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A NOVEL CONTROLLER FOR A CLASS OF NONLINEAR SYSTEMS VIA IMMERSION AND INVARIANCE

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Abstract. *In this paper, we address the problem of immersion and invariance (I&I) control for a class of nonlinear systems. Based on selection of target system, a novel controller is obtained, which efficiently avoids choosing Lyapunov function. The designed controller guarantees that all trajectories of the closed-loop systems are asymptotically stable. The efficiency and effectiveness of the proposed method is demonstrated by a practical example.*

Keywords: Nonlinear system, Immersion and invariance, State-feedback stabilization

1. **Introduction.** There are many kinds of practical systems which can be described by a class of three-order nonlinear systems, such as inverted pendulum systems [1], power systems [2-5], aircraft wing rock systems [6], electro hydraulic systems [7]. The practical systems mentioned above can be written in a unified form as follows:

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= f_{21}(x_1, x_3) + f_{22}(x_1, x_2)\, x_2, \\ \dot{x}_3 &= f_3(x) + ku, \end{aligned} \tag{1}$$

where $x = (x_1, x_2, x_3)^{\mathrm{T}} \in \mathrm{R}^3$, $u \in \mathrm{R}$ are the measurable state and the input of the systems, respectively. $k \in \mathrm{R}$ is a known constant; $f_{21}(x_1, x_3)$ is left-invertible (with respect to second argument)[1]. Throughout the paper it is assumed that all functions and mappings are C^∞.

In the past years, considerable research efforts have been made for the stabilization of nonlinear systems [8-18]. Several constructive methods such as feedback linearization, backstepping, sliding mode control, and passivity-based control have been established to stabilize nonlinear systems; see [19-22] and the references therein.

A new method to design asymptotically stabilizing control law for nonlinear systems was presented in [23]. The method relies upon the notions of systems immersion and manifold invariance and does not require the knowledge of a Lyapunov function. The method has been further developed in a series of publications that have been recently summarized in

[1]A mapping $f_{21}(x_1, x_3) : \mathrm{R} \times \mathrm{R} \to \mathrm{R}$ is left-invertible (with respect to x_3) if there exists a mapping $f_{21}^L : \mathrm{R} \times \mathrm{R} \to \mathrm{R}$ such that $f_{21}^L(x_1, f_{21}(x_1, x_3)) = x_3$, for all $x_3 \in \mathrm{R}$ (and for all x_1).

[6]. And in the past few years, there were new research results on applications of the new method [24-30]. And the new method is called as I&I method.

The basic idea of I&I methodology is to achieve stabilization by immersing the plant dynamics into a stable (lower-order) target systems. This methodology, as it is based on the concept of asymptotic model matching, offers greater flexibility. In [6,23], it was rigorously proven that I&I technique is fundamentally different from sliding mode control (SMC) and singular perturbation methods. The three of them, of course, rely on the concept of invariant manifolds, but they are used in different ways. In SMC the invariant manifold is created switching vector fields and it is reached in finite time. In I&I, there is no switching, and convergence to the manifold is asymptotic. The great advantage of the latter is, of course, to avoid the deleterious chattering effect, which is unavoidable in SMC. The composite control of P. Kokotovic, and related singularly perturbed techniques, do not create a manifold, but simply ensure the convergence to the slow manifold that already exists in the system. This gives very little room to use this technique in a constructive way.

The main contribution of this paper is to propose a state-feedback controller for nonlinear systems (1) via I&I. The paper is organized as follows. In Section 2 a brief introduction to the I&I control synthesis is given. In Section 3 we give main result of designing I&I controller for (1). Comparative simulation result of wing rock system proves the improved performance in terms of faster convergence in Section 4. Finally, Section 5 concludes the paper.

2. **Immersion and Invariance.**

The present section reviews the basic theoretical results of [23], namely a set of sufficient conditions for the construction of globally asymptotically stabilizing state feedback control laws for general nonlinear systems.

Theorem 2.1. [23] *Consider the system on*

$$\dot{x} = f(x) + g(x)u, \tag{2}$$

with state $x \in \mathrm{R}^n$ and control $u \in \mathrm{R}^m$, and an equilibrium point $x_* \in \mathrm{R}^n$ to be stabilized. Let $p < n$ and assume we can find mappings

$$\begin{aligned} &\alpha : \mathrm{R}^p \to \mathrm{R}^p, \quad \pi : \mathrm{R}^p \to \mathrm{R}^n, \quad c : \mathrm{R}^p \to \mathrm{R}^m, \\ &\phi : \mathrm{R}^n \to \mathrm{R}^{n-p}, \quad \psi : \mathrm{R}^{n\times(n-p)} \to \mathrm{R}^m \end{aligned}$$

such that the followings hold.
(H1) (Target system)*. The system*

$$\dot{\xi} = \alpha(\xi), \tag{3}$$

with state $\xi \in \mathrm{R}^p$, has an asymptotically stable equilibrium at $\xi_* \in \mathrm{R}^p$ and $x_* = \pi(\xi_*)$.
(H2) (Immersion condition)*. For all $\xi \in \mathrm{R}^p$*

$$f(\pi(\xi)) + g(\pi(\xi))c(\pi(\xi)) = \frac{\partial \pi}{\partial \xi}\alpha(\xi) \tag{4}$$

(H3) (Implicit manifold)*. The set identity (4) in reference [23] holds.*
(H4) (Manifold attractivity and trajectory boundedness)*. All trajectories of the system*

$$\dot{z} = \frac{\partial \phi}{\partial x}\left[f(x) + g(x)\psi(x,z)\right], \tag{5}$$

$$\dot{x} = f(x) + g(x)\psi(x,z), \tag{6}$$

are bounded and satisfy

$$\lim_{t\to\infty} z(t) = 0. \tag{7}$$

Then x_* is an asymptotically stable equilibrium of the closed loop system

$$\dot{x} = f(x) + g(x)\,\psi(x, \phi(x)).$$

Remark 2.1. *The result in Theorem 2.1 lends itself to the following interpretation. The stabilization problem of system (2) can be divided into two subproblems. First, given the system (2) and the target dynamical system (3) find, if possible, a manifold M, described implicitly by $\{x \in \mathrm{R}^n \,|\phi(x) = 0\}$, and in parameterized form by $\{x \in \mathrm{R}^n | x = \pi(\xi), \xi \in \mathrm{R}^p\}$, which can be rendered invariant and attractive, and such that the restriction of the closed-loop system to M is described by $\dot{\xi} = \alpha(\xi)$. Second, we do not propose to apply the control $u = c(\pi(\xi))$ that renders the manifold invariant; instead we design a control law $u = \psi(x, z)$ that drives to zero the coordinate z and keeps the system trajectories bounded.*

Remark 2.2. *The convergence condition (7) can be relaxed, i.e., to prove asymptotic stability of the equilibrium x_* it suffices to require*

$$\lim_{t\to\infty} f(x, \psi(x, z)) - f(x, \psi(x, 0)) = 0$$

In other words, it is not necessary to reach the manifold M, in order to stabilize the equilibrium x_*.

Remark 2.3. *If we can find a partition of $x = col(x_1, x_2)$ with $x_1 \in \mathrm{R}^p$ and $x_2 \in \mathrm{R}^{n-p}$ and a corresponding partition of $\pi(.) = col(\pi_1(.), \pi_2(.))$ such that $x_1 = \pi_1(\xi)$ is a global change of coordinates, then (H3) is satisfied with $z = \phi(x) = x_2 - \pi_2\left(\pi_1^{-1}(x_1)\right)$. As a result, instead of considering the trajectories of the extended system (5)-(6) in (H4), it suffices to study the trajectories of the system with state (x_1, z).*

3. I&I-Based Controller Design for Nonlinear Systems.

3.1. Control objective.

As mentioned earlier, x_* denotes the operating stable equilibrium. We assume that x_* is known to us and state the control objective as 'to design a control law $u = \psi(x, z)$ in order to make the system (1) asymptotically stable at x_*, and to improve the transient response of the closed-loop system'.

3.2. Controller design.

We proceed to verify the H1-H4 of Theorem 2.1.

(H1) (Target system). The key idea is to immerse a plant dynamics into a stable target system. Thus, we define the target dynamics as

$$\Sigma_T : \begin{cases} \dot{\xi}_1 = \xi_2, \\ \dot{\xi}_2 = -V'(\xi_1) - \mathrm{R}(\xi_1, \xi_2)\,\xi_2, \end{cases} \tag{8}$$

where $\xi_1, \xi_2 \in \mathrm{R}$, $V(\xi_1)$:$\mathrm{R} \to \mathrm{R}$ is the potential energy of the system and $\mathrm{R}(\xi_1, \xi_2)$:$\mathrm{R}\times\mathrm{R} \to \mathrm{R}$ is a damping function which are to be chosen. The target system (8) has a stable equilibrium at ξ_* with energy function $H(\xi_1, \xi_2) = \frac{1}{2}\xi_2^2 + V(\xi_1)$.

To ensure the stability at the equilibrium we introduce the following assumption.

Assumption 3.1. (i) The potential energy function $V(\xi_1)$, satisfies $V'(\xi_{1*}) = 0$ and $V''(\xi_{1*}) > 0$.

(ii) The damping matrix is $\mathrm{R}(\xi_*) \geq 0$.

(H2) (Immersion condition). Given the control objectives and our choice of target dynamics a natural selection of the mapping π is

$$\pi(\xi) = \begin{bmatrix} \xi_1 \\ \xi_2 \\ \pi_3(\xi) \end{bmatrix}, \tag{9}$$

where $\pi_3(\xi)$ is a function to be defined. With this choice of $\pi(\xi)$ and the target dynamics above, Equation (4) becomes

$$\begin{aligned}&\begin{bmatrix} \xi_2 \\ f_{21}(\xi_1, \pi_3(\xi)) + f_{22}(\xi_1, \xi_2)\xi_2 \\ f_3(\pi(\xi)) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ k \end{bmatrix} c(\pi(\xi)) \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \frac{\partial \pi_3(\xi)}{\partial \xi_1} & \frac{\partial \pi_3(\xi)}{\partial \xi_2} \end{bmatrix} \begin{bmatrix} \xi_2 \\ -V'(\xi_1) - \mathrm{R}(\xi_1, \xi_2)\xi_2 \end{bmatrix}\end{aligned} \tag{10}$$

Next we choose $\pi_3(\xi)$ and $c(\pi(\xi))$ to satisfy the above equation as follows: the first row of (10) is already satisfied. From the second row we have

$$f_{21}(\xi_1, \pi_3(\xi)) + f_{22}(\xi_1, \xi_2)\xi_2 = -V'(\xi_1) - \mathrm{R}(\xi_1, \xi_2)\xi_2 \tag{11}$$

Choose $\mathrm{R}(\xi_1, \xi_2) = -f_{22}(\xi_1, \xi_2) + d$ and $V(\xi_1) = -\beta \cos \tilde{\xi}_1$ for some $\beta > 0$ (to be chosen). We use $\tilde{\xi}_1$ to denote $\xi_1 - \xi_{1*}$. Then the above equation becomes

$$f_{21}(\xi_1, \pi_3(\xi)) = -\beta \sin \tilde{\xi}_1 - d\xi_2, \tag{12}$$

and from (12) we get

$$\pi_3(\xi) = f_{21}^L\left(\xi_1, -\beta \sin \tilde{\xi}_1 - d\xi_2\right). \tag{13}$$

From the third row we have

$$\begin{aligned} c(\pi(\xi)) = k^{-1} \Bigg[& \frac{\partial \pi_3(\xi)}{\partial \xi_1}\xi_2 - f_3(\pi(\xi)) \\ & - \frac{\partial \pi_3(\xi)}{\partial \xi_2}\left(\beta \sin \tilde{\xi}_1 + (d - f_{22}(\xi_1, \xi_2))\xi_2\right)\Bigg] \end{aligned} \tag{14}$$

(H3) (Implicit manifold). The manifold is implicitly described by $\mathcal{M} = \{x \in \mathrm{R}^3 | \phi(x) = 0\}$, with

$$\begin{aligned} \phi(x) &= x_3 - \pi_3(x_1, x_2) \\ &= x_3 - f_{21}^L(x_1, -\beta \sin \tilde{x}_1 - dx_2) \end{aligned} \tag{15}$$

where $\tilde{x}_1$ denotes $x_1 - x_{1*}$.

(H4) (Manifold attractivity and trajectory boundedness). The off-the-manifold coordinates are $z = \phi(x)$ and straightforward calculations show that

$$\begin{aligned} \dot{z} &= \dot{x}_3 - \dot{\pi}_3(x_1, x_2) \\ &= f_3(x) + k\psi(x, z) - \frac{\partial \pi_3(x_1, x_2)}{\partial x_1}x_2 \\ &\quad - \frac{\partial \pi_3(x_1, x_2)}{\partial x_2}(f_{21}(x_1, x_3) + f_{22}(x_1, x_2)x_2) \end{aligned} \tag{16}$$

To ensure the boundedness of the trajectories of the off-the-manifold coordinate z and also that $\lim_{t\to\infty} z(t) = 0$ we take

$$\dot{z} = -\gamma z, \quad \gamma > 0, \tag{17}$$

and then we have

$$\begin{aligned} \psi(x, z) = k^{-1} \Bigg[& -\gamma z + \frac{\partial \pi_3(x_1, x_2)}{\partial x_1}x_2 - f_3(x) \\ & + \frac{\partial \pi_3(x_1, x_2)}{\partial x_2}(f_{21}(x_1, x_3) + f_{22}(x_1, x_2)x_2)\Bigg] \end{aligned} \tag{18}$$

We calculate the control law as

$$\begin{aligned} u &= \psi\left(x, \phi\left(x\right)\right) \\ &= k^{-1}\left[-\gamma\phi\left(x\right) + \frac{\partial\pi_3\left(x_1, x_2\right)}{\partial x_1}x_2 - f_3\left(x\right)\right. \\ &\quad\left. + \frac{\partial\pi_3\left(x_1, x_2\right)}{\partial x_2}\left(f_{21}\left(x_1, x_3\right) + f_{22}\left(x_1, x_2\right)x_2\right)\right] \\ &= k^{-1}\left[-\gamma\left(x_3 - f_{21}^L\left(x_1, -\beta\sin\tilde{x}_1 - dx_2\right)\right) + \frac{\partial f_{21}^L\left(x_1, -\beta\sin\tilde{x}_1 - dx_2\right)}{\partial x_1}x_2\right. \\ &\quad\left. + \frac{\partial f_{21}^L\left(x_1, -\beta\sin\tilde{x}_1 - dx_2\right)}{\partial x_2}\left(f_{21}\left(x_1, x_3\right) + f_{22}\left(x_1, x_2\right)x_2\right) - f_3\left(x\right)\right] \end{aligned} \quad . \tag{19}$$

3.3. **Stability result.** We establish boundedness of the trajectories of the closed-loop system (1) with the control law (19) and the off-the-manifold coordinate z.

From (8) and (9), we can obtain that states x_1, x_2 are bounded and converge to equilibrium x_{1*}, x_{2*}. We know that the off-the-manifold coordinate z is bounded and $\lim\limits_{t\to\infty} z\left(t\right) = 0$ from (17). Next we have $x_3 = z + \pi_3\left(x_1, x_2\right)$, from (12) and (13) we have $\pi_3\left(x_1, x_2\right)$ is bounded for all $\left(x_1, x_2\right)$, and hence we can conclude boundedness of x_3.

The above discussion on the control synthesis can be summarized in the following proposition which is the main result of this paper.

Proposition 3.1. *The closed-loop system (1) with the control law (19) is asymptotically stable at* x_*.

Proof: From the derivations above it is clear that Proposition 3.1 can be easily proved, but omitted here for brevity.

4. **A Practical Example.** In this section, we use a practical example which is taken from [6] to illustrate the effectiveness and merit of our result.

Consider the system of aircraft wing rock

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= x_3 + \varphi_2\left(x_1, x_2\right)^{\mathrm{T}}\theta, \\ \dot{x}_3 &= \tfrac{1}{\tau}u - \tfrac{1}{\tau}x_3, \end{aligned} \tag{20}$$

where the states x_1, x_2 and x_3 represent the roll angle, roll rate and aileron deflection angle respectively, τ is the aileron time constant, u is the control input, $\theta \in \mathrm{R}^5$ is a known constant vector and $\varphi_2\left(x_1, x_2\right) = \left[1, x_1, x_2, \left|x_1\right| x_2, \left|x_2\right| x_2\right]^{\mathrm{T}}$. The system has an equilibrium at zero and the control objective is to regulate all the state trajectories to the equilibrium.

Follow the design methodology of the previous section and apply the control law

$$u = \tau\left[-\gamma\left(x_3 + \beta\sin x_1 + \theta_2 x_1 + dx_2\right) - \left(\beta\cos x_1 + \theta_2\right)x_2 - d\dot{x}_2 + \frac{1}{\tau}x_3\right] \tag{21}$$

The system (20) in closed-loop with controller is simulated by using the proposed method and backstepping in [20], $\theta = [0, -26.67, 0.76485, -2.9225, 0]$ and $1/\tau = 15$. The design parameters are set $\beta = 20$, $\gamma = 50$. Simulation is done with $d = 6$ and result is shown in Figure 1 for the initial conditions $x\left(0\right) = \left(0.4, 0, 0\right)$.

As seen from Figure 1, we achieve slightly faster convergence speed with the proposed method.

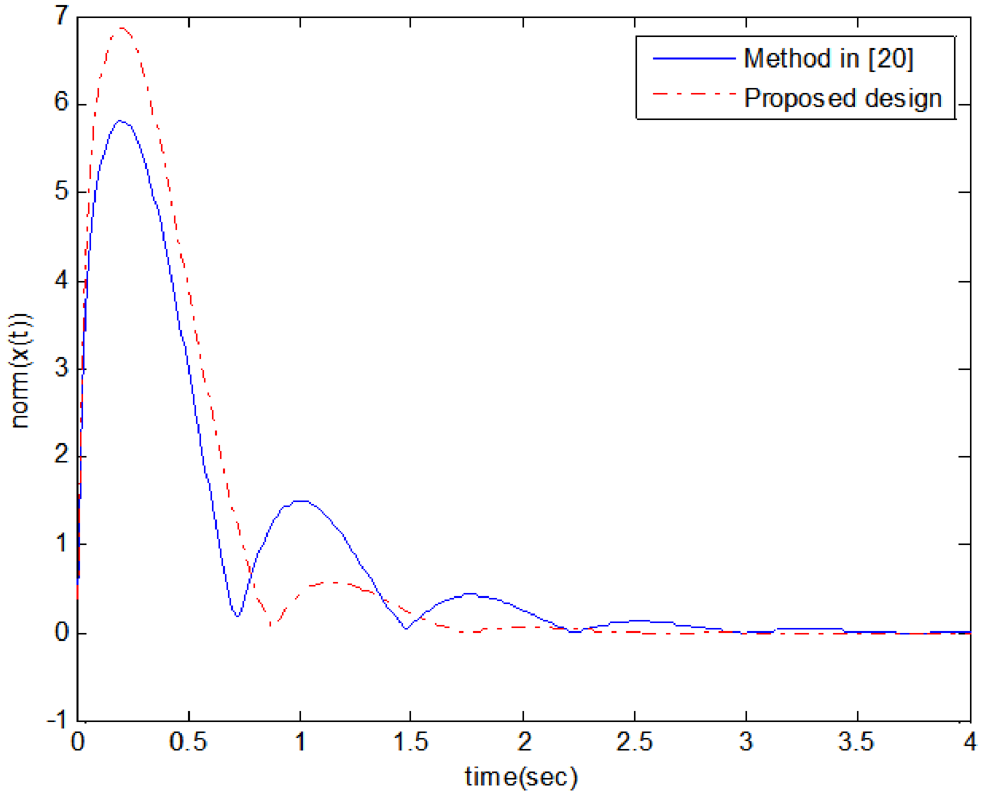

Figure 1. Comparative results for different controllers

5. **Conclusions.** We have presented a constructive algorithm for designing state feedback controller which relies on the tools of I&I to stabilize a class of nonlinear systems at equilibrium. We choose a manifold such that the closed-loop system restricted to the manifold is the target dynamics. The control law is synthesized in order to render the manifold invariant and attractive. Comparative result demonstrates the superior performance of the proposed technique in convergence speed.

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