International Journal of Innovative Computing, Information and Control
Volume **11**, Number **1**, February **2015**
ICIC International ©2015 ISSN 1349-4198
pp. 295–307

STABILIZATION FOR A CLASS OF MARKOVIAN JUMP LINEAR SYSTEMS WITH LINEAR FRACTIONAL UNCERTAINTIES

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Received March 2014; revised July 2014

Abstract. *In this paper, the problem of stochastic stabilization for a class of continuous-time Markovian jump linear systems with linear fractional uncertainties is considered. By multiple Lyapunov functional technique, the state feedback controller and dynamic output feedback controller are derived in terms of linear matrix inequalities (LMIs), which guarantees the consider system is stochastic stabilization. Two numerical examples are presented to illustrate the effectiveness and the potential of the obtain results.*
Keywords: Markovian jump linear systems, Stochastic stabilization, Linear fractional uncertainties, LMIs

1. Introduction.

During the past decades, the robust control problem for linear time-invariant systems has been extensively investigated, and many problems have been successfully solved such as the stability problem, the stabilization problem, the filtering problem and their robustness. However, some industrial systems cannot be represented by such a class of linear time-invariant model since the mode varieties of systems with abrupt changes in their structure. These changes may be a consequence of random component failures or repairs, abrupt environmental disturbances, and can be found in communications systems, aircraft control systems, robotic manipulator systems, manufacturing systems, large flexible structures for space stations, etc. Such classes of dynamical systems can be described by the Markovian jump linear systems (MJLS) [1]. Therefore, it is very important and significance to investigate the control and design for the Markovian jump linear systems.

Many important results on Markovian jump systems have been researched, the stability analysis and control design were addressed in [1-6]. On H_∞ filter design, [7] investigated the H_∞ filtering problem for singular Markovian jump systems, [8] dealt with the problem of H_∞ filtering design for nonlinear Markovian jump systems. When time delays appear, the delay H_∞ filtering problem was discussed in [9], and [10] investigated the delay-dependent filtering problem for Markovian jump linear systems. Significant progress on neural network has been made in [11,12].

On the other hand, uncertainties have been introduced in many literatures to overcome the negative effects of external disturbance, such as the norm bounded uncertainties [13], polytopic uncertainties [14], exponential uncertainties [15] and linear fractional uncertainties [16]. Linear fractional uncertainties, which represents the terms like

$F_{Ar(t)}(t)[I - G_{Ar(t)}F_{Ar(t)}(t)]^{-1}$ that depend on an unknown, possibly time-varying and parameter $r(t)$, to describe the neglected dynamics, prevent performance degradations and any other phenomena such as aging. These problems are important and challenging in both theory and practice, and there has very little literature on stochastic stabilization for continuous-time Markovian jump linear systems with linear fractional uncertainties, which has motivated us for this study.

This paper focuses on the robust control design for continuous-time Markovian jump linear systems with linear fractional uncertainties. Two criteria are proposed to ensure the stochastic stabilization of the considered closed-loop systems. By multiple Lyapunov functional technique, the state feedback controller and dynamic output feedback controller are also given. All the given results are formulated by LMIs, which can be easily checked by using the MATLAB LMI toolbox. Finally, two numerical examples are provided to show the effectiveness of the proposed techniques.

Notations: The symmetric term in a symmetric matrix are denoted by $*$, matrix P^T stands for the transpose of the matrix P; $(\Omega, \mathbb{F}, \mathbb{P})$ is a probability space, where Ω is the sample space, $\mathbb{F}$ is the σ-algebra of subsets of the sample space, and $\mathbb{P}$ is the probability measure on $\mathbb{F}$; I and 0 represent the identity matrix and a zero matrix, respectively.

2. Problem Statement and Preliminaries.

Let $\{r(t), t \geq 0\}$ be a continuous-time Markovian process on the probability space $(\Omega, \mathbb{F}, \mathbb{P})$ with a right continuous trajectory taking values in a finite set $\overline{\mathbb{N}} = \{1, 2, \cdots, N\}$ with transition probability matrix $\Lambda = [\lambda_{ij}]$ being given by

$$P[r(t+h) = j \mid r(t) = i] = \begin{cases} \lambda_{ij}h + o(h), & i \neq j, \\ 1 + \lambda_{ii}h + o(h), & \text{otherwise}, \end{cases}$$

where $h > 0$; $\lim\limits_{h\to 0} \frac{o(h)}{h} = 0$ and $\lambda_{ij} \geq 0$ is the transition probability rate from the mode i to the mode j at time t when $i \neq j$ and $\lambda_{ii} = -\sum\limits_{j=1, j\neq i}^{N} \lambda_{ij}$.

Fix a probability space $(\Omega, \mathbb{F}, \mathbb{P})$ and consider the following continuous-time Markovian jump linear systems (1) and (2) with linear fractional uncertainties.

$$\begin{cases} \dot{x}(t) = A_{r(t)}(t)x(t) + B_{r(t)}(t)u(t) \\ x(0) = x_0 \end{cases} \tag{1}$$

$$\begin{cases} \dot{x}(t) = A_{r(t)}(t)x(t) + B_{r(t)}(t)u(t), \quad x(0) = x_0 \\ y(t) = C_{r(t)}(t)x(t) \end{cases} \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $u(t) \in \mathbb{R}^m$ is the control input; $y(t) \in \mathbb{R}^p$ is the measured output; we note $r(t) = i$ implies the i-th subsystem being activated. $A_i(t)$, $B_i(t)$, $C_i(t)$ are time-varying matrices with linear fractional uncertainties being given by

$$\begin{aligned} &A_i(t) = A_i + M_{Ai}H_{Ai}(t)N_{Ai}, \quad B_i(t) = B_i + M_{Bi}H_{Bi}(t)N_{Bi}, \\ &C_i(t) = C_i + M_{Ci}H_{Ci}(t)N_{Ci} \end{aligned} \tag{3}$$

with

$$\begin{aligned} &H_{Ai}(t) = F_{Ai}(t)[I - G_{Ai}F_{Ai}(t)]^{-1}, \quad H_{Bi}(t) = F_{Bi}(t)[I - G_{Bi}F_{Bi}(t)]^{-1}, \\ &H_{Ci}(t) = F_{Ci}(t)[I - G_{Ci}F_{Ci}(t)]^{-1} \end{aligned} \tag{4}$$

where A_i, B_i, C_i, M_{Ai}, M_{Bi}, M_{Ci}, N_{Ai}, N_{Bi}, N_{Ci} and G_{Ai}, G_{Bi}, G_{Ci} are constant matrices with appropriate dimensions, $F_{Ai}(t)$, $F_{Bi}(t)$, $F_{Ci}(t)$ are time-varying matrices with norm-bounded uncertainties satisfying:

$$F_{Ai}^T(t)F_{Ai}(t) \le I, \;\; F_{Bi}^T(t)F_{Bi}(t) \le I, \;\; F_{Ci}^T(t)F_{Ci}(t) \le I$$

$$I - G_{Ai}^T G_{Ai} > 0, \;\; I - G_{Bi}^T G_{Bi} > 0, \;\; I - G_{Ci}^T G_{Ci} > 0$$

Consider the continuous-time MJLS (5), we give the following concepts and lemmas.

$$\begin{cases} dx(t) = A_i(t)x(t)dt \\ x(0) = x_0 \end{cases} \tag{5}$$

Lemma 2.1. [13] : *The MJLS (5) is stochastically stable if there exists a set of symmetric and positive definite matrices $P_i > 0$ such that the following holds for all admissible linear fractional uncertainties and for every $i \in \overline{\mathbb{N}}$:*

$$A_i^T(t)P_i + P_iA_i(t) + \sum_{j=1}^{N} \lambda_{ij}P_j < 0$$

Lemma 2.2. [17] : *Let $I - G_i^T G_i > 0$, setting:*

$$\xi = \{H_i(t) = F_i(t)[I - G_iF_i(t)]^{-1}, F_i^T(t)F_i(t) \le I\}$$

Then, ξ can be rewritten as:

$$\xi = \{H_i(t) = (I - G_i^T G_i)^{-1}G_i^T + (I - G_i^T G_i)^{-1/2}\pi(t)\}$$

Lemma 2.3. [18] : *Let H, E be given matrices with appropriate dimensions and F satisfying $F^TF \le I$, for any $\varepsilon > 0$, we have*

$$HFE + E^TF^TH^T \le \varepsilon HH^T + \varepsilon^{-1}E^TE$$

Definition 2.1. [13] : *The MJLS (5) is said to be stochastic stabilization if there exists a finite positive constant $T(x_0, r_0)$ such that the following holds for any initial condition (x_0, r_0) and for all admissible linear fractional uncertainties.*

$$\mathbf{E}\left[\int_0^\infty \|x(t)\|^2 dt | x_0, r_0\right] \le T(x_0, r_0)$$

3. **Robust Control via State Feedback.** In this section, we will give a solution to the stochastic stabilization problem for MJLS (1) with linear fractional uncertainties via switched state feedback, the state feedback controller takes the following form:

$$u(t) = K_ix(t) \tag{6}$$

where K_i is a state feedback controller gain to be determined for every mode $i \in \overline{\mathbb{N}}$.

Theorem 3.1. *If there exist symmetric and positive-definite matrices $X_i > 0$, matrices Y_i, positive scalars $\varepsilon_{Ai} > 0$ and $\varepsilon_{Bi} > 0$ such that the following LMIs holds for any $i \in \overline{\mathbb{N}}$ and for all linear fractional uncertainties:*

$$\begin{bmatrix} J_{ui} & * & * & * & * & * \\ \varepsilon_{Ai}^{-1}N_{Ai}X_i & -I & 0 & 0 & 0 & 0 \\ \varepsilon_{Bi}^{-1}N_{Bi}Y_i & 0 & -I & 0 & 0 & 0 \\ R_{1Ai}^T & 0 & 0 & -I + G_{Ai}^TG_{Ai} & 0 & 0 \\ R_{1Bi}^T & 0 & 0 & 0 & -I + G_{Bi}^TG_{Bi} & 0 \\ \mathbb{S}_i^T(X) & 0 & 0 & 0 & 0 & -\mathbb{X}_i(X) \end{bmatrix} < 0 \tag{7}$$

where

$$J_{ui} = X_iA_i^T + A_iX_i + Y_i^TB_i^T + B_iY_i + \lambda_{ii}X_i$$

$$R_{1Ai} = \varepsilon_{Ai}M_{Ai} + \varepsilon_{Ai}^{-1}X_iN_{Ai}^TG_{Ai}, \quad R_{1Bi} = \varepsilon_{Bi}M_{Bi} + \varepsilon_{Bi}^{-1}Y_i^TN_{Bi}^TG_{Bi}$$

Then the MJLS (1) is stochastic stabilization. Moreover, the state feedback controller gain matrix in (6) can be given by $K_i = Y_i X_i^{-1}$.

Proof: Combining the controller $u(t) = K_i x(t)$ with the MJLS (1), we can get

$$\dot{x}(t) = [A_i(t) + B_i(t)K_i]x(t), \quad x(0) = x_0 \tag{8}$$

Based on Lemma 2.1, system (1) is stochastically stable if there exist symmetric and positive-definite matrices P_i such that the following (9) is satisfied for each $i \in \overline{\mathbb{N}}$:

$$[A_i(t) + B_i(t)K_i]^T P_i + P_i[A_i(t) + B_i(t)K_i] + \sum_{j=1}^{N} \lambda_{ij} P_j < 0 \tag{9}$$

Set $X_i = P_i^{-1}$, $i \in \overline{\mathbb{N}}$, pre- and post-multiplying (9) by X_i, we obtain

$$X_i[A_i(t) + B_i(t)K_i]^T + [A_i(t) + B_i(t)K_i]X_i + X_i \left[\sum_{j=1}^{N} \lambda_{ij} X_j^{-1} \right] X_i < 0 \tag{10}$$

Using the linear fractional uncertainties of the matrices A and B, the inequality (10) becomes

$$\begin{aligned} &X_i A_i^T + A_i X_i + X_i K_i^T B_i^T + B_i K_i X_i + M_{Ai} H_{Ai}(t) N_{Ai} X_i + X_i N_{Ai}^T H_{Ai}^T(t) M_{Ai}^T \\ &+ M_{Bi} H_{Bi}(t) N_{Bi} K_i X_i + X_i K_i^T N_{Bi}^T H_{Bi}^T(t) M_{Bi}^T + X_i \left[\sum_{j=1}^{N} \lambda_{ij} X_j^{-1} \right] X_i < 0 \end{aligned} \tag{11}$$

According to Lemma 2.2, Lemma 2.3 and the linear fractional uncertainties (3) and (4), the following two matrix inequalities are obvious:

$$\begin{aligned} &M_{Ai} H_{Ai}(t) N_{Ai} X_i + X_i N_{Ai}^T H_{Ai}^T(t) M_{Ai}^T \\ &\leq M_{Ai}(I - G_{Ai}^T G_{Ai})^{-1} G_{Ai}^T N_{Ai} X_i + X_i N_{Ai}^T G_{Ai}(I - G_{Ai}^T G_{Ai})^{-1} M_{Ai}^T \\ &\quad + \varepsilon_{Ai}^2 M_{Ai}(I - G_{Ai}^T G_{Ai})^{-1} M_{Ai}^T + \varepsilon_{Ai}^{-2} X_i N_{Ai}^T N_{Ai} X_i \\ &\quad + \varepsilon_{Ai}^{-2} X_i N_{Ai}^T G_{Ai}(I - G_{Ai}^T G_{Ai})^{-1} G_{Ai}^T N_{Ai} X_i \\ &= R_{1Ai}(I - G_{Ai}^T G_{Ai})^{-1} R_{1Ai}^T + \varepsilon_{Ai}^{-2} X_i N_{Ai}^T N_{Ai} X_i \end{aligned} \tag{12}$$

$$\begin{aligned} &M_{Bi} H_{Bi}(t) N_{Bi} K_i X_i + X_i K_i^T N_{Bi}^T H_{Bi}^T(t) M_{Bi}^T \\ &\leq M_{Bi}(I - G_{Bi}^T G_{Bi})^{-1} G_{Bi}^T N_{Bi} K_i X_i + X_i K_i^T N_{Bi}^T G_{Bi}(I - G_{Bi}^T G_{Bi})^{-1} M_{Bi}^T \\ &\quad + \varepsilon_{Bi}^2 M_{Bi}(I - G_{Bi}^T G_{Bi})^{-1} M_{Bi}^T + \varepsilon_{Bi}^{-2} X_i K_i^T N_{Bi}^T N_{Bi} K_i X_i \\ &\quad + \varepsilon_{Bi}^{-2} X_i K_i^T N_{Bi}^T G_{Bi}(I - G_{Bi}^T G_{Bi})^{-1} G_{Bi}^T N_{Bi} K_i X_i \\ &= R_{1Bi}(I - G_{Bi}^T G_{Bi})^{-1} R_{1Bi}^T + \varepsilon_{Bi}^{-2} X_i K_i^T N_{Bi}^T N_{Bi} K_i X_i \end{aligned} \tag{13}$$

where $R_{1Ai} = \varepsilon_{Ai} M_{Ai} + \varepsilon_{Ai}^{-1} X_i N_{Ai}^T G_{Ai}$, $R_{1Bi} = \varepsilon_{Bi} M_{Bi} + \varepsilon_{Bi}^{-1} X_i K_i^T N_{Bi}^T G_{Bi}$.

Let

$$\mathbb{S}_i(X) = \left[\sqrt{\lambda_{i1}} X_i, \cdots, \sqrt{\lambda_{ii-1}} X_i, \sqrt{\lambda_{ii+1}} X_i, \cdots, \sqrt{\lambda_{iN}} X_i \right] \tag{14}$$

$$\mathbb{X}_i(X) = diag[X_1, \cdots, X_{i-1}, X_{i+1}, \cdots, X_N] \tag{15}$$

Then we can get

$$X_i \left[\sum_{j=1}^{N} \lambda_{ij} X_j^{-1} \right] X_i = \lambda_{ii} X_i + \mathbb{S}_i(X) \mathbb{X}_i^{-1}(X) \mathbb{S}_i^T(X) \tag{16}$$

Setting $Y_i = K_i X_i$, the system (1) will be stochastic stabilization if the following holds:

$$\begin{aligned} &X_i A_i^T + A_i X_i + Y_i^T B_i^T + B_i Y_i + \lambda_{ii} X_i + R_{1Ai}(I - G_{Ai}^T G_{Ai})^{-1} R_{1Ai}^T \\ &+ \varepsilon_{Ai}^{-2} X_i N_{Ai}^T N_{Ai} X_i + R_{1Bi}(I - G_{Bi}^T G_{Bi})^{-1} R_{1Bi}^T \\ &+ \varepsilon_{Bi}^{-2} Y_i^T N_{Bi}^T N_{Bi} Y_i + \mathbb{S}_i(X) \mathbb{X}_i^{-1}(X) \mathbb{S}_i^T(X) < 0 \end{aligned} \tag{17}$$

with $R_{1Ai} = \varepsilon_{Ai} M_{Ai} + \varepsilon_{Ai}^{-1} X_i N_{Ai}^T G_{Ai}$, $R_{1Bi} = \varepsilon_{Bi} M_{Bi} + \varepsilon_{Bi}^{-1} Y_i^T N_{Bi}^T G_{Bi}$.

After using the Schur complement lemma for (17), we get LMIs (7) of Theorem 3.1.

This completes the proof.

4. **Robust Control via Dynamic Output Feedback.** In this section, we will give a solution to the stochastic stabilization problem for MJLS (2) with linear fractional uncertainties via switched dynamic output feedback, the dynamic output feedback controller is described by the following structure:

$$\begin{cases} \dot{x}_c(t) = K_{Ai} x_c(t) + K_{Bi} y(t), \quad x_c(0) = 0 \\ u(t) = K_{Ci} x_c(t) \end{cases} \tag{18}$$

where $x_c(t) \in \mathbb{R}^n$ is the controller state; K_{Ai}, K_{Bi} and K_{Ci} are the dynamic output feedback gains to be determined for every mode $i \in \overline{\mathbb{N}}$.

Theorem 4.1. *If there exist symmetric and positive-definite matrices $X_i > 0$, $Y_i > 0$, k_{Bi}, k_{Ci} and positive scalars $\varepsilon_{Ai} > 0$, $\varepsilon_{Bi} > 0$, $\varepsilon_{Ci} > 0$ such that, for any $i \in \overline{\mathbb{N}}$ and for all linear fractional uncertainties, the following LMIs (19), (20) and (21) are satisfied*

$$\begin{bmatrix} J_{\tilde{\mathbb{H}}_{1i}} & * & * & * & * & * & * & * \\ \varepsilon_{Ai}^{-1} N_{Ai} Y_i & -I & 0 & 0 & 0 & 0 & 0 & 0 \\ \varepsilon_{Bi}^{-1} N_{Bi} \mathrm{k}_{Ci} & 0 & -I & 0 & 0 & 0 & 0 & 0 \\ \varepsilon_{Ci}^{-1} N_{Ci} Y_i & 0 & 0 & -I & 0 & 0 & 0 & 0 \\ R_{2Ai}^T & 0 & 0 & 0 & -Q_{Ai} & 0 & 0 & 0 \\ R_{2Bi}^T & 0 & 0 & 0 & 0 & -Q_{Bi} & 0 & 0 \\ \varepsilon_{Ci}^{-1} G_{Ci}^T N_{Ci} Y_i & 0 & 0 & 0 & 0 & 0 & -Q_{Ci} & 0 \\ \mathbb{S}_i^T(Y) & 0 & 0 & 0 & 0 & 0 & 0 & -\mathbb{Y}_i(Y) \end{bmatrix} < 0 \tag{19}$$

$$\begin{bmatrix} J_{\tilde{\mathbb{H}}_{3i}} & * & * & * \\ \varepsilon_{Ai} M_{Ai}^T X_i + \varepsilon_{Ai}^{-1} G_{Ai}^T N_{Ai} & -Q_{Ai} & 0 & 0 \\ \varepsilon_{Bi} M_{Bi}^T X_i & 0 & -Q_{Bi} & 0 \\ \varepsilon_{Ci} M_{Ci}^T \mathrm{k}_{Bi} + \varepsilon_{Ci}^{-1} G_{Ci}^T N_{Ci} & 0 & 0 & -Q_{Ci} \end{bmatrix} < 0 \tag{20}$$

$$\begin{bmatrix} Y_i & I \\ I & X_i \end{bmatrix} > 0 \tag{21}$$

with

$$\mathbb{S}_i(Y) = [\sqrt{\lambda_{i1}} Y_i, \cdots, \sqrt{\lambda_{ii-1}} Y_i, \sqrt{\lambda_{ii+1}} Y_i, \cdots, \sqrt{\lambda_{iN}} Y_i]$$
$$\mathbb{Y}_i(Y) = diag[Y_1, \cdots, Y_{i-1}, Y_{i+1}, \cdots, Y_N]$$
$$J_{\tilde{\mathbb{H}}_{1i}} = A_i Y_i + Y_i A_i^T + B_i \mathrm{k}_{Ci} + \mathrm{k}_{Ci}^T B_i^T + \lambda_{ii} Y_i$$
$$J_{\tilde{\mathbb{H}}_{3i}} = X_i A_i + A_i^T X_i + \mathrm{k}_{Bi} C_i + C_i^T \mathrm{k}_{Bi}^T + \sum_{j=1}^{N} \lambda_{ij} X_j + \varepsilon_{Ai}^{-2} N_{Ai}^T N_{Ai} + \varepsilon_{Ci}^{-2} N_{Ci}^T N_{Ci}$$
$$R_{2Ai} = \varepsilon_{Ai} M_{Ai} + \varepsilon_{Ai}^{-1} Y_i^T N_{Ai}^T G_{Ai}, \quad R_{2Bi} = \varepsilon_{Bi} M_{Bi} + \varepsilon_{Bi}^{-1} \mathrm{k}_{Ci}^T N_{Bi}^T G_{Bi}$$
$$Q_{Ai} = I - G_{Ai}^T G_{Ai}, \quad Q_{Bi} = I - G_{Bi}^T G_{Bi}, \quad Q_{Ci} = I - G_{Ci}^T G_{Ci}$$
$$\mathrm{k}_{Bi} = [Y_i^{-1} - X_i] K_{Bi}, \quad \mathrm{k}_{Ci} = K_{Ci} Y_i$$

Then the MJLS (2) is stochastic stabilization. Furthermore, the dynamic output feedback controller gain matrix in (18) can be given by:

$$\begin{cases} K_{Ai} = [X_i - Y_i^{-1}]^{-1}[A_i^T + X_i A_i Y_i + \mathrm{k}_{Bi} C_i Y_i + X_i B_i \mathrm{k}_{Ci} + \sum_{j=1}^{N} \lambda_{ij} Y_j^{-1} Y_i \\ \qquad + N_{Ai}^T G_{Ai} Q_{Ai}^{-1} M_{Ai}^T + X_i M_{Ai} Q_{Ai}^{-1} G_{Ai}^T N_{Ai} Y_i + X_i M_{Bi} Q_{Bi}^{-1} G_{Bi}^T N_{Bi} \mathrm{k}_{Ci} \\ \qquad + \mathrm{k}_{Bi} M_{Ci} Q_{Ci}^{-1} N_{Ci} Y_i + \varepsilon_{Ai}^2 X_i M_{Ai} Q_{Ai}^{-1} M_{Ai}^T + \varepsilon_{Ai}^{-2} N_{Ai}^T G_{Ai} Q_{Ai}^{-1} G_{Ai}^T N_{Ai} Y_i \\ \qquad + \varepsilon_{Ai}^{-2} N_{Ai}^T N_{Ai} Y_i + \varepsilon_{Bi}^2 X_i M_{Bi} Q_{Bi}^{-1} M_{Bi}^T \\ \qquad + \varepsilon_{Ci}^{-2} N_{Ci}^T G_{Ci} Q_{Ci}^{-1} G_{Ci}^T N_{Ci} Y_i + \varepsilon_{Ci}^{-2} N_{Ci}^T N_{Ci} Y_i] Y_i^{-1} \\ K_{Bi} = [Y_i^{-1} - X_i]^{-1} \mathrm{k}_{Bi} \\ K_{Ci} = \mathrm{k}_{Ci} Y_i^{-1} \end{cases} \tag{22}$$

Proof: Basing on the system (2) and the controller dynamics (18) we get the following systems:

$$\dot{\eta}(t) = \left[\widetilde{A}_i + \Delta\widetilde{A}_i(t)\right] \eta(t) \tag{23}$$

where

$$\eta(t) = \begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix}, \quad \widetilde{A}_i = \begin{bmatrix} A_i & B_i K_{Ci} \\ K_{Bi} C_i & K_{Ai} \end{bmatrix},$$

$$\Delta\widetilde{A}_i(t) = \Delta\widetilde{A}_{Ai}(t) + \Delta\widetilde{B}_{Bi}(t) + \Delta\widetilde{C}_{Ci}(t),$$

$$\Delta\widetilde{A}_{Ai}(t) = \begin{bmatrix} M_{Ai} H_{Ai}(t) N_{Ai} & 0 \\ 0 & 0 \end{bmatrix}, \quad \Delta\widetilde{B}_{Bi}(t) = \begin{bmatrix} 0 & M_{Bi} H_{Bi}(t) N_{Bi} K_{Ci} \\ 0 & 0 \end{bmatrix},$$

$$\Delta\widetilde{C}_{Ci}(t) = \begin{bmatrix} 0 & 0 \\ K_{Ci} M_{Ci} H_{Ci}(t) N_{Ci} & 0 \end{bmatrix}.$$

Applying Lemma 2.1, the system (23) is stochastically stable if there exists a set of symmetric and positive-definite matrices P_i such that the following LMIs hold for each $i \in \overline{\mathbb{N}}$:

$$\left[\widetilde{A}_i + \Delta\widetilde{A}_i(t)\right]^T P_i + P_i \left[\widetilde{A}_i + \Delta\widetilde{A}_i(t)\right] + \sum_{j=1}^{N} \lambda_{ij} P_j < 0 \tag{24}$$

Using the expression for $\Delta\widetilde{A}_i(t)$, we get

$$\begin{aligned} &\widetilde{A}_i^T P_i + P_i \widetilde{A}_i + P_i \Delta\widetilde{A}_{Ai}(t) + \Delta\widetilde{A}_{Ai}^T(t) P_i + P_i \Delta\widetilde{B}_{Bi}(t) \\ &+ \Delta\widetilde{B}_{Bi}^T(t) P_i + P_i \Delta\widetilde{C}_{Ci}(t) + \Delta\widetilde{C}_{Ci}^T(t) P_i + \sum_{j=1}^{N} \lambda_{ij} P_j < 0 \end{aligned} \tag{25}$$

Setting

$$Q_{Ai} = I - G_{Ai}^T G_{Ai}, \quad Q_{Bi} = I - G_{Bi}^T G_{Bi}, \quad Q_{Ci} = I - G_{Ci}^T G_{Ci}$$

and using (3) and (4), we can get

$$\begin{aligned} \Delta\widetilde{A}_{Ai}(t) = &\begin{bmatrix} M_{Ai} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} (I - G_{Ai}^T G_{Ai})^{-1} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} G_{Ai}^T & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} N_{Ai} & 0 \\ 0 & 0 \end{bmatrix} \\ &+ \begin{bmatrix} M_{Ai} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} (I - G_{Ai}^T G_{Ai})^{-1/2} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_A(t) & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} N_{Ai} & 0 \\ 0 & 0 \end{bmatrix} \\ &= \widetilde{M}_{Ai} \widetilde{Q}_{Ai}^{-1} \widetilde{G}_{Ai}^T \widetilde{N}_{Ai} + \widetilde{M}_{Ai} \widetilde{Q}_{Ai}^{-1/2} \widetilde{\pi}_A(t) \widetilde{N}_{Ai} \end{aligned} \tag{26}$$

$$\begin{aligned}\Delta\widetilde{B}_{Bi}(t) &= \begin{bmatrix} 0 & M_{Bi} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & (I-G_{Bi}^TG_{Bi})^{-1} \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & G_{Bi}^T \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & N_{Bi}K_{Ci} \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 & M_{Bi} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & (I-G_{Bi}^TG_{Bi})^{-1/2} \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & \pi_B(t) \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & N_{Bi}K_{Ci} \end{bmatrix} \\ &= \widetilde{M}_{Bi}\widetilde{Q}_{Bi}^{-1}\widetilde{G}_{Bi}^T\widetilde{N}_{Bi} + \widetilde{M}_{Bi}\widetilde{Q}_{Bi}^{-1/2}\widetilde{\pi}_B(t)\widetilde{N}_{Bi}\end{aligned} \tag{27}$$

$$\begin{aligned}\Delta\widetilde{C}_{Ci}(t) &= \begin{bmatrix} 0 & 0 \\ 0 & K_{Bi}M_{Ci} \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & (I-G_{Ci}^TG_{Ci})^{-1} \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & G_{Ci}^T \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & N_{Ci} \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 & 0 \\ 0 & K_{Bi}M_{Ci} \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & (I-G_{Ci}^TG_{Ci})^{-1/2} \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & \pi_C(t) \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & N_{Ci} \end{bmatrix} \\ &= \widetilde{M}_{Ci}\widetilde{Q}_{Ci}^{-1}\widetilde{G}_{Ci}^T\widetilde{N}_{Ci} + \widetilde{M}_{Ci}\widetilde{Q}_{Ci}^{-1/2}\widetilde{\pi}_C(t)\widetilde{N}_{Ci}\end{aligned} \tag{28}$$

Based on Lemma 2.2, Lemma 2.3 and (26), (27), (28) we have

$$\begin{aligned}&P_i\Delta\widetilde{A}_{Ai}(t) + \Delta\widetilde{A}_{Ai}^T(t)P_i \\ &= P_i\widetilde{M}_{Ai}\widetilde{Q}_{Ai}^{-1}\widetilde{G}_{Ai}^T\widetilde{N}_{Ai} + \widetilde{N}_{Ai}^T\widetilde{G}_{Ai}\widetilde{Q}_{Ai}^{-1}\widetilde{M}_{Ai}^TP_i \\ &\quad + P_i\widetilde{M}_{Ai}\widetilde{Q}_{Ai}^{-1/2}\widetilde{\pi}_A(t)\widetilde{N}_{Ai} + \widetilde{N}_{Ai}^T\widetilde{\pi}_A^T(t)\widetilde{Q}_{Ai}^{-1/2}\widetilde{M}_{Ai}^TP_i \\ &\le P_i\widetilde{M}_{Ai}\widetilde{Q}_{Ai}^{-1}\widetilde{G}_{Ai}^T\widetilde{N}_{Ai} + \widetilde{N}_{Ai}^T\widetilde{G}_{Ai}\widetilde{Q}_{Ai}^{-1}\widetilde{M}_{Ai}^TP_i \\ &\quad + \varepsilon_{Ai}^2P_i\widetilde{M}_{Ai}\widetilde{Q}_{Ai}^{-1}\widetilde{M}_{Ai}^TP_i + \varepsilon_{Ai}^{-2}\widetilde{N}_{Ai}^T\widetilde{\pi}_A^T(t)\widetilde{\pi}_A(t)\widetilde{N}_{Ai} \\ &\le P_i\widetilde{M}_{Ai}\widetilde{Q}_{Ai}^{-1}\widetilde{G}_{Ai}^T\widetilde{N}_{Ai} + \widetilde{N}_{Ai}^T\widetilde{G}_{Ai}\widetilde{Q}_{Ai}^{-1}\widetilde{M}_{Ai}^TP_i \\ &\quad + \varepsilon_{Ai}^2P_i\widetilde{M}_{Ai}\widetilde{Q}_{Ai}^{-1}\widetilde{M}_{Ai}^TP_i + \varepsilon_{Ai}^{-2}\widetilde{N}_{Ai}^T\widetilde{G}_{Ai}\widetilde{Q}_{Ai}^{-1}\widetilde{G}_{Ai}^T\widetilde{N}_{Ai} + \varepsilon_{Ai}^{-2}\widetilde{N}_{Ai}^T\widetilde{N}_{Ai} \\ &= \widetilde{R}_{2Ai}\widetilde{Q}_{Ai}^{-1}\widetilde{R}_{2Ai}^T + \varepsilon_{Ai}^{-2}\widetilde{N}_{Ai}^T\widetilde{N}_{Ai}\end{aligned} \tag{29}$$

$$\begin{aligned}&P_i\Delta\widetilde{B}_{Bi}(t) + \Delta\widetilde{B}_{Bi}^T(t)P_i \\ &= P_i\widetilde{M}_{Bi}\widetilde{Q}_{Bi}^{-1}\widetilde{G}_{Bi}^T\widetilde{N}_{Bi} + \widetilde{N}_{Bi}^T\widetilde{G}_{Bi}\widetilde{Q}_{Bi}^{-1}\widetilde{M}_{Bi}^TP_i \\ &\quad + P_i\widetilde{M}_{Bi}\widetilde{Q}_{Bi}^{-1/2}\widetilde{\pi}_B(t)\widetilde{N}_{Bi} + \widetilde{N}_{Bi}^T\widetilde{\pi}_B^T(t)\widetilde{Q}_{Bi}^{-1/2}\widetilde{M}_{Bi}^TP_i \\ &\le P_i\widetilde{M}_{Bi}\widetilde{Q}_{Bi}^{-1}\widetilde{G}_{Bi}^T\widetilde{N}_{Bi} + \widetilde{N}_{Bi}^T\widetilde{G}_{Bi}\widetilde{Q}_{Bi}^{-1}\widetilde{M}_{Bi}^TP_i \\ &\quad + \varepsilon_{Bi}^2P_i\widetilde{M}_{Bi}\widetilde{Q}_{Bi}^{-1}\widetilde{M}_{Bi}^TP_i + \varepsilon_{Bi}^{-2}\widetilde{N}_{Bi}^T\widetilde{\pi}_B^T(t)\widetilde{\pi}_B(t)\widetilde{N}_{Bi} \\ &\le P_i\widetilde{M}_{Bi}\widetilde{Q}_{Bi}^{-1}\widetilde{G}_{Bi}^T\widetilde{N}_{Bi} + \widetilde{N}_{Bi}^T\widetilde{G}_{Bi}\widetilde{Q}_{Bi}^{-1}\widetilde{M}_{Bi}^TP_i \\ &\quad + \varepsilon_{Bi}^2P_i\widetilde{M}_{Bi}\widetilde{Q}_{Bi}^{-1}\widetilde{M}_{Bi}^TP_i + \varepsilon_{Bi}^{-2}\widetilde{N}_{Bi}^T\widetilde{G}_{Bi}\widetilde{Q}_{Bi}^{-1}\widetilde{G}_{Bi}^T\widetilde{N}_{Bi} + \varepsilon_{Bi}^{-2}\widetilde{N}_{Bi}^T\widetilde{N}_{Bi} \\ &= \widetilde{R}_{2Bi}\widetilde{Q}_{Bi}^{-1}\widetilde{R}_{2Bi}^T + \varepsilon_{Bi}^{-2}\widetilde{N}_{Bi}^T\widetilde{N}_{Bi}\end{aligned} \tag{30}$$

$$\begin{aligned}&P_i\Delta\widetilde{C}_{Ci}(t) + \Delta\widetilde{C}_{Ci}^T(t)P_i \\ &= P_i\widetilde{M}_{Ci}\widetilde{Q}_{Ci}^{-1}\widetilde{G}_{Ci}^T\widetilde{N}_{Ci} + \widetilde{N}_{Ci}^T\widetilde{G}_{Ci}\widetilde{Q}_{Ci}^{-1}\widetilde{M}_{Ci}^TP_i \\ &\quad + P_i\widetilde{M}_{Ci}\widetilde{Q}_{Ci}^{-1/2}\widetilde{\pi}_C(t)\widetilde{N}_{Ci} + \widetilde{N}_{Ci}^T\widetilde{\pi}_C^T(t)\widetilde{Q}_{Ci}^{-1/2}\widetilde{M}_{Ci}^TP_i \\ &\le P_i\widetilde{M}_{Ci}\widetilde{Q}_{Ci}^{-1}\widetilde{G}_{Ci}^T\widetilde{N}_{Ci} + \widetilde{N}_{Ci}^T\widetilde{G}_{Ci}\widetilde{Q}_{Ci}^{-1}\widetilde{M}_{Ci}^TP_i \\ &\quad + \varepsilon_{Ci}^2P_i\widetilde{M}_{Ci}\widetilde{Q}_{Ci}^{-1}\widetilde{M}_{Ci}^TP_i + \varepsilon_{Ci}^{-2}\widetilde{N}_{Ci}^T\widetilde{\pi}_C^T(t)\widetilde{\pi}_C(t)\widetilde{N}_{Ci} \\ &\le P_i\widetilde{M}_{Ci}\widetilde{Q}_{Ci}^{-1}\widetilde{G}_{Ci}^T\widetilde{N}_{Ci} + \widetilde{N}_{Ci}^T\widetilde{G}_{Ci}\widetilde{Q}_{Ci}^{-1}\widetilde{M}_{Ci}^TP_i \\ &\quad + \varepsilon_{Ci}^2P_i\widetilde{M}_{Ci}\widetilde{Q}_{Ci}^{-1}\widetilde{M}_{Ci}^TP_i + \varepsilon_{Ci}^{-2}\widetilde{N}_{Ci}^T\widetilde{G}_{Ci}\widetilde{Q}_{Ci}^{-1}\widetilde{G}_{Ci}^T\widetilde{N}_{Ci} + \varepsilon_{Ci}^{-2}\widetilde{N}_{Ci}^T\widetilde{N}_{Ci} \\ &= \widetilde{R}_{2Ci}\widetilde{Q}_{Ci}^{-1}\widetilde{R}_{2Ci}^T + \varepsilon_{Ci}^{-2}\widetilde{N}_{Ci}^T\widetilde{N}_{Ci}\end{aligned} \tag{31}$$

where

$$\widetilde{R}_{2Ai} = \varepsilon_{Ai} P_i \widetilde{M}_{Ai} + \varepsilon_{Ai}^{-1} \widetilde{N}_{Ai}^T \widetilde{G}_{Ai},$$

$$\widetilde{R}_{2Bi} = \varepsilon_{Bi} P_i \widetilde{M}_{Bi} + \varepsilon_{Bi}^{-1} \widetilde{N}_{Bi}^T \widetilde{G}_{Bi},$$

$$\widetilde{R}_{2Ci} = \varepsilon_{Ci} P_i \widetilde{M}_{Ci} + \varepsilon_{Ci}^{-1} \widetilde{N}_{Ci}^T \widetilde{G}_{Ci}.$$

The previous stability condition will be satisfied if the following holds:

$$\begin{aligned}
&\widetilde{A}_i^T P_i + P_i \widetilde{A}_i + \sum_{j=1}^{N} \lambda_{ij} P_j + P_i \widetilde{M}_{Ai} \widetilde{Q}_{Ai}^{-1} \widetilde{G}_{Ai}^T \widetilde{N}_{Ai} + \widetilde{N}_{Ai}^T \widetilde{G}_{Ai} \widetilde{Q}_{Ai}^{-1} \widetilde{M}_{Ai}^T P_i \\
&+ \varepsilon_{Ai}^2 P_i \widetilde{M}_{Ai} \widetilde{Q}_{Ai}^{-1} \widetilde{M}_{Ai}^T P_i + \varepsilon_{Ai}^{-2} \widetilde{N}_{Ai}^T \widetilde{G}_{Ai} \widetilde{Q}_{Ai}^{-1} \widetilde{G}_{Ai}^T \widetilde{N}_{Ai} + \varepsilon_{Ai}^{-2} \widetilde{N}_{Ai}^T \widetilde{N}_{Ai} \\
&+ P_i \widetilde{M}_{Bi} \widetilde{Q}_{Bi}^{-1} \widetilde{G}_{Bi}^T \widetilde{N}_{Bi} + \widetilde{N}_{Bi}^T \widetilde{G}_{Bi} \widetilde{Q}_{Bi}^{-1} \widetilde{M}_{Bi}^T P_i + \varepsilon_{Bi}^2 P_i \widetilde{M}_{Bi} \widetilde{Q}_{Bi}^{-1} \widetilde{M}_{Bi}^T P_i \\
&+ \varepsilon_{Bi}^{-2} \widetilde{N}_{Bi}^T \widetilde{G}_{Bi} \widetilde{Q}_{Bi}^{-1} \widetilde{G}_{Bi}^T \widetilde{N}_{Bi} + \varepsilon_{Bi}^{-2} \widetilde{N}_{Bi}^T \widetilde{N}_{Bi} + P_i \widetilde{M}_{Ci} \widetilde{Q}_{Ci}^{-1} \widetilde{G}_{Ci}^T \widetilde{N}_{Ci} + \varepsilon_{Ci}^{-2} \widetilde{N}_{Ci}^T \widetilde{N}_{Ci} \\
&+ \widetilde{N}_{Ci}^T \widetilde{G}_{Ci} \widetilde{Q}_{Ci}^{-1} \widetilde{M}_{Ci}^T P_i + \varepsilon_{Ci}^2 P_i \widetilde{M}_{Ci} \widetilde{Q}_{Ci}^{-1} \widetilde{M}_{Ci}^T P_i + \varepsilon_{Ci}^{-2} \widetilde{N}_{Ci}^T \widetilde{G}_{Ci} \widetilde{Q}_{Ci}^{-1} \widetilde{G}_{Ci}^T \widetilde{N}_{Ci} < 0
\end{aligned} \tag{32}$$

Setting

$$P_i = \begin{bmatrix} P_{1i} & P_{2i} \\ P_{2i}^T & P_{3i} \end{bmatrix}, \quad L_i = [P_{1i} - P_{2i} P_{3i}^{-1} P_{2i}^T]^{-1}, \quad U_i = \begin{bmatrix} L_i & I \\ L_i & 0 \end{bmatrix}, \quad V_i = \begin{bmatrix} I & 0 \\ 0 & -P_{3i}^{-1} P_{2i}^T \end{bmatrix}$$

where $P_{1i} > 0$, $P_{3i} > 0$ are symmetric and positive-define matrices, we get

$$V_i U_i = \begin{bmatrix} L_i & I \\ -P_{3i}^{-1} P_{2i}^T L_i & 0 \end{bmatrix}, \qquad U_i^T V_i^T = \begin{bmatrix} L_i^T & -L_i^T P_{2i} P_{3i}^{-1} \\ I & 0 \end{bmatrix}$$

The set of coupled matrix inequalities (32) that guarantees robust stochastically stable is nonlinear in P_i and the controller gains K_{Ai}, K_{Bi} and K_{Ci}, To cast it into an LMI form, let us pre-and-post-multiply the inequality (32) by $U_i^T V_i^T$ and $V_i U_i$, respectively, using all these transformation and since $\sum_{j=1}^{N} \lambda_{ij} L_i^T [P_{2i} P_{3i}^{-1} P_{3j} - P_{2j}] P_{3j}^{-1} [P_{2i} P_{3i}^{-1} P_{3j} - P_{2j}]^T L_i \geq 0$, the previous stochastically stable condition (32) becomes:

$$\begin{bmatrix} \tilde{H}_{1i} & \tilde{H}_{2i} \\ \tilde{H}_{2i}^T & \tilde{H}_{3i} \end{bmatrix} < 0, \tag{33}$$

with

$$\begin{aligned}
\tilde{H}_{1i} = {} & A_i L_i + L_i^T A_i^T - B_i K_{Ci} P_{3i}^{-1} P_{2i}^T L_i - L_i^T P_{2i} P_{3i}^{-1} K_{Ci}^T B_i^T + \sum_{j=1}^{N} \lambda_{ij} L_i^T L_j^{-1} L_i \\
&+ M_{Ai} Q_{Ai}^{-1} G_{Ai}^T N_{Ai} L_i + L_i^T N_{Ai}^T G_{Ai} Q_{Ai}^{-1} M_{Ai}^T - M_{Bi} Q_{Bi}^{-1} G_{Bi}^T N_{Bi} K_{Ci} P_{3i}^{-1} P_{2i}^T L_i \\
&- L_i^T P_{2i} P_{3i}^{-1} K_{Ci}^T N_{Bi}^T G_{Bi} Q_{Bi}^{-1} M_{Bi}^T + \varepsilon_{Ai}^2 M_{Ai} Q_{Ai}^{-1} M_{Ai}^T + \varepsilon_{Ai}^{-2} L_i^T N_{Ai}^T G_{Ai} Q_{Ai}^{-1} G_{Ai}^T N_{Ai} L_i \\
&+ \varepsilon_{Ai}^{-2} L_i^T N_{Ai}^T N_{Ai} L_i + \varepsilon_{Bi}^2 M_{Bi} Q_{Bi}^{-1} M_{Bi}^T \\
&+ \varepsilon_{Bi}^{-2} L_i^T P_{2i} P_{3i}^{-1} K_{Ci}^T N_{Bi}^T G_{Bi} Q_{Bi}^{-1} G_{Bi}^T N_{Bi} K_{Ci} P_{3i}^{-1} P_{2i}^T L_i \\
&+ \varepsilon_{Bi}^{-2} L_i^T P_{2i} P_{3i}^{-1} K_{Ci}^T N_{Bi}^T N_{Bi} K_{Ci} P_{3i}^{-1} P_{2i}^T L_i \\
&+ \varepsilon_{Ci}^{-2} L_i^T N_{Ci}^T G_{Ci} Q_{Ci}^{-1} G_{Ci}^T N_{Ci} L_i + \varepsilon_{Ci}^{-2} L_i^T N_{Ci}^T N_{Ci} L_i
\end{aligned}$$

$$\begin{aligned}\tilde{H}_{2i} = {} & A_i + L_i^T A_i^T P_{1i} + L_i^T C_i^T K_{Bi}^T P_{2i}^T - L_i^T P_{2i} P_{3i}^{-1} K_{Ci}^T B_i^T P_{1i} - L_i^T P_{2i} P_{3i}^{-1} K_{Ai}^T P_{2i}^T \\ & + \sum_{j=1}^{N} \lambda_{ij} L_i^T [P_{1j} - P_{2i} P_{3i}^{-1} P_{2j}^T] + M_{Ai} Q_{Ai}^{-1} G_{Ai}^T N_{Ai} + L_i^T N_{Ai}^T G_{Ai} Q_{Ai}^{-1} M_{Ai}^T P_{1i} \\ & - L_i^T P_{2i} P_{3i}^{-1} K_{Ci}^T N_{Bi}^T G_{Bi} Q_{Bi}^{-1} M_{Bi}^T P_{1i} + L_i^T N_{Ci}^T G_{Ci} Q_{Ci}^{-1} M_{Ci}^T K_{Bi}^T P_{2i}^T \\ & + \varepsilon_{Ai}^2 M_{Ai} Q_{Ai}^{-1} M_{Ai}^T P_{1i} + \varepsilon_{Ai}^{-2} L_i^T N_{Ai}^T G_{Ai} Q_{Ai}^{-1} G_{Ai}^T N_{Ai} + \varepsilon_{Ai}^{-2} L_i^T N_{Ai}^T N_{Ai} \\ & + \varepsilon_{Bi}^2 M_{Bi} Q_{Bi}^{-1} M_{Bi}^T P_{1i} + \varepsilon_{Ci}^{-2} L_i^T N_{Ci}^T G_{Ci} Q_{Ci}^{-1} G_{Ci}^T N_{Ci} + \varepsilon_{Ci}^{-2} L_i^T N_{Ci}^T N_{Ci}\end{aligned}$$

$$\begin{aligned}\tilde{H}_{3i} = {} & P_{1i} A_i + A_i^T P_{1i} + P_{2i} K_{Bi} C_i + C_i^T K_{Bi}^T P_{2i}^T + \sum_{j=1}^{N} \lambda_{ij} P_{1j} + P_{1i} M_{Ai} Q_{Ai}^{-1} G_{Ai}^T N_{Ai} \\ & + N_{Ai}^T G_{Ai} Q_{Ai}^{-1} M_{Ai}^T P_{1i} + P_{2i} K_{Bi} M_{Ci} Q_{Ci}^{-1} G_{Ci}^T N_{Ci} + N_{Ci}^T G_{Ci} Q_{Ci}^{-1} M_{Ci}^T K_{Bi}^T P_{2i}^T \\ & + \varepsilon_{Ai}^2 P_{1i} M_{Ai} Q_{Ai}^{-1} M_{Ai}^T P_{1i} + \varepsilon_{Ai}^{-2} N_{Ai}^T G_{Ai} Q_{Ai}^{-1} G_{Ai}^T N_{Ai} \\ & + \varepsilon_{Ai}^{-2} N_{Ai}^T N_{Ai} + \varepsilon_{Bi}^2 P_{1i} M_{Bi} Q_{Bi}^{-1} M_{Bi}^T P_{1i} \\ & + \varepsilon_{Ci}^2 P_{2i} K_{Bi} M_{Ci} Q_{Ci}^{-1} M_{Ci}^T K_{Bi}^T P_{2i}^T + \varepsilon_{Ci}^{-2} N_{Ci}^T G_{Ci} Q_{Ci}^{-1} G_{Ci}^T N_{Ci} + \varepsilon_{Ci}^{-2} N_{Ci}^T N_{Ci}\end{aligned}$$

Setting

$$P_{1i} = X_i, \quad P_{2i} = Y_i^{-1} - X_i, \quad P_{3i} = X_i - Y_i^{-1}$$

Then we get

$$L_i = [P_{1i} - P_{2i} P_{3i}^{-1} P_{2i}^T]^{-1} = Y_i, \quad P_{3i}^{-1} P_{2i}^T = -I$$

Define

$$\mathrm{k}_{Bi} = P_{2i} K_{Bi} = [Y_i^{-1} - X_i] K_{Bi}, \quad \mathrm{k}_{Ci} = -K_{Ci} P_{3i}^{-1} P_{2i}^T L_i = K_{Ci} Y_i$$

Basing on all the previous algebraic manipulations, we can get:

$$\begin{bmatrix} \tilde{\mathbb{H}}_{1i} & \tilde{\mathbb{H}}_{2i} \\ \tilde{\mathbb{H}}_{2i}^T & \tilde{\mathbb{H}}_{3i} \end{bmatrix} < 0, \tag{34}$$

with

$$\begin{aligned}\tilde{\mathbb{H}}_{1i} = {} & A_i Y_i + Y_i^T A_i^T + B_i \mathrm{k}_{Ci} + \mathrm{k}_{Ci}^T B_i^T + \sum_{j=1}^{N} \lambda_{ij} Y_i^T Y_j^{-1} Y_i \\ & + M_{Ai} Q_{Ai}^{-1} G_{Ai}^T N_{Ai} Y_i + Y_i^T N_{Ai}^T G_{Ai} Q_{Ai}^{-1} M_{Ai}^T + M_{Bi} Q_{Bi}^{-1} G_{Bi}^T N_{Bi} \mathrm{k}_{Ci} \\ & + \mathrm{k}_{Ci}^T N_{Bi}^T G_{Bi} Q_{Bi}^{-1} M_{Bi}^T + \varepsilon_{Ai}^2 M_{Ai} Q_{Ai}^{-1} M_{Ai}^T + \varepsilon_{Ai}^{-2} Y_i^T N_{Ai}^T G_{Ai} Q_{Ai}^{-1} G_{Ai}^T N_{Ai} Y_i \\ & + \varepsilon_{Ai}^{-2} Y_i^T N_{Ai}^T N_{Ai} Y_i + \varepsilon_{Bi}^2 M_{Bi} Q_{Bi}^{-1} M_{Bi}^T + \varepsilon_{Bi}^{-2} \mathrm{k}_{Ci}^T N_{Bi}^T G_{Bi} Q_{Bi}^{-1} G_{Bi}^T N_{Bi} \mathrm{k}_{Ci} \\ & + \varepsilon_{Bi}^{-2} \mathrm{k}_{Ci}^T N_{Bi}^T N_{Bi} \mathrm{k}_{Ci} + \varepsilon_{Ci}^{-2} Y_i^T N_{Ci}^T G_{Ci} Q_{Ci}^{-1} G_{Ci}^T N_{Ci} Y_i + \varepsilon_{Ci}^{-2} Y_i^T N_{Ci}^T N_{Ci} Y_i\end{aligned}$$

$$\begin{aligned}\tilde{\mathbb{H}}_{2i} = {} & A_i + Y_i^T A_i^T X_i + Y_i^T C_i^T \mathrm{k}_{Bi}^T + \mathrm{k}_{Ci}^T B_i^T X_i + Y_i^T K_{Ai}^T [Y_i^{-1} - X_i]^T \\ & + \sum_{j=1}^{N} \lambda_{ij} Y_i^T Y_j^{-1} + M_{Ai} Q_{Ai}^{-1} G_{Ai}^T N_{Ai} + Y_i^T N_{Ai}^T G_{Ai} Q_{Ai}^{-1} M_{Ai}^T X_i \\ & + \mathrm{k}_{Ci}^T N_{Bi}^T G_{Bi} Q_{Bi}^{-1} M_{Bi}^T X_i + Y_i^T N_{Ci}^T G_{Ci} Q_{Ci}^{-1} M_{Ci}^T \mathrm{k}_{Bi}^T \\ & + \varepsilon_{Ai}^2 M_{Ai} Q_{Ai}^{-1} M_{Ai}^T X_i + \varepsilon_{Ai}^{-2} Y_i^T N_{Ai}^T G_{Ai} Q_{Ai}^{-1} G_{Ai}^T N_{Ai} + \varepsilon_{Ai}^{-2} Y_i^T N_{Ai}^T N_{Ai} \\ & + \varepsilon_{Bi}^2 M_{Bi} Q_{Bi}^{-1} M_{Bi}^T X_i + \varepsilon_{Ci}^{-2} Y_i^T N_{Ci}^T G_{Ci} Q_{Ci}^{-1} G_{Ci}^T N_{Ci} + \varepsilon_{Ci}^{-2} Y_i^T N_{Ci}^T N_{Ci}\end{aligned}$$

$$\begin{aligned}\tilde{\mathbb{H}}_{3i} &= X_iA_i + A_i^TX_i + \mathrm{k}_{Bi}C_i + C_i^T\mathrm{k}_{Bi}^T + \sum_{j=1}^{N}\lambda_{ij}X_j + X_iM_{Ai}Q_{Ai}^{-1}G_{Ai}^TN_{Ai} \\ &+ N_{Ai}^TG_{Ai}Q_{Ai}^{-1}M_{Ai}^TX_i + \mathrm{k}_{Bi}M_{Ci}Q_{Ci}^{-1}G_{Ci}^TN_{Ci} + N_{Ci}^TG_{Ci}Q_{Ci}^{-1}M_{Ci}^T\mathrm{k}_{Bi}^T \\ &+ \varepsilon_{Ai}^2X_iM_{Ai}Q_{Ai}^{-1}M_{Ai}^TX_i + \varepsilon_{Ai}^{-2}N_{Ai}^TG_{Ai}Q_{Ai}^{-1}G_{Ai}^TN_{Ai} \\ &+ \varepsilon_{Ai}^{-2}N_{Ai}^TN_{Ai} + \varepsilon_{Bi}^2X_iM_{Bi}Q_{Bi}^{-1}M_{Bi}^TX_i \\ &+ \varepsilon_{Ci}^2\mathrm{k}_{Bi}M_{Ci}Q_{Ci}^{-1}M_{Ci}^T\mathrm{k}_{Bi}^T + \varepsilon_{Ci}^{-2}N_{Ci}^TG_{Ci}Q_{Ci}^{-1}G_{Ci}^TN_{Ci} + \varepsilon_{Ci}^{-2}N_{Ci}^TN_{Ci}\end{aligned}$$

Using the expression of the controller gains given by (22), we set $\tilde{\mathbb{H}}_{2i} = 0$, and this implies that the stability condition (34) is equivalent to the following conditions:

$$\tilde{\mathbb{H}}_{1i} < 0; \qquad \tilde{\mathbb{H}}_{3i} < 0 \tag{35}$$

Let

$$\begin{aligned}\mathbb{S}_i(Y) &= [\sqrt{\lambda_{i1}}Y_i, \cdots, \sqrt{\lambda_{ii-1}}Y_i, \sqrt{\lambda_{ii+1}}Y_i, \cdots, \sqrt{\lambda_{iN}}Y_i] \\ \mathbb{Y}_i(Y) &= diag[Y_1, \cdots, Y_{i-1}, Y_{i+1}, \cdots, Y_N]\end{aligned}$$

Then we have

$$\sum_{j=1}^{N}\lambda_{ij}Y_i^TY_j^{-1}Y_i = \lambda_{ii}Y_i + \mathbb{S}_i(Y)\mathbb{Y}_i^{-1}(Y)\mathbb{S}_i^{\mathrm{T}}(Y)$$

According to the Schur complement lemma, the previous stability conditions (35) becomes (19) and (20) of Theorem 4.1.

Finally, notice that $U_i^TV_i^TP_iV_iU_i = \begin{bmatrix} Y_i & I \\ I & X_i \end{bmatrix}$, we get (21).

This completes the proof.

5. **Illustrative Example.** In this section, we shall present two examples to show the applicability of the proposed approaches.

Example 5.1. *Consider the MJLS (1) with $N = 2$, $r(t) : [0, \infty) \to \{1, 2\}$ with the following parameters:*

Subsystem 1:

$$A_1 = \begin{bmatrix} 1.2 & -0.4 \\ 0.3 & 2.0 \end{bmatrix}, \quad M_{A1} = \begin{bmatrix} 0.2 \\ 0.7 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1.2 & 0.0 \\ 0.0 & 1.2 \end{bmatrix}, \quad M_{B1} = \begin{bmatrix} 0.3 \\ 0.8 \end{bmatrix},$$

$$N_{A1} = \begin{bmatrix} 0.1 & 0.5 \end{bmatrix}, \quad G_{A1} = \begin{bmatrix} 0.2 \end{bmatrix}, \quad N_{B1} = \begin{bmatrix} 0.2 & 0.6 \end{bmatrix}, \quad G_{B1} = \begin{bmatrix} 0.2 \end{bmatrix}.$$

Subsystem 2:

$$A_2 = \begin{bmatrix} -0.3 & 0.6 \\ 0.1 & -0.23 \end{bmatrix}, \quad M_{A2} = \begin{bmatrix} 0.12 \\ 0.15 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1.2 & 0.0 \\ 0.0 & 1.2 \end{bmatrix}, \quad M_{B2} = \begin{bmatrix} 0.14 \\ 0.17 \end{bmatrix},$$

$$N_{A2} = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}, \quad G_{A2} = \begin{bmatrix} 0.2 \end{bmatrix}, \quad N_{B2} = \begin{bmatrix} 0.15 & 0.18 \end{bmatrix}, \quad G_{B2} = \begin{bmatrix} 0.2 \end{bmatrix}.$$

The transition rate matrices are chosen as:

$$\Lambda = \begin{bmatrix} -3.0 & 3.0 \\ 4.0 & -4.0 \end{bmatrix}$$

Setting $\varepsilon_{A1} = \varepsilon_{A2} = 0.5$ and $\varepsilon_{B1} = \varepsilon_{B2} = 0.5$, and solving the LMI (7) in Theorem 3.1 by using the Matlab LMI Toolbox, we get

$$X_1 = \begin{bmatrix} 0.1602 & -0.0246 \\ -0.0246 & 0.0956 \end{bmatrix}, \quad X_2 = \begin{bmatrix} 0.1393 & -0.0095 \\ -0.0095 & 0.1176 \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} -0.4820 & 0.0155 \\ 0.0155 & -0.5083 \end{bmatrix}, \quad Y_2 = \begin{bmatrix} -0.2348 & -0.0477 \\ -0.0477 & -0.2761 \end{bmatrix}.$$

The corresponding state feedback controller gains matrix can be given as

$$K_1 = \begin{bmatrix} -3.1074 & 0.6385 \\ -0.7506 & -5.5094 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -1.7230 & -0.5454 \\ -0.5060 & -2.3885 \end{bmatrix}.$$

Consequently, the state feedback controller (6) is feasible and can guarantee the stochastic stabilization of the continuous-time MJLS (1) with linear fractional uncertainties.

Example 5.2. *Consider the MJLS (2) with $N = 2$, $r(t) : [0, \infty) \to \{1, 2\}$ with the following parameters:*

Subsystem 1:

$$A_1 = \begin{bmatrix} 0.3 & -0.5 \\ 0.7 & 1.4 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1.3 & 0.0 \\ 0.0 & 1.3 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix},$$

$$M_{A1} = \begin{bmatrix} 0.14 \\ 0.28 \end{bmatrix}, \quad M_{B1} = \begin{bmatrix} 0.12 \\ 0.26 \end{bmatrix}, \quad M_{C1} = \begin{bmatrix} 0.13 \\ 0.25 \end{bmatrix},$$

$$N_{A1} = \begin{bmatrix} 0.27 & 0.13 \end{bmatrix}, \quad G_{A1} = \begin{bmatrix} 0.3 \end{bmatrix}, \quad N_{B1} = \begin{bmatrix} 0.25 & 0.12 \end{bmatrix},$$

$$G_{B1} = \begin{bmatrix} 0.3 \end{bmatrix}, \quad N_{C1} = \begin{bmatrix} 0.26 & 0.14 \end{bmatrix}, \quad G_{C1} = \begin{bmatrix} 0.3 \end{bmatrix}.$$

Subsystem 2:

$$A_2 = \begin{bmatrix} -0.5 & 0.20 \\ 0.3 & -0.12 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1.4 & 0.0 \\ 0.0 & 1.4 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix},$$

$$M_{A2} = \begin{bmatrix} 0.16 \\ 0.22 \end{bmatrix}, \quad M_{B2} = \begin{bmatrix} 0.18 \\ 0.15 \end{bmatrix}, \quad M_{C2} = \begin{bmatrix} 0.12 \\ 0.17 \end{bmatrix},$$

$$N_{A2} = \begin{bmatrix} 0.14 & 0.28 \end{bmatrix}, \quad G_{A2} = \begin{bmatrix} 0.3 \end{bmatrix}, \quad N_{B2} = \begin{bmatrix} 0.15 & 0.26 \end{bmatrix},$$

$$G_{B1} = \begin{bmatrix} 0.3 \end{bmatrix}, \quad N_{C2} = \begin{bmatrix} 0.14 & 0.25 \end{bmatrix}, \quad G_{C2} = \begin{bmatrix} 0.3 \end{bmatrix}.$$

The transition rate matrices are chosen as:

$$\Lambda = \begin{bmatrix} -3.0 & 3.0 \\ 4.0 & -4.0 \end{bmatrix}$$

Setting $\varepsilon_{A1} = \varepsilon_{A2} = 0.5$, $\varepsilon_{B1} = \varepsilon_{B2} = 0.4$ and $\varepsilon_{C1} = \varepsilon_{C2} = 0.3$, and solving the LMI (19), (20) and (21) in Theorem 4.1 by using the Matlab LMI Toolbox, we get

$$X_1 = \begin{bmatrix} 0.9611 & -0.0317 \\ -0.0317 & 0.9340 \end{bmatrix}, \quad X_2 = \begin{bmatrix} 0.9196 & -0.0533 \\ -0.0533 & 0.9066 \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} 0.2918 & -0.0267 \\ -0.0267 & 0.3040 \end{bmatrix}, \quad Y_2 = \begin{bmatrix} 0.2691 & -0.0206 \\ -0.0206 & 0.2661 \end{bmatrix},$$

$$\mathrm{k}_{B1} = \begin{bmatrix} -1.2604 & -0.3233 \\ -0.3233 & -1.9411 \end{bmatrix}, \quad \mathrm{k}_{B2} = \begin{bmatrix} -0.2868 & -0.5934 \\ -0.5934 & -0.9698 \end{bmatrix},$$

$$\mathrm{k}_{C1} = \begin{bmatrix} -0.3422 & -0.0024 \\ -0.0024 & -0.6054 \end{bmatrix}, \quad \mathrm{k}_{C2} = \begin{bmatrix} -0.0943 & -0.0603 \\ -0.0603 & -0.1574 \end{bmatrix}.$$

The corresponding dynamic output feedback controller gains matrix can be given as

$$K_{A1} = \begin{bmatrix} 0.0570 & -0.4709 \\ -0.4790 & -1.1466 \end{bmatrix}, \quad K_{B1} = \begin{bmatrix} 1.4855 & -0.3156 \\ -0.2534 & 1.4466 \end{bmatrix},$$

$$K_{C1} = \begin{bmatrix} -1.1829 & -0.1116 \\ -0.1916 & -2.0079 \end{bmatrix}, \quad K_{A2} = \begin{bmatrix} 1.3827 & -0.2908 \\ -0.2364 & 1.2817 \end{bmatrix},$$

$$K_{B2} = \begin{bmatrix} -0.0778 & -0.1720 \\ -0.1972 & -0.3169 \end{bmatrix}, \quad K_{C2} = \begin{bmatrix} -0.3700 & -0.2552 \\ -0.2709 & -0.2168 \end{bmatrix}.$$

Consequently, the dynamic output feedback controller (18) is feasible and can guarantee the stochastic stabilization of the continuous-time MJLS (2) with linear fractional uncertainties.

6. **Conclusions.** The stochastic stabilization problem for continuous-time Markovian jump linear systems with linear fractional uncertainties via switched state feedback and switched dynamic output feedback have been investigated in this paper. Two sufficient conditions have been established that guarantee the stochastic stabilization of the systems. The method presented in this paper could also be extended to the robust H_∞ control problem and filtering problem for continuous-time MJLS with linear fractional uncertainties.

Acknowledgment. This work was partially supported by the Talents Foundation of Guizhou Institute of Technology (XJZK.20131207).

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