International Journal of Innovative Computing, Information and Control
Volume **9**, Number **2**, February **2013**

ICIC International ©2013 ISSN 1349-4198
pp. **625–640**

OBSERVER BASED REGULATOR PROBLEM FOR WWTP WITH CONSTRAINTS ON THE CONTROL

Fouad Mesquine and Othman Bakka

Faculty of Sciences-Semlalia
Cadi Ayyad University
P.O.Box 2390, Av. Pr. My. Abdellah, Marrakesh 40000, Morocco
mesquine@uca.ma; bakka.othman@gmail.com

Received November 2011; revised March 2012

Abstract. *The aim of this work is to control a non linear biological nitrogen removal process. The paper illustrates the design steps of an observer based control scheme applied to the linearized model of a phenomenological model of the process. The estimation algorithm is combined with the control technique to monitor the process. The goal of the control is the removal or at least the reduction of organic waste. The control law is based on positive invariance concept that had shown efficiency in handling control constraints. The efficiency of both the control and the estimation is demonstrated via computer simulations.*
Keywords: Activated sludge, WasteWater Treatment Plants (WWTP), Constrained control, Positive invariance, Observers

1. **Introduction.** The modeling and control of activated sludge process, which is recognized as the most common and major unit process for reduction of organic waste, has become a subject of great interest. Researchers [2, 9, 20, 21, 28] have investigated different control strategies for the monitoring of such processes. The development of effective control strategies on this kind of WasteWater Treatment Plants (WWTP) is hampered by the inherent nonlinearites, the time-varying dynamics and the lack of suitable instrumentation.

WasteWater treatment is just one component in the urban water cycle. However, it is an important one since it ensures that the environmental impact of water human usage is significantly reduced. The treatment consists of several processes: biological, chemical and physical. WasteWater treatment aims to reduce the amount of nitrogen, phosphorous, organic matter and suspended solids. To reduce the amount of these substances, WWTP consisting of four treatment steps have been designed. These steps are a primarily mechanical pre-treatment step, a biological treatment step, a chemical treatment step and a final step of sludge treatment.

The purpose of the mechanical pre-treatment step is to remove various types of suspended solids from the incoming WasteWater. The aim of the biological treatment step has originally been solely to remove organic matter. However, today many WWTP are also designed for the biological removal of nitrogen and phosphorous. The most common type of biological treatment step is based on the activated sludge process. In the biological treatment of WasteWater, the sedimentation process enables to separate the treated wastewater from the biomass sludge and produces a clear treated effluent. In addition to clarification, secondary settler tanks or clarifiers have the function of thickening the activated sludge for returning to the bioreactor and even to storage tank. The settling process can take place in the same reactor or in a secondary settler. By all these reasons, secondary settling tanks have been considered essential and often they can be limiting factors for good removal efficiencies of the activated sludge system. The purpose of the sludge treatment step is to prepare the sludge for end disposal. Anaerobic digestion is probably one of the most used processes for reducing the amount of sludge. At the same time, the digestion process produces gaz, providing a significant source of energy, which is usually used at the WWTP. For control process the WWTP is modeled by ordinary differential equations derived from mass-balance considerations, which involve nonlinear terms. The most important parameter is the specific growth rate which is a complex nonlinear function of plant states and several uncertain biological parameters.

On the other hand, the state space representation is frequently used to form multivariable approach to linear control. The most common control schemes are based on availability of the state for feedback. In the same real process, it is either impossible or inappropriate to measure all elements of system state. To overcome this problem, an auxiliary dynamical system, known as observer, driven by the inputs and outputs of the original system, is designed [13]. Another problem that arises when considering real process is the limitation of state or control of the process. In fact, processes are naturally non linear and to obtain linear useful model, approximation of small variations around steady state is used. Hence, validity of such linear model is limited to a neighborhood of the steady point leading to constraints on some variables. Further, inherent physical limitations may be source of limited variables. The respect of these constraints can be accomplished by designing suitable feedback control laws. In many cases, this can be done by constructing positively invariant domains inside the set of the constraints, [1, 5, 7, 18]. Other important applications were derived from this concept. In particular, the observers in the framework of positive invariant sets are given in [15, 16].

During the last decades, many investigations have been focused on the control of the nitrogen and dissolved oxygen in an activated sludge reactor within a WWTP with different strategies. One may quote predictive control, optimal control and adaptive control, etc. [3, 23]. Note here that constraints on the control are handled and further all required measurements are assumed available. A part from this, one may also cite works about the same topic but limited to estimation [8, 11] and not the control. Furthermore, works combining estimation to control for monitoring such processes can also be found [26]. However, constraints are not taken into account during the design steps. Therefore, these works may be thought as a generalization where constraints, estimation and control are considered using the positive invariance concept together at the design stage.

The objective of this work is to apply positive invariance concept techniques to a WWTP. The obtained linearized model combines the problems of non availability of the state to measure with the limitations of some variables. The control is achieved by an observer based controller that can take into account constraints on the control and on the error. The obtained linear model is worked out to meet all design required conditions. The efficiency of the process monitoring is showed via simulations with the real plant.

The remainder of the paper is organized as follows. Section 2 provides the modeling of the WWTP through the modeling of the aerated, anoxic and settler basins. Further, the obtained model is linearized around a steady state point. Section 3 is devoted to the presentation of the control scheme. It consists of an observer based controller that respects constraints on the control and on the observation error and achieves asymptotic stability of the system. Application to the WWTP is achieved in Section 4. First, the model of the WWTP is worked out to obtain the controllable and observable parts of the system. Second, a reduced order observer is applied to the obtained controllable and observable part. Finally, the simulated non linear system parameters and the ones obtained from our control scheme are compared. Section 5 reports and discusses the obtained results and Section 6 concludes the paper.

Notations:

- For two vectors x, y of $\mathbb{R}^n$, $x \leq y$ (respectively, $x < y$) means $x_i \leq y_i$ (respectively, $x_i < y_i$), $i = 1, \ldots, n$.
- $x_i^+ = \sup(x_i, 0)$, $x_i^- = \sup(-x_i, 0)$, and for $x(t)$ a function of time, $\dot{x} = \frac{dx}{dt}$.
- For $A \in \mathbb{R}^{n\times n}$, $\sigma(A)$ denotes its spectrum and

$$\widetilde{A} = \begin{pmatrix} A_1 & A_2 \\ A_2 & A_1 \end{pmatrix}, \quad A_1(i,j) = \begin{cases} a_{ij}, & i = j \\ a_{ij}^+, & i \neq j \end{cases} \quad \text{and} \quad A_2(i,j) = \begin{cases} 0, & i = j \\ a_{ij}^-, & i \neq j \end{cases}$$

- $\mathbb{I}_n$ and $int\mathbb{R}_+^n$ are respectively the identity matrix of dimension n and the interior of $\mathbb{R}_+^n$.

2. **Process Modeling.** A typical, conventional activated sludge plant for the removal of carbonaceous and nitrogen materials consists of an anoxic basin followed by an aerated one, and a settler; see the figure below. In the presence of dissolved oxygen, wastewater, that is mixed with the returned activated sludge, is biodegraded in the aerated reactor. Treated effluent is separated from the sludge and wasted while a large fraction is returned to the anoxic reactor to maintain an appropriate substrate-to-biomass ratio.

In this work, six basic components are present in the wastewater: autotrophic bacteria X_A, heterotrophic bacteria X_H, readily biodegradable carbonaceous substrates S_S, nitrogen substrates S_{NH}, S_{NO} and dissolved oxygen S_O, where X_A, X_H, S_S, S_{NH}, S_{NO} and S_O represent the concentrations of these elements. In the modeling of the process, the following assumptions are considered: first, the physical properties of fluid are constant and there is no concentration gradient across the vessel. Second, substrates and dissolved oxygen are considered as rate-limiting with a bi-substrate Monod-type Kinetic. Finally, no bioreaction takes place in the settler that is considered perfect.

Based on the above description and assumptions, the full set of ordinary differential equations (mass balance equations), making up the IAWQ (ASM1) Model NO.1 are obtained [10, 22].

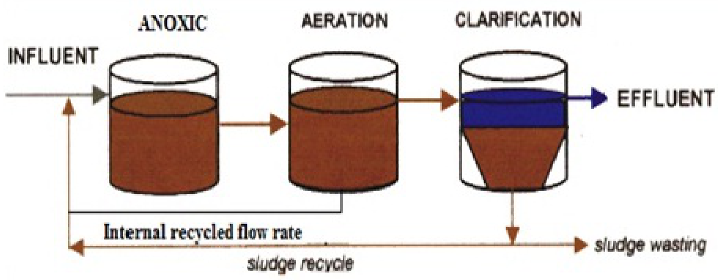

FIGURE 1. W.W.T. Plant

2.1. **Modeling of the aerated basin.** In the aerated basin, writing the mass balance equations leads to the following:

$$\dot{X}_{A,nit}(t) = (1 + r_1 + r_2)\, D_{nit}\, (X_{A,denit} - X_{A,nit}) + (\mu_{A,nit} - b_A)\, X_{A,nit} \tag{1}$$

$$\dot{X}_{H,nit}(t) = (1 + r_1 + r_2)\, D_{nit}\, (X_{H,denit} - X_{H,nit}) + (\mu_{H,nit} - b_H)\, X_{H,nit} \tag{2}$$

$$\dot{S}_{S,nit}(t) = (1 + r_1 + r_2)\, D_{nit}\, (S_{S,denit} - S_{S,nit}) + (\mu_{H,nit} + \mu_{Ha,nit})\, X_{H,nit}/Y_H \tag{3}$$

$$\begin{aligned}\dot{S}_{NH,nit}(t) = {} & (1+r_1+r_2)\, D_{nit}\,(S_{NH,denit} - S_{NH,nit}) + (i_{xb} + 1/Y_A)\, \mu_{A,nit} X_{A,nit} \\ & - (\mu_{H,nit} + \mu_{Ha,nit})\, i_{xb} X_{H,nit}\end{aligned} \tag{4}$$

$$\begin{aligned}\dot{S}_{NO,nit}(t) = {} & (1+r_1+r_2)\, D_{nit}\,(S_{NO,denit} - S_{NO,nit}) + \mu_{A,nit}\frac{X_{A,nit}}{Y_A} \\ & - \frac{1-Y_H}{2.86 Y_A}\mu_{Ha,nit} X_{H,nit}\end{aligned} \tag{5}$$

$$\begin{aligned}\dot{S}_{O,nit}(t) = {} & (1+r_1+r_2)\, D_{nit}\,(S_{O,denit} - S_{O,nit}) + a_0 Q_{air}(C_S - S_{O,nit}) \\ & - \frac{4.57-Y_A}{Y_A}\mu_{A,nit} X_{A,nit} - \frac{1-Y_H}{Y_H}\mu_{Ha,nit} X_{H,nit}\end{aligned} \tag{6}$$

where

$$\mu_{A,nit} = \mu_{\max,A}\frac{S_{NH,nit}}{(K_{NH,A}+S_{NH,nit})}\frac{S_{O,nit}}{(K_{O,A}+S_{O,nit})}$$

$$\mu_{H,nit} = \mu_{\max,H}\frac{S_{S,nit}}{(K_S+S_{S,nit})}\frac{S_{NH,nit}}{(K_{NH,H}+S_{NH,nit})}\frac{S_{O,nit}}{(K_{O,H}+S_{O,nit})}$$

$$\mu_{Ha,nit} = \mu_{\max,H}\frac{S_{S,nit}}{(K_S+S_{S,nit})}\frac{S_{NH,nit}}{(K_{NH,H}+S_{NH,nit})}\frac{K_{O,H}}{(K_{O,H}+S_{O,nit})}\frac{S_{NO,nit}}{(K_{NO}+S_{NO,nit})}\eta_{NO}$$

$\mu_{A,nit}$ and $\mu_{H,nit}$ are the growth rates of autotrophs and heterotrophs in aerobic conditions and $\mu_{Ha,nit}$ is the growth rate of heterotrophs in anoxic conditions.

2.2. **Modeling of the anoxic basin.** In the anoxic basin, mass balance equations lead to the following:

$$\begin{aligned}\dot{X}_{A,denit}(t) = {} & D_{denit}(X_{A,in} + r_1 X_{A,nit}) - (1+r_1+r_2) D_{denit} X_{A,denit} + \\ & + \alpha . r_2 D_{denit} X_{rec} + (\mu_{A,denit} - b_A) X_{A,denit}\end{aligned} \tag{7}$$

$$\begin{aligned}\dot{X}_{H,denit}(t) = {} & D_{denit}(X_{H,in} + r_1 X_{H,nit}) - (1+r_1+r_2)\, D_{denit} X_{H,denit} + \\ & + (1-\alpha) r_2 D_{denit} X_{rec} + (\mu_{H,denit} - b_H) X_{H,denit}\end{aligned} \tag{8}$$

$$\begin{aligned}\dot{S}_{S,denit}(t) = {} & -(\mu_{H,denit} + \mu_{Ha,denit})\frac{X_{H,denit}}{Y_H} - (1+r_1+r_2)\, D_{denit} S_{S,denit} \\ & + D_{denit}(S_{S,in} - r_1 S_{S,nit})\end{aligned} \tag{9}$$

$$\begin{aligned}\dot{S}_{NH,denit}(t) = {} & D_{denit}(S_{NH,in} - r_1 S_{NH,nit}) - (1+r_1+r_2)\, .D_{denit} S_{NH,denit} \\ & - (i_{xb} + 1/Y_A)\, \mu_{A,denit} X_{A,denit} - (\mu_{H,denit} + \mu_{Ha,denit})\, i_{xb} X_{H,denit}\end{aligned} \tag{10}$$

$$\begin{aligned}\dot{S}_{NO,denit}(t) = {} & D_{denit}(S_{NO,in} - r_1 S_{NO,nit}) - (1+r_1+r_2)\, D_{denit} S_{NO,denit} + \\ & + \frac{\mu_{A,denit} X_{A,denit}}{Y_A} - \frac{1-Y_H}{2.86 Y_H}\mu_{Ha,denit} X_{H,denit}\end{aligned} \tag{11}$$

where

$$\mu_{A,denit} = \mu_{\max,A} \cdot \frac{S_{NH,denit}}{(K_{NH,A}+S_{NH,denit})}$$

$$\mu_{H,denit} = \mu_{\max,H} \cdot \frac{S_{S,denit}}{(K_S+S_{S,denit})} \cdot \frac{S_{NH,denit}}{(K_{NH,H}+S_{NH,denit})}$$

$$\mu_{Ha,denit} = \mu_{\max,H} \cdot \frac{S_{S,denit}}{(K_S+S_{S,denit})} \cdot \frac{S_{NH,denit}}{(K_{NH,H}+S_{NH,denit})} \cdot \frac{S_{NO,denit}}{(K_{NO}+S_{NO,denit})} . \eta_{NO}$$

TABLE 1. Process characteristics

Variable	Value	Description
V_{nit}	1333 m^3	volume of nitrification basin
V_{denit}	1000 m^3	volume of denitrification basin
V_{dec}	6000 m^3	volume of settler
Q_{in}	18446 m^3/j	influent flow rate
Q_w	385 m^3/j	waste flow rate
$X_{A,in}$	0 mg/l	autotrophs in the influent
$X_{H,in}$	30 mg/l	hetertrophs in the influent
$S_{S,in}$	200 mg/l	substrate in the influent
$S_{NH,in}$	30 mg/l	ammonium in the influent
$S_{NO,in}$	2 mg/l	nitrate in the influent
$S_{O,in}$	0 mg/l	oxygen in the influent

TABLE 2. Kinetic parameters and stochiometric coefficient characteristic

Variable	Value	Description
Y_A	0.24	yield of autotroph mass
Y_H	0.67	yield of heterotroph mass
i_{xb}	0.086	
K_S	20 mg/l	affinity constant
$K_{NH,A}$	1 mg/l	affinity constant
$K_{NH,H}$	0.05 mg/l	affinity constant
K_{NO}	0.5 mg/l	affinity constant
$K_{O,A}$	0.4 mg/l	affinity constant
$K_{O,H}$	0.2 mg/l	affinity constant
$\mu_{A\max}$	0.8 l/j	maximum specific growth rate
$\mu_{H\max}$	0.6 l/j	maximum specific growth rate
b_A	0.2 l/j	decay coefficient of autotrophs
b_H	0.68 l/j	decay coefficient of heterotrophs
η_{NO}	0.8 l/j	correction factor for anoxic growth
α	0.5	

2.3. **Modeling of the setller.** In the setller, the mass balance equations enable us to write:

$$\dot{X}_{rec} = (1 + r_2)D_{dec}(X_{A,nit} + X_{H,nit}) - (r_2 + w)D_{dec}X_{rec} \tag{12}$$

Above, r_1, r_2 and w represent, respectively, the ratios of the internal recycled flow Q_{r1}, the recycled flow Q_{r2} and the waste flow Q_w to influent flow Q_{in}. That is $Q_{r1} = r_1Q_{in}$, $Q_{r2} = r_2Q_{in}$ and $Q_w = wQ_{in}$. Further, C_S is the maximum dissolved oxygen concentration and X_{rec} is the concentration of the recycled biomass. Finally, $D_{nit} = \frac{Q_{in}}{V_{nit}}$, $D_{denit} = \frac{Q_{in}}{V_{denit}}$ and $D_{dec} = \frac{Q_{in}}{V_{dec}}$; where D_{nit}, D_{denit} and D_{dec} are the dilution rates in respectively, nitrification, denitrification, basins and settler tank. All remaining involved variables and parameters of the system (1)-(12) have been directly taken from [25], and are defined in Tables 1 and 2. To obtain a model in the state space, the state vector is considered as

$$\begin{aligned} X(t) = [&X_{A,nit}(t)\ X_{H,nit}(t)\ S_{S,nit}(t)\ S_{NH,nit}(t)\ S_{NO,nit}(t)\ S_{O,nit}(t)\ X_{A,denit}(t) \\ &X_{H,denit}(t)\ S_{S,denit}(t)\ S_{NH,denit}(t)\ S_{NO,denit}(t)\ X_{rec}(t)]^T. \end{aligned} \tag{13}$$

Further, to complete the model, the following input and output vectors are used

$$U(t) = [Q_{r1}\ Q_{r2}\ Q_{air}]^T, \tag{14}$$

$$Y(t) = [S_{NH,nit}(t)\ S_{NO,nit}(t)\ S_{O,nit}(t)]^T \tag{15}$$

The constraints on the control are given by the following limitations:

$$\begin{cases} -\bar{Q}_{r1} \leq Q_{r1} \leq 4\bar{Q}_{r1} \\ -\bar{Q}_{r2} \leq Q_{r2} \leq \bar{Q}_{r2} \\ -\bar{Q}_{air} \leq Q_{air} \leq 2\bar{Q}_{air} \end{cases} \tag{16}$$

Linearizing the system around the equilibrium point computed from the non linear equations leads to the new variables (x, u, y) that are now deviation variables. That is, they are deviations from the point the model is linearized about, not their original absolute values. The equilibrium point is given by

$$\bar{x} = [69.6\ \ 623\ \ 13.5\ \ 3.2\ \ 10.4\ \ 2.4\ \ 68.9\ 624.6\ \ 20.9\ \ 8.9\ \ 5.3\ \ 1356.8]^T \tag{17}$$

and $\bar{Q}_{r1} = 2300\ m^3/j$, $\bar{Q}_{r2} = 18446\ m^3/j$ and $\bar{Q}_{air} = 100\ m^3/j$ which leads to the following matrices for the system:

$$A = \begin{pmatrix} -29.07 & 0 & 0 & 2.65 & 0 & 2.17 & 29.40 & 0 & 0 & 0 & 0 & 0 \\ 0 & -29.48 & 6.04 & 0.64 & 0 & 4.40 & 0 & 29.40 & 0 & 0 & 0 & 0 \\ 0 & -0.34 & -38.99 & -1.02 & -0.05 & 5.00 & 0 & 0 & 29.40 & 0 & 0 & 0 \\ -2.22 & -0.02 & -0.55 & -40.74 & 0 & 12.23 & 0 & 0 & 0 & 29.40 & 0 & 0 \\ 2.18 & 0 & -0.06 & 11.05 & -29.40 & 9.64 & 0 & 0 & 0 & 0 & 29.40 & 0 \\ -9.45 & 0 & -0.18 & -47.90 & -0.01 & -167.00 & 0 & 0 & 00 & 0 & 0 & \\ 2.30 & 0 & 0 & 0 & 0 & 0 & -38.67 & 0 & 0 & 0.55 & 0 & 1.84 \\ 0 & 2.30 & 0 & 0 & 0 & 0 & 0 & -39.18 & 4.44 & 0.11 & 0 & 16.60 \\ 0 & 0 & 2.30 & 0 & 0 & 0 & 0 & -0.78 & -50.67 & -0.30 & -3.42 & 0 \\ 0 & 0 & 0 & 2.30 & 0 & 0 & -3.06 & -0.04 & -0.66 & -39.32 & -0.19 & 0 \\ 0 & 0 & 0 & 0 & 2.30 & 0 & 2.99 & -0.03 & -0.55 & 1.13 & -39.58 & 0 \\ 6.14 & 6.14 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3.13 \end{pmatrix}$$

$$B = 10^4 \begin{pmatrix} -0.0011 & -0.0011 & 0 \\ 0.0023 & 0.0023 & 0 \\ 0.0102 & 0.0102 & 0 \\ 0.0079 & 0.0079 & 0 \\ -0.0071 & -0.0071 & 0 \\ -0.0033 & -0.0033 & 0.0008 \\ 0.0014 & 0.1233 & 0 \\ -0.0031 & 1.1003 & 0 \\ -0.0386 & -0.0386 & 0 \\ -0.0105 & -0.0165 & 0 \\ 0.0094 & -0.0097 & 0 \\ 0 & -0.2042 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Remark 2.1. *It is worth noting here that the obtained state space representation is not controllable nor observable. In fact, matrices of controllability and observability are respectively of rank 10 and 9. Further, the matrix A of the system has a spectrum that contains stable eigenvalues, let say $n - m = 9$ stable eigenvalues.*

3. **The Control Scheme.** In this section, we present the control scheme that will be applied to the linearized obtained system. The control startegie is based on the positive invariance concept that had shown efficiency in handling constrained control systems. Moreover, as the system state is composed by non measurable quantities, observers as

software sensors are introduced. Let us first recall the observer based regulator with constraints. To this end, consider the linear constrained system

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{18}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control vector, $y(t) \in \mathbb{R}^p$ is the output vector, A and B are constant matrices of appropriate dimension and (A, B) is controllable. It is assumed that A possesses at last $(n - m)$ stable eigenvalues. The control u is constrained in the set Ω defined as follows:

$$\Omega = \{u \in \mathbb{R}^m \backslash -u_{\min} \leq u \leq u_{\max} u_{\min}, u_{\max} \in int\mathbb{R}^m_+\}. \tag{19}$$

Using a state feedback control:

$$u(t) = sat(Fx(t)), \quad F \in \mathbb{R}^{m\times n}, \tag{20}$$

where the saturation function is as follows:

$$sat(Fx(t)) = \begin{cases} u_{\max} & \text{if } Fx \geq u_{\max} \\ u & \text{if } -u_{\min} < Fx < u_{\max} \\ -u_{\min} & \text{if } Fx \leq -u_{\min} \end{cases}$$

leads to a domain of linear behavior for the closed loop system that is given by

$$D(F, u_{\min}, u_{\max}) = \{x \in \mathbb{R}^n \backslash -u_{\min} \leq Fx \leq u_{\max}\}, \tag{21}$$

and the closed loop system in this case

$$\dot{x}(t) = (A + BF)x(t). \tag{22}$$

Hence, if the domain (21) is positively invariant, in the sense of the definition given below, one guarantees the respect of the control constraints for all $t \geq 0$.

Definition 3.1. *A subset D of $\mathbb{R}^m$ is said to be positively invariant with respect to system (22) if the condition $x(t_0) \in D$ implies that $x(t) \in D$ $\forall t \geq t_0$.*

At this level, one may introduce the observer for this class of systems. Note that the proposed observer is a reduced order one as the measurable part of the output is a linear combination of the states. To not neglect this information, only a part of the state is reconstructed via the reduced order observer [24]. Let this part be noted as

$$z(t) = Tx(t) \quad z \in \mathbb{R}^{n-p}, \tag{23}$$

where matrix $T \in \mathbb{R}^{n-p\times n}$ is chosen in such a way that the matrix $\begin{pmatrix} C^T & T^T \end{pmatrix}^T$ is invertible. $z(\cdot)$ is the state of the observer dynamics and may be generated from an auxiliary dynamical system as follows:

$$\dot{z}(t) = Dz(t) + Ey(t) + Gu(t) \tag{24}$$

At this stage, our problem may be stated as finding matrices F, D, E and G such that the closed loop system with the control $u(t) = sat(F\hat{x}(t))$ is asymptotically stable and the input constraints are respected. The observed state is given by

$$\hat{x}(t) = \begin{pmatrix} C \\ T \end{pmatrix}^{-1} \begin{pmatrix} y(t) \\ z(t) \end{pmatrix} = \begin{pmatrix} V & P \end{pmatrix} \begin{pmatrix} y(t) \\ z(t) \end{pmatrix} \tag{25}$$

where the matrices V, C, T and P satisfy

$$V C + P T = \mathbb{I}. \tag{26}$$

Recall that the minimal order observer matrices, as proposed, are given by [24]

$$D = T\,A\,P, \quad E = T\,A\,V \;\text{ and }\; G = T\,B \tag{27}$$

which is equivalent to

$$T\,A - E\,C = D\,T. \tag{28}$$

Matrix P is chosen to ensure asymptotic stability of the matrix D. In fact, matrix D defines the dynamics of the errors and this guarantees vanishing errors [16]. Indeed,

$$\begin{aligned}\dot{\epsilon}(t) &= \dot{z}(t) - T\dot{x}(t)\\ &= Dz(t) + Ey(t) + Gu(t) - T(Ax(t) + Bu(t))\\ &= Dz(t) + ECx(t) - TAx(t)\\ &= Dz(t) - DTx(t)\\ &= D\epsilon(t).\end{aligned}$$

For the observation error, we define the field $D(\mathbb{I}, \epsilon_{\max}, \epsilon_{\min})$ in which we allow the error $\epsilon(t)$ to evolve. Further, define the reconstruction error as $e(t) = \hat{x}(t) - x(t)$. Note that, it is related to the observation error as follows:

$$\begin{aligned}e(t) &= Vy(t) + Pz(t) - x(t)\\ &= VCx(t) + Pz(t) - (VC + PT)x(t)\\ &= P(z(t) - Tx(t))\\ &= P\epsilon(t)\end{aligned}$$

Furthermore, one may prove that the control dynamics are as follows [17]:

$$\begin{aligned}\dot{u}(t) &= F\dot{\hat{x}}(t)\\ &= FP\dot{z}(t) + FVC\dot{x}(t)\\ &= FP(Dz(t) + Ey(t) + Gu(t)) + FVC(Ax(t) + Bu(t))\\ &= FP(TAPz(t) + TAVy(t)) + (FPTB + FVCB)u(t) + FVCAx(t)\\ &= FPTA(Pz(t) + Vy(t)) + F(PT + VC)Bu(t) + FVCAx(t)\\ &= FPTA\hat{x}(t) + FBu(t) + FVCA(\hat{x}(t) - e(t))\\ &= F(PT + VC)A\hat{x}(t) + FBF\hat{x}(t) - FVCAe(t)\\ &= (FA + FBF)\hat{x}(t) - FVCAe(t)\\ &= HF\hat{x}(t) - FVCAP\epsilon(t)\\ &= Hu(t) + L_r\epsilon(t)\end{aligned}$$

Therefore, the system formed by the control $u(t)$ and the error $\epsilon(t)$ is obtained as

$$\begin{pmatrix}\dot{u}(t)\\ \dot{\epsilon}(t)\end{pmatrix} = \begin{pmatrix}H & L_r\\ 0 & D\end{pmatrix}\begin{pmatrix}u(t)\\ \epsilon(t)\end{pmatrix} \tag{29}$$

This background enables one to recall the theorem [17] giving conditions for computing the controller that respects all the needed requirements:

Theorem 3.1. *The field $D(\mathbb{I}, u_{\max}, u_{\min}) \times D(\mathbb{I}, \epsilon_{\max}, \epsilon_{\min})$ is positively invariant with respect to the trajectory of system (29) if and only if, there exists a matrix $H \in R^{m\times m}$ such that:*

$$\begin{cases} H\,F = F\,A + F\,B\,F\\ \widetilde{M}q_\epsilon \le 0\end{cases} \tag{30}$$

where

$$M = \begin{pmatrix}H & L_r\\ 0 & D\end{pmatrix};\quad q_\epsilon = \begin{pmatrix}u_{\max}\\ \epsilon_{\max}\\ u_{\min}\\ \epsilon_{\min}\end{pmatrix};\quad L_r = -FVCAP \tag{31}$$

for every pair $(u(0), \epsilon(0)) \in D(\mathbb{I}, u_{\max}, u_{\min}) \times D(\mathbb{I}, \epsilon_{\max}, \epsilon_{\min})$.

To compute the feedback gain, the inverse procedure is used [5, 6]. Hence, matrix H satisfying all required conditions such that a solution exists is chosen and the feedback F is obtained as a solution to the equation:

$$F\ A + F\ B\ F = H\ F. \tag{32}$$

The controllers proposed here are shown to be robust with respect to parametric uncertainties within given sets for the system matrices. For more details about robustness and sensitivity of such controllers, the reader is refereed to [19].

Remark 3.1. *Note here that all computation effort is handled off line. Choice of an adequate matrix H with all required conditions is studied in* [18]*, solution of Equation (32) is the detailed subject of the work* [5].

4. **Application to the WWTP.**

In this section, the control scheme presented in the previous section is adapted to the special case of the WWTP which is our initial aim. In order to apply the concept of positive invariance, the studied system must be controllable and observable. However, the obtained model do not completely satisfy the two later conditions because, as pointed out in Remark 2.1 the controllability and the observability matrices have respectively ranks 10 and 9. First, the model is worked out to extract controllable and observable parts. Hence, a matrix H satisfying the required conditions is chosen and a controller is computed as solution to Equation (32). Finally, the reduced order observer is designed.

4.1. **Working out the model.**

Any representation in the state space can be transformed into the equivalent form by using the transformation $Z = Mx$ [12]:

$$\begin{cases} \dot{Z} = \bar{A}Z + \bar{B}u \\ y(t) = \bar{C}Z \end{cases} \tag{33}$$

with

$$\bar{A} = M\ A\ M^{-1},\ \ \bar{B} = M\ B\ \text{ and }\ \bar{C} = C\ M^{-1}$$

$$\bar{A} = \begin{pmatrix} A_{c\bar{o}} & A_{12} \\ 0 & A_{co} \end{pmatrix},\ \ \bar{B} = \begin{pmatrix} B_{c\bar{o}} \\ B_{co} \end{pmatrix},\ \ \bar{C} = \begin{pmatrix} 0 & C_{co} \end{pmatrix} \text{ and } Z = \begin{pmatrix} Z_{c\bar{o}} \\ Z_{co} \end{pmatrix}.$$

Hence, the system may be re-written as

$$\begin{cases} \dot{Z}_{c\bar{o}} = A_{c\bar{o}}Z_{c\bar{o}} + A_{12}Z_{co} + B_{c\bar{o}}u \\ \dot{Z}_{co} = A_{co}Z_{co} + B_{co}u \\ y = C_{co}Z_{co} \end{cases} \tag{34}$$

where (A_{co}, B_{co}, C_{co}) is controllable and observable. Further, computing the spectrum $\sigma(A_{c\bar{o}}) = \{-0.3373, -35.9885 + 1.2899i, -35.9885 - 1.2899i\}$ shows that it is stable and hence stabilizing the system matrix A_{co} suffices to stabilizes the hole system [12].

For the WWTP, the states $(S_{NH,nit}(t)\quad S_{NO,nit}(t)\quad S_{O,nit}(t))$ are measurable, so the matrix M is chosen like

$$M = \begin{pmatrix} 0.0001 & 0.9975 & -0.0395 & 0 & 0 & 0 & 0 & -0.0561 & 0.0034 & -0.0001 & 0 & 0.0185 \\ -0.0014 & 0.0589 & 0.0680 & 0 & 0 & 0 & 0.0009 & 0.9943 & -0.0573 & 0.0012 & 0.0003 & -0.0003 \\ -0.0000 & -0.0184 & 0.0007 & 0 & 0 & 0 & 0 & 0.0013 & -0.0001 & 0 & 0 & 0.9998 \\ -0.0200 & 0.0355 & 0.9965 & 0 & 0 & 0 & 0.0064 & -0.0703 & 0.0001 & 0.0172 & 0.0037 & 0 \\ -0.0001 & 0.0000 & 0.0041 & 0 & 0 & 0 & -0.0214 & 0.0572 & 0.9981 & 0.0001 & 0.0000 & 0 \\ 0.0001 & -0.0002 & -0.0064 & 0 & 0 & 0 & 0.9997 & 0.0008 & 0.0214 & -0.0001 & 0 & 0 \\ -0.9929 & -0.0007 & -0.0188 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0842 & 0.0821 & 0 \\ -0.0017 & 0.0005 & 0.0146 & 0 & 0 & 0 & 0 & 0 & 0 & -0.6895 & -0.7242 & 0 \\ 0.1173 & 0.0004 & 0.0122 & 0 & 0 & 0 & 0 & 0 & 0 & -0.7192 & 0.6847 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

which leads to the following decomposition for the system:

$$Z_{co}^T = \begin{pmatrix} Z_4 & Z_5 & Z_6 & Z_7 & Z_8 & Z_9 & -x_6 & -x_5 & -x_4 \end{pmatrix}; \quad Z_{c\bar{o}} = \begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \end{pmatrix} \tag{35}$$

with $Z_i = \sum_{j=1}^n M_{ij} x_i$, $i = 1, \ldots, n$. Finally, the vector Z_{co} may be decomposed into the available and unavailable parts as

$$Z_{co} = \begin{pmatrix} \xi_e \\ \xi_m \end{pmatrix}; \quad \text{with} \quad \xi_e^T = \begin{pmatrix} Z_4 & Z_5 & Z_6 & Z_7 & Z_8 & Z_9 \end{pmatrix}; \quad \xi_m = \begin{pmatrix} Z_{10} \\ Z_{11} \\ Z_{12} \end{pmatrix}$$

ξ_e is the vector of unmeasured variables and ξ_m is the vector of available states. Matrices of the decomposed system are given by

$$A_{co} = \begin{pmatrix} -38.85 & 28.99 & -0.00 & 0.17 & 0.01 & -0.01 & -5.10 & 0.04 & 1.00 \\ 2.33 & -50.29 & -0.26 & -0.24 & 2.72 & -2.0948 & -0.0205 & 0.0002 & 0.004 \\ 0.01 & -0.44 & -38.67 & -2.33 & -0.32 & -0.17 & 0.0327 & -0.0003 & -0.0064 \\ 0.01 & 0.07 & -28.69 & -29.23 & -0.04 & -1.26 & 2.25 & -0.1898 & 2.81 \\ -0.01 & 1.29 & -0.08 & 0.11 & -38.99 & 0.81 & -0.07 & 1.6664 & 1.60 \\ 0.04 & 0.28 & 7.71 & -1.26 & -0.51 & -39.77 & -0.31 & -1.5741 & 1.35 \\ 0 & 0 & 0 & -9.38 & -0.01 & 1.11 & -167.007 & -0.0190 & -47.90 \\ 0 & -0.00 & -0.00 & -0.24 & 21.29 & -20.38 & 9.6467 & -29.4080 & 11.05 \\ 0.00 & 0 & -0.00 & 0.25 & 20.27 & 21.41 & 12.23 & -0.0033 & -40.74 \end{pmatrix}$$

$$B_{co} = 10^3 \begin{pmatrix} 0.1040 & -0.6657 & 0 \\ -0.3872 & 0.2181 & 0 \\ 0.0052 & 1.2322 & 0 \\ 0.0252 & 0.0145 & 0 \\ 0.0057 & 0.1856 & 0 \\ 0.1403 & 0.0522 & 0 \\ 0.0332 & 0.0332 & -0.0076 \\ 0.0708 & 0.0708 & 0 \\ -0.0789 & -0.0789 & 0 \end{pmatrix}$$

$$\text{and} \quad C_{co} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \end{pmatrix}$$

with the following control constraints:

$$u_{\max} = \begin{pmatrix} 9200 \\ 18446 \\ 200 \end{pmatrix}, \quad u_{\min} = \begin{pmatrix} 2300 \\ 18446 \\ 100 \end{pmatrix} \tag{36}$$

The observer may be designed, at this stage, for the decomposed system. To this end, matrix T is chosen such that only the part $z(t) = TZ_{co}(t)$ is estimated. Further, matrix P is chosen to ensure asymptotic stability of matrix $D = TA_{co}P$. In fact, in this case matrix T is given by

$$T = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \text{ and } P = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

According to Equation (27), the matrices D, G and E are computed.

$$D = \begin{pmatrix} -38.858 & 28.9925 & -0.007 & 0.1784 & 0.0119 & -0.0185 \\ 2.3306 & -50.2968 & -0.2666 & -0.2478 & 2.7231 & -2.0948 \\ 0.0147 & -0.4418 & -38.678 & -2.3376 & -0.3296 & -0.1756 \\ 0.0119 & 0.0735 & -28.6943 & -29.2308 & -0.0443 & -1.2648 \\ -0.0105 & 1.2947 & -0.0824 & 0.1128 & -38.9935 & 0.813 \\ 0.0442 & 0.2873 & 7.7115 & -1.263 & -0.5140 & -39.7722 \end{pmatrix}$$

$$G = 10^3 \begin{pmatrix} 0.1040 & -0.6657 & 0 \\ -0.3872 & 0.2181 & 0 \\ 0.0052 & 1.2322 & 0 \\ 0.0252 & 0.0145 & 0 \\ 0.0057 & 0.1856 & 0 \\ 0.1403 & 0.0522 & 0 \end{pmatrix}$$

$$E = \begin{pmatrix} -1.0078 & -0.0489 & 5.1034 \\ -0.0042 & -0.0002 & 0.0205 \\ 0.0064 & 0.0003 & -0.0327 \\ -2.8104 & 0.1898 & -2.2577 \\ -1.6047 & -1.6664 & 0.0718 \\ -1.3549 & 1.5741 & 0.3185 \end{pmatrix}$$

For the reconstruction error, one may choose the limits as:

$$\epsilon_{\max} = \begin{pmatrix} 1 & 1 & 0.5 & 1 & 1 & 1 \end{pmatrix}; \quad \epsilon_{\min} = \begin{pmatrix} 0.5 & 0.5 & 0.25 & 0.8250 & 0.5 & 0.5 \end{pmatrix}$$

For the matrix H, we choose to assign the following closed loop eigenvalues $\{-170; -55; -51\}$, which leads to the following choice of matrix H:

$$H = \begin{pmatrix} -170 & 0 & 0 \\ 0 & -55 & 0 \\ 0 & 0 & -51 \end{pmatrix}$$

It is worth noting here that the remaining closed loop eigenvalues are the $n-m$ stable ones coming from the open loop system [14]. Hence, solving Equation (32) leads to:

$$F = \begin{pmatrix} 0.0010 & -0.0044 & 0.0319 & 0.0725 & -0.0791 & -0.0944 & 1.0349 & -0.0000 & 0.4162 \\ 0.0001 & -0.0005 & 0.0000 & 0.0001 & 0.0000 & -0.0002 & 0.0017 & -0.0000 & 0.0007 \\ 0.0148 & -0.0774 & 0.1421 & 0.0036 & -0.1591 & -0.2023 & -0.0352 & -0.0024 & 0.0864 \end{pmatrix}$$

Conditions of Theorem 3.1 are easily checked and are given by the vector $\tilde{M}q_\epsilon = 10^4(-3.3981\ -1.1\ -0.1018\ -0.0010\ -0.0044 - 0.0017\ -0.0014\ -0.0037\ -0.0034\ -1.6974\ -0.55\ -0.0506\ -0.0005 - 0.0020\ -0.0016\ -0.0008\ -0.0018\ -0.0014) < 0$, which is a strictly negative vector. One may conclude that all required conditions are satisfied; and hence, the observer based controller as proposed is able to monitor the WWTP guaranteeing asymptotic stability and respect of all constraints for the control and the observation error.

5. **Simulation Results.** Figures 2-15 below are devoted to present the evolution of all variables of the system. In fact, the observer based controller, as defined in the section above, is applied to the WWTP. Estimated values are compared with the simulated ones from the non linear model. As general remarks asymptotic stability is obtained, all

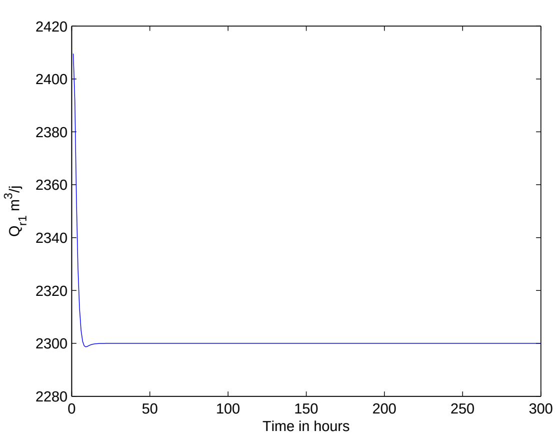

FIGURE 2. Evolution of the internal recycled flow Q_{r1}

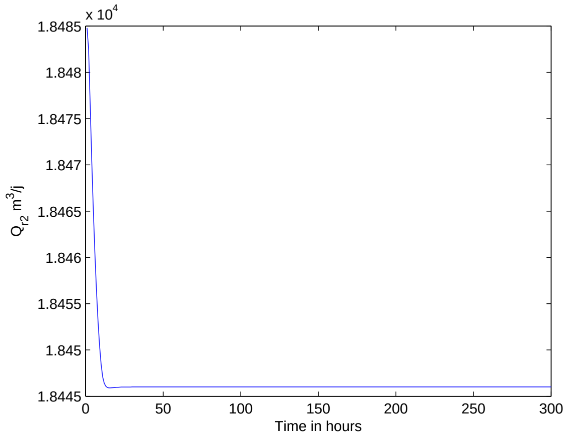

FIGURE 3. Evolution of the internal recycled flow Q_{r2}

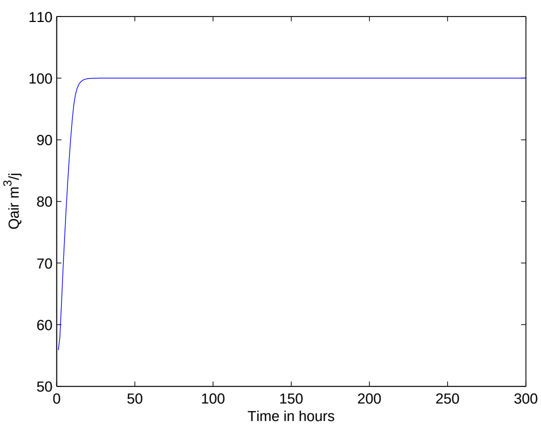

FIGURE 4. Evolution of the dissolved oxygen Q_{air}

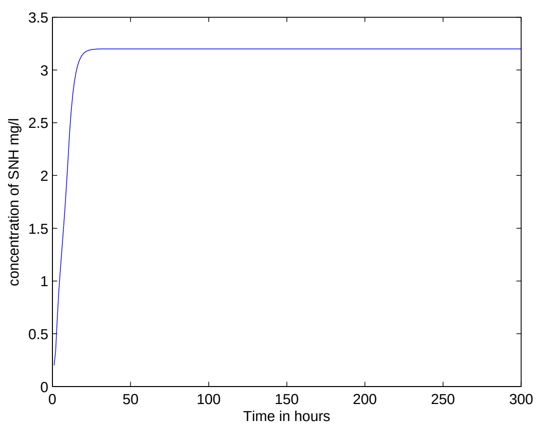

FIGURE 5. Evolution of the ammonium S_{NH}

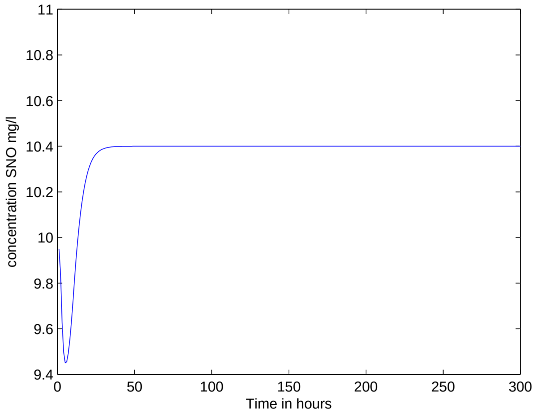

FIGURE 6. Evolution of the nitrate S_{NO}

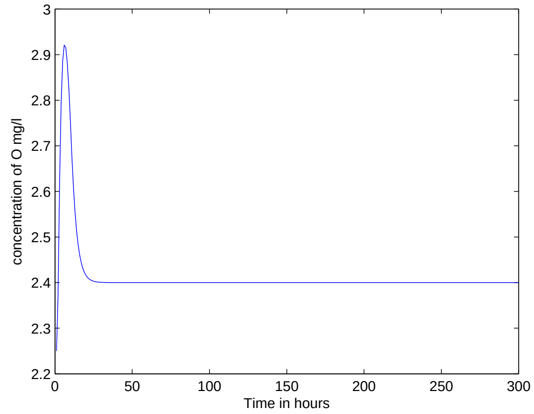

FIGURE 7. Evolution of the oxygen O_2

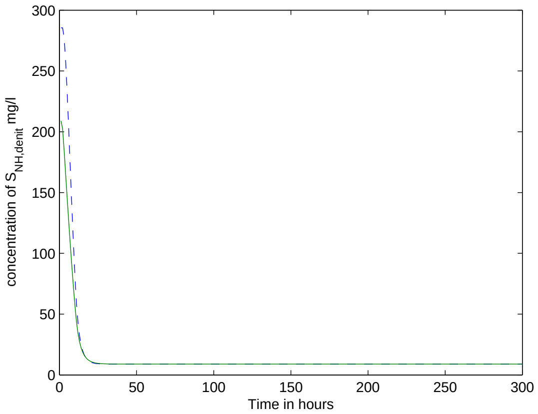

FIGURE 8. The concentration of $S_{NH,denit}$ and its estimate

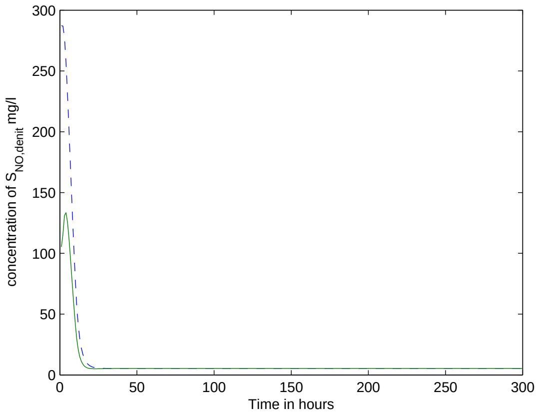

FIGURE 9. The concentration of $S_{NO,denit}$ and its estimate

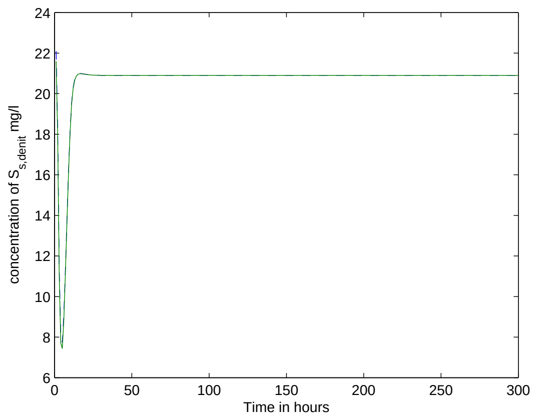

FIGURE 10. The concentration of $S_{S,denit}$ and its estimate

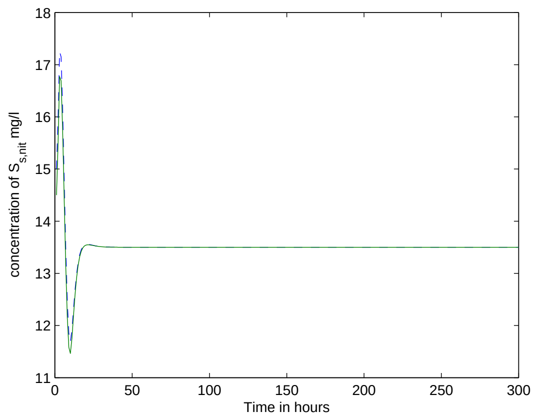

FIGURE 11. The concentration of $S_{S,nit}$ and its estimate

constraints are respected and the amount of all non desired organic matter is reduced in the output to the desired values. Further, the limits imposed to the estimation errors are

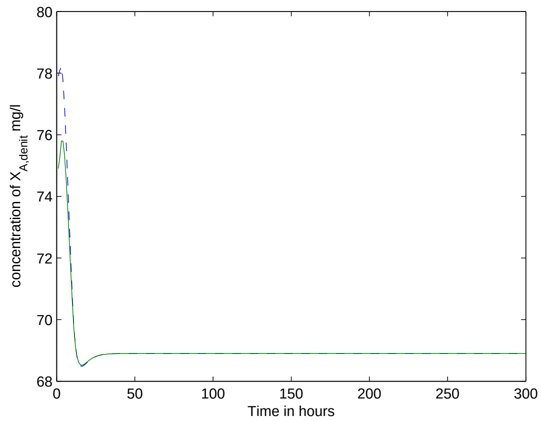

FIGURE 12. The concentration of $X_{A,denit}$ and its estimate

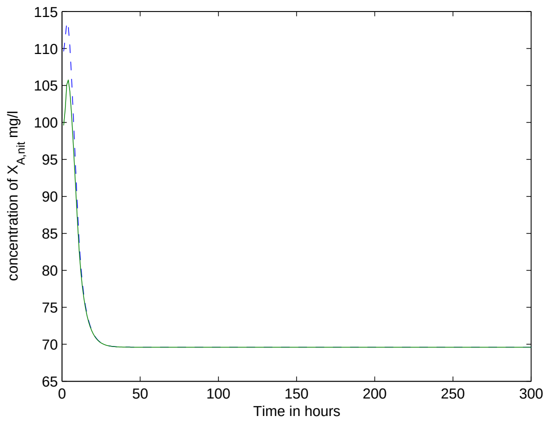

FIGURE 13. The concentration of $X_{A,nit}$ and its estimate

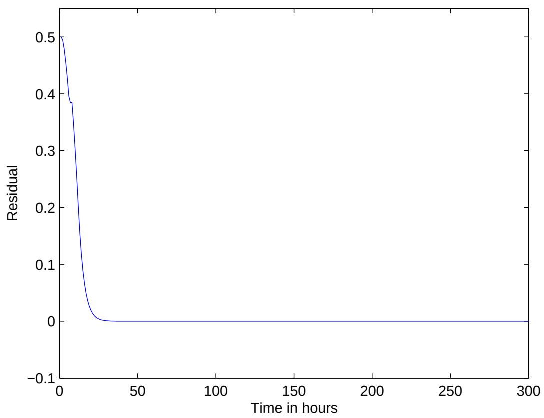

FIGURE 14. Estimation error ($S_{S,nit}$)

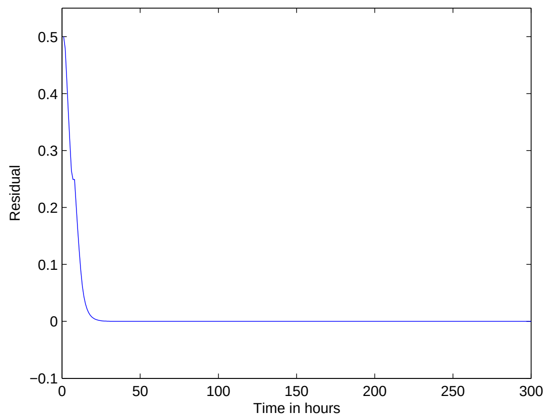

FIGURE 15. Estimation error ($S_{S,denit}$)

also respected. First, the control evolution is presented in Figures 2-4. Respect of all given control constraints is clearly noticed. Second, Figures 5-7 show the output evolution from the initial values $y_0 = Cx_o = C[100\,300\,15\,1\,10\,2.25\,74\,200\,22\,200\,100\,300]^T = [15\,1\,10]^T$. Finally, from Figures 8-13 one may note that the convergence, for all estimated states, is obtained. Hence, and in practice, the concentrations of the organic matter are reduced and converge to the desired values. Furthermore, Figures 14 and 15 are devoted to show that the reconstruction errors limits are really respected and this is clear from the figures.

6. **Conclusion.** In this paper, the minimal order observer in the control loop of a non linear system with input constraints is introduced. In fact, observer, as software sensor, in the framework of positive invariance techniques is used to control the linearized model of a WWTP. For this process, linearization leads to some constraints on the control. Further, state variables are unavailable to measure and more than that no adequate sensor exists. Hence, the introduction of the observer is of great interest. The positive invariance techniques that had emerged as very efficient to handle similar problems of constrained control is successfully used to control the nitrogen removal process. The observer based constrained control, as presented above may compete with approaches

in easiness, applicability and computing effort. In fact, all the needed computation is achieved off line and once the design finished, the control law is easy to implement on the process. Further, it is true that in the computational steps some trial and error tests are necessary; however, with the background available for the choice of the observer and the matrix H assigning the closed loop poles [18], the computation effort is sensitively reduced. On the other hand, the evolution of the closed loop system, as presented in the figures above, with the designed control law shows its efficiency and the success of the controller to the reduction of the organic waste.

REFERENCES

[1] M. A. Rami, H. Ayad and F. Mesquine, Enlarging ellipsoidal invariant sets for constrained linear systems, *International Journal of Innovative Computing, Information and Control*, vol.3, no.5, pp.1097-1108, 2007.

[2] G. Bastin and D. Dochain, On line estimation and adaptive control of bioreactors, *Analytica Chimica Acta*, vol.243, no.324, 1990.

[3] C. A. Belchiora, R. A. Araujo and J. A. Landeckb, Dissolved oxygen control of the activated sludge wastewater treatment process using stable adaptive fuzzy control, *Computers and Chemical Engineering*, vol.37, pp.152-162, 2012.

[4] A. Benzaouia and A. Hmamed, Regulator problem for continuous time systems with nonsymmetrical constrained control, *IEEE Trans. Automat. Control*, vol.38, no.6, pp.596-602, 1993.

[5] A. Benzaouia, The resolution of equation XA + XBX = HX and the pole assignment problem, *IEEE Trans. Automat. Control*, vol.39, no.10, pp.2091-2095, 1994.

[6] A. Benzaouia and A. Baddou, Piecewise linear constrained control for continuous-time systems, *IEEE Trans. Automat. Control*, vol.44, no.7, pp.1477-1481, 1999.

[7] A. Benzaouia, F. Tadeo and F. Mesquine, The regulator problem for linear systems with saturations on the control and its increments or rate: An LMI approach, *IEEE Circuit and Systems I*, vol.53, pp.2673-2680, 2006.

[8] B. Boulkroune, M. Darouach, M. Zasadzinski, S. Gille and D. Fiorelli, A nonlinear observer design for an activated sludge wastewater treatment process, *Journal of Process Control*, vol.19, pp.1558-1565. 2009.

[9] D. Dochain, Design of adaptive controller for nonlinear stirred tank bioreactors: Extension to the MlMO situation, *Journal of Process Control*, vol.1, pp.41-48, 1991.

[10] M. Henze, C. P. L. Grady, W. Gujerm, G. V. R. Maraism and T. Matsuo, Activated sludge model No.1. I.A.W.Q., *Scientific and Technical Report No.1*, 1987.

[11] F. Lafont, E. Busvelleb and J. P. Gauthier, An adaptive high-gain observer for waste water treatment systems, *Journal of Process Control*, vol.21, pp.893-900, 2011.

[12] J. B. Lassere, Reachable controllable sets and stabilizing control of constrained linear systems, *Automatica*, vol.29, no.2, pp.534-536, 1993.

[13] D. G. Luenberger, An introduction to observers, *IEEE Trans. Automat. Control*, vol.16, no.6, pp.596-602, 1971.

[14] F. Mesquine, *Contribution à La Commande des Systèmes Dynamiques Discrets Avec Contraintes sur les Entrées Par Application du Concept d'invariance Positive*, Cadi Ayyad University, thèse de doctorat de 3ème cycle, 1992.

[15] F. Mesquine, *Contribution à la Commande des Systèmes Linéaires à entrées Contraintes Par les Observateurs et Nouvelles Méthodologie de Placement de Pôles*, Cadi Ayyad University, thèse de doctorat d'état, 1997.

[16] F. Mesquine and D. Mehdi, Constrained observer for linear continuous time systems, *Int. J. Syst. Sci.*, vol.27, no.12, pp.1363-1369, 1996.

[17] F. Mesquine, D. Mehdi and A. Benzaouia, Constrained based full and minimal order observer control for linear systems, *The 6th CMMNI*, Tunis, 1998.

[18] F. Mesquine, F. Tadeo and A. Benzaouia, Regulator problem for linear systems with constraints on control and its increment or rate, *Automatica*, vol.40, no.8, pp.1387-1395, 2004.

[19] F. Mesquine, F. Tadeo and A. Benlamkadem, Constrained regulator problem for linear uncertain systems: Control of a PH process, *Mathematical Problems in Engineering*, 2006.

[20] F. Nejjari, A. Benhammou, B. Dahhou and G. Roux, Nonlinear multivariable control of a biological wastewater treatment process, *The 4th European Control Conference*, Bruxelles, Belgique, 1997.

[21] F. Nejjari, E. Dahhou, A. Benhammou and G. Roux, Nonlinear multivariable adaptive control of an activated sludge wastewater treatment process, *Int. J Adapt. Control Signal Process*, vol.13, pp.347-365, 1999.

[22] F. Nejjari and J. Quevedo, Predective control of a nutrient removal biological plant, *American Control Conference Boston*, Massachusetts, 2004.

[23] M. O'Brien, J. Mack, B. Lennox, D. Lovett and A. Wall, Model predictive control of an activated sludge process: A case study, *Control Engineering Practice*, vol.19, pp.54-61, 2011.

[24] J. Oreilly, *Observer for Linear Systems*, Academic-Press Edition Contributed Works, 1983.

[25] Y. S. Ilse, J. V. Haegebaerta, R. Carretteb and J. F. Van Impea, Linearization of the activated sludge model ASM1 for fast and reliable predictions, *Water Research*, vol.37, pp.1831-1851, 2003.

[26] A. Stare, N. Hvala and D. Vrecko, Modeling, identification, and validation of models for predictive ammonia control in a wastewater treatment plant – A case study, *ISA Transactions*, vol.45, no.2, pp.159-174, 2006.

[27] G. F. Wredenhagen and P. R. Belanger, Piecewise linear LQ control for systems with input constraints, *Automatica*, vol.30, no.3, pp.403-416, 1994.

[28] H. Zhao, S. H. Issacs, H. Soeberg and M. Kummel, Nonlinear optimal control of an allmating activated sludge process in a pilot plant, *Journal of Process Control*, vol.4, pp.33-43, 1994.