II fuzzy	*164943*	*155123*	*140831*	*119244*	*78852*	*39846*
No. of pixels advised for kriging by type-I fuzzy	*206215*	*190083*	*165535*	*124774*	*54927*	*13857*

Table 4. Comparison of different approaches in terms of computational cost (in seconds)

Test Images	*De-noising Methods*				
	PWFK	*AWF*	*SAFK*	*SAFK-II*	*Proposed Approach*
Blood Cells	*13.375*	*0.125*	*0.938*	*0.81*	*0.83*
Cameraman	*10.813*	*0.125*	*1.14*	*0.992*	*1.02*
Boat	*50.047*	*0.328*	*3.391*	*2.91*	*3.08*
Lena	*52.313*	*0.343*	*2.968*	*2.58*	*2.62*
Baboon	*47.765*	*0.36*	*5.906*	*4.71*	*4.93*

4.5. **Performance comparison in terms of computational cost.** Performance of the proposed approach in terms of computational cost has also been made with AWF, PWFK, SAFK and SAFK-II. Results regarding time taken by various approaches to restore various noisy images are presented in Table 4. From Table 4, it may be observed that the proposed approach out performs PWFK and SAFK. Further, it offers almost same computational performance as compared with SAFK-II. Furthermore, it is slightly expensive in computation as compared with AWF like other fuzzy kriging based image restorations approaches (PWFK, SAFK and SAFK-II).

5. **Conclusions.** An intelligent and improved image denoising method based on the concept of punctual kriging is analyzed. More improved Fuzzy type-II IF THEN rules based on region homogeneity and deviations, are used to intelligently decide the fate of a pixel whether it needs to be estimated or not in view of edge preservation. The performance of type-II fuzzy sets with local neighborhood information has been analyzed for this purpose. The overall kriging procedure is coupled with a fuzzy smoothing filter. Due to the use of type-II Fuzzy Decider and local information about the status of pixels in window, punctual kriging is employed to estimate pixels along region boundaries and isolated discontinuities. However, for pixels inside the regions, away from the region boundaries, fuzzy smoothing is used. The results show a marked improvement in the performance of image restoration scheme as compared with the existing fuzzy kriging and adaptive Wiener filter approaches. A total of 450 images are used to test the effectiveness of the proposed fuzzy type-II based punctual kriging image restoration approach. The proposed technique outperforms at all determined image quality measures at all noise levels. In future work, we intend to use evolutionary algorithms in addition to fuzzy sets for developing a majority voting based composite predictor. The composite predictor thus formed would decide the fate of pixel under consideration for the better exploitation of the intensity variation on the edges, lines and object boundaries in the image.

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