International Journal of Innovative
Computing, Information and Control
Volume **11**, Number **1**, February **2015**

ICIC International ©2015 ISSN 1349-4198
pp. 247–256

ON THE MEAN SQUARE EXPONENTIAL STABILITY FOR A STOCHASTIC FUZZY CELLULAR NEURAL NETWORK WITH DISTRIBUTED DELAYS AND TIME-VARYING DELAYS

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Received February 2014; revised June 2014

Abstract. *This paper is devoted to be concerned with the mean square exponential stability of a stochastic fuzzy cellular neural network with distributed delays. With the help of the stochastic analysis approach and Itô differential formula, a set of sufficient conditions on mean square exponential stability have been established. Finally, an example is given to demonstrate that the proposed criterion is useful and effective.*
Keywords: Stochastic fuzzy cellular neural network, Mean square exponential stability, Distributed delay, Time-varying

1. **Introduction.** Over the past decades, since the important applications of cellular neural networks (CNNS) in various fields such as signal process, combinatorial optimization, image processing and recognition problems, they have been extensively studied [1,2]. In CNNS, processing of moving images requires the introduction of delay in the signals transmitted among the cells [3]. It is well known that time delays are often a source of instability of cellular neural networks, therefore, considerable attention has been paid to the problem of stability analysis of cellular neural networks with delays, and a lot of research results have been reported, see for example, [4-13]. In real nervous systems, stochastic disturbances are nearly inevitable and affect the stability of cellular neural networks [14,15]. Recently, stability analysis of stochastic cellular neural networks with time delays has received much attention, see, for example, [5-13]. In mathematical modelling of real word problems, we encounter two inconveniences, i.e., the complexity and the uncertainty or vagueness. In order to take vagueness into consideration, fuzzy theory is considered as a suitable setting [16]. There are many papers that deal with this topic, and one can see [17-24,33-36].

In 2006, Huang [25] had investigated the exponential stability of the following fuzzy neural networks with distributed delay

$$
\begin{aligned}
\frac{dx_i}{dt} &= -d_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} c_{ij}\mu_j + I_i \\
&\quad + \bigwedge_{j=1}^{n} \alpha_{ij} \int_{t-\tau}^{t} k_j(t-s) f_j(x_j(s))ds + \bigwedge_{j=1}^{n} T_{ij}\mu_j \\
&\quad + \bigvee_{j=1}^{n} \beta_{ij} \int_{t-\tau}^{t} k_j(t-s) f_j(x_j(s))ds + \bigvee_{j=1}^{n} H_{ij}\mu_j, \\
x_i(t) &= \varphi_i(t), \quad -\tau \le t \le 0,
\end{aligned} \tag{1}
$$

where $i = 1, 2, \cdots, x_i, \mu_i, I_i$ denote the state,input and bias of the ith neuron, respectively. The integer n corresponds to the number of units in a neural networks. f_j denotes the signal propagation function of the jth unit. $d_i > 0$ represents the rate with which ith neuron will reset its potential to the resting state in isolation when disconnected from the network and external inputs. a_{ij}, b_{ij} are elements of feedback template and feed-forward template. α_{ij}, β_{ij}, T_{ij} and H_{ij} are elements of fuzzy feedback MIN template, fuzzy feedback MAX template, fuzzy feed-forward MIN template and fuzzy feed-forward MAX template, respectively. $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operation, respectively. $0 \le \tau(t) \le \tau$, $\tau(t)$ represents transmission delay at time t and $\tau(t) : R_+ \to [0, \tau]$ is continuously differentiable function such that $\dot{\tau}(t) \le k < 1$. $k_i(s) \ge 0$ is the feedback kernel, defined on the interval $[0, \tau]$ when τ is a positive finite number or $[0, +\infty)$ while τ is infinite. Kernels satisfy $\int_0^{\tau} k_j(s)ds = 1$, $j = 1, 2, \cdots, n$. The initial conditions of (1.1) are of the form $x_i(t) = \varphi_i(t)$, $-\tau \le t \le 0$, where φ_i is bounded and continuous on $[-\tau, 0]$ when τ is finite or $(-\infty, 0]$ when τ is infinite, $i = 1, 2, \cdots, n$.

Here we shall point out that in practical implementation of neural networks, stochastic phenomenon usually appears in the electrical circuit design of neural networks, for example, the synaptic transmission is a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes. Moreover, stochastic fuzzy cellular neural networks have important applications in processing moving images and associative memories, specifically for circuit design and construction. Yu et al. [37] had revealed that a neural network could be stabilized or destabilized by certain stochastic inputs. Thus, it has important theoretical value and tremendous potential for application in running mechanism of neural networks. However, Huang [25] did not consider this aspect. Therefore, we think that it is worthwhile to investigate the dynamical behavior of the stochastic version of model (2). Motivated by the analysis above, we will consider the following system

$$
\begin{aligned}
dx_i = &\Bigg[-d_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t) - \tau_j(t)) \\
&+ \sum_{j=1}^{n} c_{ij}\mu_j + I_i + \bigwedge_{j=1}^{n} \alpha_{ij} \int_{t-\tau}^{t} k_j(t-s) f_j(x_j(s))ds + \bigwedge_{j=1}^{n} T_{ij}\mu_j \\
&+ \bigvee_{j=1}^{n} \beta_{ij} \int_{t-\tau}^{t} k_j(t-s) f_j(x_j(s))ds + \bigvee_{j=1}^{n} H_{ij}\mu_j \Bigg] dt
\end{aligned}
$$

$$+\sum_{j=1}^{n}\sigma_{ij}(t,x_j(t),x_j(t-\tau_j(t)))d\omega_j(t),$$
$$x_i(t)=\varphi_i(t),\quad -\tau\le t\le 0, \tag{2}$$

where $\sigma(t,x(t),x(t-\tau(t)))=(\sigma_{ij}t,x(t),x(t-\tau(t)))_{n\times n}$ is the diffusion coefficient matrix and $\omega(t)=(\omega_1(t),\cdots,\omega_n(t))$ is an n-dimensional Brownian motion defined on a complete probability space (Ω,F,P) with the natural filtration $\{F_t\}_{t\ge 0}$ (i.e., $F_t=\sigma\{\omega(s):0\le s\le t\}$). $\phi(t)\in L^p_{F_0}([-\tau,0],R^n)$ is the initial function vector, where $\phi(t)\in L^p_{F_0}([-\tau,0],R^n)$ denotes the family of all C-valued random processes $\xi(s)$ such that $\xi(s)$ is F_0-measurable and $\int_{-\tau}^{0}E||\xi(s)||^p ds<\infty$. Throughout this paper, we assume that $f_j(.)$ and $\sigma_{ij}(t,.,.)$ are locally Lipschitz continuous and satisfy the linear growth condition as well. Therefore, it is known that system (2) has a unique global solution on $t\ge 0$, which is denoted by $x(t)=(x_1(t),\cdots,x_n(t))^T$.

We know that there are at least three different types of stochastic stability to describe limiting behaviors of stochastic differential equations: stability in probability, moment stability and almost sure stability. Mao [26] pointed out that the mean square exponential stability is one of the most useful concepts because it is closer to the real situation. For example, mean square exponential stability implies that the second moment of the solution will tend to the trivial solution exponentially fast [27]. In recent years, there are some papers which is concerned with this topic, one can see [28-30].

The main object of this paper is to investigate the mean square exponentially stability of system (2). By employing a Lyapunov function, stochastic analysis and inequality technique, the criteria ensuring mean square exponential stability are established. The proposed results generalize and improve some of the earlier publications.

The remainder of this paper is organized as follows. In Section 2, the basic notations and assumptions are introduced. The criterion for checking the mean square exponentially stability for (2) is given in Section 3. An illustrative example is included in Section 4. We conclude this paper in Section 5.

2. **Preliminaries.**

Let $C\triangleq C([-\tau,0],R^n)$ be the Banach space of continuous functions which map $[-\tau,0]$ into R^n with the topology of uniform convergence. For any $(x_1(t),x_2(t),\cdots,x_n(t))^T\in R^n$, we define $||x(t)||=(\sum_{i=1}^n|x_i(t)|^2)^{\frac12}$.

Let $C^{2,1}([-\tau,+\infty)\times R^n,R_+)$ denote the family of all nonnegative functions $V(t,x)$ on $[-\tau,\infty)\times R^n$ which are continuous twice differentiable in x and once differentiable in t. If $V\in C^{2,1}([-\tau,+\infty)\times R^n,R_+)$, define an operator

$$\begin{aligned}LV(t,x) \;=\; & V_t(t,x)+V_x(t,x)\Bigg[-d_ix_i(t)+\sum_{j=1}^n a_{ij}f_j(x_j(t))+\sum_{j=1}^n b_{ij}f_j(x_j(t)-\tau_j(t))\\ &+\bigwedge_{j=1}^n\alpha_{ij}\int_{t-\tau}^t k_j(t-s)f_j(x_j(s))ds+\bigvee_{j=1}^n\beta_{ij}\int_{t-\tau}^t k_j(t-s)f_j(x_j(s))ds\Bigg]\\ &+\frac12\text{trace}\left[\sigma_i^TV_{xx}\sigma_i\right],\end{aligned}\tag{3}$$

where

$$V_t=\frac{\partial V(t,x)}{\partial t},\quad V_x=\left(\frac{\partial V(t,x)}{\partial x_1},\cdots,\frac{\partial V(t,x)}{\partial x_n}\right),$$
$$V_{xx}=\left(\frac{\partial^2V(t,x)}{\partial x_i\partial x_j}\right)_{n\times n},\quad \sigma_i=\sigma_i(t,x(t),x(t-\tau(t))).$$

We assume that the following conditions hold true:
(H1) There exist positive constants $\nu_i, i = 1, 2, \cdots, n,$ such that

$$|f_i(u) - f_i(v)| \leq \nu_i |u - v|, \ f_i(0) = 0, \ \text{for } u, v \in R.$$

(H2) $\sigma_i : R^+ \times R^n \times R^n \to R^{n \times n}$ is locally Lipschitz continuous and satisfies the linear growth condition. Moreover, σ_i satisfies

$$\text{trace}[\sigma_i(t, x(t), x(t - \tau_j(t))) \leq \Theta_{1i}|x_i(t)|^2 + \Theta_{2i}|x_i(t - \tau_i(t))|^2, \ \ \sigma_i(t, 0, 0) = 0,$$

where Θ_{1i} and Θ_{2i} are known constant entries with n dimension.

Remark 2.1. *The assumptions* (H1) *and* (H2) *are reasonable and acceptable in practice. In* (H1), *for example, we can let the signal propagation function of the jth unit be $f_j = \frac{1}{2}(|u+1| - |u-1|)$, which is often used in the practical implementation of neural networks. In* (H2), *$\sigma_i(t, x(t), x(t - \tau_j(t)))$ is the diffusion coefficient matrix which is reasonable.*

If the assumptions (H1) and (H2) hold, then (2) has a unique global solution on $t \geq 0$ (the proof similar to those in [2,26,31]). It is obvious that (2) admits an equilibrium solution $x^*(t) = 0$.

Definition 2.1. [26] *The trivial solution of (2) is said to be mean square exponentially stable if there is a pair of positive constants λ and G such that*

$$E||x(t; t_0, x_0)|| < G||x_0||e^{-\lambda(t-t_0)},$$

on $t \geq t_0$ for all $x_0 \in R^n$, where λ is also called convergence rate.

Lemma 2.1. [32] *Let $x = (x_1(t), x_2(t), \cdots, x_n(t))^T$ and $y = (y_1(t), y_2(t), \cdots, y_n(t))^T$ be two states of system (2). Then*

$$\left| \bigwedge_{j=1}^{n} \alpha_{ij}(t) f_j(x) - \bigwedge_{j=1}^{n} \alpha_{ij}(t) f_j(y) \right| \leq \sum_{j=1}^{n} |\alpha_{ij}(t)||f_j(x) - f_j(y)|$$

and

$$\left| \bigvee_{j=1}^{n} \beta_{ij}(t) f_j(x) - \bigvee_{j=1}^{n} \beta_{ij}(t) f_j(y) \right| \leq \sum_{j=1}^{n} |\beta_{ij}(t)||f_j(x) - f_j(y)|.$$

3. **Exponential Stability of the Global Solution.**

In this section, we present some results on exponential stability of the global solution of system (2).

Theorem 3.1. *Assume that* (H1) *and* (H2) *hold. Further suppose that there exist constants $\sigma_j > 0$ $(j = 1, 2, \cdots, n)$ such that*

$$\begin{aligned} &\sigma_i \Bigg[-2d_i + \sum_{j=1}^{n} |a_{ij}|\nu_j + \sum_{j=1, j\neq i}^{n} \frac{\sigma_j}{\sigma_i}|a_{ji}|\nu_i \\ &+ \sum_{j=1}^{n} |b_{ij}|\nu_j + \sum_{j=1}^{n} (|\alpha_{ij}| + |\beta_{ij}|)\nu_j + \Theta_{1i} \Bigg] \\ &+ \sum_{j=1, j\neq i}^{n} \frac{\sigma_j}{\sigma_i}|b_{ji}|\nu_i + \sum_{j=1, j\neq i}^{n} \frac{\sigma_j}{\sigma_i}(|\alpha_{ji}| + |\beta_{ji}|)\nu_i + \Theta_{2i} < 0, \end{aligned}$$

then the equilibrium solution x^* of system (2) is exponentially stable in the mean square.

Proof: Suppose that $x^* = (x_1^*, x_2^*, \cdots, x_n^*)^T$ is the equilibrium point of system (2). Let $z_i(t) = x_i(t) - x_i^*$ $(i = 1, 2, \cdots, n)$. Then

$$\begin{aligned} dz_i = & \Bigg[-d_i z_i(t) + \sum_{j=1}^n a_{ij}[f_j(z_j(t)+x_j^*) - f_j(x_j^*)] \\ & + \sum_{j=1}^n b_{ij}[f_j(z_j(t) - \tau_j(t) + x_j^*) - f_j(x_j^*)] \\ & + \bigwedge_{j=1}^n \alpha_{ij} \int_{t-\tau}^t k_j(t-s) f_j(z_j(s)+x_j^*) ds - \bigwedge_{j=1}^n \alpha_{ij} \int_{t-\tau}^t k_j(t-s) f_j(x_j^*) ds \\ & + \bigvee_{j=1}^n \beta_{ij} \int_{t-\tau}^t k_j(t-s) f_j(z_j(s)+x_j^*) ds - \bigvee_{j=1}^n \beta_{ij} \int_{t-\tau}^t k_j(t-s) f_j(x_j^*) ds \Bigg] dt \\ & + \sum_{j=1}^n \sigma_{ij}(t, z_j(t)+x_j^*, z_j(t-\tau_j(t)) + x_j^*) d\omega_j(t), \end{aligned} \tag{4}$$

where $z_i(t) = \Psi_i(t)$, $\Psi_i(t) = \varphi_i(t) - x_i^*$, $t \in [-\tau, 0]$.

Since

$$\begin{aligned} & \sigma_i \Bigg[-2d_i + \sum_{j=1}^n |a_{ij}|\nu_j + \sum_{j=1, j\neq i}^n \frac{\sigma_j}{\sigma_i} |a_{ji}| \nu_i \\ & + \sum_{j=1}^n |b_{ij}|\nu_j + \sum_{j=1}^n (|\alpha_{ij}| + |\beta_{ij}|)\nu_j + \Theta_{1i} \Bigg] \\ & + \sum_{j=1, j\neq i}^n \frac{\sigma_j}{\sigma_i} |b_{ji}|\nu_i + \sum_{j=1, j\neq i}^n \frac{\sigma_j}{\sigma_i} (|\alpha_{ji}| + |\beta_{ji}|)\nu_i + \Theta_{2i} < 0, \end{aligned}$$

we can choose a small positive constant $\varepsilon > 0$ such that

$$\begin{aligned} & \sigma_i \Bigg[\varepsilon - 2d_i + \sum_{j=1}^n |a_{ij}|\nu_j + \sum_{j=1, j\neq i}^n \frac{\sigma_j}{\sigma_i} |a_{ji}| \nu_i \\ & + \sum_{j=1}^n |b_{ij}|\nu_j + \sum_{j=1}^n (|\alpha_{ij}| + |\beta_{ij}|)\nu_j + \Theta_{1i} \Bigg] \\ & + \sum_{j=1, j\neq i}^n \frac{\sigma_j}{\sigma_i} |b_{ji}|\nu_i + \sum_{j=1, j\neq i}^n \frac{\sigma_j}{\sigma_i} (|\alpha_{ji}| + |\beta_{ji}|)\nu_i + \Theta_{2i} < 0. \end{aligned}$$

Now we consider the Lyapunov function

$$V(t, z) = \sum_{i=1}^n \sigma_i |z_i(t)|^2 e^{\varepsilon t}. \tag{5}$$

Calculating the differential operator $LV(t, x)$ of V along the solution $x(t)$ of (2), we have

$$LV(t,x) = \sum_{i=1}^n \sigma_i \Bigg\{ e^{\varepsilon t} \Bigg[\varepsilon |z_i(t)|^2 + 2|z_i(t)| \Bigg(-d_i z_i(t) + \sum_{j=1}^n a_{ij}(f_j(z_j(t)+x_j^*) - f_j(x_j^*)) \Bigg)$$

$$
\begin{aligned}
&+\sum_{j=1}^{n} b_{ij}(f_j(z_j(t)-\tau_j(t)+x_j^*)-f_j(x_j^*))\\
&+\bigwedge_{j=1}^{n}\alpha_{ij}\int_{t-\tau}^{t}k_j(t-s)f_j(z_j(s)+x_j^*)ds\\
&-\bigwedge_{j=1}^{n}\alpha_{ij}\int_{t-\tau}^{t}k_j(t-s)f_j(x_j^*)ds+\bigvee_{j=1}^{n}\beta_{ij}\int_{t-\tau}^{t}k_j(t-s)f_j(z_j(s)+x_j^*)ds\\
&-\bigvee_{j=1}^{n}\beta_{ij}\int_{t-\tau}^{t}k_j(t-s)f_j(x_j^*)ds)\Bigg]\Bigg\}\\
&+\sum_{i=1}^{n}\sigma_i e^{\varepsilon t}\mathrm{trace}[\sigma_i^T(t,z_i(t),z_i(t-\tau_i(t)))\sigma_i(t,z_i(t),z_i(t-\tau_i(t)))]\\
\le\ & e^{\varepsilon t}\sum_{i=1}^{n}\sigma_i\Bigg\{(\varepsilon-2d_i)|z_i(t)|^2+\sum_{j=1}^{n}|a_{ij}|2\nu_j|z_i(t)||z_j(t)|\\
&+\sum_{j=1}^{n}|b_{ij}|2\nu_j|z_i(t)||z_j(t-\tau_j(t))|+\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)2\nu_j|z_i(t)||z_j(t)|\\
&+\Theta_{1i}|z_i(t)|^2+\Theta_{2i}|z_i(t-\tau_i(t))|^2\Bigg\}.
\end{aligned}
$$

Estimating the right of inequality above by basic inequality $2ab\le a^2+b^2$, we have

$$
\begin{aligned}
LV(t,x)\le\ & e^{\varepsilon t}\sum_{i=1}^{n}\sigma_i\Bigg\{(\varepsilon-2d_i)|z_i(t)|^2+\sum_{j=1}^{n}|a_{ij}|\nu_j(|z_i(t)|^2+|z_j(t)|^2)\\
&+\sum_{j=1}^{n}|b_{ij}|\nu_j(|z_i(t)|^2+|z_j(t-\tau_j(t))|^2)\\
&+\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)\nu_j(|z_i(t)|^2+|z_j(t)|^2)\\
&+\Theta_{1i}|z_i(t)|^2+\Theta_{2i}|z_i(t-\tau_i(t))|^2\Bigg\}\\
\le\ & e^{\varepsilon t}\sum_{i=1}^{n}\sigma_i\Bigg\{\Bigg[(\varepsilon-2d_i)+\sum_{j=1}^{n}|a_{ij}|\nu_j+\sum_{j=1,j\neq i}^{n}\frac{\sigma_j}{\sigma_i}|a_{ij}|\nu_i\\
&+\sum_{j=1}^{n}|b_{ij}|\nu_j+\sum_{j=1}^{n}(|\alpha_{ij}|+|\beta_{ij}|)\nu_i+\Theta_{1i}\Bigg]|z_i(t)|^2\\
&+\Bigg[\sum_{j=1,j\neq i}^{n}\frac{\sigma_j}{\sigma_i}|b_{ji}|\nu_i+\sum_{j=1,j\neq i}^{n}\frac{\sigma_j}{\sigma_i}(|\alpha_{ji}|+|\beta_{ji}|)\nu_i+\Theta_{2i}\Bigg]\\
&\times|z_i(t-\tau_i(t))|^2\Bigg\}
\end{aligned}
$$

$$\leq \Theta_0 e^{\varepsilon t} \sup_{-\tau<s<0} \sum_{i=1}^{n} |z_i(t+s)|^2 < 0,$$

where

$$\begin{aligned}\Theta_0 = \sigma_i \Bigg[&\varepsilon - 2d_i + \sum_{j=1}^{n} |a_{ij}|\nu_j + \sum_{j=1,j\neq i}^{n} \frac{\sigma_j}{\sigma_i}|a_{ji}|\nu_i \\ &+ \sum_{j=1}^{n} |b_{ij}|\nu_j + \sum_{j=1}^{n}(|\alpha_{ij}| + |\beta_{ij}|)\nu_j + \Theta_{1i}\Bigg] \\ &+ \sum_{j=1,j\neq i}^{n} \frac{\sigma_j}{\sigma_i}|b_{ji}|\nu_i + \sum_{j=1,j\neq i}^{n} \frac{\sigma_j}{\sigma_i}(|\alpha_{ji}| + |\beta_{ji}|)\nu_i + \Theta_{2i} < 0.\end{aligned}$$

Then

$$E[V(t,z)] < E[V(0,\varphi)], \quad t > 0. \tag{6}$$

On the other hand,

$$E[V(0,\Psi)] = E\left[\sum_{i=1}^{n} \sigma_i |\Psi_i(0)|^2\right] < \max_{1<i<n}\{\sigma_i\} E\left[\sum_{i=1}^{n} |\Psi_i(0)|^2\right], \tag{7}$$

$$E[V(t,z)] \geq e^{\varepsilon t} \min_{1<i<n}\{\sigma_i\} E\left[\sum_{i=1}^{n} |z_i(t)|^2\right], \quad t > 0. \tag{8}$$

It follows from (6)-(8) that

$$E[||z(t,\Psi)||^2] < G_0 E[||\Psi(0)||^2] e^{\varepsilon t}, \tag{9}$$

where

$$G_0 = \frac{\max_{1<i<n}\{\sigma_i\}}{\min_{1<i<n}\{\sigma_i\}} \geq 1.$$

Thus, the equilibrium point of system (2) $x^* = (x_1^*, x_2^*, \cdots, x_n^*)^T$ is exponentially stable in the mean square. The proof of Theorem 3.1 is complete.

4. **Illustrative Examples.**

Example 4.1. *Consider the following stochastic fuzzy cellular neural network with distributed delays and time-varying delays*

$$\begin{aligned} dx_i = \Bigg[&-d_i x_i(t) + \sum_{j=1}^{2} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{2} b_{ij} f_j(x_j(t) - \tau_j(t)) \\ &+ \sum_{j=1}^{2} c_{ij}\mu_j + I_i + \bigwedge_{j=1}^{2} \alpha_{ij} \int_{t-\tau}^{t} k_j(t-s) f_j(x_j(s)) ds + \bigwedge_{j=1}^{2} T_{ij}\mu_j \\ &+ \bigvee_{j=1}^{2} \beta_{ij} \int_{t-\tau}^{t} k_j(t-s) f_j(x_j(s)) ds + \bigvee_{j=1}^{2} H_{ij}\mu_j \Bigg] dt \\ &+ \sum_{j=1}^{n} \sigma_{ij}(t, x_j(t), x_j(t-\tau_j(t))) d\omega_j(t), \end{aligned} \tag{10}$$

where

$$a_{11} = 1, \quad a_{12} = 1, \quad a_{21} = 1, \quad a_{22} = 2,$$

$$
\begin{aligned}
&b_{11} = 1, \quad b_{12} = 1, \quad b_{21} = 1.2, \quad b_{22} = 1.2,\\
&c_{11} = 1, \quad c_{12} = 1, \quad c_{21} = 1, \quad c_{22} = 1,\\
&\alpha_{11} = 0.5, \quad \alpha_{12} = 0.5, \quad \alpha_{21} = 0.5, \quad \alpha_{22} = 0.5,\\
&\beta_{11} = 0.5, \quad \beta_{12} = 0.5, \quad \beta_{21} = 0.5, \quad \beta_{22} = 0.5,\\
&\sigma_{11} = 1, \quad \sigma_{12} = 1, \quad \sigma_{21} = 1, \quad \sigma_{22} = 1,\\
&T_{11} = 1, \quad T_{12} = 1, \quad T_{21} = 1, \quad T_{22} = 1,\\
&H_{11} = 1, \quad H_{12} = 1, \quad H_{21} = 1, \quad H_{22} = 1,\\
&d_1 = 10, \quad d_2 = 9, \quad I_1 = 2, \quad I_2 = 2.
\end{aligned}
$$

Here $I_1 = 2$ and $I_2 = 2$ denote the input of the 1th neuron and 2th neuron being 2, respectively; the number of units in the neural networks (10) is 2. f_j denotes the signal propagation function of the jth unit; the rate with which 1th neuron will reset its potential to the resting state in isolation when disconnected from the network and external inputs is $d_1 = 10$, the rate with which 2th neuron will reset its potential to the resting state in isolation when disconnected from the network and external inputs is $d_2 = 9$; $a_{11} = 1$, $a_{12} = 1$, $a_{21} = 1$ and $a_{22} = 2$ are all the elements of feedback template; $b_{11} = 1$, $b_{12} = 1$, $b_{21} = 1.2$ and $b_{22} = 1.2$ are all the elements of feed-forward template; $\alpha_{11} = 0.5$, $\alpha_{12} = 0.5$, $\alpha_{21} = 0.5$, $\alpha_{22} = 0.5$ are all the elements of fuzzy feedback MIN template; $\beta_{11} = 0.5$, $\beta_{12} = 0.5$, $\beta_{21} = 0.5$, $\beta_{22} = 0.5$ are all the elements of fuzzy feedback MAX template; $T_{11} = 1$, $T_{12} = 1$, $T_{21} = 1$ and $T_{22} = 1$ are all the elements of fuzzy feed-forward MIN template; $H_{11} = 1$, $H_{12} = 1$, $H_{21} = 1$ and $H_{22} = 1$ are all the elements of fuzzy feed-forward MAX template; $\tau(t)$ represents transmission delay at time t. Obviously, all the parameters in model (10) have concrete physical meaning.

Let

$$
\Theta_{11} = 1.2, \quad \Theta_{12} = 1.5, \quad \Theta_{21} = 2, \quad \Theta_{22} = 2,
$$
$$
f_j(x) = \tfrac{1}{2}(|x+1| - |x-1|), \quad \tau_j(t) = 1 + \tfrac{1}{4}\cos 2t \quad (j = 1, 2).
$$

Obviously, $\nu_j = 1$ $(j = 1, 2)$ and the conditions (H1)-(H3) hold. By simple computation, we have

$$
\begin{aligned}
&\sigma_1 \left[-2d_1 + \sum_{j=1}^{2} |a_{1j}|\nu_j + \sum_{j=1, j\neq 1}^{2} \frac{\sigma_j}{\sigma_1} |a_{j1}|\nu_i + \sum_{j=1}^{2} |b_{1j}|\nu_j + \sum_{j=1}^{2} (|\alpha_{1j}| + |\beta_{1j}|)\nu_j + \Theta_{11} \right]\\
&+ \sum_{j=1, j\neq 1}^{2} \frac{\sigma_j}{\sigma_1} |b_{j1}|\nu_1 + \sum_{j=1, j\neq 1}^{2} \frac{\sigma_j}{\sigma_1} (|\alpha_{j1}| + |\beta_{j1}|)\nu_1 + \Theta_{21} = -7.8 < 0
\end{aligned}
$$

and

$$
\begin{aligned}
&\sigma_2 \left[-2d_2 + \sum_{j=1}^{2} |a_{2j}|\nu_j + \sum_{j=1, j\neq 2}^{2} \frac{\sigma_j}{\sigma_2} |a_{j2}|\nu_i + \sum_{j=1}^{2} |b_{2j}|\nu_j + \sum_{j=1}^{2} (|\alpha_{2j}| + |\beta_{ij}|)\nu_j + \Theta_{12} \right]\\
&+ \sum_{j=1, j\neq 2}^{2} \frac{\sigma_j}{\sigma_2} |b_{j2}|\nu_2 + \sum_{j=1, j\neq 2}^{2} \frac{\sigma_j}{\sigma_2} (|\alpha_{j2}| + |\beta_{j2}|)\nu_2 + \Theta_{22} = -1.9 < 0.
\end{aligned}
$$

It follows from Theorem 3.1 that the equilibrium solution x^* of system (2) is exponentially stable in the mean square.

5. **Conclusions.** In this paper, we have investigated the exponential stability exponential stability in the mean square for a stochastic fuzzy cellular neural network with distributed delays and time-varying delays. By means of the stochastic analysis approach and Itô

differential formula, we obtain the sufficient condition ensuring the exponential stability in the mean square for the stochastic fuzzy cellular neural networks with distributed delays and time-varying delays. The methods used in this paper are novel and can be extended to numerous other types of neural networks. These problems will be discussed in near future. In addition, one example illustrates the effectiveness of our main results.

Acknowledgment. This work is partially supported by National Natural Science Foundation of China (No. 11261010, No. 11101126, No. 11201138), Soft Science and Technology Program of Guizhou Province (No. 2011LKC2030), Natural Science and Technology Foundation of Guizhou Province (J[2012]2100), Governor Foundation of Guizhou Province ([2012]53), Natural Science and Technology Foundation of Guizhou Province (2014), Natural Science Innovation Team Project of Guizhou Province ([2013]14) and Scientific Research Fund of Hunan Provincial Education Department (No. 12B034). The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

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