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DESIGN OF ROBUST BROADBAND BEAMFORMERS WITH MINIMAX CRITERION

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Abstract. *Broadband beamformers are known to be sensitive to errors and mismatches in their array elements. This paper proposes a robust minimax beamformer design which takes into account sensor element characteristics. The design formulation minimizes robust integral squared error subject to relaxation constraints on the peak deviation error for the beamformer. The advantage of the proposed formulation is that the design allows a trade-off between the peak deviation error and the sensitivity of the minimax error with respect to variations in sensor element characteristics. This improvement has been achieved with almost no increase in complexity compared with the original minimax problem. Design examples demonstrate that the proposed beamformer is less sensitive with respect to variations in sensor element characteristics when compared to the traditional minimax beamformer with a minor degradation in beamformer performance.*
Keywords: Robust beamformer design, Minimax, Second order cone programming

1. **Introduction.** Broadband beamformers have been extensively studied due to their wide application in many areas including radar, sonar, imaging, wireless communications, speech and acoustics [1-8]. The two best-known multi-microphone speech enhancement techniques are: fixed and adaptive beamformers. In this paper, we concentrate on the design of fixed beamformers [7,8].

Traditionally, the design of fixed beamformers is formulated using a least squares or a minimax criterion. The least squares beamformer in general has a low integral squared error and a large peak deviation error. The minimax beamformer, on the other hand, has a low peak deviation error and a large integral squared error. In practice, it is important to obtain a trade-off between the integral squared error and the peak deviation error.

In general, fixed broadband beamformers using small-size microphone arrays are sensitive to errors in the sensor element characteristics. Thus, a robust beamformer design has been developed for the least squares criterion to reduce the sensitivity of the beamformers. Robust least squares beamformers incorporate the array characteristics model into the design [7, 8, 9]. These beamformers, however, often have large peak deviations errors which make them less useful for applications that require low peak deviation error. In [10], the design of robust broadband beamformers with minimax criterion was investigated. The formulated problem, however, is more complicated than the original minimax problem as additional minimax constraints are added to ensure low sensitivity to model errors for the beamformer.

In this paper, we investigate the design of a robust minimax beamformer to reduce its sensitivity with respect to amplitude and phase variations in sensor element characteristics. A design formulation is proposed in which the formulated robust integral squared error is minimized subject to constraints on the peak deviation error. The advantage of the proposed formulation is that the complexity for solving the proposed problem is approximately the same as for the traditional minimax problem. In addition, a trade-off between the peak error deviation and the sensitivity of the beamformer subject to variations can be achieved by adjusting the peak deviation error for the beamformer. The proposed problem is formulated as a second order cone programming problem (SOCP), which can be solved efficiently using a SOCP technique such as SeDuMi [12]. Design examples show that the proposed robust beamformer significantly reduces the sensitivity of the minimax beamformer to variations in the sensor element characteristics. In addition, the proposed method performs better with a greater degree of freedom to vary the peak deviation error than the method developed in [11], which considers the worse case performance.

The paper is organized as follows. In Section 2, we present the array geometry and the frequency response of the broadband beamformer. From this, the design of beamformers with least squares and minimax criterions are presented in Section 3. The design of robust least squared beamformer is discussed in Section 4. A new design of robust minimax beamformer using SOCP technique is developed in Section 5. Design examples are given in Section 6 and the conclusions are in Section 7.

2. **Array Geometry.** Consider the design of a microphone array with N elements. For simplicity, we consider far-field signaling modeling and a linear microphone array. The optimization method, however, is applicable for more general formulations. Here, each microphone is connected to an L-dimensional FIR filter with real coefficients,

$$\mathbf{h}_n = [h_n(0), \ldots, h_n(L-1)]^\top, \quad 0 \le n \le N-1. \tag{1}$$

The response of the broadband beamformer for a normalized frequency $\omega \in \Omega$ and an angle $\phi \in \Phi$ is given by

$$G(\omega, \phi) = \sum_{n=0}^{N-1} \sum_{\ell=0}^{L-1} h_n(\ell) e^{-j\ell\omega} \cdot e^{-j\omega\tau_n(\phi)} \tag{2}$$

where $\tau_n(\phi)$ is the delay in number of samples,

$$\tau_n(\phi) = \frac{f_s d_n \cos\phi}{c}. \tag{3}$$

The constant $c = 343\,\mathrm{ms}^{-1}$ is the speed of sound, f_s is the sampling frequency and d_n is the distance between the n^{th} microphone and the center of the microphone array. Denote by $\mathbf{h}$ an $NL \times 1$ real valued coefficient vector,

$$\mathbf{h} = [\mathbf{h}_0, \ldots, \mathbf{h}_{N-1}]^\top \tag{4}$$

then the beamformer response in (2) can be expressed as

$$G(\omega, \phi) = \mathbf{h}^\top \mathbf{g}(\omega, \phi) \tag{5}$$

where $\mathbf{g}(\omega, \phi)$ is an $NL \times 1$ vector,

$$\mathbf{g}(\omega, \phi) = \left[e^{-j\omega\tau_0(\phi)}, \ldots, e^{-j\omega\tau_{N-1}(\phi)}\right]^\top \otimes \left[1, \ldots, e^{-j(L-1)\omega}\right]^\top$$

and $\otimes$ represents the Kronecker product.

3. **Least Squared and Minimax Beamformers.** Denote by $H_d(\omega,\phi)$ the desired beamformer response at a frequency ω and an angle ϕ. The design of non-robust beamformer with least squared criterion can be formulated as minimizing the integral squares error [7],

$$\begin{aligned} J(\mathbf{h}) &= \int_\Omega\int_\Phi |H(\omega,\phi)-H_d(\omega,\phi)|^2 d\phi\, d\omega \\ &= \mathbf{h}^\top\Big(\int_\Omega\int_\Phi \mathbf{Q}(\omega,\phi)d\phi\, d\omega\Big)\mathbf{h} - 2\mathbf{h}^\top\int_\Omega\int_\Phi \Re\big(\mathbf{p}(\omega,\phi)\big)d\phi\, d\omega + \int_\Omega\int_\Phi |H_d(\omega,\phi)|^2 d\phi\, d\omega \end{aligned} \tag{6}$$

where

$$\begin{aligned} \mathbf{Q}(\omega,\phi) &= \Re\big(\mathbf{g}(\omega,\phi)\mathbf{g}^H(\omega,\phi)\big) \\ \mathbf{p}(\omega,p) &= \mathbf{g}(\omega,\phi)H_d^*(\omega,\phi) \end{aligned}$$

and $\Re\big(\cdot\big)$ denotes the real part. Thus,

$$J(\mathbf{h}) = \mathbf{h}^\top\mathbf{Q}\mathbf{h} - 2\mathbf{h}^\top\Re\big(\mathbf{p}\big) + c \tag{7}$$

where

$$\begin{aligned} \mathbf{Q} &= \int_\Omega\int_\Phi \mathbf{Q}(\omega,\phi)d\phi\, d\omega \\ \mathbf{p} &= \int_\Omega\int_\Phi \mathbf{p}(\omega,\phi)d\phi\, d\omega \end{aligned}$$

and c is a constant. The least squares solution is given by

$$\mathbf{h}_{LS} = \mathbf{Q}^{-1}\Re\big(\mathbf{p}\big). \tag{8}$$

The design of a beamformer with minimax criterion, on the other hand, can be formulated as:

$$\begin{cases} \min\limits_{\mathbf{h}} \varepsilon \\ |H(\omega,\phi)-H_d(\omega,\phi)| \le \varepsilon,\ \forall\omega\in\Omega,\ \phi\in\Phi. \end{cases} \tag{9}$$

This problem can be written as

$$\begin{cases} \min\limits_{\mathbf{h}} \varepsilon \\ \sqrt{e_r^2(\omega,\phi)+e_i^2(\omega,\phi)} \le \varepsilon,\ \forall\omega\in\Omega,\ \phi\in\Phi \end{cases} \tag{10}$$

where

$$\begin{aligned} e_r(\omega,\phi) &= \mathbf{h}^\top\mathbf{g}_r(\omega,\phi) - H_{d,r}(\omega,\phi) \\ e_i(\omega,\phi) &= \mathbf{h}^\top\mathbf{g}_i(\omega,\phi) - H_{d,i}(\omega,\phi). \end{aligned}$$

Here, $\mathbf{g}_r(\omega,\phi)$, $\mathbf{g}_i(\omega,\phi)$, $H_{d,r}(\omega,\phi)$ and $H_{d,i}(\omega,\phi)$ denote the real and the imaginary parts of $\mathbf{g}(\omega,\phi)$ and $H_d(\omega,\phi)$, respectively. The optimization problem (10) is equivalent to

$$\begin{cases} \min\limits_{\mathbf{h}} \varepsilon \\ \left(\varepsilon, \begin{bmatrix} e_r(\omega,\phi) \\ e_i(\omega,\phi) \end{bmatrix}\right) \in \mathrm{Q}_{\mathrm{cone}},\ \forall\omega\in\Omega,\ \phi\in\Phi \end{cases} \tag{11}$$

where the quadratic cone is defined as

$$\mathrm{Q}_{\mathrm{cone}} := \big\{(\varepsilon,\mathbf{x})\in R\times R^2 : \varepsilon \ge \|\mathbf{x}\|\big\} \tag{12}$$

and $\|\cdot\|$ is the Euclidean norm. The problem (12) is a SOCP which can be solved efficiently using a SOCP software such as SeDuMi [12].

4. **Robust Least Squared Beamformer.** Fixed beamformer design is highly sensitive to errors in microphone characteristics such as gain, phase and microphone position, especially for small size microphone arrays [8]. Thus, when designing a robust beamformer the characteristics of the microphones must be taken into account. Here, we assume that each microphone n, $1 \leq n \leq N-1$, has a variation of the form $c_n e^{-j\gamma_n}$ where c_n and γ_n are independent random variations for the amplitude and the phase, respectively. As such, robust integral squared error $J_r(\mathbf{h})$ is obtained by integrating the squared error deviation over the possible regions of the microphone characteristics. To simplify the model, we assume that all microphones characteristics have the same probability density functions (pdf), $f_\alpha(c)$ and $f_G(\gamma)$, for their magnitude and phase, respectively. In general, the model for the pdf can be obtained from the microphone manufacturers.

As with [8], the robust integral squared error can be given as

$$J_r(\mathbf{h}) = \mathbf{h}^\top \mathbf{Q}_r \mathbf{h} - 2\mathbf{h}^\top \mathbf{p}_r + c_r \tag{13}$$

where

$$\mathbf{Q}_r = \Big(\big(\mu_c^2 \sigma_\gamma^c \mathbf{1}_N + (\sigma_c^2 - \mu_c^2 \sigma_\gamma^c) \mathbf{I}_N \big) \otimes \mathbf{1}_L \Big) \odot \mathbf{Q}$$

and

$$\mathbf{p}_r = \mu_c \big(\mu_\gamma^c \Re(\mathbf{p}) + \mu_\gamma^s \Im(\mathbf{p}) \big).$$

Here,

$$\mu_c = \int_c c f_\alpha(c)\, dc, \quad \sigma_c^2 = \int_c c^2 f_\alpha(c)\, dc,$$

and

$$\mu_\gamma^c = \int_\gamma \cos(\gamma) f_G(\gamma)\, d\gamma, \quad \mu_\gamma^s = \int_\gamma \sin(\gamma) f_G(\gamma)\, d\gamma.$$

Also

$$\sigma_\gamma^c = (\mu_\gamma^c)^2 + (\mu_\gamma^s)^2$$

where $\mathbf{1}_N$ represents an $N \times N$ matrix with all elements being one and $\mathbf{I}_N$ is an $N \times N$ identity matrix. In addition, $\odot$ represents matrix element-to-element multiplication, $\Im(\cdot)$ denotes the imaginary part of a complex number and c_r is a constant. The robust least square solution is given by

$$\mathbf{h}_{r,LS} = (\mathbf{Q}_r)^{-1} \mathbf{p}_r. \tag{14}$$

5. **Robust Minimax Beamformer.** Least squares and minimax beamformers are very sensitive to variations in the microphone characteristics. In [10], the design of robust broadband beamformer with minimax criterion is investigated. The formulated problems, however, are more complicated than the original minimax problem as additional minimax constraints are required to be satisfied within the variation regions. In the following, we will discuss: (i) the proposed robust minimax beamformer which requires approximately the same complexity to solve as the original minimax problem; and (ii) the approach develop in [11] which takes into account the worse case performance optimization for comparison purpose.

5.1. **Proposed robust minimax beamformer.** We propose a formulation for robust minimax beamformer that minimizes the robust integral squared error subject to relaxation constraints on the peak deviation error. The advantage of this formulation is that the complexity when solving the proposed problem is approximately the same as the original minimax problem as the number of constraints remains the same. In addition, the formulation allows a trade-off between the peak deviation error and the amount of sensitivity in the beamformer with respect to variation in beamformer element characteristics. Accordingly, the optimization problem can be formulated as

$$\begin{cases} \min\limits_{\mathbf{h}} J_r(\mathbf{h}) = \mathbf{h}^\top \mathbf{Q}_r \mathbf{h} - 2\mathbf{h}^\top \mathbf{p}_r + c \\ |H(\omega,\phi) - H_d(\omega,\phi)| \leq \alpha, \ \forall \omega \in \Omega, \ \phi \in \Phi \end{cases} \tag{15}$$

where α is an upper bound on the frequency response deviation for all ω and ϕ. For (15) to have a feasible solution, α is chosen such that $\alpha \geq \varepsilon_{MM}$, where ε_{MM} is the optimal peak deviation error for the minimax optimization problem (11). Since the matrix $\mathbf{Q}_r$ is symmetric and positive definite, we can do a Cholesky factorization

$$\mathbf{Q}_r = \mathbf{R}^\top \mathbf{R}.$$

Let

$$\tilde{\mathbf{p}} = (\mathbf{R}^\top)^{-1} \mathbf{p}_r,$$

then the robust integral squared error $J_r(\mathbf{h})$ can be expressed as

$$J_r(\mathbf{h}) = ||\mathbf{R}\mathbf{h} - \tilde{\mathbf{p}}||^2 - \mathbf{p}_r^\top \mathbf{Q}_r^{-1} \mathbf{p}_r + c. \tag{16}$$

As the term $-\mathbf{p}_r^\top \mathbf{Q}_r^{-1} \mathbf{p}_r + c$ is a constant, minimizing $J_r(\mathbf{h})$ is equivalent to minimizing the norm $||\mathbf{R}\mathbf{h} - \tilde{\mathbf{p}}||$. As such, the problem (15) can be reformulated as:

$$\begin{cases} \min\limits_{\mathbf{h}} \gamma \\ ||\mathbf{R}\mathbf{h} - \tilde{\mathbf{p}}|| \leq \gamma \\ \sqrt{e_r^2(\omega,\phi) + e_i^2(\omega,\phi)} \leq \alpha, \ \forall \omega \in \Omega, \ \phi \in \Phi. \end{cases} \tag{17}$$

The problem (17) is solved by using discretization. Denote by N_ω and N_ϕ and the number of discretized points for ω and Φ, respectively. Let $\mathbf{x} = [\gamma; \mathbf{h}]$, then (17) can be expressed in the standard SOCP formulation as

$$\begin{cases} \min\limits_{\mathbf{x}} \mathbf{b}^\top \mathbf{x} \\ ||\mathbf{A}_0 \mathbf{x} - \tilde{\mathbf{p}}|| \leq \mathbf{b}^\top \mathbf{x} \\ ||\mathbf{A}_{i,k} \mathbf{x} - \mathbf{c}_{i,k}|| \leq \alpha, \ \forall 1 \leq i \leq N_\omega, \ 1 \leq k \leq N_\phi, \end{cases}$$

where $\mathbf{b} = [1, \ 0, \ldots, \ 0]^\top, \ \mathbf{A}_0 = [\mathbf{0} \ \mathbf{R}]$ and

$$\mathbf{A}_{i,k} = \begin{bmatrix} 0 \ \mathbf{c}_r^\top(\omega_i,\phi_k) \\ 0 \ \mathbf{c}_i^\top(\omega_i,\phi_k) \end{bmatrix}, \ \mathbf{c}_{i,k} = \begin{bmatrix} H_{d,r}(\omega_i,\phi_k) \\ H_{d,i}(\omega_i,\phi_k) \end{bmatrix}.$$

Alternatively, the problem can be written as

$$\begin{cases} \min\limits_{\mathbf{h}} \gamma \\ (\gamma, \mathbf{R}\mathbf{h} - \tilde{\mathbf{p}}) \in \mathrm{Q}_{\text{cone}} \\ \left(\alpha, \begin{bmatrix} e_r(\omega_i,\phi_k) \\ e_i(\omega_i,\phi_k) \end{bmatrix}\right) \in \mathrm{Q}_{\text{cone}}, \\ \qquad \forall \ 1 \leq i \leq N_\omega, 1 \leq k \leq N_\phi. \end{cases} \tag{18}$$

The above optimization problem can be solved accurately using SOCP software such as SeDuMi. In the design example, the robust minimax optimization problem (18) will be solved for different levels of α and the sensitivity of the obtained solution with respect to variations in the sensor element characteristics will be investigated.

5.2. **Robust minimax with worst case performance optimization.** The performance of the proposed robust beamformer is compared with the approach developed in [11] that takes into account the worse case performance optimization. As in [11], for the case that the norm of the distortion vector $\Delta\mathbf{g}(\omega,\phi)$ of $\mathbf{g}(\omega,\phi)$ is bounded by some known positive constant γ, i.e.,

$$\|\Delta\mathbf{g}(\omega,\phi)\| \leq \gamma, \tag{19}$$

the minimax design with worst case performance optimization can be formulated as

$$\min_{\mathbf{h}} \max_{\omega,\phi} \max_{\|\Delta\mathbf{g}(\omega,\phi)\|\leq\gamma} |\mathbf{h}^T(\mathbf{g}(\omega,\phi)+\Delta\mathbf{g}(\omega,\phi)) - H_d(\omega,\phi)|. \tag{20}$$

The optimization problem (20) can be formulated as

$$\left\{\begin{array}{l} \min\limits_{\mathbf{h}} \varepsilon \\ |\mathbf{h}^T(\mathbf{g}(\omega,\phi)+\Delta\mathbf{g}(\omega,\phi)) - H_d(\omega,\phi)| \leq \varepsilon,\ \forall\omega\in\Omega,\ \phi\in\Phi,\ \|\Delta\mathbf{g}(\omega,\phi)\|\leq\gamma. \end{array}\right. \tag{21}$$

By using (19) and the triangle inequality, the constraint in (21) can be bounded as

$$\begin{aligned} |\mathbf{h}^T(\mathbf{g}(\omega,\phi)+\Delta\mathbf{g}(\omega,\phi)) - H_d(\omega,\phi)| &\leq |\mathbf{h}^T\mathbf{g}(\omega,\phi) - H_d(\omega,\phi)| + |\mathbf{h}^T\Delta\mathbf{g}(\omega,\phi)| \\ &\leq |\mathbf{h}^T\mathbf{g}(\omega,\phi) - H_d(\omega,\phi)| + \gamma\|\mathbf{h}\|. \end{aligned}$$

As such, the optimization problem (21) can be approximated as

$$\left\{\begin{array}{l} \min\limits_{\mathbf{h}} \varepsilon \\ |\mathbf{h}^T\mathbf{g}(\omega,\phi) - H_d(\omega,\phi)| + \gamma\|\mathbf{h}\| \leq \varepsilon,\ \forall\omega\in\Omega,\ \phi\in\Phi. \end{array}\right. \tag{22}$$

This problem can be reformulated as a SOCP as

$$\left\{\begin{array}{l} \min\limits_{\tau,\mathbf{h}} \varepsilon \\ |\mathbf{h}^T\mathbf{g}(\omega,\phi) - H_d(\omega,\phi)| \leq \tau,\ \forall\omega\in\Omega,\ \phi\in\Phi \\ \gamma\|\mathbf{h}\| \leq \varepsilon - \tau. \end{array}\right. \tag{23}$$

We now estimate γ in (19) for the farfield case with variations in the microphone array characteristics such as gain and phase errors. Denote by δ_g and δ_ϕ the maximum absolute deviation for the gain and the phase, respectively. By extending the derivation in [11] for the nearfield case, γ can be obtained as:

$$\gamma = \sqrt{NL((1+\delta_g)(1+\delta_g - 2\cos\delta_\phi)+1)}. \tag{24}$$

6. **Design Examples.** Consider the design of an equispaced linear broadband beamformer with $N = 10$, $L = 40$ and the common spacing of 0.04 m between microphone elements. The desired beamformer has a spectral passband $[300, 3800]$ Hz and a spectral stopband $[0, 200] \cup [3950, 4000]$ Hz with a sampling frequency $f_s = 8000$ Hz. Also, the spatial passband is $\Phi_p = [0, 15^\circ]$ and the spatial stopband is $\Phi_s = [25^\circ, 180^\circ]$.

The beamformer desired response $H_d(\omega,\phi)$ is given by

$$H_d(\omega,\phi) = \left\{\begin{array}{ll} e^{-j\omega N_d}, & (\omega,\phi)\in\Omega_p\times\Phi_p \\ 0, & (\omega,\phi)\in\Omega_s\times\Phi_s\cup\Omega_p\times\Phi_s\cup\,\Omega_s\times\Phi_p \end{array}\right. \tag{25}$$

where N_d is the desired delay, $N_d = L/2$. The magnitude and the phase of the microphone array are assumed to follow uniform distributions in the intervals $[0.998, 0.102]$ and $[-0.002, 0.002]$, respectively. The number of discretization points for ω and ϕ are 256 and 128, respectively.

The robust minimax optimization problem is optimized for 14 equispaced points α in the range $[\alpha_{MM}, \alpha_{LS}]$ where α_{LS} is the peak error deviation for the robust least squares solution. Figure 1 shows robust integral squared error $J_r(\mathbf{h})$ for different solutions in the trade-off curve. The robust integral squared error is reduced significantly by 73.6 dB from the minimax solution with an increase of only 0.3 dB in the peak error deviation. Also, the peak error deviation can be reduced by 3 dB from the robust least squares solution with an increase of 1.6 dB in the robust integral squared error.

Table 1 shows the performance of the: (i) least squares beamformer, (ii) robust least squares; (iii) minimax beamformer; (iv) trade-off robust beamformers, (v) and the minimax beamformer design with the worse case performance optimization [11]. To calculate the peak deviation error with variation, Monte Carlo simulation is employed with 500

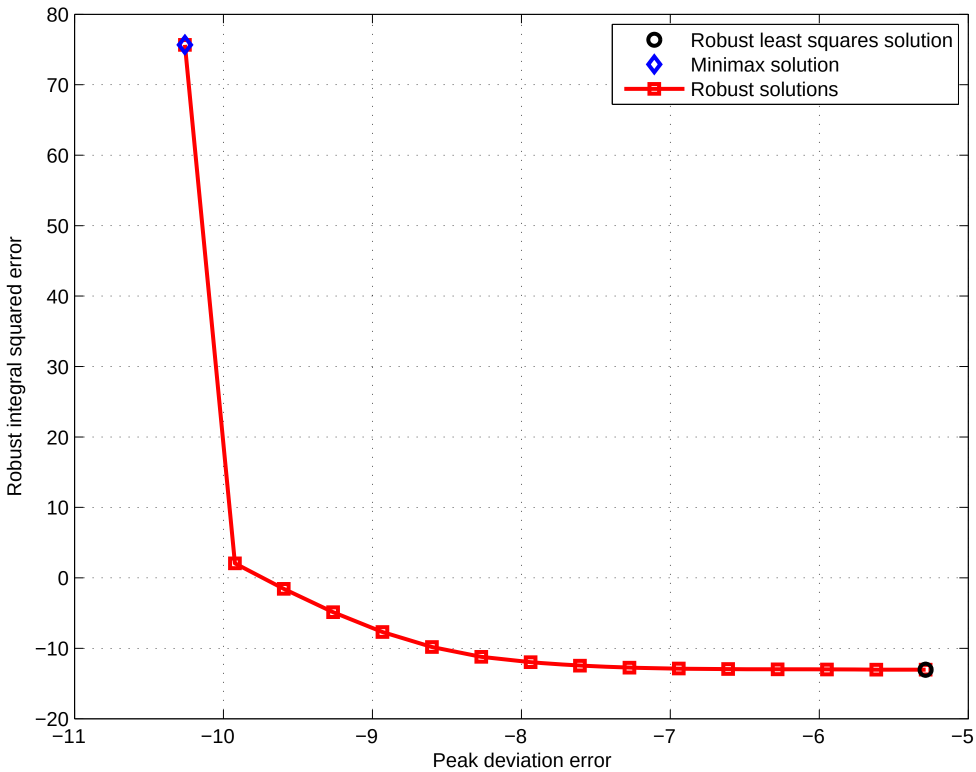

FIGURE 1. Robust integral squared error with different peak variation errors

TABLE 1. Performance of robust and non-robust beamformers

Beamformers	Peak deviation error	Robust integral squared error $J_r(\mathbf{h})$	Peak deviation error with variations
Least squares	−6.22 dB	28.15 dB	29.53 dB
Robust least squares	−5.29 dB	−13.02 dB	−5.01 dB
Minimax	−10.26 dB	75.65 dB	76.80 dB
Trade-off beamformer 1	−9.0 dB	−7.21 dB	−4.46 dB
Trade-off beamformer 2	−8.5 dB	−10.31 dB	−6.49 dB
Trade-off beamformer 3	−8.0 dB	−11.85 dB	−7.00 dB
Robust minimax [11] worst case optimization	−6.9 dB	−3.94 dB	−6.90 dB

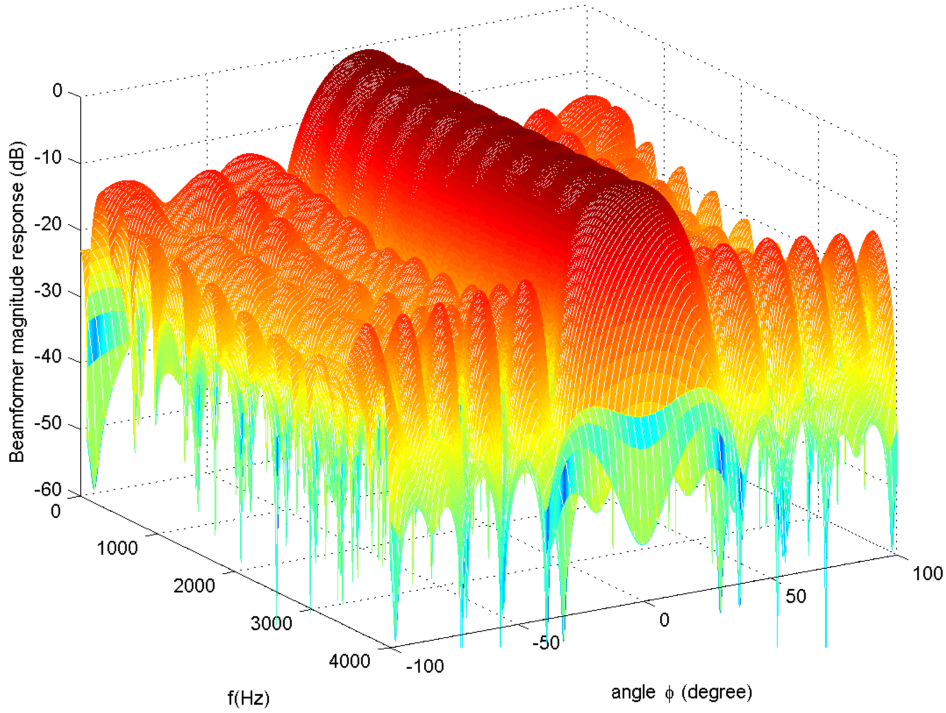

FIGURE 2. Beampattern for minimax beamformer

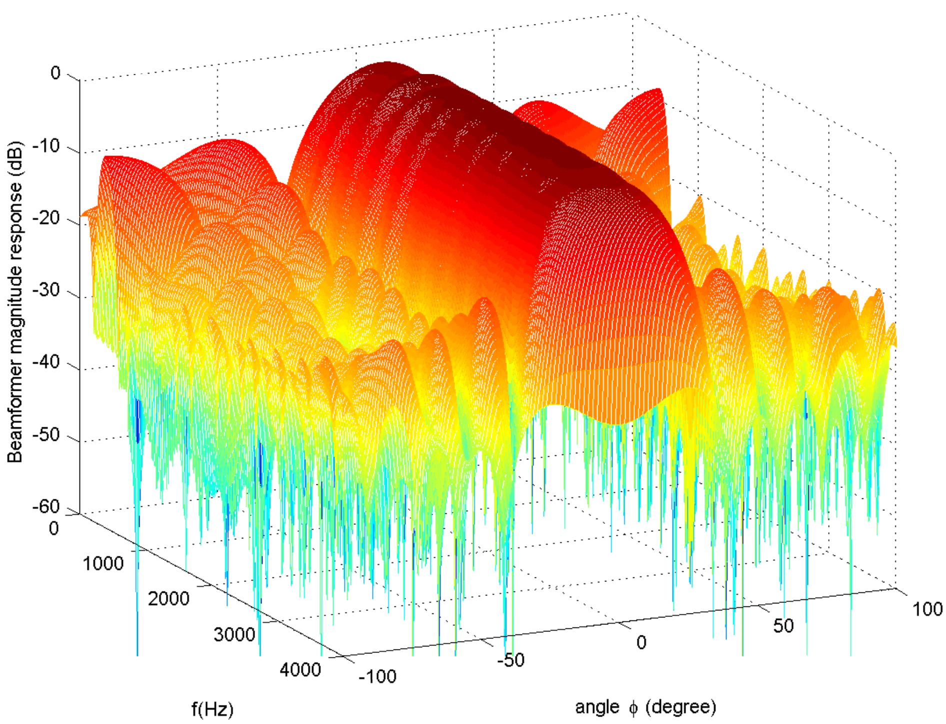

FIGURE 3. Beampattern for the trade-off beamformer 2 in Table 1

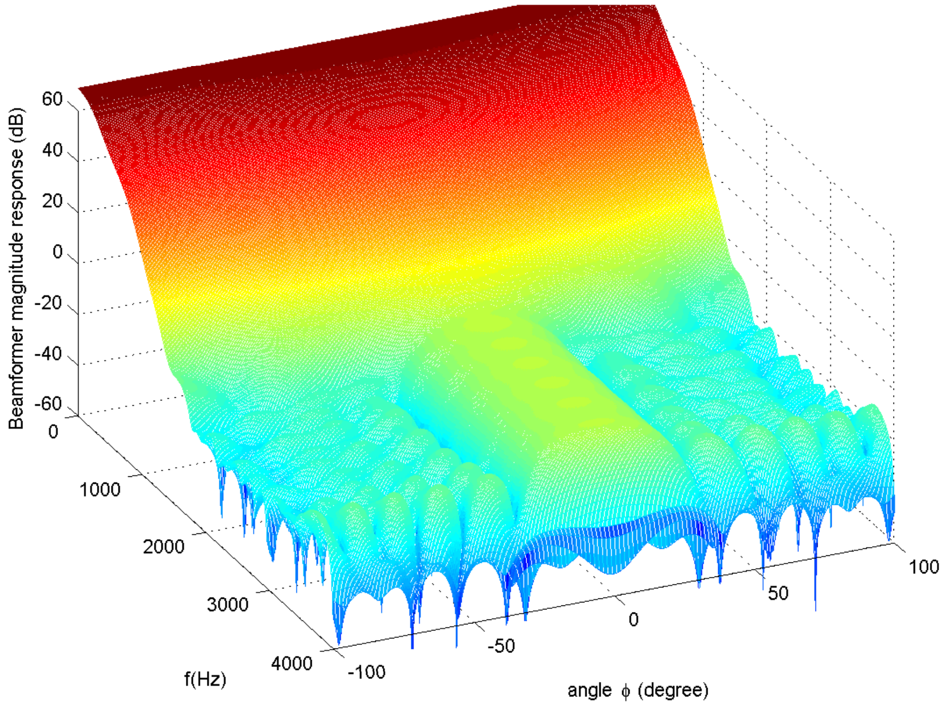

FIGURE 4. Beampattern for minimax beamformer with variations in the sensor element characteristics

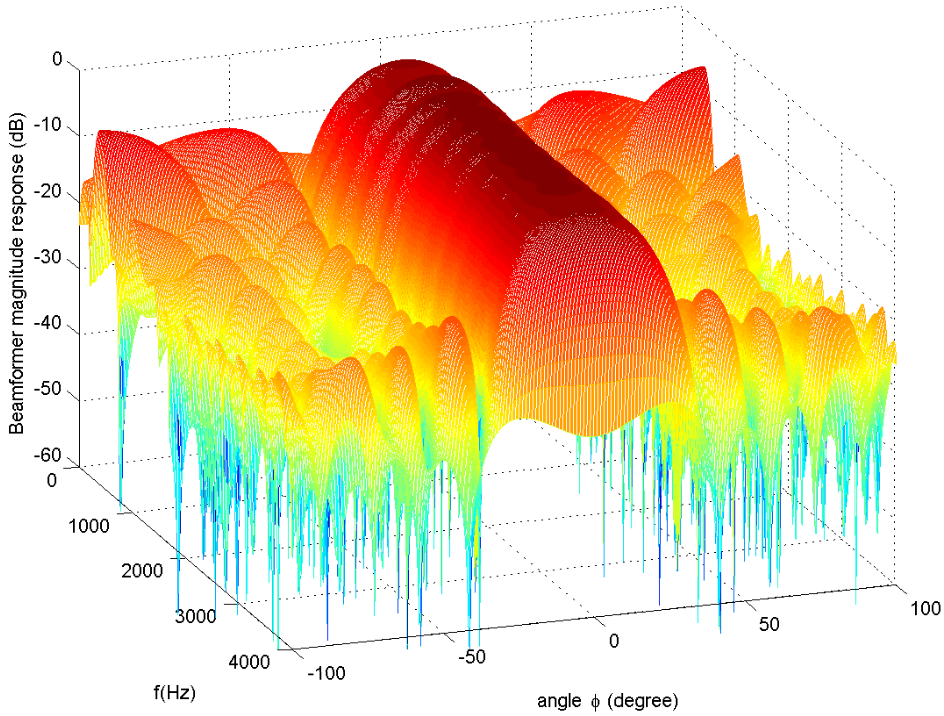

FIGURE 5. Beampattern for the trade-off beamformer 2 in Table 1 with variations in the sensor element characteristics

variations in the magnitude and the phase of the microphone characteristics. The table shows (i) the peak deviation error, (ii) the robust integral squared error $J_r(\mathbf{h})$ and (iii) the peak deviation error from Monte Carlo simulation with variations in the magnitude and the phase of the sensor element characteristics.

From Table 1, it can be seen that the least squares and minimax beamformers are very sensitive with respect to deviation in the amplitude and the phase of the sensor element characteristics. The robust least squares beamformer has a low sensitivity with respect to deviation in the sensor element characteristics, but the peak deviation error remains high. The table also exhibits three solutions in the trade-off curve with the lower bound on the peak deviation error γ ranges from $-9\,\text{dB}$ to $-8\,\text{dB}$. As such, the peak deviation errors for the trade-off beamformers deviate less than $3\,\text{dB}$ from the minimax solution. It can be seen from the table that the trade-off beamformers are less sensitivity with respect to variations. Also, an increase in the peak deviation error bound results in a reduction in the sensitivity in the beamformer. For the trade-off beamformer with a bound $\gamma = -9\,\text{dB}$, the beamformer is $1.26\,\text{dB}$ worse off in the peak deviation error when compared to the minimax beamformer with an improvement of $81.26\,\text{dB}$ in the peak deviation error with variation.

The last row in the table displays the solution obtained from [11] for the minimax solution with the worse case performance optimization. It can be seen from the table that the robust integral squared error for the solution is higher than the trade-off solutions as it considers only the worst case situation. Also, the solution has approximately $1.1\,\text{dB}$ higher in the peak deviation error than the trade-off solution 3 with a slightly higher in peak deviation error with variations. In addition, the solution obtained from [11] does not allow a control on the peak deviation error for the robust solution.

Figures 2 and 3 show the beampatterns for the minimax beamformer and a trade-off robust beamformer in Figure 1 with $\gamma = -8.6\,\text{dB}$. The peak deviation error is slightly increased for the trade-off robust beamformer when compared with the minimax beamformer. Figures 4 and 5 show the beampatterns for the minimax beamformer and the trade-off robust beamformer for the case with variations in the magnitude and the phase of the sensor element characteristics. It is clear that the beampattern for the robust beamformer maintains in the presence of errors especially at low frequencies.

7. **Conclusions.** This paper investigates the performance of a robust minimax beamformer that is less sensitive to variations in sensor element characteristics. The proposed design minimizes the robust integral squared error for the beamformer subject to constraints on the peak deviation error. The advantage of the proposed formulation is that the complexity for solving the proposed problem is approximately the same as for the minimax problem while allowing a control in the trade-off between the peak deviation error and the sensitivity of the beamformer to variation errors. Design examples demonstrate that the proposed robust beamformer has significantly reduced the sensitivity of the minimax beamformer towards sensor element errors.

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