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ADAPTIVE CONTROL OF BILATERAL TELEOPERATION WITH UNSYMMETRICAL TIME-VARYING DELAYS

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Abstract. *In this paper, adaptive control is proposed for master-slave teleoperation systems with dynamical uncertainties and unsymmetric time-varying delays in communication channels. With the technique of force-motion transformation based on impedance model, we can convert the objective of force reflection into motion synchronization. Using partial feedback linearization, the whole teleoperation dynamics including both master and slave robots are transformed into two subsystems. Then, a novel adaptive control is proposed to deal with the unsymmetric time-varying delays and the dynamical uncertainties. The stability of two subsystems is proved with LMIs (linear matrix inequalities) based on Lyapunov stability synthesis. Extensive simulations and experiments are conducted to validate the effectiveness of the proposed control and illustrate the performance under this control law.*

1. Introduction.

Bilateral teleoperation system is considered as a system of combination technologies of both network communication and robotic technologies [1]. Wide range of applications using bilateral teleoperation systems can be found in areas such as outer space exploration [2], toxic materials handling [3] and minimally invasive surgery [14]. However, because of the existence of time-varying delays [4] in the communication channels between the master and slave robots, teleoperation system has been one of the most challenging research areas.

In bilateral teleoperation, the information between the master and slave is transmitted via a communication network, long distance and unexpected disturbance may cause some delays. The existence of such communication time delays may destabilize the whole master-slave teleoperation system [5]. Using the passivity theory and scattering approach, the stability analysis and controller design were extensively studied. In [7], passivity-based teleoperation method was proposed where passivity and scattering theory were used to analyze mechanisms responsible for loss of stability, and a time delay compensation scheme was derived to guarantee stability independent of the constant delay.

Other works such as [6, 10], assumed that the time delays in two directions are symmetric, which means the backward and forward time delays are equal. However, in reality, time delays in communication channels are often time-varying and the backward and forward time delays are not always the same. The reason exists in two facts which are mentioned in networked control systems. The varying network bandwidth for teleoperation causes the delays time varying, while the different network paths of forward and backward data packets make the delays unsymmetric. Another factor we consider in this paper is the dynamics of bilateral teleoperation system. Predictive methods [8, 9] require the precise knowledge of the environment, the operator, or both are required to achieve a good performance. Actually, the precise knowledges of robots, the environment, and the operator, are difficult to acquire; therefore, we need to develop an adaptive control law to deal with this problem [15-19].

Due to the complexity of the communication network, the delays of data packets are not only time varying but also unsymmetric. It is very important to investigate the unsymmetry of time delays and its impact on the stability of network-based teleoperation systems. Therefore, in this paper, in order to achieve haptic fidelity objectives and robust stability in bilateral teleoperation with unsymmetric time-varying delays and dynamical uncertainties, adaptive control is investigated for bilateral teleoperation system. First of all, the objective of force control is converted into motion synchronization using the impedance model; then, we transform the dynamics of the teleoperation system, which contain both the master and slave dynamics, into two subsystems; then, adaptive control is proposed to deal with the unsymmetric time-varying delays and the dynamical uncertainties. The stabilities of two subsystems are also proved with LMIs (Linear Matrix Inequalities) based on Lyapunov stability synthesis. Finally, simulations and experiments are conducted to verify the effectiveness of the proposed control.

2. **Dynamics Description.** The dynamics of bilateral teleoperation system consisting of both master and salve manipulators can be described as

$$M_m(q_m)\ddot{q}_m + C_m(q_m,\dot{q}_m)\dot{q}_m + G_m(q_m) + f_m(\dot{q}_m) = F_h + \tau_m \tag{1}$$

$$M_s(q_s)\ddot{q}_s + C_s(q_s,\dot{q}_s)\dot{q}_s + G_s(q_s) + f_s(\dot{q}_s) = \tau_s - F_e \tag{2}$$

where q_m, q_s are the n dimension vectors of joint displacement, $\dot{q}_m$, $\dot{q}_s$ are the n dimension vectors of joint velocity, τ_m, τ_s are the n dimension vectors of input torque, $M_j(q)$ is the $n \times n$ symmetric positive definite manipulator inertia matrix, $C_j(q_j,\dot{q}_j)$ is the $n \times n$ matrix of Centripetal and Coriolis torques and $G_j(q_j)$ is the gradient of the gravitational potential energy, where $j = m, s$, and $f_m(\dot{q}_m)$ and $f_s(\dot{q}_s)$ are the external friction force vectors. Also, F_h is the human operator force and F_e is the environmental force acting on the slave robot when it contacts the environment. Figure 1 shows the teleoperation system.

For the teleoperation system (1) and (2), we consider the human operator force F_h and the environmental force F_e given by

$$F_h = K_h q_m + C_h \dot{q}_m \tag{3}$$

$$F_e = K_e q_s + C_e \dot{q}_s \tag{4}$$

where K_h, C_h, K_e and C_e are known positive scalars.

The control objective can be described as

$$F_h = F_e \tag{5}$$

with $\ddot{q}_m$, $\ddot{q}_s$, $\dot{q}_m$ and $\dot{q}_s$ converging to zero under the time delays. In this paper, we will mainly focus on time delays, especially those unsymmetric ones existing in communication channels. So, we can assume that there are time delays $d_1(t)$ and $d_2(t)$ in forward and backward directions separately, which satisfy assumption as follows:

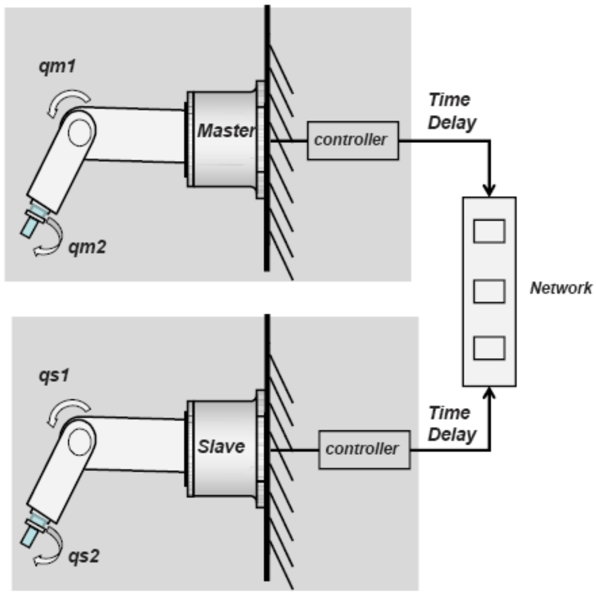

FIGURE 1. Motion synchronization of teleoperation system

Assumption 2.1. *The time-varying delays of the system are time-differentiable for all time and satisfy* $0 \leq d_1(t) \leq h_m$, $0 \leq d_2(t) \leq h_s$, $|\dot{d}_1(t)| \leq \mu_1 \leq k_d$, $|\dot{d}_2(t)| \leq \mu_2 \leq k_d$, *where* h_m, h_s, μ_1, μ_2 *and* k_d *are positive scalars. Furthermore, we define* $h = \max[h_m, h_s]$ *and* $\mu = \max[\mu_1, \mu_2]$, *which will be used in the proof of stabilities.*

Therefore, considering the force expressions in (3) and (4), together with the time delays existing in both directions of the communication channels, we can change the objective of force control into motion synchronization, which is expressed as follows:

$$\lim_{t\to\infty} \|K_h q_m(t) - K_e q_s(t - d_2(t))\| = 0 \tag{6}$$

$$\lim_{t\to\infty} \|K_h q_m(t - d_1(t)) - K_e q_s(t)\| = 0 \tag{7}$$

where $d_1(t)$ stands for the forward time delay while $d_2(t)$ represents the backward time delay.

3. **Position Coordination of Master-Slave Teleoperation System.** Master-slave position coordination is clearly described in (6) and (7); however, we still have to define new variables r_j and $\dot{q}_{jr}$ with $j = m, s$ to help us design the control law. These two variables are defined as

$$r_j = \dot{q}_j + \Lambda q_j \tag{8}$$

$$\dot{q}_{jr} = -\Lambda q_j \tag{9}$$

where Λ is a positive diagonal matrix.

Let $\mu_m = M_m\ddot{q}_{mr} + C_m\dot{q}_m + G_m + f_m(\dot{q}_m) - F_h$, $\mu_s = M_s\ddot{q}_{sr} + C_s\dot{q}_s + G_s + f_s(\dot{q}_s) + F_e$, since $\dot{q}_j = -\Lambda q_j + r_j$ and $\ddot{q}_j = -\Lambda\dot{q}_j + \dot{r}_j$, $j = m,\ s$, Equations (1) and (2) become

$$M_m(q_m)\dot{r}_m = \tau_m - \mu_m \tag{10}$$

$$M_s(q_s)\dot{r}_s = \tau_s - \mu_s \tag{11}$$

Define the following nonlinear feedback

$$\tau_m = M_m(q)(U_m + M_m^{-1}(q)\mu_m) \tag{12}$$

$$\tau_s = M_s(q)(U_s + M_s^{-1}(q)\mu_s) \tag{13}$$

where U_m and U_s are auxiliary control inputs defined as

$$U_m = K_1 r_m(t) + K_2 r_s(t - d_2(t)) \tag{14}$$
$$U_s = K_3 r_m(t - d_1(t)) + K_4 r_s(t) \tag{15}$$

where $K_i \in R^{n\times n}$ is a constant matrix, $i = 1 \dots 4$.

Therefore, the closed-loop system for q_m and q_s sub-system becomes

$$\dot{r} = U \tag{16}$$

where $r = [r_m^T, r_s^T]^T$, $U = [U_m^T, U_s^T]^T$.

Considering the unsymmetric time delays in the forward and backward communication channels, we define the coordination errors between the master and slave robots as

$$e_m(t) = K_h q_m(t) - K_e q_s(t - d_2(t)) \tag{17}$$
$$e_s(t) = K_h q_m(t - d_1(t)) - K_e q_s(t) \tag{18}$$

Therefore, the master and slave robots states synchronize if the coordination errors and their derivatives approach the origin asymptotically.

The derivatives of the coordination errors can be written as

$$\dot{e}_m(t) = -\Lambda e_m(t) - K_e\Lambda q_s(t - d_2(t))\dot{d}_2(t) - K_e r_s(t - d_2(t))[1 - \dot{d}_2(t)] + K_h r_m(t) \tag{19}$$
$$\dot{e}_s(t) = -\Lambda e_s(t) + K_h\Lambda q_m(t - d_1(t))\dot{d}_1(t) + K_h r_m(t - d_1(t))[1 - \dot{d}_1(t)] - K_e r_s(t) \tag{20}$$

Let $e = [e_m^T, e_s^T]^T$, then we have

$$\begin{aligned}\dot{e} = \begin{bmatrix} \dot{e}_m \\ \dot{e}_s \end{bmatrix} &= \begin{bmatrix} -\Lambda & 0 \\ 0 & -\Lambda \end{bmatrix}\begin{bmatrix} e_m \\ e_s \end{bmatrix} + \begin{bmatrix} K_h r_m(t) - K_e r_s(t - d_2(t))[1 - \dot{d}_2(t)] \\ K_h r_m(t - d_1(t))[1 - \dot{d}_1(t)] - K_e r_s(t) \end{bmatrix} \\ &+ \begin{bmatrix} -K_e\Lambda q_s(t - d_2(t))\dot{d}_2(t) \\ K_h\Lambda q_m(t - d_1(t))\dot{d}_1(t) \end{bmatrix}\end{aligned} \tag{21}$$

Furthermore, we can define a variable $X = [X_1^T, X_2^T]^T$ with $X_1 = e$ and $X_2 = r$, considering (16), we can rewrite (21) briefly as

$$\dot{X} = A_1 X + A_2 X(t - d_1(t), t - d_2(t)) + \bar{W}(t - d_1(t), t - d_2(t)) \tag{22}$$

where

$$A_1 = \begin{bmatrix} -\Lambda & 0 & K_h & 0 \\ 0 & -\Lambda & 0 & -K_e \\ 0 & 0 & K_1 & 0 \\ 0 & 0 & 0 & K_4 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & -K_e[1 - \dot{d}_2(t)]I \\ 0 & 0 & K_h[1 - \dot{d}_1(t)]I & 0 \\ 0 & 0 & 0 & K_2 \\ 0 & 0 & K_3 & 0 \end{bmatrix},$$

$$\bar{W}(t - d_1(t), t - d_2(t)) = \begin{bmatrix} -K_e\Lambda q_s(t - d_2(t))\dot{d}_2(t) \\ K_h\Lambda q_m(t - d_1(t))\dot{d}_1(t) \\ 0 \\ 0 \end{bmatrix},$$

$$X(t - d_1(t), t - d_2(t)) = \begin{bmatrix} e_m(t - d_1(t)) \\ e_s(t - d_2(t)) \\ r_m(t - d_1(t)) \\ r_s(t - d_2(t)) \end{bmatrix}.$$

Obviously, the above Equation (22) can be decoupled into two subsystems as

X_1 subsystem:

$$\dot{e} = A_{11}e + A_{12}r + A_{13}r(t-d_1(t), t-d_2(t)) + W(t-d_1(t), t-d_2(t)) \tag{23}$$
$$A_{11} = \begin{bmatrix} -\Lambda & 0 \\ 0 & -\Lambda \end{bmatrix}, \quad A_{13} = \begin{bmatrix} 0 & -K_e[1-\dot{d}_2(t)]I \\ K_h[1-\dot{d}_1(t)]I & 0 \end{bmatrix}$$
$$A_{12} = \begin{bmatrix} K_h & 0 \\ 0 & -K_e \end{bmatrix}, \quad W(t-d_1(t), t-d_2(t)) = \begin{bmatrix} K_e\Lambda q_s(t-d_2(t))\dot{d}_2(t) \\ K_h\Lambda q_m(t-d_1(t))\dot{d}_1(t) \end{bmatrix}$$

X_2 subsystem:

$$\dot{r} = A_{21}r + A_{22}r(t-d_1(t), t-d_2(t)) \tag{24}$$
$$A_{21} = \begin{bmatrix} K_1 & 0 \\ 0 & K_4 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} 0 & K_2 \\ K_3 & 0 \end{bmatrix}$$

In bilateral teleoperation systems, dynamical uncertainty is one of the most important factors that could effect the synchronization performance. Parameters M_j, C_j, G_j, f_j can be separated as nominal parts denoted by M_j^0, C_j^0, G_j^0, f_j^0 and uncertain parts denoted by ΔM_j, ΔC_j, ΔG_j, Δf_j, respectively. These variables satisfy the following relationships: $M_j = M_j^0 + \Delta M_j$, $C_j = C_j^0 + \Delta C_j$, $G_j = G_j^0 + \Delta G_j$, $f_j = f_j^0 + \Delta f_j$.

We propose the adaptive control law as

$$\tau_m = \tau_m^0 + \Delta\tau_m \tag{25}$$
$$\tau_s = \tau_s^0 + \Delta\tau_s \tag{26}$$

The input torques for nominal system are

$$\tau_m^0 = M_m^0(K_1 r_m(t) + K_2 r_s(t-d_2(t))) + \mu_m^0 \tag{27}$$
$$\tau_s^0 = M_s^0(K_3 r_m(t-d_1(t)) + K_4 r_s(t)) + \mu_s^0 \tag{28}$$

where $\mu_m^0 = M_m^0\ddot{q}_{mr} + C_m^0\dot{q}_m + G_m^0 + f_m^0(\dot{q}_m) - F_h$, $\mu_s^0 = M_s^0\ddot{q}_{sr} + C_s^0\dot{q}_s + G_s^0 + f_s^0(\dot{q}_s) + F_e$, $\mu_m = \mu_m^0 + \Delta\mu_m$, $\mu_s = \mu_s^0 + \Delta\mu_s$.

To simplify the control problem in this paper, some assumptions related to the dynamical uncertainties have to be proposed.

Assumption 3.1. *For positive defined matrix P_j with appropriate dimension, we have $\|P_j\|\|M_j^{-1}(q_j)\|\|\Delta M_j\| \le \Theta_{j1}$, $\|P_j\|\|M_j^{-1}(q_j)\|\|\Delta C_j\| \le \Theta_{j2} + \Theta_{j3}\|\dot{q}_j\|$, $\|P_j\|\|M_j^{-1}(q_j)\|$ $\|\Delta G_j\| \le \Theta_{j4}$, $\|P_j\|\|M_j^{-1}(q_j)\|\|\Delta f_j(\dot{q}_j)\| \le \Theta_{j5} + \Theta_{j6}\|\dot{q}_j\|$, $\|P_j\|\|M_j^{-1}M_j^0 - I\| \le \Theta_{j7}$, with unknown constants $\Theta_{j1}, \ldots, \Theta_{j7}$, $j = m, s$.*

The control inputs $\Delta\tau_m$ and $\Delta\tau_s$ are used to compensate the dynamical uncertainties and they are defined as follows:

$$\Delta\tau_m = -\sum_{i=1}^{7} \frac{b_m}{p_m} \frac{r_m\hat{\Theta}_{mi}\Psi_{mi}^2}{\|r_m\|\Psi_{mi} + \delta_{mi}} \tag{29}$$
$$\Delta\tau_s = -\sum_{i=1}^{7} \frac{b_s}{p_s} \frac{r_s\hat{\Theta}_{si}\Psi_{si}^2}{\|r_s\|\Psi_{si} + \delta_{si}} \tag{30}$$

where $\Psi_j = [\|\ddot{q}_{jr}\|, \|\dot{q}_j\|, \|\dot{q}_j\|^2, 1, 1, \|\dot{q}_j\|, \|U_j\|]$, and $\hat{\Theta}_j = [\hat{\Theta}_{j1}, \ldots, \hat{\Theta}_{j7}]^T$ is the estimation of Θ_j, b_m, b_s, p_m, p_s are four known positive parameters which will be defined later. In developing control laws (29) and (30), the parameters $\hat{\Theta}_m$ and $\hat{\Theta}_s$ are estimations and cannot be obtained easily. Therefore, we choose the following adaptive law to update the estimations:

$$\dot{\hat{\Theta}}_{mi} = -\alpha_{mi}\hat{\Theta}_{mi} + \frac{\omega_{mi}\|r_m\|^2\Psi_{mi}^2}{\|r_m\|\Psi_{mi} + \delta_{mi}}, \quad \hat{\Theta}_{mi}(0) > 0 \tag{31}$$

$$\dot{\hat{\Theta}}_{si} = -\alpha_{si}\hat{\Theta}_{si} + \frac{\omega_{si}\|r_s\|^2\Psi_{si}^2}{\|r_s\|\Psi_{si} + \delta_{si}}, \quad \hat{\Theta}_{si}(0) > 0 \tag{32}$$

with $\alpha_{ji} > 0$ and $\delta_{ji} > 0$ being designed parametrical functions and satisfying $\lim_{t\to\infty} \alpha_{ji} = 0$, $\int_0^\infty \alpha_{ji}(t)dt = \varrho_{ji} < \infty$, $\int_0^\infty \delta_{ji}ds = \epsilon_{ji} < \infty$, with finite constant ϱ_{ji}, ϵ_{ji} and $\omega_{ji} > 0$ is a designed parameter. The validity of the adaptive control law defined above will be proved later.

4. Stability Analysis.

4.1. $\boldsymbol{X_2}$ subsystem.

Considering (12), it is easy to rewrite the term as

$$-U_m + M_m^{-1}\tau_m - M_m^{-1}(q_m)\mu_m = (M_m^{-1}M_m^0 - I)U_m + M_m^{-1}\Delta\tau_m - M_m^{-1}(q_m)\Delta\mu_m$$

Similarly, we can obtain

$$-U_s + M_s^{-1}\tau_s - M_s^{-1}(q_s)\mu_s = (M_s^{-1}M_s^0 - I)U_s + M_s^{-1}\Delta\tau_s - M_s^{-1}(q_s)\Delta\mu_s$$

Add the items listed above into X_2 subsystem dynamics (24), we can get

$$\dot{r} = A_{21}r + A_{22}r(t - d_1(t), t - d_2(t)) + \Xi \tag{33}$$

$$A_{21} = \begin{bmatrix} K_1 & 0 \\ 0 & K_4 \end{bmatrix}, \quad r = \begin{bmatrix} r_m \\ r_s \end{bmatrix}, \quad r(t - d_1(t), t - d_2(t)) = \begin{bmatrix} r_m(t - d_1(t)) \\ r_s(t - d_2(t)) \end{bmatrix}$$

$$A_{22} = \begin{bmatrix} 0 & K_2 \\ K_3 & 0 \end{bmatrix}, \quad \Xi = \begin{bmatrix} \Xi_m \\ \Xi_s \end{bmatrix} = \begin{bmatrix} (M_m^{-1}M_m^0 - I)U_m + M_m^{-1}\Delta\tau_m - M_m^{-1}(q_m)\Delta\mu_m \\ (M_s^{-1}M_s^0 - I)U_s + M_s^{-1}\Delta\tau_s - M_s^{-1}(q_s)\Delta\mu_s \end{bmatrix}$$

To prove the stability of this subsystem, some lemmas and assumptions will be introduced first:

Assumption 4.1. *The known positive parameters b_m, b_s, p_m and p_s satisfy $b_m \le \lambda_{\min}(M_m^{-1})$ and $\lambda_{\max}(P_m) \le p_m$, $b_s \le \lambda_{\min}(M_s^{-1})$ and $\lambda_{\max}(P_s) \le p_s$, that is $x^Tb_mIx \le x^TM_m^{-1}x$, $x^Tp_mIx \ge x^TP_mx$, $x^Tb_sIx \le x^TM_s^{-1}x$, $x^Tp_sIx \ge x^TP_sx$ with any vectors.*

Theorem 4.1. *The time-delay system (24) is asymptotically stable for any time delay $d_1(t)$ and $d_2(t)$ satisfying Assumption 2.1, if there exist matrices $P = \mathrm{diag}[P_m, P_s] > 0$, $Q = \mathrm{diag}[Q_m, Q_s] > 0$, $Q_3 = \mathrm{diag}[Q_{m3}, Q_{s3}] > 0$, $Z = \mathrm{diag}[Z_m, Z_s] > 0$, $N_j = \mathrm{diag}[N_{mj}, N_{sj}]$, $S_j = \mathrm{diag}[S_{mj}, S_{sj}]$, $j = 1, 2$, such that the LMI shown in (34) holds.*

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & -S_1 & hN_1 & hS_1 & A_{21}^ThZ \\ * & \Phi_{22} & -S_2 & hN_2 & hS_2 & A_{22}^ThZ \\ * & * & -Q & 0 & 0 & 0 \\ * & * & * & -hZ & 0 & 0 \\ * & * & * & * & -hZ & 0 \\ * & * & * & * & * & -hZ \end{bmatrix} < 0 \tag{34}$$

where $\Phi_{11} = PA_{21} + A_{21}^TP + Q + Q_3 + N_1 + N_1^T$, $\Phi_{12} = PA_{22} + N_2^T - N_1 + S_1$, $\Phi_{22} = -(1-\mu)Q_3 + S_2 + S_2^T - N_2 - N_2^T$.

Proof: Define the Lyapunov-Krasovskii functionals as

$$V = V_1 + V_2 + V_3 \tag{35}$$

where $V_1 = r^TPr + \tilde{\Theta}^T\Omega^{-1}\tilde{\Theta}$, $V_2 = \int_{t-h_m}^t r_m^TQ_mr_mds + \int_{t-d_1(t)}^t r_m^TQ_{m3}r_mds + \int_{t-h_s}^t r_s^TQ_sr_sds + \int_{t-d_2(t)}^t r_s^TQ_{s3}r_sds$, $V_3 = \int_{-h_m}^0 \int_{t+\theta}^t \dot{r}_m^TZ_m\dot{r}_mdsd\theta + \int_{-h_s}^0 \int_{t+\theta}^t \dot{r}_s^TZ_s\dot{r}_sdsd\theta$, where $P = P^T >$

0, $Q_{ji} = Q_{ji}^T > 0$, $Z_{jk} = Z_{jk}^T > 0$, $i = 1, 2, 3$; $j = m, s$; $k = 1, 2$ and $\tilde{\Theta} = \Theta - \hat{\Theta}$, $\Omega = \mathrm{diag}[\omega_{ji}]$ with $j = m, s$, $i = 1 \ldots 7$.

Considering the derivative of V_1, we have

$$\begin{aligned}
\dot{V}_1 &= r^T P \dot{r} + \dot{r}^T P r + 2\tilde{\Theta}^T \Omega^{-1} \dot{\tilde{\Theta}} \\
&= r^T (A_{21}^T P + P A_{21}) r + r^T (t - d_1(t), t - d_2(t))(A_{22}^T P + A_{22}^T P^T) r \\
&\quad + 2 r_m^T P_m ((M_m^{-1} M_m^0 - I) U_m + M_m^{-1} \Delta\tau_m - M_m^{-1}(q_m) \Delta\mu_m) \\
&\quad + 2 r_s^T P_s ((M_s^{-1} M_s^0 - I) U_s + M_s^{-1} \Delta\tau_s - M_s^{-1}(q_s) \Delta\mu_s) + 2\tilde{\Theta}^T \Omega^{-1} \dot{\tilde{\Theta}} \\
&\le r^T (A_{21}^T P + P A_{21}) r + r^T (t - d_1(t), t - d_2(t))(A_{22}^T P + A_{22}^T P^T) r + 2\tilde{\Theta}^T \Omega^{-1} \dot{\tilde{\Theta}} \\
&\quad + 2\|r_m\| \|P_m\| \|M_m^{-1} M_m^0 - I\| \|U_m\| + 2 r_m^T P_m M_m^{-1} \Delta\tau_m \\
&\quad + 2\|r_m\| \|P_m\| \|M_m^{-1}(q_m)\| \|\Delta\mu_m\| \\
&\quad + 2\|r_s\| \|P_s\| \|M_s^{-1} M_s^0 - I\| \|U_s\| + 2 r_s^T P_s M_s^{-1} \Delta\tau_s \\
&\quad + 2\|r_s\| \|P_s\| \|M_s^{-1}(q_s)\| \|\Delta\mu_s\|
\end{aligned} \tag{36}$$

Since P_j and M_j are positive definite, using Assumption 4.1 and the adaptive controls defined in (29) and (30), it is easy to have

$$r_m^T P_m M_m^{-1} \Delta\tau_m = -r_m^T P_m M_m^{-1} \sum_{i=1}^{7} \frac{b_m}{p_m} \frac{r_m \hat{\Theta}_{mi} \Psi_{mi}^2}{\|r_m\| \Psi_{mi} + \delta_{mi}} \le -\sum_{i=1}^{7} \frac{\|r_m\|^2 \hat{\Theta}_{mi} \Psi_{mi}^2}{\|r_m\| \Psi_{mi} + \delta_{mi}} \tag{37}$$

$$r_s^T P_s M_s^{-1} \Delta\tau_s = -r_s^T P_s M_s^{-1} \frac{b_s}{p_s} \frac{r_s \hat{\Theta}_{si} \Psi_{si}^2}{\|r_s\| \Psi_{si} + \delta_{si}} \le -\sum_{i=1}^{7} \frac{\|r_s\|^2 \hat{\Theta}_{si} \Psi_{si}^2}{\|r_s\| \Psi_{si} + \delta_{si}} \tag{38}$$

Considering $\|P_j\| \|M_j^{-1}(q_j)\| \|\Delta\mu_j\| \le \|P_j\| \|M_j^{-1}(q_j)\| \|\Delta M_j\| \|\ddot{q}_{jr}\| + \|P_j\| \|M_j^{-1}(q_j)\| \|\Delta C_j\| \|\dot{q}_j\| + \|P_j\| \|M_j^{-1}(q_j)\| \|\Delta G_j\| + \|P_j\| \|M_j^{-1}(q_j)\| \|\Delta f_j(\dot{q}_j)\|$, $j = m, s$, together with Assumption 3.1, we have

$$\begin{aligned}
\dot{V}_1 &\le r^T (A_{21}^T P + P A_{21}) r + r^T (t - d_1(t), t - d_2(t))(A_{22}^T P + A_{22}^T P^T) r \\
&\quad + 2\left[\|r_m\| \Theta_m^T \Psi_m - \sum_{i=1}^{7} \frac{\|r_m\|^2 \hat{\Theta}_{mi} \Psi_{mi}^2}{\|r_m\| \Psi_{mi} + \delta_{mi}}\right] + 2\sum_{i=1}^{7} \tilde{\Theta}_{mi} \left[\frac{\alpha_{mi}}{\omega_{mi}} \hat{\Theta}_{mi} - \frac{\|r_m\|^2 \Psi_{mi}^2}{\|r_m\| \Psi_{mi} + \delta_{mi}}\right] \\
&\quad + 2\left[\|r_s\| \Theta_s^T \Psi_s - \sum_{i=1}^{7} \frac{\|r_s\|^2 \hat{\Theta}_{si} \Psi_{si}^2}{\|r_s\| \Psi_{si} + \delta_{si}}\right] + 2\sum_{i=1}^{7} \tilde{\Theta}_{si} \left[\frac{\alpha_{si}}{\omega_{si}} \hat{\Theta}_{si} - \frac{\|r_s\|^2 \Psi_{si}^2}{\|r_s\| \Psi_{si} + \delta_{si}}\right] \\
&\le r^T (A_{21}^T P + P A_{21}) r + r^T (t - d_1(t), t - d_2(t))(A_{22}^T P + A_{22}^T P^T) r \\
&\quad + \sum_{j=m,s} \sum_{i=1}^{7} 2\Theta_{ji} \delta_{ji} - 2\sum_{j=m,s} \sum_{i=1}^{7} \frac{\alpha_{ji}}{\omega_{ji}} \left(\hat{\Theta}_{ji} - \frac{1}{2}\Theta_{ji}\right)^2 + 2\sum_{j=m,s} \sum_{i=1}^{7} \frac{\alpha_{ji}}{4\omega_{ji}} \Theta_{ji}^2
\end{aligned} \tag{39}$$

Considering the derivative of V_2, we have $\dot{V}_2 \le r_m^T (Q_m + Q_{m3}) r_m - r_m^T (t - h_m) Q_m r_m (t - h_m) - (1 - \mu_1) r_m^T (t - d_1(t)) Q_{m3} r_m (t - d_1(t)) + r_s^T (Q_s + Q_{s3}) r_s - r_s^T (t - h_s) Q_s r_s (t - h_s) - (1 - \mu_2) r_s^T (t - d_2(t)) Q_{s3} r_s (t - d_2(t))$. Consider Assumption 2.1, the derivative of V_3 satisfies $\dot{V}_3 = h_m \dot{r}_m^T Z_m \dot{r}_m - \int_{t-h_m}^{t} \dot{r}_m^T Z_m \dot{r}_m ds + h_s \dot{r}_s^T Z_s \dot{r}_s - \int_{t-h_s}^{t} \dot{r}_s^T Z_s \dot{r}_s ds$. To form the appropriate LMIs, we will use some equations from the Leibniz-Newton formula as follows:

$2[r_m^T N_{m1} + r_m^T (t - d_1(t)) N_{m2}][r_m - r_m(t - d_1(t)) - \int_{t-d_1(t)}^{t} \dot{r}_m(s) ds] = 0$, $2[r_m^T S_{m1} + r_m^T (t - d_1(t)) S_{m2}][r_m(t - d_1(t)) - r_m(t - h_m) - \int_{t-h_m}^{t-d_1(t)} \dot{r}_m(s) ds] = 0$, $2[r_s^T N_{s1} + r_s^T (t - d_2(t)) N_{s2}][r_s - r_s(t - d_2(t)) - \int_{t-d_2(t)}^{t} \dot{r}_s(s) ds] = 0$, $2[r_s^T S_{s1} + r_s^T (t - d_2(t)) S_{s2}][r_s(t - d_2(t)) - r_s(t - h_s) - \int_{t-h_s}^{t-d_2(t)} \dot{r}_s(s) ds] = 0$, where N_{j1}, N_{j2}, S_{j1}, S_{j2}, $j = m, s$ are matrices with appropriate dimensions. Finally, combining the derivatives $\dot{V}_1$, $\dot{V}_2$, $\dot{V}_3$, and adding the above Leibniz-Newton formulas, we have

$$
\begin{aligned}
\dot{V} \leq\ & r(t)^T(A_{21}^T P + PA_{21})r(t) + r^T(t-d_1(t), t-d_2(t))(A_{22}^T P + A_{22}^T P^T)r(t) \\
& + \sum_{j=m,s}\sum_{i=1}^{7} 2\Theta_{ji}\delta_{ji} - 2\sum_{j=m,s}\sum_{i=1}^{7}\frac{\alpha_{ji}}{\omega_{ji}}\left(\hat{\Theta}_{ji} - \frac{1}{2}\Theta_{ji}\right)^2 + 2\sum_{j=m,s}\sum_{i=1}^{7}\frac{\alpha_{ji}}{4\omega_{ji}}\Theta_{ji}^2 \\
& + r_m^T(Q_m + Q_{m3})r_m - r_m^T(t-h_m)Q_m r_m(t-h_m) \\
& - (1-\mu_1)r_m^T(t-d_1(t))Q_{m3}r_m(t-d_1(t)) \\
& + r_s^T(Q_s + Q_{s3})r_s - r_s^T(t-h_s)Q_s r_s(t-h_s) \\
& - (1-\mu_2)r_s^T(t-d_2(t))Q_{s3}r_s(t-d_2(t)) \\
& + h_m\dot{r}_m^T Z_m\dot{r}_m - \int_{t-h_m}^{t}\dot{r}_m^T Z_m\dot{r}_m ds + h_s\dot{r}_s^T Z_s\dot{r}_s - \int_{t-h_s}^{t}\dot{r}_s^T Z_s\dot{r}_s ds \\
& + 2[r_m^T N_{m1} + r_m^T(t-d_1(t))N_{m2}] * \left[r_m - r_m(t-d_1(t)) - \int_{t-d_1(t)}^{t}\dot{r}_m(s)ds\right] \\
& + 2[r_s^T N_{s1} + r_s^T(t-d_2(t))N_{s2}] * \left[r_s - r_s(t-d_2(t)) - \int_{t-d_2(t)}^{t}\dot{r}_s(s)ds\right] \\
& + 2[r_m^T S_{m1} + r_m^T(t-d_1(t))S_{m2}] * \left[r_m(t-d_1(t)) - r_m(t-h_m) - \int_{t-h_m}^{t-d_1(t)}\dot{r}_m(s)ds\right] \\
& + 2[r_s^T S_{s1} + r_s^T(t-d_2(t))S_{s2}] * \left[r_s(t-d_2(t)) - r_s(t-h_s) - \int_{t-h_s}^{t-d_2(t)}\dot{r}_s(s)ds\right]
\end{aligned}
\tag{40}
$$

To simplify the form of LMIs, we consider the constants h_2, h_{12} and μ defined in Assumption 2.1, then

$$
\begin{aligned}
\dot{V} \leq\ & \zeta^T\Upsilon\zeta - \int_{t-d_1(t)}^{t}[\zeta_m^T N_m + \dot{r}_m^T(s)Z_m] * Z_m^{-1}[N_m^T\zeta_m + Z_m\dot{r}_m(s)]ds \\
& - \int_{t-h_m}^{t-d_1(t)}[\zeta_m^T S_m + \dot{r}_m^T(s)(Z_m)] * (Z_m)^{-1}[S_m^T\zeta_m + (Z_m)\dot{r}_m(s)]ds \\
& - \int_{t-d_2(t)}^{t}[\zeta_s^T N_s + \dot{r}_s^T(s)Z_{s1}] * Z_s^{-1}[N_s^T\zeta_s + Z_s\dot{r}_s(s)]ds \\
& - \int_{t-h_s}^{t-d_2(t)}[\zeta_s^T S_s + \dot{r}_s^T(s)(Z_s)] * Z_s^{-1}[S_s^T\zeta_s + (Z_s)\dot{r}_s(s)]ds \\
& + \sum_{j=m,s}\sum_{i=1}^{7} 2\Theta_{ji}\delta_{ji} - 2\sum_{j=m,s}\sum_{i=1}^{7}\frac{\alpha_{ji}}{\omega_{ji}}\left(\hat{\Theta}_{ji} - \frac{1}{2}\Theta_{ji}\right)^2 + 2\sum_{j=m,s}\sum_{i=1}^{7}\frac{\alpha_{ji}}{4\omega_{ji}}\Theta_{ji}^2
\end{aligned}
\tag{41}
$$

where $\Upsilon = \Pi + \bar{A}^T hZ\bar{A} + hNZ^{-1}N^T + hSZ^{-1}S^T$, $\Pi = \begin{bmatrix} \Phi_{11} & \Phi_{12} & -S_1 \\ * & \Phi_{22} & -S_2 \\ * & * & -Q \end{bmatrix}$, $\zeta = [r^T(t), r^T(t-d_1(t), r(t-d_2(t))), r^T(t-h_m, r-h_s)]^T$. $\zeta_m = [r_m^T(t), r^T(t-d_1(t)), r^T(t-h_m)]$, $\zeta_s = [r_s^T(t), r^T(t-d_2(t)), r^T(t-h_s)]^T$, $N_j = [N_{j1}^T, N_{j2}^T, 0]^T$, $S_j = [S_{j1}^T, S_{j2}^T, 0]$, $j = m, s$. $N = [N_1^T, N_2^T, 0]^T$, $S = [S_1^T, S_2^T, 0]^T$, $\bar{A} = [A_{21}, A_{22}, 0]$, $\Phi_{11} = PA_{21} + A_{21}^T P + Q + Q_3 + N_1 + N_1^T$, $\Phi_{12} = PA_{22} + N_2^T - N_1 + S_1$, $\Phi_{22} = -(1-\mu)Q_3 + S_2 + S_2^T - N_2 - N_2^T$, $r(t-h_m, r-h_s) = [r_m(t-h_m)^T, r_s(t-h_s)^T]^T$.

Therefore, $\dot{V} \le \lambda_{\min}(\Upsilon)\|\zeta\|^2 + \sum_{j=m,s}\sum_{i=1}^{7} 2\Theta_{ji}\delta_{ji} + 2\sum_{j=m,s}\sum_{i=1}^{7} \frac{\alpha_{ji}}{4\omega_{ji}}\Theta_{ji}^2$. By Schur complements [11], $\Upsilon < 0$ is equivalent to (34). Since $\sum_{j=m,s}\sum_{i=1}^{7}\left(2\Theta_{ji}\delta_{ji} + 2\frac{\alpha_{ji}}{4\omega_{ji}}\Theta_{ji}^2\right)$ is bounded, there exists $t > t_1$, $\sum_{j=m,s}\sum_{i=1}^{7} 2\Theta_{ji}\delta_{ji} + 2\sum_{j=m,s}\sum_{i=1}^{7} \frac{\alpha_{ji}}{4\omega_{ji}}\Theta_{ji}^2 \le \rho$, when $\|\zeta\| \ge \sqrt{\frac{\rho}{-\lambda_{\min}(\Upsilon)}}$, $\dot{V} \le 0$, from above all, ζ, that is, r and $r(t-d_1(t), t-d_2(t))$, converge to a small set containing the origin as $t \to \infty$. Integrating both sides of the above equation gives $V(t) - V(0) \le \int_0^t \zeta^T\Upsilon\zeta ds + 2\sum_{j=m,s}\sum_{i=1}^{7}\int_0^t \Theta_{ji}\delta_{ji}ds + 2\sum_{j=m,s}\sum_{i=1}^{7}\int_0^t \frac{\alpha_{ji}}{4\omega_{ji}}\Theta_{ji}^2 ds$. Since Θ_{ji} and ω_{ji} are constants, $\int_0^\infty \alpha_{ji}ds = \varrho_{ji}$, $\int_0^\infty \delta_{ji}ds = \epsilon_{ji}$, we can rewrite as $V(t) - V(0) \le \int_0^t \zeta^T\Upsilon\zeta ds + 2\sum_{j=m,s}\sum_{i=1}^{7}\Theta_{ji}\int_0^t \delta_{ji}ds + 2\sum_{j=m,s}\sum_{i=1}^{7}\frac{\int_0^t \alpha_{ji}(s)ds}{4\omega_{ji}}\Theta_{ji}^2 \le \int_0^t \zeta^T\Upsilon\zeta ds + 2\sum_{j=m,s}\sum_{i=1}^{7}\Theta_{ji}\epsilon_{ji} + 2\sum_{j=m,s}\sum_{i=1}^{7}\frac{\varrho_{ji}}{4\omega_{ji}}\Theta_{ji}^2 < \infty$. Thus, V is bounded, which implies that $r \in L_\infty$. From the above equation, we have $\int_0^t \zeta^T\Upsilon\zeta ds \le V(t) - V(0) - 2\sum_{j=m,s}\sum_{i=1}^{7}\Theta_{ji}\epsilon_{ji} - 2\sum_{j=m,s}\sum_{i=1}^{7}\frac{\varrho_{ji}}{4\omega_{ji}}\Theta_{ji}^2$, which leads to $r \in L_2$, as a result, $q_s, q_m \in L_2$. The characteristic mentioned above shows that $\dot{V}$ is negative, which implies that the system (24) is asymptotically stable. As the X_2 subsystem we considered here is simple as the coefficients are all constant, so obviously, the X_2 subsystem is asymptotically stable.

4.2. **X_1 subsystem.** From previous stability proof of subsystem X_2, we know that, the signals $r_m(t)$, $r_s(t) \in L_\infty[0,\infty)$, and from the definition of $r_m(t)$ and $r_s(t)$, we know $\dot{q}_m$, $\dot{q}_s$ and q_m, q_s are bounded. Therefore, we have $W(t-d_1(t), t-d_2(t))$ is bounded. In the former subsection, we have proved that $r \in L_2$, so we can derive the following remark.

Remark 4.1. *As $r(t) \in L_2$, q_m and q_s are bounded, considering the definition of $r(t)$ and $W(t-d_1(t), t-d_2(t))$, we have $W(t-d_1(t), t-d_2(t)) \in L_2$.*

Similar to X_2 subsystem, the time-varying delays also can be handled with LMI. We can define a variable $\xi(t) \triangleq \begin{bmatrix} r^T(t) & r^T(t-d_1(t), t-d_2(t)) & W^T(t-d_1(t), t-d_2(t)) \end{bmatrix}^T$, with Remark 4.1, we know that $\xi(t) \in L_2$. To prove the stability of X_1 subsystem, the following theorem is proposed.

Theorem 4.2. *If there exists $2n \times 2n$ positive matrix R, positive scalar γ such that the following LMI holds*

$$\Pi = \begin{bmatrix} A_{11}^T R + RA_{11} + I & RA_{12} & 0 & R & R \\ * & -\gamma^2 I & 0 & 0 & 0 \\ * & * & (-\gamma^2 + (1+k_d)^2 K_e K_h)I & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & I \end{bmatrix} < 0 \quad (42)$$

where I denotes the identity matrix of appropriate dimension, γ is a positive constant defined beforehand, then we could conclude that the variable $e(t)$ in system (23) converges to zero with disturbance attenuation level γ.

Proof: Considering Lyapunov function as $V(e(t), t) = e^T(t)Re(t)$, we have $\dot{V}(e(t), t) = e^T(t)[A_{11}^T R + RA_{11}]e(t) + 2e^T(t)RA_{12}r(t) + 2e^T(t)RA_{13}r(t-d_1(t), t-d_2(t)) + 2e^T(t)RW(t-d_1(t), t-d_2(t))$. Noting that $|\dot{d}_i(t)| \le k_d$, $|1-\dot{d}_i(t)| < 1 + k_d$, we obviously have $\|A_{13}\|^2 < (1+k_d)^2 K_e K_h$. Further more, we have $2e^T(t)RA_{13}r(t-d_t) \le \|Re(t)\|^2 + \|A_{13}r(t-d_1(t), t-d_2(t))\|^2 \le \|Re(t)\|^2 + (1+k_d)^2 K_e K_h \cdot \|r(t-d_1(t), t-d_2(t))\|^2 = e^T(t) \cdot R \cdot (Re(t)) + (Re(t))^T \cdot R^T \cdot e(t) - (Re(t))^T(Re(t)) + (1+k_d)^2 K_e K_h \cdot \|r(t-d_1(t), t-d_2(t))\|^2$.

Consider the following equation: $\int_0^t [e^T(\alpha)e(\alpha) - \gamma^2\zeta^T(\alpha)\zeta(\alpha)]d\alpha = \int_0^t [e^T(\alpha)e(\alpha) - \gamma^2\zeta^T(\alpha)\zeta(\alpha) + \dot{V}(e(\alpha),\alpha)]d\alpha - V(e(t)) \le \int_0^t \eta^T(\alpha)\Pi\eta(\alpha)d\alpha - e^T(t)Re(t)$, where the variable $\eta(\alpha)$ is defined as $\eta(\alpha) = [e(\alpha), r(\alpha), r(\alpha - d_1(\alpha), \alpha - d_2(\alpha)), W(\alpha - d_1(\alpha), \alpha - d_2(\alpha)), Re(\alpha)]^T$. Owing to (42), we obtain $\int_0^t [e^T(\alpha)e(\alpha) - \gamma^2\zeta^T(\alpha)\zeta(\alpha)]d\alpha \le 0$. The equality exists at time $t = 0$. Then, we have $\int_0^t e^T(\alpha)e(\alpha)d\alpha \le \int_0^t \gamma^2\zeta^T(\alpha)\zeta(\alpha)d\alpha < \infty$. Further more, we can know that $e(t) \in L_2$, that is $e(t) \to 0$ as $t \to \infty$. This completes the proof.

5. **Simulation and Experiment Studies.** The simulations are performed on 2-Degree-Of-Freedom (2-DOF) robotic manipulators shown in Figure 2. We assume that there exists a pair of master and slave manipulators in the teleoperation system. The dynamics of 2-DOF robotic manipulators are described as $M_m(q_m)\ddot{q}_m + G_m(q_m) = \tau_m + f_m + F_h$, $M_s(q_s)\ddot{q}_s + G_s(q_s) = \tau_s + f_s - F_e$, where $M_j = \text{diag}[M_{j11}, M_{j22}]$, $G_j(q) = [G_{j1}, G_{j2}]^T$, $M_{j11} = m_{j1}l_{cj1}^2 + m_{j2}(l_{j1} + l_{cj2})^2$, $M_{j22} = I_2$, $G_{j1} = m_{j1}gl_{cj1}\sin(q_{j1}) + m_{j2}g(l_{j1} + l_{cj2})\sin(q_{j1})$, $G_{j2} = 0$, $j = m, s$. The human force F_h and the environmental force F_e are defined in (3) and (4), the frictions f_m and f_s are considered using Coulomb and Viscous model [12] and defined as $f_j = \alpha_1 sign(\dot{q}_j) + \alpha_2\dot{q}_j$, $j = m, s$. In the simulation, we choose the physical parameters as $m_{m1} = m_{s1} = 0.5$ kg, $m_{m2} = m_{s2} = 0.5$ kg, $l_{m1} = l_{s1} = 0.6$ m, $l_{m2} = l_{s2} = 0.8$ m, $l_{cm1} = l_{cs1} = 0.2$ m, $l_{cm2} = l_{cs2} = 0.3$ m, $I_2 = 0.2$ Nm^2, $g = 9.8$ m/s^2, $\alpha_1 = 0.05$ and $\alpha_2 = 0.1$. The feedback gain parameters in (14) and (15) are chose as $K_1 = \begin{bmatrix} -4.3 & -0.1 \\ 0.3 & -4.9 \end{bmatrix}$, $K_2 = \begin{bmatrix} -0.5 & 0.4 \\ 0.1 & -0.2 \end{bmatrix}$, $K_3 = \begin{bmatrix} 0.3 & 0.4 \\ 0.1 & -0.2 \end{bmatrix}$, $K_4 = \begin{bmatrix} -4.8 & 0.2 \\ 0.1 & -3.8 \end{bmatrix}$. The time delays in the simulation are chosen as $d_1(t) = 1.6\sin^2(t)$, $d_2(t) = 1.5\cos^2(t)$, and the upper and lower bounds of the values and the derivatives of the time-varying delays are $h_m = 1.6$, $\mu_1 = 1.6$, $h_s = 1.5$, $\mu_2 = 1.5$ respectively. From the system parameters chosen above, we can solve (34) using the LMI toolbox in the MATLAB and obtain as $P = \begin{bmatrix} 102.9650 & 7.6525 & 0 & 0 \\ 7.6525 & 111.2570 & 0 & 0 \\ 0 & 0 & 106.3623 & -0.9639 \\ 0 & 0 & -0.9639 & 129.9028 \end{bmatrix}$, $Q = \begin{bmatrix} 168.2905 & 0.4537 & 0 & 0 \\ 0.4537 & 159.0173 & 0 & 0 \\ 0 & 0 & 168.0812 & -3.9383 \\ 0 & 0 & -3.9383 & 161.9334 \end{bmatrix}$,

$$Q_3 = \begin{bmatrix} 14.7393 & 0.3949 & 0 & 0 \\ 0.3949 & 14.1235 & 0 & 0 \\ 0 & 0 & 12.0400 & 4.2515 \\ 0 & 0 & 4.2515 & 22.3434 \end{bmatrix}, \quad Z = \begin{bmatrix} 17.2944 & 2.3108 & 0 & 0 \\ 2.3108 & 19.4525 & 0 & 0 \\ 0 & 0 & 17.4662 & 0.4574 \\ 0 & 0 & 0.4574 & 27.4831 \end{bmatrix}.$$

We can choose $p_m = p_s = 130.0$, which is obviously greater than the maximum eigenvalue of P. We choose other parameters in this simulation as $\omega_{ji} = 5.5$, $\delta_{ji} = \alpha_{ji} = 1/(t+1)^3$, where $j = m, s$, $i = 1 \dots 7$, $b_m = b_s = 1.0$ and $k_d = 1.6$. The initial states for X_2 are assumed to be $r(t) = [0.8\sin\ t, 0.7\cos\ t, 0.2\sin^2\ t + 1, 0.3\cos^2\ t + 1]^T$. The state trajectories of subsystem X_2 are shown in Figure 3.

Remark 5.1. *As the true value of* Θ *is unavailable in this paper, we choose the initial estimation value of* Θ *as* $\hat{\Theta}_{ji}(0) = 1.0$, $j = m, s$, $i = 1 \dots 7$, *which will finally converge to the true value of* Θ. *In the simulation, we do not know the precise dynamics beforehand, and only the initial boundeness of dynamical parameters are chosen, using the adaptive law, the initial boundedness converged to the actual values, even if the external disturbance and unknown time-delays exist.*

The parameters of X_1 subsystem are selected as $\Lambda = \text{diag}[5.5, 4.5]$, $K_h = K_e = 5.0$ N/degree, and $C_h = C_e = 0.1$ N/degree/s. The disturbance attenuation level $\gamma = 16.2822$. Using the LMI toolbox to solve (42), we obtain $R_i = \text{diag}[4.4696, 3.5002, 4.4696, 3.5002]$.

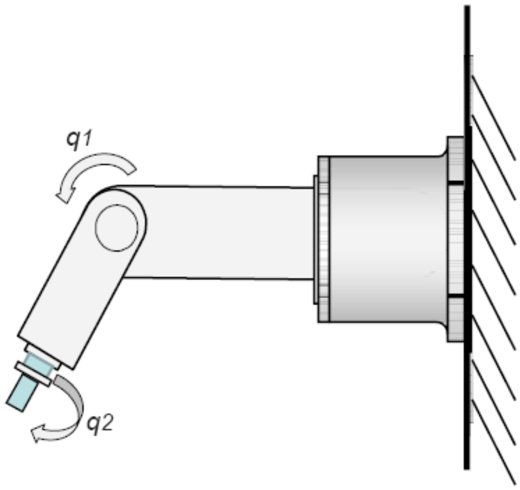

FIGURE 2. A 2-DOF robotic manipulator

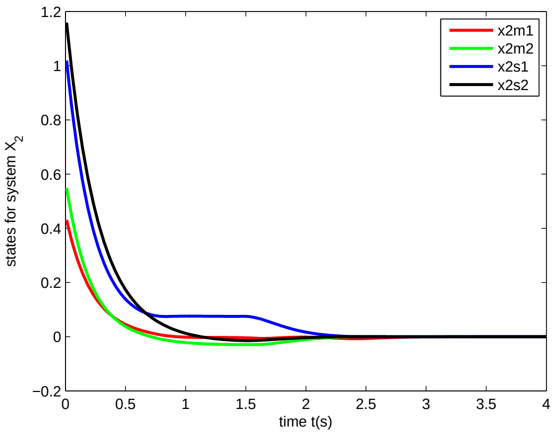

FIGURE 3. The states of X_2 subsystem

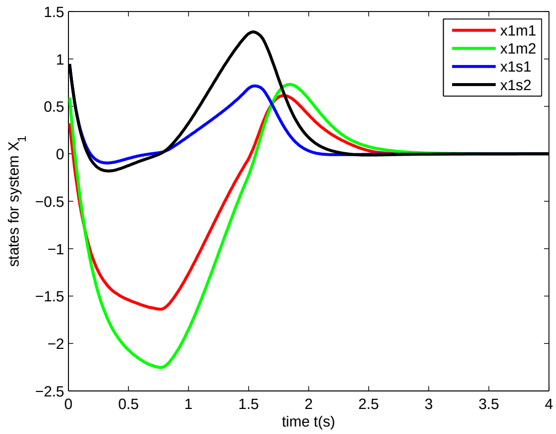

FIGURE 4. The states of X_1 subsystem

The initial states of X_1 are $X_1(t) = [0.8 \sin\ t, 0.9 \cos\ t, 0.2 \sin^2\ t + 1, 0.1 \cos^2\ t + 1]^T$. The trajectories of X_1 subsystem are shown in Figure 4.

The synchronization performances are listed from Figures 5-8. Figure 5 shows the joint position trajectories of both master and slave robots. Input torques for the master and slave robots are shown in Figure 7. As we can see from Figure 4, the synchronizing errors in (6) and (7) converge to the zero quickly, that is, the motion synchronization of master and slave robots is achieved and stable. From Figure 5, we can see that, although the initial positions of the master and slave robots are different and both are not zero, the joint trajectories of slave robot quickly track the joint trajectories of master robot. Finally, from Figure 8, we can see that the human force F_h tracks environmental force F_e quickly, which means we can see F_h as the environmental force F_e. From these figures, we can see that the designed controller is also effective.

The teleoperation system for the experiment consists of two identical robotic manipulators (the master and slave) shown in Figure 14. The master and slave robotic manipulator consists of an actuator, an controller and corresponding sensors. Both the master and the slave are driven by the DC motors. The encoders with up to 2048 pulses per revolution are mounted at the end of the motor shafts to measure the angular position. The MMT-4Q

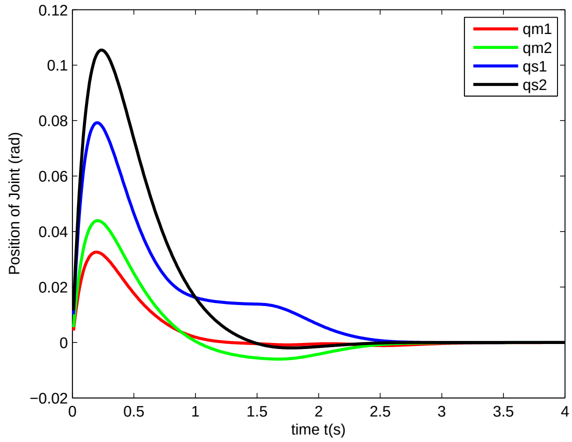

FIGURE 5. Master and slave joint positions

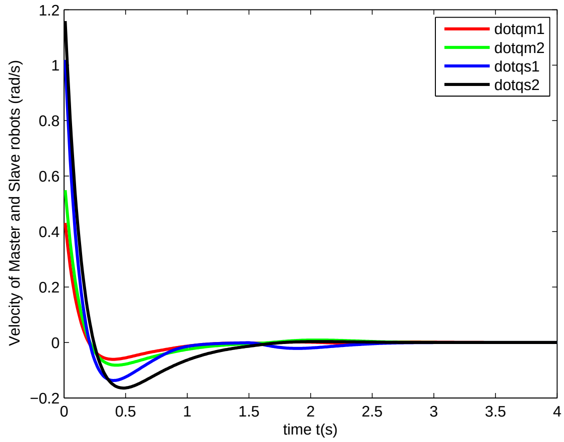

FIGURE 6. Master and slave joint velocities

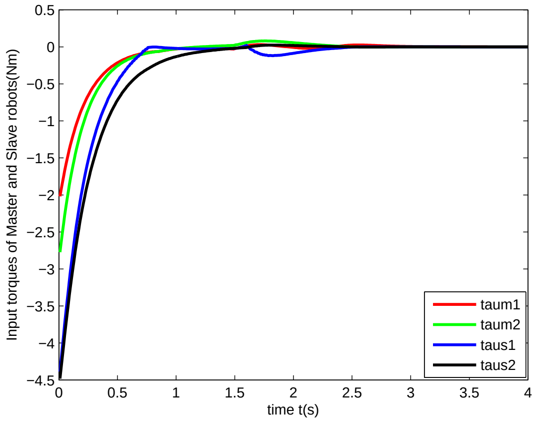

FIGURE 7. Input torques

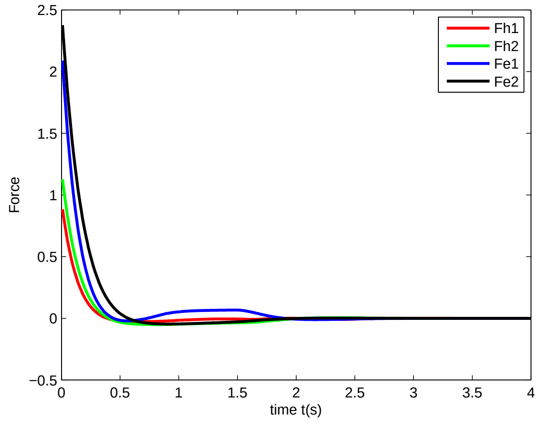

FIGURE 8. The human forces F_h and and the environmental force F_e

DC motor drivers are used to output current commands to the motors. Both master and slave robots are equipped with a pair of optical encoders that measure link angular position and velocity (via digital estimation). The link length of the both arms is 0.3 m. The forces sensed/exerted by the operator/environment is obtained using known impedance models (3) and (4). The master and the slave robot communicate using unblocked User Datagram Protocol (UDP) sockets over a 100 Mbps LAN, which is a popular Internet protocol among real-time applications for teleoperation [13].

The network-induced time delays are measured to determine the key statistical characteristics of local network. In our experiments, the average round-trip time delay induced by the LAN is 0.1 ms and its standard deviation is 0.1 ms. Because the sampling period of our teleoperation is 10 ms, the network induced sporadic time delay in our lab is too much smaller than the sampling period, which is not long enough to verify the effectiveness of the proposed control algorithm. Thus we introduced longer artificial time delays as $d_1(t) = 0.16\sin^2(t)$ s and $d_2(t) = 0.15\cos^2(t)$ s in for the forward and backward paths separately for the purpose of demonstrating the effectiveness of our strategies, which are

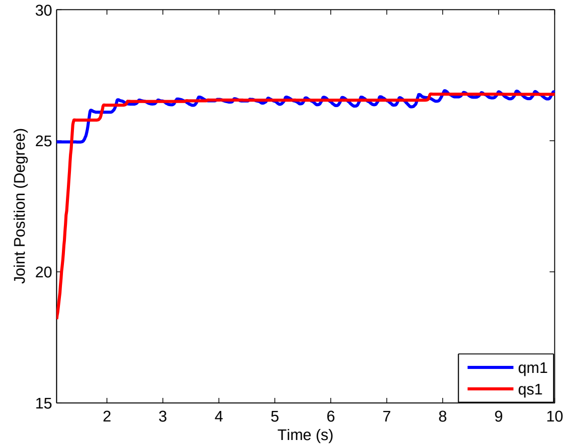

FIGURE 9. The trajectories of q_{m1} and q_{s1} in the experiment

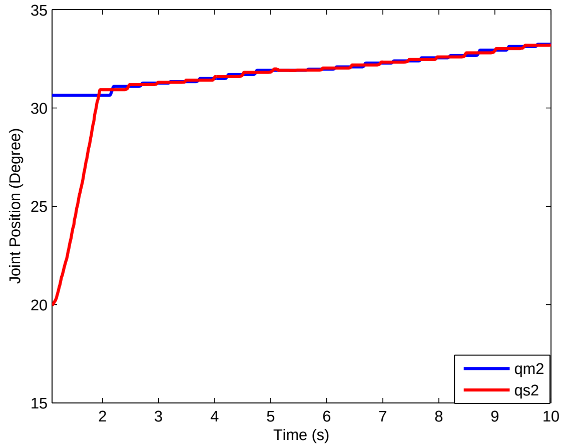

FIGURE 10. The trajectories of q_{m2} and q_{s2} in the experiment

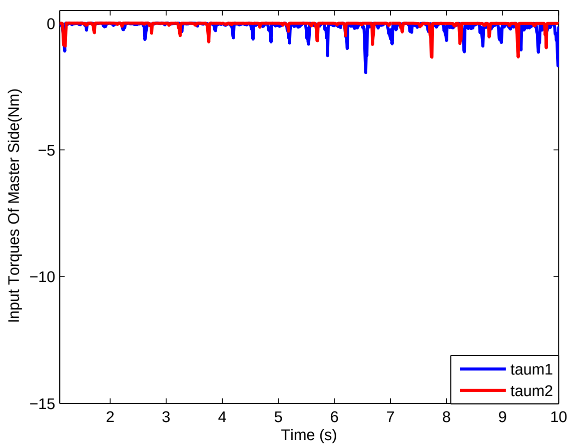

FIGURE 11. The input torques of the master robot

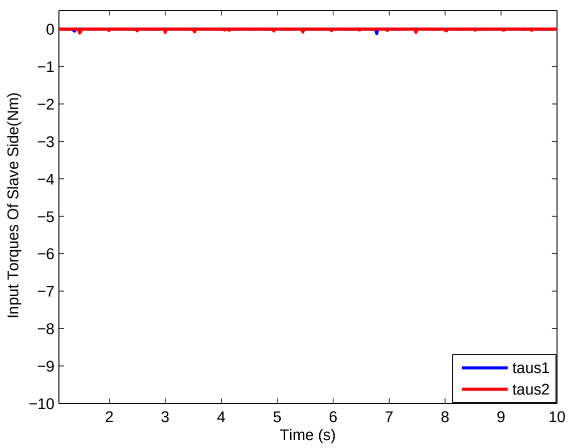

FIGURE 12. The input torques of the slave robot

much larger than the network time delays almost all the time. The time delay is realized by software. The signals are buffered in memory for the time of the delay.

The experimental results of the teleoperation system with the time-varying time delays are presented in Figures 9-13. As we can see in Figure 14, we define the right arm as the master and the left arm as the slave. Both the master and the slave trajectories as well as the contact forces due to environment are shown. In our experiment, the initial states of the master and slave robots are set as $q_{m1}(0) = 25.0$ degree, $q_{m2}(0) = 30.0$ degree, $q_{s1}(0) = 18.0$ degree, $q_{s2}(0) = 20.0$ degree. The initial value of estimated parameters $\hat{\Theta}_{ji}(0) = 0.0$ and we choose parameters in adaptive laws as $\delta_{ji} = \alpha_{ji} = 1/(t+1)^3$, $j = m, s$, $i = 1 \ldots 7$. The coefficients in (14) and (15) is set as $K_1 = K_4 = 1$, $K_2 = K_3 = -1$ for simplicity and $\Lambda = 5$.

The impedance coefficient for the master and slave robots as $K_h = K_e = 0.5$ N/degree and $C_h = C_e = 2.0$ N $\cdot$ s/rad. Figure 9 and Figure 10 show that both the joint positions of one robot track that of the other one, that is to say the motion synchronization is achieved. From Figure 13, we can see that the human force F_h and the environment

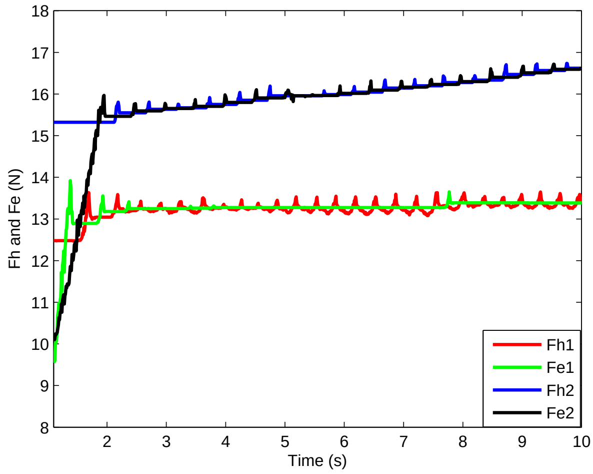

FIGURE 13. The human force F_h and the environmental force F_e

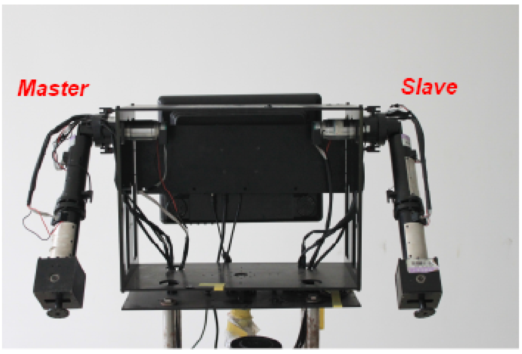

FIGURE 14. The master and slave robots in the experiments

force F_e track each other, which means F_h can reflect environment force F_e exactly. The experiment results show that the adaptive control algorithm proposed in this paper is also applicable in actual systems and the performances of both stability and transparency of the teleoperation are achieved. The transparency has now been demonstrated by the results.

6. **Conclusions.** In this paper, adaptive control of bilateral teleoperation system with unsymmetric time-varying delays and dynamical uncertainties is investigated. A novel adaptive control method is proposed to deal with the dynamical uncertainties. The stability of this two subsystems is proved with LMIs (linear matrix inequalities) based on Lyapunov stability synthesis. Simulation and experiment show the effectiveness of adaptive control law proposed in this paper.

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