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SYNCHRONIZATION FOR A CLASS OF CHAOTIC SYSTEMS BASED ON ADAPTIVE CONTROL DESIGN OF INPUT-TO-STATE STABILITY

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Abstract. *This paper proposes a novel adaptive controller designed method for global asymptotical synchronization of master-slave systems. The new method is based on input-to-state stability and small-gain theorem. The proposed approach gives the flexibility to construct two adaptive control laws. Compared with existing works, the merit of the adaptive controller with few parameters is that synchronization errors can reach zero field at fast speed. Finally, numerical simulation results are given to validate the design and analysis.*
Keywords: Synchronization, Adaptive control, ISS (Input-to-state stability)

1. **Introduction.** The research on the synchronization of various complex dynamical systems and chaos systems has advanced significantly in the past two decades [1-5]. For the master-slave systems, the synchronization objective is that the slave system mimics the motion of the master system [6]. Up to now, many works on the synchronization of the master-slave systems have been presented. For example, a discrete-time sliding mode control scheme is a proposed scheme for a class of chaotic synchronization systems in [7]. In [8], the sampled-data controller is designed for master-slave synchronization of Chaotic Lur'e systems with time delays by linear matrix inequality approach. The authors in [9] have designed quantized sampled-data controllers to guarantee global exponential asymptotical synchronization of master-slave systems. For robust exponential synchronization problem of a class of uncertain delay master-slave systems, adaptive method is given in [10]. Linear feedback controllers are designed for master-slave synchronization of Yassen's chaotic system in [11]. Besides, more results on the synchronization of master-slave systems can be found in [12,13].

In these papers, linear and nonlinear feedback control methods to synchronize the master-slave systems are presented. It is concluded that linear feedback control method is an effective method for synchronizing chaotic systems based on synchronization cost and error. It is well known that the synchronization cost is an important quality index in judging control method for the synchronization of master-slave systems. To the best of our knowledge, chaotic systems can be strongly sensitive to initial conditions [14], and synchronization cost can be affected greatly by different initial conditions. The above observation shows the open problem to be worth researching that how to find a design control method such that the synchronization cost and errors can be reduced. The idea of ISS (input-to-state stability) is proposed in [15], and the research about input-states stability gained many researchers favor and come up with achievements for this problem [16-20]. From these research works, we know that the states of system must be bounded with small input without considering initial states. In order to reduce synchronization cost and error, so we take measures of ISS control design idea to control the synchronization of master-slave systems. Our contributions are as follows. Firstly, the structure of controller is simplified by adopting state feedback. Secondly, our method could eliminate the synchronization cost and error of the master-slave systems. Finally, the parameters in adaptive controller are too less such that they can be implemented in engineering applications.

The paper is organized as follows. In Section 2, some definitions and problem statement are given. In Section 3, the methodology of synchronization for master-slave systems by adaptive control method is developed. Simulation example is used to demonstrate the effectiveness of proposed schemes in Section 4. Section 5 gives the conclusions of this paper.

2. **Problem Statement and Preliminaries.** Consider the nonlinear dynamical system described as follows:

$$\dot{x} = g(x, u) \tag{1}$$

where state vector $x \in R^n$, $u \in R^m$ denote input vector, and $u : [0, \infty) \to R^m$ is piecewise continuous bounded function. $\|u(\cdot)\|_\infty = \sup\limits_{t \geq 0} \|u(t)\|$, in which, L_∞^m is a set of all input vector. $g(0, 0) = 0$, and $g(x, u)$ is locally Lipschitz on $R^n \times R^m$.

Definition 2.1. [21] *System (1) is said to be input-to-state stable if there exists a class KL function $\beta(\cdot, \cdot)$ and a class K function $\gamma(\cdot)$, which is called a gain function, such that, for any input $u(\cdot) \in L_\infty^m$ and any $x^0 \in R^n$, the response $x(t)$ of system (1) in the initial state $x(0) = x^0$ for all $t \geq 0$ satisfies*

$$\|x(t)\| \leq \max\left\{\beta(\left\|x^0\right\|, t), \gamma(\|u(\cdot)\|_\infty)\right\} \tag{2}$$

Definition 2.2. [21] *For system (1), and function $V(\cdot) \in C^1$, $x \in R^n$, if there exist class K_∞ functions $\underline{\alpha}(\cdot)$, $\bar{\alpha}(\cdot)$, $\alpha(\cdot)$, and a class K function $\chi(\cdot)$ such that*

$$\underline{\alpha}(\|x\|) \leq V(x) \leq \bar{\alpha}(\|x\|) \tag{3a}$$

$$\|x\| \geq \chi(\|u\|) \Rightarrow \frac{\partial V}{\partial x} g(x, u) \leq -\alpha(\|x\|) \tag{3b}$$

the function $V(\cdot)$ is called an ISS-Lyapunov function for system (1).

Theorem 2.1. [21] *System (1) is input-to-state stable if and only if there exist two class K functions $\gamma_0(\cdot)$ and $\gamma(\cdot)$ such that, for any input $u(\cdot) \in L_\infty^m$ and any $x^0 \in R^n$, the response $x(t)$ in the initial state $x(0) = x^0$ satisfies*

$$\|x(\cdot)\|_\infty \leq \max\left\{\gamma_0(\left\|x^0\right\|), \gamma(\|u(\cdot)\|_\infty)\right\} \tag{4a}$$

$$\limsup_{t\to\infty} \|x(t)\| \leq \gamma\left(\limsup_{t\to\infty} \|u(t)\|\right) \tag{4b}$$

Remark 2.1. *By the above analysis, if system (1) is ISS stable, then there exist K_∞ functions $\underline{\alpha}(\cdot)$, $\bar{\alpha}(\cdot)$, $\alpha(\cdot)$, and K function $\chi(\cdot)$, which can guarantee Theorem 2.2 hold, gain function $\gamma(\cdot)$ can be defined as follows:*

$$\gamma(r) = \underline{\alpha}^{-1} \circ \bar{\alpha} \circ \chi(r) \tag{5}$$

Theorem 2.2. [21] ***(Small-gain Theorem)**: If $\gamma_1(\gamma_2(r)) < r$ for all $r > 0$, system (1), viewed as a system with state $x = (x_1, x_2)$ and input u, is input-to-state stable.*

Master system is considered in this paper as follows:

$$\begin{cases} \dot{x}_1 = ax_1 - x_2x_3 \\ \dot{x}_2 = -bx_2 + x_1x_3 \\ \dot{x}_3 = -cx_3 + x_1x_2 \end{cases} \tag{6}$$

where, $a > 0$, $b > 0$, $c > 0$ are control parameters, $|x_1| \le M_1$, $|x_2| \le M_2$, $|x_3| \le M_3$, and M_1, M_2, M_3 are three known constants states bounded of (6).

Slave system is chosen as:

$$\begin{cases} \dot{y}_1 = ay_1 - y_2y_3 + u_1 \\ \dot{y}_2 = -by_2 + y_1y_3 + u_2 \\ \dot{y}_3 = -cy_3 + y_1y_2 + u_3 \end{cases} \tag{7}$$

If we define synchronization errors as $e_1 = y_1 - x_1$, $e_2 = y_2 - x_2$, $e_3 = y_3 - x_3$, from master system (6) and slave system (7), the synchronization errors dynamical system are obtained:

$$\begin{cases} \dot{e}_1 = ae_1 - x_3e_2 - x_2e_3 - e_2e_3 + u_1 \\ \dot{e}_2 = x_3e_1 - be_2 + x_1e_3 + e_1e_3 + u_2 \\ \dot{e}_3 = x_2e_1 + x_1e_2 - ce_3 + e_1e_2 + u_3 \end{cases} \tag{8}$$

we denote $e = (e_1, e_2, e_3)^T$, the controller $u = (u_1, u_2, u_3)^T$ in system (8) will be designed by the following control objective.

Control objective: The main goal in this paper is to construct the adaptive controller $u = (u_1, u_2, u_3)^T$ such that the synchronization error vector $e = (e_1, e_2, e_3)^T$ is $\lim\limits_{t\to\infty} e(t) = 0$.

Assumption 2.1. Assume that function $f(e) = e_1e_2$ satisfies Lipschitz condition on a bounded set $\tilde{V}$, that is, there exists a positive constant L (maybe unknown) such that $|f(e^1) - f(e^2)| \le L\,\|e^1 - e^2\|$ for e^1, $e^2 \in \tilde{V}$.

3. **Main Results.** In this section, the following adaptive controller (9)-(11) will be used to guarantee the master system (6) and slave (7) to be synchronization.

$$u = \begin{cases} (0,0,0)^T, & \|e\| > |\rho|\,\varpi \\ (u_1, u_2, u_3)^T, & \|e\| \le |\rho|\,\varpi \end{cases} \tag{9}$$

where $u_1 = k_1e_1$, $u_2 = k_2e_2$, $u_3 = -\frac{e_1e_2}{\rho^2}$, parameters k_1, k_2 are to be designed.

Updated laws:

$$\dot{\rho} = \begin{cases} \frac{1}{2\rho\varpi^2}[\lambda + 2\,\|e\|^2\,(|a| + |b| + |c|) + 2(M_2 + |e_1|)\,|e_2| \cdot |e_3|], & \|e\| > |\rho|\,\varpi \\ -\frac{\mu_1\varpi\hat{L}}{\rho|\rho|}\,|\rho - 1|\,|e_3|\,, & \|e\| \le |\rho|\,\varpi \end{cases} \tag{10}$$

$$\dot{\hat{L}} = \begin{cases} 0, & \|e\| > |\rho|\,\varpi \\ \frac{\mu_2\varpi}{|\rho|}\,|\rho - 1|\,|e_3|\,, & \|e\| \le |\rho|\,\varpi \end{cases} \tag{11}$$

in which, μ_1, μ_2, and ϖ are given positive constants.

Theorem 3.1. *If Assumption 2.1 holds, the parameters k_1, k_2 of control gain satisfy $k_1 < -a - \frac{(\sqrt{ac}+\sqrt{bc})^2}{c}$, $k_2 < b - \frac{(\sqrt{ac}+\sqrt{bc})^2}{c}$, system (8) is global asymptotical stable by employing controller (9) with updated laws (10)-(11) if there exists a small scalar $\sigma > 0$ satisfying inequality $c_1 = c - 2\sigma$, $\delta + 2c - c_1^2 < 2$, $\varepsilon + 2c - c_1^2 < 2$, $2c - 2 - c_1^2 > 0$.*

Proof: We give the two cases controller design according to control objective.

Case (1): $\|e\| > |\rho|\,\varpi$

We adopt open-loop control and consider $s = s(e, \rho, \tilde{L}) = \|e\|^2 - \rho^2\varpi^2 + 0.5\delta^{-1}\tilde{L}^2$. It is obviously shown that $s > 0$. Consider the positive function $V = \frac{1}{2}s^2$, the derivative of V about t along (8) is obtained:

$$\begin{aligned}\dot{V} &= s\dot{s} \le s\Big\{2\|e\|^2(|a| + |b| + |c| + 2|\bar{x}_1| + \|\tilde{x}\|) \\ &\quad + 2\Big[(M_1 + |e_1|)|e_2| \cdot |e_3| - 2\rho\dot{\rho}\varpi^2 + \delta^{-1}\tilde{L}\dot{\tilde{L}}\Big]\Big\}\end{aligned}$$

From Theorem 3.1, we can get

$$\dot{V} \le -\lambda s \tag{12}$$

From the result of [22], (12) implies that the error state of the (8) can reach on the sliding surface $s = 0$ in finite times.

Case (2): $\|e\| \le |\rho|\varpi$

If Assumption 2.1 holds, the controller $(u_1, u_2, u_3)^T$ is employed in this case, then system (8) can be written as:

$$\begin{cases} \dot{e}_1 = (a + k_1)e_1 - x_3e_2 - x_2e_3 - e_2e_3 \\ \dot{e}_2 = x_3e_1 - (b - k_2)e_2 + x_1e_3 + e_1e_3 \\ \dot{e}_3 = x_2e_1 + x_1e_2 - ce_3 + e_1e_2 - \frac{e_1e_2}{\rho^2} \end{cases} \tag{13}$$

System (13) can be seen as two sub-systems, in which $\dot{e}_1$ and $\dot{e}_2$ are the first sub-system, $\dot{e}_3$ is the second sub-system. Now, the first sub-system is considered:

$$\begin{cases} \dot{e}_1 = (a + k_1)e_1 - x_3e_2 - x_2e_3 - e_2e_3 \\ \dot{e}_2 = x_3e_1 - (b - k_2)e_2 + x_1e_3 + e_1e_3 \end{cases} \tag{14}$$

where, e_1 and e_2 are to be seen as states, e_3 to be seen as input, taking the following Lyapunov function candidate:

$$V_1(e_1, e_2) = \frac{1}{2}(e_1^2 + e_2^2) \tag{15}$$

then the time derivative of $V_1(e_1, e_2)$ about t along (14), we can get

$$\begin{aligned}\dot{V}_1(e_1, e_2) &= (a + k_1)e_1^2 + (k_2 - b)e_2^2 + |e_3|(|x_1| \cdot |e_2| + |x_2||e_1|) \\ &\le (a + k_1)e_1^2 + (k_2 - b)e_2^2 + (M_1|e_2| + M_2|e_1|)|e_3|\end{aligned} \tag{16}$$

Let $\chi_1(r) = \frac{c-2\sigma}{M_1+M_2}r$, when $\|(e_1, e_2)\| = \sqrt{e_1^2 + e_2^2} \ge \chi_1(|e_3|)$ holds, then we can get the following inequality:

$$\begin{aligned}(M_1|e_2| + M_2|e_1|)|e_3| &\le \frac{M_1 + M_2}{c - 2\sigma}\sqrt{e_1^2 + e_2^2}(M_1|e_2| + M_2|e_1|) \\ &\le \frac{M_1 + M_2}{c - 2\sigma}\sqrt{e_1^2 + e_2^2}\left(M_1\sqrt{e_1^2 + e_2^2} + M_2\sqrt{e_1^2 + e_2^2}\right) \\ &= \frac{M_1 + M_2}{c - 2\sigma}(e_1^2 + e_2^2)\end{aligned} \tag{17}$$

From (16) and (17), we obtain

$$\begin{aligned}\dot{V}_1(e_1, e_2) &\le (a + k_1)e_1^2 + (k_2 - b)e_2^2 + \frac{M_1 + M_2}{c - 2\sigma}(e_1^2 + e_2^2) \\ &= \left[a + k_1 + \frac{(M_1 + M_2)^2}{c - 2\sigma}\right]e_1^2 + \left[-b + k_2 + \frac{(M_1 + M_2)^2}{c - 2\sigma}\right]e_2^2\end{aligned} \tag{18}$$

Assume that $k_1 < -a - \frac{(M_1+M_2)^2}{c-2\sigma}$, $k_2 < b - \frac{(M_1+M_2)^2}{c-2\sigma}$, then we can find a small scalar $\sigma > 0$, and a constant $\eta > 0$, which satisfies as following inequality:

$$a + k_1 + \frac{(M_1+M_2)^2}{c-2\sigma} < -\eta, \quad -b + k_2 + \frac{(M_1+M_2)^2}{c-2\sigma} < -\eta \tag{19}$$

We arrive at

$$\dot{V}_1(e_1, e_2) \leq -\eta(e_1^2 + e_2^2) \tag{20}$$

We know that K_∞ function can be chosen as $\underline{\alpha}(r) = \bar{\alpha}(r) = \frac{1}{2}r^2$, $\alpha(r) = \eta r^2$, and function $V_1(e_1, e_2)$ is defined as Definition 2.2, then $V_1(e_1, e_2)$ is called as an ISS-Lyaounov function of (14), such that sub-system (14) is stable.

Now, the second sub-system is considered:

$$\dot{e}_3 = x_2 e_1 + x_1 e_2 - c e_3 + e_1 e_2 - \frac{e_1 e_2}{\rho^2} \tag{21}$$

in which, e_3 is viewed as state, e_1 and e_2 are viewed as input, and we choose the following function:

$$V_2(e_3, \rho, \tilde{L}) = \frac{1}{2}e_3^2 + \frac{1}{2}\mu_1^{-1}\rho^2 + \frac{1}{2}\mu_2^{-1}\tilde{L}^2 \tag{22}$$

Its time derivative is

$$\begin{aligned} \dot{V}_2(e_3, \rho, \tilde{L}) &= e_3\left[x_2 e_1 + x_1 e_2 - c e_3 + e_1 e_2 - \frac{e_1 e_2}{\rho^2}\right] + \mu_1^{-1}\rho\dot{\rho} + \mu_2^{-1}\tilde{L}\dot{\hat{L}} \\ &\leq -c e_3^2 + |e_3|\left[M_2|e_1| + M_1|e_2| + \frac{\varpi\hat{L}}{|\rho|}|\rho - 1|\right] \\ &\quad + \mu_1^{-1}\rho\dot{\rho} + \mu_2^{-1}\tilde{L}\dot{\hat{L}} - \frac{\varpi\tilde{L}}{|\rho|}|\rho - 1| \cdot |e_3| \end{aligned} \tag{23}$$

From undated laws (10) and (11), the following inequality is obtained

$$\dot{V}_2(e_3, \rho, \tilde{L}) \leq -c e_3^2 + |e_3|\left(M_2|e_1| + M_1|e_2|\right) \tag{24}$$

Because of inequality $|e_1| \leq \sqrt{e_1^2 + e_2^2}$, $|e_2| \leq \sqrt{e_1^2 + e_2^2}$, let $\chi_2(r) = \frac{c-2\sigma}{M_1+M_2}r$, since $\frac{c-2\sigma}{M_2+M_1}\sqrt{e_3^2 + \rho^2 + \tilde{L}^2} \geq \sqrt{e_1^2 + e_2^2}$ implies the following inequalities hold:

$$\begin{cases} |e_1| \leq \frac{c-2\sigma}{M_2+M_1}\sqrt{e_3^2 + \rho^2 + \tilde{L}^2} \\ |e_2| \leq \frac{c-2\sigma}{M_2+M_1}\sqrt{e_3^2 + \rho^2 + \tilde{L}^2} \\ |e_3| \leq \sqrt{e_3^2 + \rho^2 + \tilde{L}^2} \end{cases} \tag{25}$$

From (25), we know that

$$\begin{aligned} |e_3|\left(M_2|e_1| + M_1|e_2|\right) &\leq |e_3|\frac{c-2\sigma}{M_2+M_1}\sqrt{e_3^2 + \rho^2 + \tilde{L}^2}(M_2 + M_1) \\ &= |e_3|\sqrt{e_3^2 + \rho^2 + \tilde{L}^2}(c - 2\sigma) \end{aligned} \tag{26}$$

From (24) and (26) such that

$$\dot{V}_2(e_3, \rho, \tilde{L}) \leq -c e_3^2 + |e_3|\sqrt{e_3^2 + \rho^2 + \tilde{L}^2}(c - 2\sigma) \tag{27}$$

we denote $\bar{c} = c - 2\sigma$, because of

$$\bar{c}|e_3|\sqrt{e_3^2 + \rho^2 + \tilde{L}^2} \leq \frac{\bar{c}^2 e_3^2 + e_3^2 + \rho^2 + \tilde{L}^2}{2} = \frac{(\bar{c}^2 + 1)e_3^2 + \rho^2 + \tilde{L}^2}{2} \tag{28}$$

then, we can obtain

$$\begin{aligned}\dot{V}_2(e_3,\rho,\tilde{L}) \leq & -(2c-1-\bar{c}^2)V_2(e_3,\rho,\tilde{L}) \\ & +\frac{1}{2}\left[1+(2c-1-\bar{c}^2)\mu_1^{-1}\right]\rho^2+\frac{1}{2}\left[1+(2c-1-\bar{c}^2)\mu_2^{-1}\right]\tilde{L}^2\end{aligned} \tag{29}$$

According to the condition of Theorem 3.1, $\mu_1+2c-\bar{c}^2<2$, $\mu_2+2c-\bar{c}^2<2$, then we get the following inequality:

$$\dot{V}_2(e_3,\rho,\tilde{L}) \leq -(2c-2-\bar{c}^2)V_2(e_3,\rho,\tilde{L}) \tag{30}$$

If we take class K_∞ functions as $\underline{\alpha}(r)=\bar{\alpha}(r)=\frac{1}{2}r^2$, $\alpha(r)=(2c-2-\bar{c}^2)r^2$, (30) implies that function $V_2(e_3,\rho,\tilde{L})$ satisfies the condition of Theorem 3.1, so $V_2(e_3,\rho,\tilde{L})$ is an ISS-Lyaounov function for sub-system (21), and we know the conclusion that system (21) is input-to-output stable.

According to (5), the gain function can be given as:

$$\gamma_1(\gamma_2(r))=\frac{c-2\sigma}{c-\sigma}r<r \tag{31}$$

Therefore, in view of Theorem 2.2, the system (8) is globally asymptotically stable. We have proved Theorem 3.1.

4. **Simulation Example.** In this section, we use adaptive controllers (9)-(11) to reveal the adaptive control conformance, the initial values of undated laws as $\rho(0)=1$, $\hat{L}(0)=0.7$, and the parameters in (9)-(11) are selected as $\varpi=20$, $\lambda=200$, $\mu_1=0.05$, $\mu_2=0.001$. The initial values of the master system (6) are chosen as $x_1(0)=1$, $x_2(0)=1$, $x_3(0)=1$, and initial values of the slave system (7) are chosen as $y_1(0)=-10$, $y_2(0)=-17$, $y_3(0)=15$. Figure 1 shows the simulation results.

Case (a): If the parameters in master-slave system (6) and (7) are chosen as $a=0.4$, $b=12$, $c=5$, the simulation results of master system (6) without any controller are shown as Figure 1 and Figure 2.

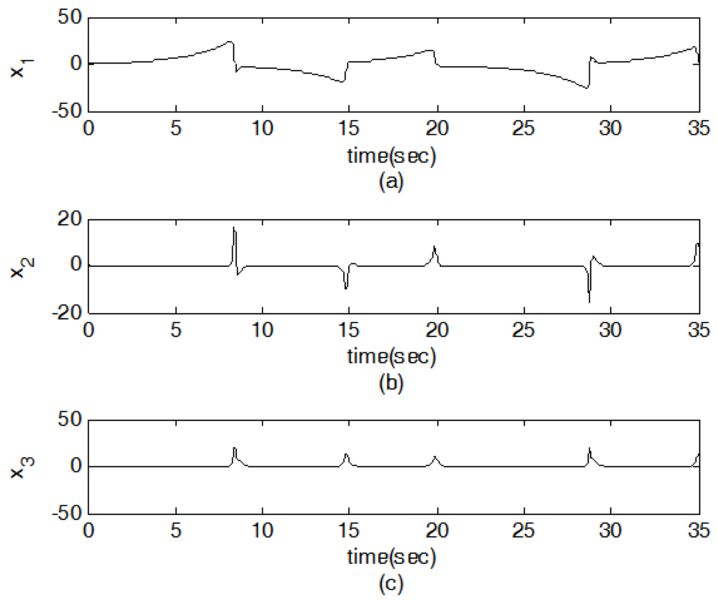

FIGURE 1. State trajectories of master system (6)

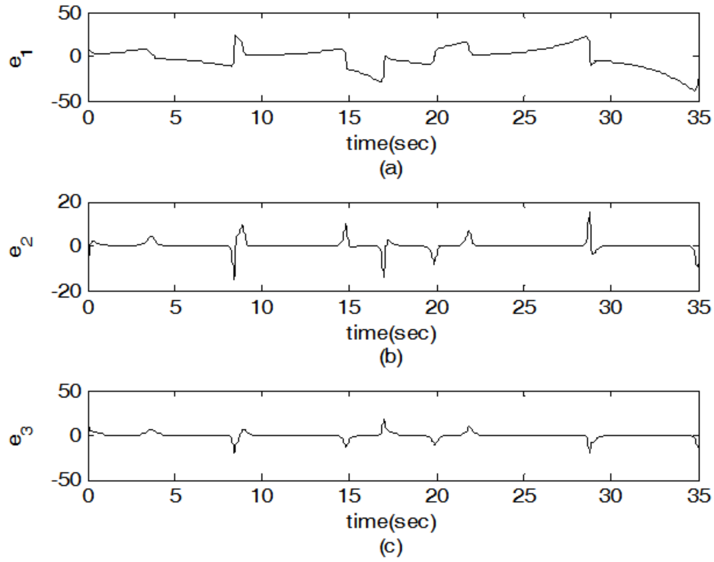

FIGURE 2. Error trajectories of master-slave system (6) and (7) without control

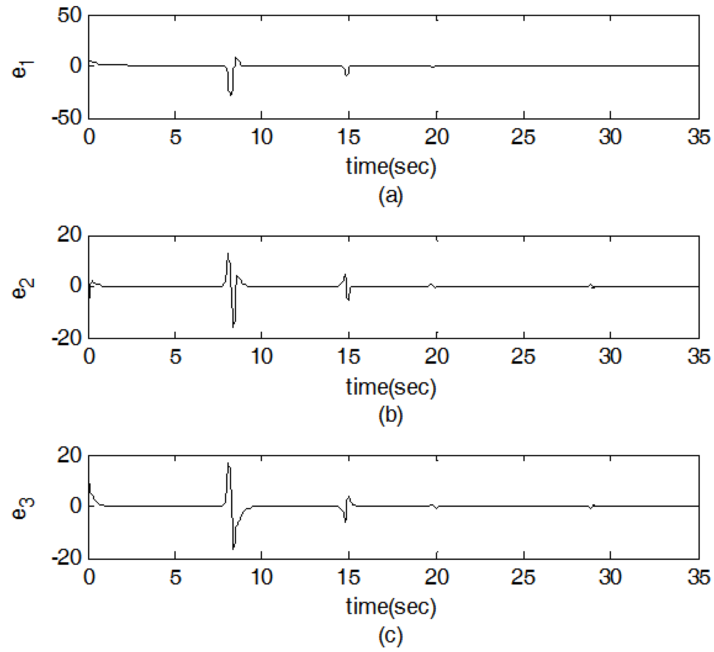

FIGURE 3. Error trajectories of master-slave system with controller in [11]

Figure 1 shows the state trajectories of the master-slave system without control input, and we know that $|x_1| \leq 50$, $|x_2| \leq 20$, $|x_3| \leq 50$. We used the first feedback controller which is proposed in [11], and the response curves of error system (8) are shown in Figure 3.

Now, the adaptive control conformance of the proposed controller (9)-(11) is employed. In view of Theorem 3.1, control gain can be chosen as $k_1 = -2500$, $k_2 = -2300$. The

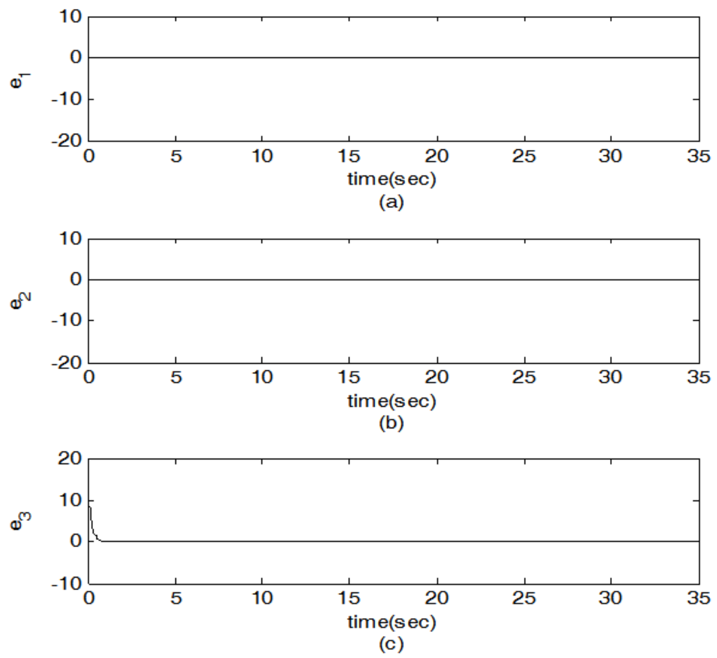

FIGURE 4. The state error trajectories of master-slave system under the action of controller (9)-(11)

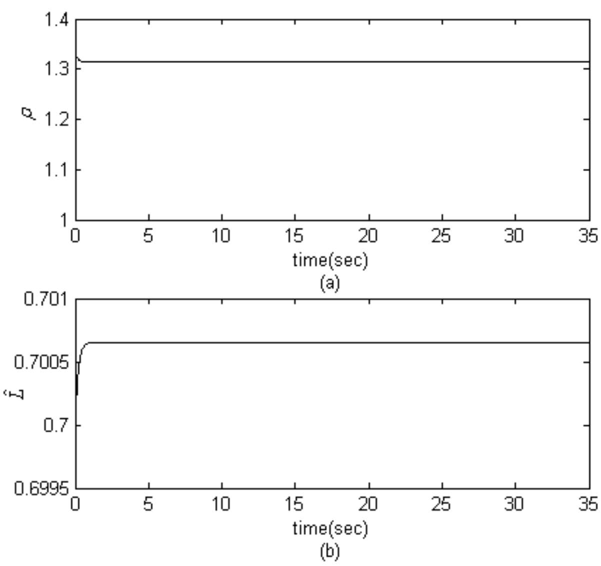

FIGURE 5. State trajectories of the updated laws in the adaptive controller (9)-(11)

synchronization errors response is shown as Figure 4. Figure 5 shows the state trajectories of the updated laws ρ and $\hat{L}$ in the adaptive controller, which are bounded.

As can be seen from Figure 3 and Figure 4, controller (9)-(11) can force the synchronization error of system (8) to reach zero at least time compared with the feedback control method in [11] under the same initial state conditions.

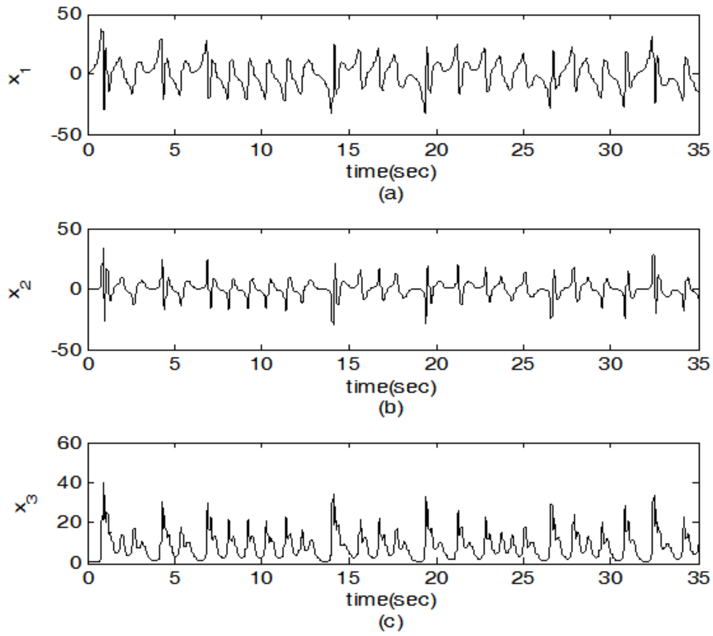

FIGURE 6. State trajectories of master system (6)

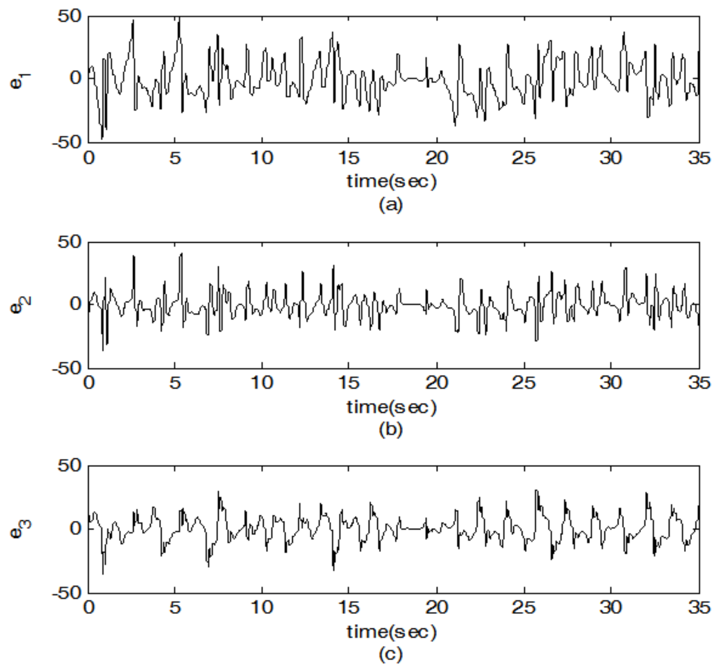

FIGURE 7. Error trajectories of master-slave system (8) without control

Case (b): In this case, we choose other parameters as $a = 4.5$, $b = 12$, $c = 5$. The simulation results of master system (6) without any controller are shown as Figure 6 and Figure 7.

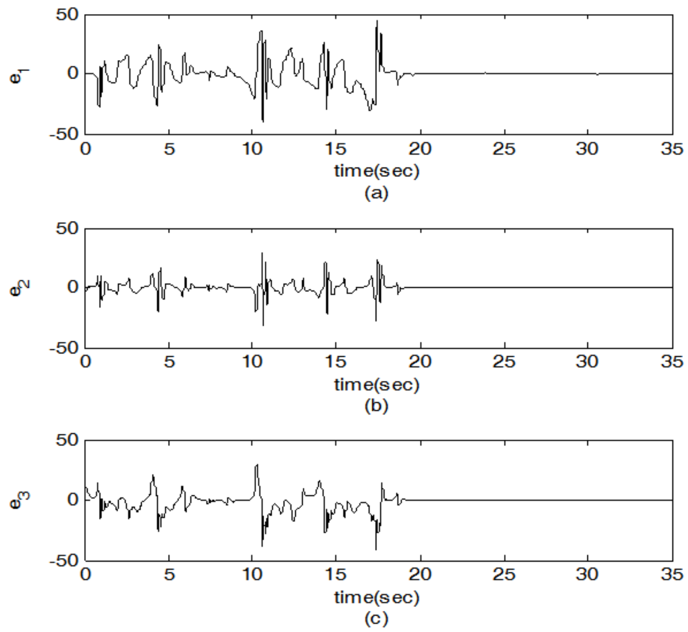

FIGURE 8. Error trajectories of master-slave system with controller in [11]

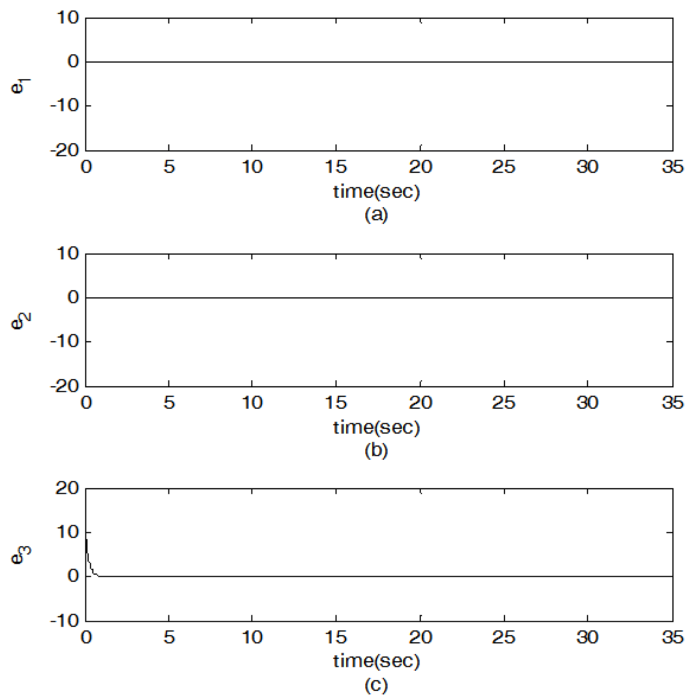

FIGURE 9. The state error trajectories of master-slave system under the action of controller (9)-(11)

The proposed feedback controller in [11] is employed at first, and Figure 3 shows the simulation results. From it we can find that the synchronization error is tending to zero at time $t = 20$.

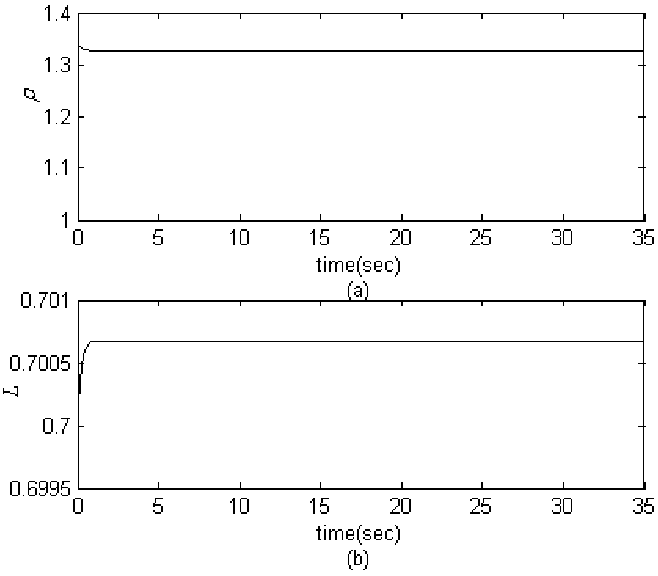

FIGURE 10. State trajectories of the updated laws in the adaptive controller (9)-(11)

From Figure 6, we know that $|x_1| \le 100$, $|x_2| \le 50$, $|x_3| \le 100$, using adaptive controller (9)-(11) with control gain $k_1 = -5000$, $k_2 = -4800$ in this paper, the synchronization errors response are given in Figure 9, and Figure 10 shows the state trajectories of the updated laws ρ and $\hat{L}$ in the adaptive controller, which are also bounded.

From Figure 8 and Figure 9, the synchronization error of system (8) with controller in [11] can tend to zero at $t = 20$, but the synchronization error of system (8) with controller (9)-(11) reach zero at fast speed in this paper, which means that the control performance is apparently improved after incorporating adaptive control method.

5. **Conclusions.** The synchronization control problem has been considered for a class of master-slave systems. We have discussed that the system possesses two kinds parameters, and used ISS stable with small-gain approach; we have proposed an adaptive control algorithm which can guarantee the master-slave system to synchronize at fast speed. The advantage of the design method in this paper lies in that the controller can reduce the synchronization cost of the master-slave systems.

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