

## STABILIZATION OF NETWORKED CONTROL SYSTEMS WITH PIECEWISE CONSTANT GENERALIZED SAMPLED-DATA HOLD FUNCTION

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**ABSTRACT.** *This paper is concerned with the control problem of networked control systems (NCSs) with piecewise constant generalized sampled-data hold function (PCGSHF). A new hybrid nonlinear NCSs model based on the PCGSHF is constructed, in which both the network-induced delay and the packet dropouts are considered in the transmission. By using a novel multi-datum point parameter method, the obtained NCSs model is divided into two parts, which are related to the given multi-datum point parameters and the bound uncertain multi-datum points, respectively. Correspondingly, the design of infinite PCGSHF can be achieved by giving finite multi-datum points. For handling the nonlinear term in system model, an effective norm bounded method is proposed based on the number of segments of PCGSHF. Next, the system stability is investigated based on Lyapunov theory and the linear matrix inequality (LMI) approach. Then, both the PCGSHF and networked feedback controller for guaranteeing the stability condition are designed. Moreover, a stability criterion based on maximum allowable network-induced delay rate is proposed for choosing a reasonable sampling period. Finally, one example is given to show the effectiveness and less conservatism of the results.*

**Keywords:** Networked control systems (NCSs), Packet dropout, Network-induced delay, PCGSHF, Zero-order holder (ZOH)

**1. Introduction.** A typical NCSs can be considered as a sampled-data feedback control system (including sampler units, digital processing units and actuator units) with communication network; see [7, 15, 26] and the references therein. Because the system units are connected through a multipurpose network, NCSs possess many advantages, such as simple installation, low cost and maintenance, and high reliability. However, networking the control system also introduces new problems caused by the packet-based data exchange between different units of the network. These problems can degrade closed-loop performance, or even worse, harm closed-loop stability of the control system. Therefore, during the last decade, a considerable attention has been devoted to the study of sampled-data based NCSs; for instance, see [4-6, 8-15, 20-22, 24, 27, 30, 32-35]. To be more specific, the stability analysis and the design of networked feedback controller were considered in [4, 6, 12-14, 20, 21, 33]. When considering some control performance, the  $H_\infty$  control methods were proposed in [5, 8, 24], and guaranteed cost control was considered in [10, 11]. Some fault detection and robust filtering problems were considered in [32-35]. The stability problem of nonuniform sampling frequency was considered in [9, 22, 27].

Although the above results studied different control problems of NCSs, it should be mentioned that they share a common characteristic: the actuator units are provided with ZOH for implementing the output signal of digital processing units. Such a consideration

raises an issue whether we can obtain better performance by introducing other types of hold functions. Obviously, if the hold function is variable instead of constant, the design of the NCSs will be more flexible which is helpful to improve the performance of NCSs. Such idea was proposed initially in [1, 2], which used GSHF instead of ZOH in traditional sampled-data control systems. [3, 16, 17, 28] studied the decentralized control of interconnected systems. In latest work [18], the authors studied the pole placement of continuous-time linear time-invariant systems and designed a GSHF for minimizing the given LQ performance index. Furthermore, it is well known that when the number of segments of PCGSHF is taken large enough, the PCGSHF can approximate the GSHF [19]. The design of PCGSHF is more significant because of its feasible structure for practical implementation. Recently, the existing studies on PCGSHF mainly focused on the property of zero point stability for sampled-data control system, in which the sampling period was limited to sufficiently small (see [23, 25]).

Although existing results are employed to design the GSHF or PCGSHF for sampled-data control system with the desired performance, an important issue that should be taken into account is the transmission delay. Moreover, these existing results on GSHF approaches cannot be used directly to design the NCSs. Specifically, the design of GSHF depends on the Gram matrix, which is based on the controllability of system; for instance, see Equations (8) and (9) in [18] and the similar equations in [1-3, 16, 17, 28]. However, the Gram matrices may be infinite because of the induced delay and packet dropout in the network. This strategy is invalid for designing the NCSs with GSHF. Similarly, the strict limiting condition in [23, 25] on sampling period may lead to the conservatism, and even fail to the design of the NCSs. To the best of our knowledge, there is no an effective and common method to design the parameters of PCGSHF for the sampled-data control system with transmission delay, which motivates our present study.

In this paper, PCGSHF technique is, for the first time, developed to solve the NCSs control problem. The main objective of this paper is to design both the PCGSHF and the networked feedback controller for ensuring the stability of the obtained hybrid nonlinear NCSs model, in which both the network-induced delay and the packet dropouts are considered in the transmission. Firstly, a novel multi-datum point parameter method is proposed. By using this method, the obtained hybrid nonlinear NCSs model is divided into two parts: one is related to the given multi-datum points; the other is related to the bounded uncertain multi-datum points. Based on the given finite multi-datum points, we can design infinite PCGSHF with one time-invariant networked feedback controller. This design method has the advantages of both higher efficiency and less computation burden. Secondly, an effective norm bounded method based on number of segments of PCGSHF is proposed to handle the nonlinear term in NCSs model, which is induced by both the network-induced delay and packet dropouts. Compared with the existing over-approximating polytypic inclusion approach (see [24]), our method efficiently reduces the amount of computation for determining the optimal upper bound of norm, which is induced by the inter-sampler behavior. Moreover, this method makes full use of the piecewise character of PCGSHF for reducing the conservatism which is induced by the large range of networked-induced delay. Thirdly, based on the above mentioned strategies, both the PCGSHF and the networked feedback controller can be designed. Finally, for choosing a reasonable sampling period in practical situation, one stability criterion based on maximum allowable network-induced delay rate is proposed.

**Notation:** The superscript  $T$  stands for transposition;  $\mathbb{R}^n$  represents the  $n$  dimensional Euclidean space;  $\mathbb{Z}^+$  represents the sets of positive integers.  $\mathbb{Z}$  denotes the set of nonnegative integers.  $I$  is the identity matrix of compatible dimensions.  $\|\bullet\|$  refers to the

Euclidean norm for vectors and induced 2-norm for matrices.  $*$  is used to describe the symmetrical element in matrix.  $sym\{\bullet\}$  describes  $(\bullet) + (\bullet)^T$ .

**2. Problem Statement and Preliminaries.** The structure of the considered NCSs is shown in Figure 1, in which the sensor is clock-driven with sampling period  $h$ , and the data is transmitted in a single packet at each time step  $(kh, k \in \mathbb{Z}^+)$ . The controller and actuator (with PCGSHF) are event-driven. The plant is described by the following continuous-time linear system model

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

where  $x(t) \in \mathbb{R}^n$  is the state and  $u(t) \in \mathbb{R}^m$  is the input.  $A$  and  $B$  are known real constant matrices with appropriate dimensions, which ensure the plant (1) is controllable. The discrete-time networked controller is of the form

$$u(kh) = Fx(kh), \tag{2}$$

where  $F \in \mathbb{R}^{m \times n}$ . The networks exist between from the sensor to the controller, and from the controller to the actuator.  $\tau_{sc}$  and  $\tau_{ca}$  are used to describe the sensor-to-controller delay and the controller-to-actuator delay, respectively. Without loss of generality, any controller delay can be absorbed into either  $\tau_{sc}$  or  $\tau_{ca}$ . Moreover, if the transmission delay  $\tau = \tau_{sc} + \tau_{ca}$  satisfies  $0 \leq \tau < h$ , the input signal  $u(kh - \tau)$  is considered as successful transmission. On the contrary, if the transmission delay  $\tau = \tau_{sc} + \tau_{ca}$  does not satisfy  $0 \leq \tau < h$ , the input signal  $u(kh - \tau)$  is considered as unsuccessful transmission, which means packet dropout. Here the number of successive packet dropout is upper bounded, and the bound is denoted by  $d_{\max}$ , which is a known constant.

According to the above consideration, a set  $\Pi = \{\alpha_i | \alpha_i \in kh; i, k \in \mathbb{Z}\}$  denotes the sequence points of successful transmissions from the sensor to the actuator, such as  $\alpha_{i-1}$ ,  $\alpha_i$  and  $\alpha_{i+1}$ . Moreover, a function  $d(\alpha_i)$  is used to describe the number of packet dropouts during two sequence points of successful transmissions, which are  $\alpha_i$  and  $\alpha_{i+1}$ . So we have the following definitions for capturing the natural of packet dropouts, network-induced delay and PCGSHF.

**Definition 2.1.** *The packet dropouts process  $d(\alpha_i)$  is defined as*

$$d(\alpha_i) = \frac{(\alpha_{i+1} - \alpha_i)}{h} - 1, \tag{3}$$

where  $d(\alpha_i) \in \Omega$  and  $\Omega = \{0, 1, 2, \dots, d_{\max}\}$ .

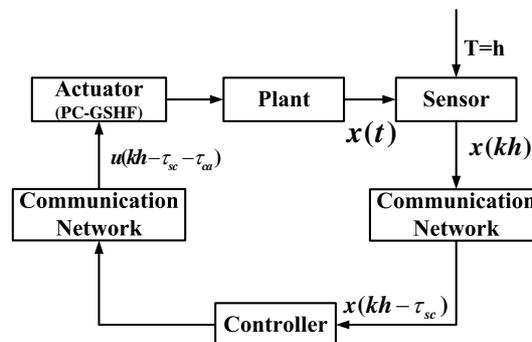


FIGURE 1. The structure of NCSs with PCGSHF

**Definition 2.2.** During one sampling period, the impulse response function of PCGSHF  $h(t)$  can be defined as

$$h(t) = \delta(i), \quad kh + \frac{(i-1)h}{\bar{a}} \leq t < kh + \frac{ih}{\bar{a}}, \tag{4}$$

where  $\bar{a}$  is the number of segments of PCGSHF,  $\delta(i) \in \mathbb{R}$ ,  $i = 1, 2, \dots, \bar{a}$  and  $h(t) = h(t+h)$ .

According to Definition 2.1 and Definition 2.2, a variable  $w_i \triangleq w(\alpha_i)$  is used to describe the case of

$$\tau(\alpha_i) \in \left[ (w_i - 1) \frac{h}{\bar{a}}, w_i \frac{h}{\bar{a}} \right), \tag{5}$$

where  $w_i \in \{1, 2, \dots, \bar{a}\}$ . Correspondingly, we can obtain the NCSs model during the two successive points,  $\alpha_i$  and  $\alpha_{i+1}$ :

$$x(\alpha_{i+1}) = G(d(\alpha_i), w_i)x(\alpha_i) + H(d(\alpha_i), w_i)x(\alpha_{i-1}), \tag{6}$$

where  $M_1(w_i) = \sum_{c=w_i+1}^{\bar{a}} \int_{(c-1)\frac{h}{\bar{a}}}^{c\frac{h}{\bar{a}}} e^{A(h-s)} ds B \delta(c) F$ ,  $\hat{N}(w_i) = \sum_{c=1}^{w_i} \int_{(c-1)\frac{h}{\bar{a}}}^{c\frac{h}{\bar{a}}} e^{A(h-s)} ds B \delta(c) F$ ,  $M(w_i) = \int_{\tau(\alpha_i)}^{w_i\frac{h}{\bar{a}}} e^{A(h-s)} ds B \delta(w_i) F$ ,  $G(d(\alpha_i), w_i) = e^{A(d(\alpha_i)+1)h} + e^{Ad(\alpha_i)h} M_1(w_i) + e^{Ad(\alpha_i)h} M(w_i) + \vartheta(d(\alpha_i)) \sum_{\beta=0}^{d(\alpha_i)-1} (e^{Ah})^\beta M_1(w_i) + \vartheta(d(\alpha_i)) \sum_{\beta=0}^{d(\alpha_i)-1} (e^{Ah})^\beta \hat{N}(w_i)$ ,  $H(d(\alpha_i), w_i) = e^{Ad(\alpha_i)h} \hat{N}(w_i) - e^{Ad(\alpha_i)h} M(w_i)$ , an indicative function  $\vartheta(d(\alpha_i))$  is defined as

$$\vartheta(d(\alpha_i)) = \begin{cases} 0 & d(\alpha_i) = 0, \\ 1 & d(\alpha_i) \neq 0. \end{cases} \tag{7}$$

The initial condition of (6) in this paper is considered as follows:  $x(0) = x(\alpha_0) = x_0$ , where  $x_0$  is the initial state. The initial process is  $d(\alpha_0)$  and  $w_0$ . The initial input  $u(t) = 0$  for  $t \in (-\infty, \tau_0)$ . Then the initial system of (6) is described as

$$x(\alpha_1) = G(d(\alpha_0), w_0)x(\alpha_0). \tag{8}$$

In this paper, the main objectives are to design both the PCGSHF  $h(t)$  and the networked feedback controller (2) to ensure the stability of NCSs (6) with the initial condition (8). In general, the  $h(t)$  and the controller (2), that satisfy the stability condition, are infinite. However, how to determine them is an important problem. Therefore, we propose a multi-datum point parameter method to achieve the joint-designing of them, which is showed in the following Lemma 2.1.

**Lemma 2.1. (Multi-datum point parameter method)** If the designer applies the multi-datum point parameter method, the parameter  $\delta(i)$  of PCGSHF  $h(t)$  can be fixed by the following formulations:

$$\delta(i) = \delta_{dp}^i + \bar{\delta}(i), \tag{9}$$

$$|\bar{\delta}(i)| \leq \delta_{\max}^i, \tag{10}$$

in which  $i \in \{1, 2, \dots, \bar{a}\}$ ,  $\delta_{dp}^i, \bar{\delta}(i) \in \mathbb{R}$ , and  $\delta_{\max}^i \in \mathbb{R}^+$ . The parameters  $\delta_{dp}^i$  ( $i \in \{1, 2, \dots, \bar{a}\}$ ) is the multi-datum point.

Correspondingly, the NCSs model (6) with the initial condition (8) can be described as

$$\begin{aligned}
 x(\alpha_{i+1}) &= \left( G_{\delta_{dp}^c}(d(\alpha_i), w_i) + G_{\bar{\delta}(c)}(d(\alpha_i), w_i) \right) x(\alpha_i) \\
 &\quad + \left( H_{\delta_{dp}^c}(d(\alpha_i), w_i) + H_{\bar{\delta}(c)}(d(\alpha_i), w_i) \right) x(\alpha_{i-1}),
 \end{aligned} \tag{11}$$

and

$$x(\alpha_1) = \left( G_{\delta_{dp}^c}(d(\alpha_0), w_0) + G_{\bar{\delta}(c)}(d(\alpha_0), w_0) \right) x(\alpha_0) \tag{12}$$

where  $G_{\delta_{dp}^c}(d(\alpha_i), w_i)$  and  $H_{\delta_{dp}^c}(d(\alpha_i), w_i)$  equal to  $G(d(\alpha_i), w_i)$  and  $H(d(\alpha_i), w_i)$  when  $\delta_{dp}^c = \delta(c)$ ,  $G_{\bar{\delta}(c)}(d(\alpha_i), w_i)$  and  $H_{\bar{\delta}(c)}(d(\alpha_i), w_i)$  equal to  $G(d(\alpha_i), w_i)$  and  $H(d(\alpha_i), w_i)$  when  $\delta(c) = \bar{\delta}(c)$ ,  $c \in \{1, 2, \dots, \bar{a}\}$ .

Furthermore, it should be noted that the NCSs (11) with (12) are hybrid nonlinear switched systems, where the nonlinear uncertain term is induced by the network-induced delay  $\tau_i$ . For handling such nonlinear term, a norm bounded method based on the segments of PCGSHF is proposed as the following Lemma 2.2.

**Lemma 2.2.** *The nonlinear uncertain term  $M(w_i)$  in (6) can be expressed as*

$$M(w_i) = e^{A(h-w_i\frac{h}{\bar{a}})} \bar{M}(v(\alpha_i), \Delta\tau_{con}) B\delta(w_i) F, \tag{13}$$

where  $\bar{M}(v(\alpha_i), \Delta\tau_{con}) = \int_0^{\Delta\tau_{con}} e^{As} ds + e^{A\Delta\tau_{con}} \int_0^{v(\alpha_i)} e^{As} ds$ ,  $\Delta\tau(\alpha_i) \in (0, \frac{h}{\bar{a}}]$ ,  $v(\alpha_i) = \Delta\tau(\alpha_i) - \Delta\tau_{con}$ ,  $\Delta\tau_{con} \in (0, \frac{h}{\bar{a}}]$  is a chosen constant. Moreover, by using Lemma 1 in [6], the following relationships can be obtained:

$$\Delta\tau_{con} = \frac{h}{2\bar{a}}, \tag{14}$$

and

$$\left\| \int_0^{v(\alpha_i)} e^{As} ds \right\| < \Lambda(\Delta\tau_{con}) = \Lambda\left(\frac{h}{2\bar{a}}\right), \tag{15}$$

where  $\Lambda(\sigma) = \frac{1}{\lambda_{\max} + \bar{a}} [e^{(\lambda_{\max} + \bar{a})|\sigma|} - 1]$ .

**Proof:** Let  $\tau(\alpha_i) = w_i\frac{h}{\bar{a}} - \Delta\tau(\alpha_i)$ , we have

$$M(w_i) = \int_{h-w_i\frac{h}{\bar{a}}}^{h-w_i\frac{h}{\bar{a}}+\Delta\tau(\alpha_i)} e^{As} ds B\delta(w_i) F = e^{A(h-w_i\frac{h}{\bar{a}})} \int_0^{\Delta\tau(\alpha_i)} e^{As} ds B\delta(w_i) F, \tag{16}$$

where  $\Delta\tau(\alpha_i) \in (0, \frac{h}{\bar{a}}]$ . Furthermore, we introduce a constant  $\Delta\tau_{con} \in (0, \frac{h}{\bar{a}}]$  and let  $v(\alpha_i) = \Delta\tau(\alpha_i) - \Delta\tau_{con}$ , the term  $\int_0^{\Delta\tau(\alpha_i)} e^{As} ds$  in (16) can be described as

$$\begin{aligned}
 \int_0^{\Delta\tau(\alpha_i)} e^{As} ds &= \int_0^{v(\alpha_i)+\Delta\tau_{con}} e^{As} ds = \int_0^{\Delta\tau_{con}} e^{As} ds - \int_{v(\alpha_i)+\Delta\tau_{con}}^{\Delta\tau_{con}} e^{As} ds \\
 &= \int_0^{\Delta\tau_{con}} e^{As} ds + e^{A\Delta\tau_{con}} \int_0^{v(\alpha_i)} e^{As} ds
 \end{aligned} \tag{17}$$

Substituting (17) into (16), we can obtain (13). Moreover, we have the following parameter index  $J(\Delta\tau_{con})$ , which is defined as

$$J(\Delta\tau_{con}) = \min_{\Delta\tau_{con} \in (0, \frac{h}{\bar{a}}]} \left\{ \max \left\{ \Lambda(-\Delta\tau_{con}), \Lambda\left(\frac{h}{\bar{a}} - \Delta\tau_{con}\right) \right\} \right\}. \tag{18}$$

According to the character of function  $\Lambda(\cdot)$  in [6], it is obvious that the minimum  $J(\Delta\tau_{con})$  can be obtained when  $\Delta\tau_{con} = \frac{h}{\bar{a}} - \Delta\tau_{con}$ . Therefore, we have (14) and (15).

**3. Main Result.** In this section, the objectives are to design both the PCGSHF (4) and the networked state feedback controller (2) for guaranteeing the stability of the NCSs (11) with initial state  $x_0$ ,  $d(\alpha_0)$  and  $w_0$ .

**Theorem 3.1.** *The NCSs (11) with the initial condition (12) are asymptotically stable, if for any scalars  $\varepsilon_c > 0$ ,  $\varepsilon > 0$ ,  $\varepsilon_{dp}^{w_j} > 0$ ,  $\hat{\varepsilon}_c > 0$ , and the given multi-datum points  $\delta_{dp}^c$ ,  $\delta_{\max}^c$  ( $c, w_i, w_j \in \{1, 2, \dots, \bar{a}\}$ ), there exist symmetric positive definite matrices  $\bar{P}(d_j, w_j)$ ,  $\hat{P}(d_i, w_i)$ ,  $\hat{Q}(d_i, d_j \in \Omega)$ , the nonsingular matrix  $E$ , and the appropriate dimensions matrix  $\hat{E}$  satisfying the following LMI:*

$$\begin{bmatrix} \hat{Y}_{11} & \hat{Y}_{12} & \hat{Y}_{13} & \hat{Y}_{14} & \hat{Y}_{15} & \hat{Y}_{16} \\ * & \hat{Y}_{22} & 0 & 0 & 0 & 0 \\ * & * & \hat{Y}_{33} & 0 & 0 & 0 \\ * & * & * & \hat{Y}_{44} & 0 & 0 \\ * & * & * & * & \hat{Y}_{55} & \hat{Y}_{56} \\ * & * & * & * & * & \hat{Y}_{66} \end{bmatrix} < 0 \tag{19}$$

where

$$\begin{aligned} \hat{Y}_{11} &= \begin{bmatrix} \bar{P}(d_i, w_i) + \bar{Q} - E - E^T & 0 & \hat{S}_{11}^T \\ * & -\bar{Q} & \hat{S}_{12}^T \\ * & * & -\bar{P}(d_j, w_j) \end{bmatrix}, \hat{S}_{11} = \Phi(d_j + 1) E \\ &+ \Phi(d_j) \sum_{c=w_j+1}^{\bar{a}} \Phi(1 - \frac{c}{\bar{a}}) \Gamma(\frac{h}{\bar{a}}) B \hat{E} \delta_{dp}^c + \Phi(d_j + 1 - \frac{w_j}{\bar{a}}) \Gamma(\Delta\tau_{con}) B \hat{E} \delta_{dp}^{w_j} \\ &+ \vartheta(d_j) \sum_{\beta=0}^{d_j-1} (\Phi(1))^\beta \sum_{c=1}^{\bar{a}} \Phi(1 - \frac{c}{\bar{a}}) \Gamma(\frac{h}{\bar{a}}) B \hat{E} \delta_{dp}^c, \hat{Y}_{12} = [ \varepsilon_{dp}^{w_j} X(d_j, w_j) \quad Z^T ], \\ \hat{S}_{12} &= \Phi(d_j) \sum_{c=1}^{w_j} \Phi(1 - \frac{c}{\bar{a}}) \Gamma(\frac{h}{\bar{a}}) B \hat{E} \delta_{dp}^c - \Phi(d_j + 1 - \frac{w_j}{\bar{a}}) \Gamma(\Delta\tau_{con}) B \hat{E} \delta_{dp}^{w_j}, X(d_j, w_j) \\ &= [ 0 \quad 0 \quad (\Phi(d_j + 1 - \frac{w_j}{\bar{a}}) e^{A\Delta\tau_{con}} \delta_{dp}^{w_j})^T ]^T, \bar{X}(d_j, c) = [ 0 \quad 0 \quad \Phi^T(d_j + 1 - \frac{c}{\bar{a}}) ]^T, \\ Z &= [ B \hat{E} \quad -B \hat{E} \quad 0 ], \hat{Y}_{16} = [ 0 \quad Z^T ], \hat{Y}_{22} = \text{diag} \left\{ -\varepsilon_{dp}^{w_j} I, -\frac{\varepsilon_{dp}^{w_j}}{\Lambda^2(\frac{h}{2\bar{a}})} I \right\}, \\ \tilde{Z}(c) &= [ \Phi(1 - \frac{c}{\bar{a}}) \Gamma(\frac{h}{\bar{a}}) B \hat{E} \quad 0 \quad 0 ], l = w_j + 1, \\ \hat{Y}_{13} &= \left[ \overbrace{\varepsilon_1 \bar{X}(d_j, 1), \dots, \varepsilon_{w_j-1} \bar{X}(d_j, w_j-1)}^{w_j-1} \quad \overbrace{\bar{Z}^T(d_j), \dots, \bar{Z}^T(d_j)}^{w_j-1} \right], \\ \hat{X}(d_j) &= \left[ 0 \quad 0 \quad \left( \Phi(d_j) + \vartheta(d_j) \sum_{\beta=0}^{d_j-1} (\Phi(1))^\beta \right)^T \right]^T, \hat{Y}_{55} = \text{diag} \left\{ -\varepsilon_{w_j} I, -\hat{\varepsilon}_{w_j} I \right\}, \\ \bar{Z}(d_j) &= \left[ \Phi(-d_j) \vartheta(d_j) \sum_{\beta=0}^{d_j-1} (\Phi(1))^\beta \Gamma(\frac{h}{\bar{a}}) B \hat{E} \quad \Gamma(\frac{h}{\bar{a}}) B \hat{E} \quad 0 \right], \\ \hat{Y}_{33} &= \text{diag} \left\{ -\varepsilon_1 I \dots - \varepsilon_{w_j-1} I, -\hat{\varepsilon}_1 I \dots - \hat{\varepsilon}_{w_j-1} I \right\}, \hat{Y}_{66} = \text{diag} \left\{ -\varepsilon I, -\frac{\varepsilon}{\Lambda^2(\frac{h}{2\bar{a}})} I \right\}, \\ \hat{Y}_{15} &= \left[ \varepsilon_{w_j} \bar{X}(d_j, w_j) \quad (\bar{Z}(d_j) + \tilde{Z})^T \right], \tilde{Z} = [ \Gamma(\Delta\tau_{con}) B \hat{E} \quad -\Gamma(\Delta\tau_{con}) B \hat{E} \quad 0 ], \\ \hat{Y}_{14} &= \left[ \overbrace{\varepsilon_l \hat{X}(d_j, l), \dots, \varepsilon_{\bar{a}} \hat{X}(d_j, \bar{a})}^{\bar{a}-w_j} \quad \overbrace{\tilde{Z}^T(l), \dots, \tilde{Z}^T(\bar{a})}^{\bar{a}-w_j} \right], \hat{Y}_{56} = \begin{bmatrix} 0 & 0 \\ e^{A\Delta\tau_{con}} \varepsilon & 0 \end{bmatrix}, \\ \hat{Y}_{44} &= \text{diag} \left\{ -\varepsilon_{w_j+1} I \dots - \varepsilon_{\bar{a}} I, -\hat{\varepsilon}_{w_j+1} I \dots - \hat{\varepsilon}_{\bar{a}} I \right\}. \end{aligned}$$

Moreover, if the inequality (19) can hold under the case of the given multi-datum point  $\delta_{dp}^c$ , the networked feedback control gain (2) can be designed as  $F = \hat{E}E^{-1}$ , the PCGSHF (4) can be designed as  $\delta_{dp}^i - \delta_{\max}^i \leq \delta(i) \leq \delta_{dp}^i + \delta_{\max}^i$ , where  $(\delta_{\max}^c)^2 = \frac{\varepsilon_a}{\varepsilon_c}$ .

**Proof:** At first, a binary Lyapunov functional is taken as

$$V(\rho) = x^T(\rho) P(d(\rho), w_\rho) x(\rho) + x^T(\alpha_i) Q x(\alpha_i), \tag{20}$$

for  $\alpha_i + h \leq \rho \leq \alpha_{i+1}$ , and yields

$$V(\alpha_i) = x^T(\alpha_i) P(d(\alpha_i), w_i) x(\alpha_i) + x^T(\alpha_{i-1}) Q x(\alpha_{i-1}), \tag{21}$$

and

$$V(\alpha_{i+1}) = x^T(\alpha_{i+1}) P(d(\alpha_{i+1}), w_{i+1}) x(\alpha_{i+1}) + x^T(\alpha_i) Q x(\alpha_i). \tag{22}$$

Then we have

$$\Delta V = V(\alpha_{i+1}) - V(\alpha_i) = \begin{bmatrix} x(\alpha_i) \\ x(\alpha_{i-1}) \end{bmatrix}^T \begin{bmatrix} \Xi_{11} & \Xi_{12} \\ * & \Xi_{22} \end{bmatrix} \begin{bmatrix} x(\alpha_i) \\ x(\alpha_{i-1}) \end{bmatrix}, \tag{23}$$

where  $\Xi_{11} = -P(d_i, w_i) + Q + G^T(d_j, w_j) P(d_j, w_j) G(d_j, w_j)$ ,  $\Xi_{22} = -Q + H^T(d_j, w_j) P(d_j, w_j) H(d_j, w_j)$ ,  $P(d(\alpha_i), w_i) \triangleq P(d_i, w_i)$ ,  $\Xi_{12} = G^T(d_j, w_j) P(d_j, w_j) H(d_j, w_j)$ , and  $P(d(\alpha_{i+1}), w_{i+1}) \triangleq P(d_j, w_j)$ .

By schur complement Lemma, the inequality (23) can be described as

$$\begin{bmatrix} -P(d_i, w_i) + Q & 0 & G^T(d_j, w_j) \\ * & -Q & H^T(d_j, w_j) \\ * & * & -P^{-1}(d_j, w_j) \end{bmatrix} < 0. \tag{24}$$

Furthermore, let  $\Gamma(\bullet) = \int_0^\bullet e^{As} ds$ ,  $\Phi(\bullet) = e^{A(\bullet)h}$ , we have the following expressions by using both the multi-datum point parameter method (see Lemma 2.1) and the (13) in Lemma 2.2,

$$\begin{aligned} & \begin{bmatrix} -P(d_i, w_i) + Q & 0 & \bar{S}_{11}^T \\ * & -Q & \bar{S}_{12}^T \\ * & * & -P^{-1}(d_j, w_j) \end{bmatrix} + \text{sym} \{ X(d_j, w_j) \Gamma(v_j) D \} \\ & + \sum_{c=1}^{w_j-1} \text{sym} \{ \bar{X}(d_j, c) \bar{\delta}(c) \bar{D}(d_j) \} + \sum_{c=w_j+1}^{\bar{a}} \text{sym} \{ \hat{X}(d_j, c) \bar{\delta}(c) \tilde{D}(c) \} \\ & + \text{sym} \{ \bar{X}(d_j, w_j) \bar{\delta}(w_j) \hat{D}(d_j) \} < 0, \end{aligned} \tag{25}$$

where

$$\begin{aligned} \bar{S}_{11} &= \Phi(d_j) \sum_{c=w_j+1}^{\bar{a}} \Phi(1 - \frac{c}{\bar{a}}) \Gamma(\frac{h}{\bar{a}}) BF \delta_{dp}^c + \Phi(d_j + 1 - \frac{w_j}{\bar{a}}) \Gamma(\Delta\tau_{con}) BF \delta_{dp}^{w_j} \\ & + \Phi(d_j + 1) + \vartheta(d_j) \sum_{\beta=0}^{d_j-1} (\Phi(1))^\beta \sum_{c=1}^{\bar{a}} \Phi(1 - \frac{c}{\bar{a}}) \Gamma(\frac{h}{\bar{a}}) BF \delta_{dp}^c, \\ \bar{S}_{12} &= \Phi(d_j) \sum_{c=1}^{w_j} \Phi(1 - \frac{c}{\bar{a}}) \Gamma(\frac{h}{\bar{a}}) BF \delta_{dp}^c - \Phi(d_j + 1 - \frac{w_j}{\bar{a}}) \Gamma(\Delta\tau_{con}) BF \delta_{dp}^{w_j}, \\ X(d_j, w_j) &= \begin{bmatrix} 0 & 0 & (\Phi(d_j + 1 - \frac{w_j}{\bar{a}}) e^{A\Delta\tau_{con}} \delta_{dp}^{w_j})^T \end{bmatrix}^T, \\ \bar{X}(d_j, c) &= \begin{bmatrix} 0 & 0 & \Phi^T(d_j + 1 - \frac{c}{\bar{a}}) \end{bmatrix}^T, D = \begin{bmatrix} BF & -BF & 0 \end{bmatrix}, \\ \bar{D}(d_j) &= \begin{bmatrix} \Phi(-d_j) \vartheta(d_j) \sum_{\beta=0}^{d_j-1} (\Phi(1))^\beta \Gamma(\frac{h}{\bar{a}}) BF & \Gamma(\frac{h}{\bar{a}}) BF & 0 \end{bmatrix}, \\ \tilde{D}(c) &= \begin{bmatrix} \Phi(1 - \frac{c}{\bar{a}}) \Gamma(\frac{h}{\bar{a}}) BF & 0 & 0 \end{bmatrix}, \end{aligned}$$

$$\hat{X}(d_j) = \begin{bmatrix} 0 & 0 & \left( \Phi(d_j) + \vartheta(d_j) \sum_{\beta=0}^{d_j-1} (\Phi(1))^\beta \right)^T \end{bmatrix}^T, \text{ and } \hat{D}(d_j) = \bar{D}(d_j) + \begin{bmatrix} \Gamma(\Delta\tau_{con})BF & -\Gamma(\Delta\tau_{con})BF & 0 \end{bmatrix} + \begin{bmatrix} e^{A\Delta\tau_{con}}\Gamma(v_j)BF & -e^{A\Delta\tau_{con}}\Gamma(v_j)BF & 0 \end{bmatrix}.$$

Here according to (14) and (15) in Lemma 2.2, we have

$$sym \{X(d_j, w_j) \Gamma(v_j) D\} \leq \varepsilon_{dp}^{w_j} X(d_j, w_j) X^T(d_j, w_j) + \frac{\Lambda^2 \left(\frac{h}{2\bar{a}}\right)}{\varepsilon_{dp}^{w_j}} D^T D \tag{26}$$

for any  $\varepsilon_{dp}^{w_j} > 0$ . Moreover, by using Lemma 2 in [30], the following inequality guarantees that (25) holds,

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} & Y_{15} & Y_{16} \\ * & Y_{22} & 0 & 0 & 0 & 0 \\ * & * & Y_{33} & 0 & 0 & 0 \\ * & * & * & Y_{44} & 0 & 0 \\ * & * & * & * & Y_{55} & Y_{56} \\ * & * & * & * & * & Y_{66} \end{bmatrix} < 0 \tag{27}$$

where  $Y_{11} = \begin{bmatrix} -P(d_i, w_i) + Q & 0 & \bar{S}_{11}^T \\ * & -Q & \bar{S}_{12}^T \\ * & * & -P^{-1}(d_j, w_j) \end{bmatrix}$ ,  $Y_{12} = [ X(d_j, w_j) \quad D^T ]$ ,  $Y_{22} = diag \left\{ -\frac{1}{\varepsilon_{dp}^{w_j}} I, -\frac{\varepsilon_{dp}^{w_j}}{\Lambda^2 \left(\frac{h}{2\bar{a}}\right)} I \right\}$ ,  $Y_{13} = \left[ \overbrace{\bar{X}(d_j, 1), \dots, \bar{X}(d_j, w_j - 1)}^{w_j-1} \quad \overbrace{\bar{D}^T(d_j), \dots, \bar{D}^T(d_j)}^{w_j-1} \right]$ ,  $Y_{33} = diag \left\{ -\frac{1}{\varepsilon_1} I \dots -\frac{1}{\varepsilon_{w_j-1}} I, -\frac{\varepsilon_1}{(\delta_{max}^1)^2} I, \dots, -\frac{\varepsilon_{w_j-1}}{(\delta_{max}^{w_j-1})^2} I \right\}$ ,  $Y_{16} = [ 0 \quad D^T ]$ ,  $Y_{14} = \left[ \overbrace{\hat{X}(d_j, w_j + 1), \dots, \hat{X}(d_j, \bar{a})}^{\bar{a}-w_j} \quad \overbrace{\tilde{D}^T(w_j + 1), \dots, \tilde{D}^T(\bar{a})}^{\bar{a}-w_j} \right]$ ,  $Y_{56} = \begin{bmatrix} 0 & 0 \\ e^{A\Delta\tau_{con}} & 0 \end{bmatrix}$ ,  $Y_{15} = [ \bar{X}(d_j, w_j) \quad (\bar{D}(d_j) + \hat{D})^T ]$ ,  $Y_{66} = diag \left\{ -\frac{1}{\varepsilon} I, -\frac{\varepsilon}{\Lambda^2 \left(\frac{h}{2\bar{a}}\right)} \right\}$ ,  $\varepsilon > 0$ ,  $Y_{44} = diag \left\{ -\frac{1}{\varepsilon_{w_j+1}} I \dots -\frac{1}{\varepsilon_{\bar{a}}} I, -\frac{\varepsilon_{w_j+1}}{(\delta_{max}^{w_j+1})^2} I, -\frac{\varepsilon_{\bar{a}}}{(\delta_{max}^{\bar{a}})^2} I \right\}$ ,  $Y_{55} = diag \left\{ -\frac{1}{\varepsilon_{w_j}} I, -\frac{\varepsilon_{w_j}}{(\delta_{max}^{w_j})^2} I \right\}$ ,  $\varepsilon_{dp}^{w_j} > 0$ ,  $\hat{D} = [ \Gamma(\Delta\tau_{con})BF \quad -\Gamma(\Delta\tau_{con})BF \quad 0 ]$ ,  $\varepsilon_c > 0$ , and  $c \in \{1, 2, \dots, \bar{a}\}$ .

Here for any nonsingular matrix  $E \in \mathbb{R}^{n \times n}$ , there exists

$$(E^{-1} - P(d_i, w_i))^T P^{-1}(d_i, w_i) (E^{-1} - P(d_i, w_i)) \geq 0, \tag{28}$$

which implies that

$$-P(d_i, w_i) \leq E^{-T} P^{-1}(d_i, w_i) E^{-1} - E^{-1} - E^{-T}. \tag{29}$$

Correspondingly, the term  $-P(d_i, w_i) + Q$  in  $Y_{11}$  can be replaced by the term  $E^{-T} P^{-1}(d_i, w_i) E^{-1} - E^{-1} - E^{-T} + Q$ . Furthermore, pre- and post-multiply the inequality (27) with this replacement by matrix  $diag \left\{ E^T, E^T, I, \varepsilon_{dp}^{w_j} I, I, \bar{E}, \tilde{E}, \varepsilon_{w_j} I, I, \varepsilon I, I \right\}$ , where  $\bar{E} = \overbrace{\varepsilon_1 I, \dots, \varepsilon_{w_j-1} I}^{w_j-1}, \overbrace{I, \dots, I}^{w_j-1}$  and  $\tilde{E} = \overbrace{\varepsilon_{w_j+1} I, \dots, \varepsilon_{\bar{a}} I}^{\bar{a}-w_j}, \overbrace{I, \dots, I}^{\bar{a}-w_j}$ , we can obtain (19) under the case of  $\bar{P}(d_i, w_i) = P^{-1}(d_i, w_i)$ ,  $\bar{P}(d_j, w_j) = P^{-1}(d_j, w_j)$ ,  $\bar{Q} = E^T Q E$ ,  $\hat{\varepsilon}_c = \frac{\varepsilon_c}{(\delta_{max}^c)^2}$  and  $F E = \hat{E}$ .

It should be mentioned that in Theorem 3.1, there exists a restriction condition of  $\tau_i \in [0, h)$ . It means that the allowable range of network-induced delay is related strictly to

the sampling period  $h$ . However, in practical application, the allowable range of network-induced delay can be predetermined, for instance,  $\tau_i \in [0, \tau_{\max})$ ,  $\tau_{\max}$  is the upper bound of network-induced delay. The sampling period  $h$  is the parameter, which needs to be designed. Therefore, the definition of maximum allowable network-induced delay rate is introduced as follows:

**Definition 3.1.** Let  $\tau_{\max} = \frac{(\bar{a}-\hat{a})\bar{h}}{\bar{a}}$  denote the maximum allowable network-induced delay, where  $\hat{a}$  is chosen from the set  $\{1, 2, \dots, \bar{a} - 1\}$ ,  $\bar{a} > 1$ , then we have

$$\theta = \frac{(\bar{a} - \hat{a})}{\bar{a}} \times 100\%. \tag{30}$$

The parameter  $\theta$  is called the maximum allowable network-induced delay rate, which denotes the relationship between the maximum allowable network-induced delay and the sampling period  $\bar{h}$ .

**Corollary 3.1.** Based on the maximum allowable network-induced delay rate  $\theta$ , the NCSs (11) with initial condition (12) are asymptotically stable, if the parameters  $w_i$  and  $w_j$  in (19) belong to the set  $\{1, 2, \dots, (\bar{a} - \hat{a})\}$ , the sampling period  $h$  is replaced by  $\bar{h}$ .

**Proof:** Because of the network-induced delay  $\tau_i \in [0, \theta\bar{h})$ , the influence of network-induced delay for the NCSs (11) during the interval  $[kh + \frac{(\bar{a}-\hat{a})}{\bar{a}}h, (k+1)h)$  should not be considered. Therefore, the parameters  $w_i$  and  $w_j$  in (19) belong to the set  $\{1, \dots, (\bar{a} - \hat{a})\}$ , the sampling period  $h$  is replaced by  $\bar{h}$ .

**4. Numerical Example.** In this section, we present a numerical simulation to show the application of the proposed methods in this paper.

**Example 4.1.** Considering the continuous-time linear time-invariant model as

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} u(t). \tag{31}$$

This system (31) was considered in [4, 21, 26, 29]. In their analysis, the networked feedback control gain is taken as  $F = [-3.75 \quad -11.5]$ . Notes that as these results mentioned, the maximum allowable network-induced delay (or the maximum allowable sampling period) 1.73 is the necessary and sufficient condition for the considered NCSs (31) with both the networked feedback controller  $F$  and the ZOH. Moreover, in [21], the maximum number of packet dropouts is less than 1 when the sampling period  $\bar{h}$  is larger than 1s. Here for indicating the less conservative of the strategy in this paper, we take the same networked feedback controller with the above mentioned results. Moreover, the maximum allowable network-induced delay  $\tau_{\max}$  is taken as 3s, the maximum number of packet dropouts is taken as 1. Correspondingly, if the sampling period  $\bar{h}$  is chosen as 6s, we have  $\bar{a} = 2$ ,  $\theta = 50\%$ , and

$$h(t) = \begin{cases} \delta(1) & k\bar{h} \leq t < k\bar{h} + 3 \\ \delta(2) & k\bar{h} + 3 \leq t < k\bar{h} + 6 \end{cases} \tag{32}$$

Furthermore, by using Corollary 3.1 with these multi-datum points  $\delta_{dp}(1) = 0.0001$  and  $\delta_{dp}(2) = 0.03$ , we have

$$h(t) = \begin{cases} \delta(1) \in [-0.0015, 0.0027] & k\bar{h} \leq t < k\bar{h} + 3 \\ \delta(2) \in [0.0291, 0.0309] & k\bar{h} + 3 \leq t < k\bar{h} + 6 \end{cases} \tag{33}$$

From (33), we can conclude that the NCSs (31) with the networked feedback controller  $F$  are asymptotically stability on the cases of  $\tau_{\max} = 3s$ ,  $d_{\max} = 1$  and  $\bar{h} = 6s$ . Obviously,

TABLE 1. The maximum allowable network-induced delay (MANID) for system (31) by existing stability conditions

<i>The method</i>	<i>MANID</i>
[4]	0.869 s
[21]	1.365 s
[29]	1.729 s
<i>this paper</i>	3 s

comparing with the current results, the obtained results in this paper are less conservative (see Table 1).

In fact, the ZOH is a special case of PCGSHF, when  $\delta(i) = 1$ . Such structure can result that the existence of limiting conditions (such as 1.73s or the maximum number of packet dropouts) when the networked feedback controller is given. By designing a desired PCGSHF, these limiting conditions can be removed. Therefore, according to above analysis, we can obtain larger stable region by using PCGSHF instead of ZOH.

**5. Conclusions.** In this paper, we have considered the control problem of NCSs with PCGSHF. A hybrid NCSs model based on the PCGSHF has been constructed, in which both the network-induced delay and the packet dropouts are considered in the transmission. The obtained system model has been divided effectively by using a multi-datum point parameter method, which was based on the given finite multi-datum point for determining the range of parameters in PCGSHF. An effective norm bounded method based on the number of segments of PCGSHF has been proposed to handle the nonlinear term in system model. By using the above strategies, the stability conditions have been obtained based on the Lyapunov theory. Correspondingly, both the PCGSHF and the networked feedback controller have been designed by solving a set of LMIs. Moreover, for choosing a reasonable sampling period in practical situation, a stability criterion based on maximum allowable network-induced delay rate has been proposed. Finally, one example has been given to show the effectiveness and less conservatism of the proposed strategies.

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