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ON-OFF INTERMITTENCY MANAGEMENT FOR PRODUCTION PROCESS IMPROVEMENT

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Abstract. *On-off intermittency can occur in physical phenomena that form spatial distributions. In physics, the spatial distributions in physics correspond to sites that perform production activities in economics. Therefore, in the equipment manufacturing business, deviations in the monthly rate of return correspond to a spatial distribution. We propose a normalized lead time utilization which orders projects that span the duration of the production process. Moreover, by considering the long-term fluctuation, we propose an easy calculation method of production capacity. From the experience of manufacturing for many years, a sudden situation occurs; that is a delay of order materials and transportation delay, only changing production schedule is difficult in order to keep the delivery date. We present a numerical example using an actual data of annual production lead time.*
Keywords: On-off intermittency, Normalized lead time, Production capacity, Fluctuation

1. **Introduction.** We have many years of experience in studying manufacturing operations associated with control equipment for general industrial machines. In particular, we have focused on the reduction of production throughput. Business style is a complete make-to-order production system and the production process is a batch process. We take order the production if the customer's offer is acceptable. At this time, we can get a final delivery date and an amount of money.

In the previous study [1], we constructed a state in which the production density of each process corresponds to the physical propagation of heat [3]. Using this approach, we showed that a diffusion equation dominates the manufacturing process. In other words, when minimizing the potential of the production field (stochastic field), the equation, which is defined by the production density function $S_i(x, t)$ and the boundary conditions, is described using the diffusion equation with advection to move in transportation speed ρ. The boundary conditions mean a closed system in the production field. The adiabatic state in thermodynamics represents the same state [1].

The fluctuation, self-similarity and on-off intermittency occur in non-equilibrium dissipative system of physics. We have tried in many years that a production process is almost equal to a non-equilibrium dissipative system.

In the previous study of fluctuation [5], we clarified the self-similarity of fluctuations in a supply chain system and presented a size-independent mathematical model of a supply chain system using Langevin-type stochastic differential equations. We also demonstrate that for this supply chain system, when the time constant of the time correlation function possesses a uniform poisson distribution, the system exhibits f^{-1} fluctuation and when this time constant possesses a uniform distribution, the system exhibits f^{-2} fluctuation. Furthermore, the supply chain system has a Lorentzian spectrum under the condition of fluctuations having spectral density. We proposed that profit can be increased when adopting a strategy that purposefully leads to a state of excessive production or one of excessive order entries.

Regarding with phase transition, by introducing the Ginzburg-Landau free energy, we defined a parameter corresponding to an order parameter as a factor of the phase transition in manufacturing processes. Because thermal diffusion equations can be applied as mathematical models in the manufacturing process, we consider the applicability of the "Edge of Chaos", which is used in complex systems, to the manufacturing industry and the extent to which it would do so. We believe that in the manufacturing industry, the "Edge of Chaos" is a phenomenon that is caused by the loss of synchronization between the production and production throughput. The phase transition phenomenon is observed as the process throughput while manufacturing certain control equipment. As a result, by not exceeding the average value of the rate-of-return, we proposed that it was possible to maintain uninterrupted production [6]. Furthermore, self-similarity is present in the packet-queuing congestion that occurs in a manufacturing communication network. Our overall goal is to eliminate bottlenecks in such a network [7].

In this study, from long term experience of manufacturing, in the severe environment, it is a difficult situation to satisfy the customers by only changing schedule. We must consider the uncertainty risks which are an order material delay and transportation delay.

We report that on-off intermittency is observed in the monthly rate-of-return deviation for a manufacturing control system. On-off intermittency is a universal phenomenon in nonlinear dynamical systems; for example, the turbulence in flowing fluids can exhibit on-off intermittency. Based on the statistical properties of fluid turbulence, the shape of the energy spectrum is based on the assumption that energy dissipation preserves spatial uniformity [8].

We analyzed the factors that cause the rate-of-return deviation to exhibit on-off intermittency in the equipment manufacturing business. First, we integrated a self-similarity infinite linear Langevin equation. As a result, we can assume that the distribution of the rate-of-return deviation is a power-law distribution. Such a distribution is characterized by a single exponent that describes the change in the amount of coarse graining due to a scale change. This phenomenon is related to strongly correlated fluctuations in price [9].

For many years, we have come to accumulate data relating to production. We present a normalized lead time utilization which is an annual production lead time. Moreover, we provide an easy calculation method of production capacity in the production process. There are pieces of long term production equipment which need in the customer's order, that order has the fluctuation in the production process. Therefore, we must plan a production capacity to keep a delivery date. We present the numerical example of an actual annual lead time data.

By managing the on-off intermittency, we presented an actual process to compare the average value production lead time and work variance in a production process over six months. Based on our actual data, we showed the reduced variance, which led to an improved production throughput. To the best of our knowledge, this is the first study on the application of self-similarity and on-off intermittency to the manufacturing industry.

2. **Production Systems in the Manufacturing Equipment Industry.** The production methods used in manufacturing equipment are briefly covered in this paper. More

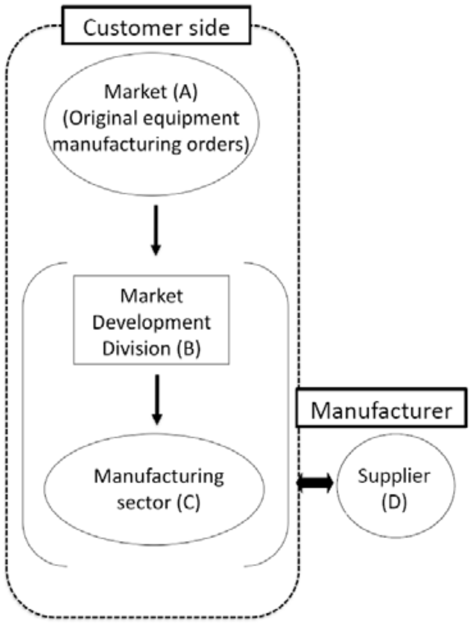

FIGURE 1. Business structure of company of research target

information is provided in our report [2]. This system is considered to be a "Make-to-order system with version control", which enables manufacturing after orders are received from clients, resulting in "volatility" according to its delivery date and lead time. In addition, there is volatility in the lead time, depending on the content of the make-to-order products (production equipment).

In Figure 1(A), the "Customer side" refers to an ordering company and "Supplier (D)" means the target company in this paper. The product manufacturer, which is the source of the ordered manufacturing equipment presents an order that takes into account the market price. In Figure 1(B), the market development department at the customer's factory receives the order through the sale contract based on the predetermined strategy.

3. **Continuous Structure of the Principal Production and Rate of Return.** Monthly revenues are deterministically calculated over each fiscal term in a company. The rate of return is also calculated over the fiscal term for the research target company. Of course, the company operates continuously in time. However, because the production itself is not independent of time, the effect is reflected in the business profitability. In this study, we consider the structure that connects the time of the principal structure and the production structure of the rate of return per period or per month.

For example, there are structures similar to the coupled electronic circuit shown in Figure 2. Assume that a voltage $E(t) = V_b + G \sin \omega t$ is applied to this circuit and that the left and right circuits are coupled by the inductance L_0. Therefore, with respect to the difference $\Delta V = V_2 - V_1$ between the voltages across the diodes V_1 and V_2, $\gamma = |\Delta V|$. Then, the coupling coefficient is defined as follows.

Definition 3.1. *Coupling coefficient*

$$\kappa = \frac{L}{L_0} \tag{1}$$

In this case, by changing the inductance L_0, γ can be confirmed using the repeated phenomenon of synchronous and asynchronous tuning. In other words, on-off intermittency is observed by breaking the tuning [10, 12, 13]. In addition, it is found that the time

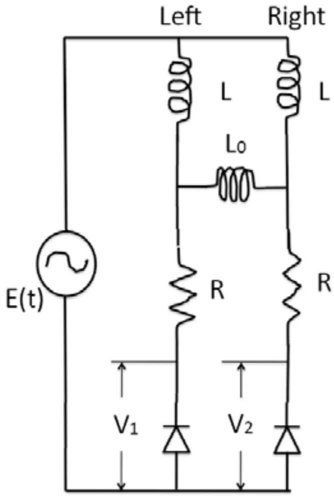

FIGURE 2. Coupled electronic circuit

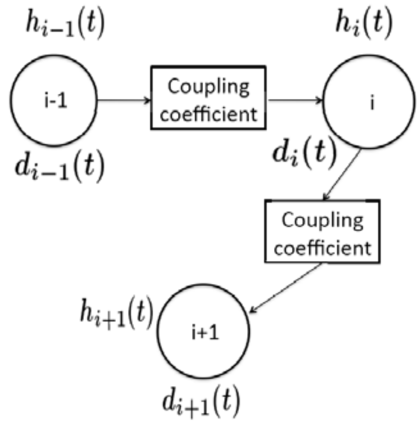

FIGURE 3. Model of continuous structure in the business period

distribution of tuning states, represented by variable r assumes a power-law distribution [10, 12, 13]. Therefore, in this study, we try to extend the coupling coefficient, represented by the above-described structure, to continuity of production and business profitability.

Figure 3 shows the continuous structure for our business model. Revenues are calculated independently on a monthly or quarterly basis. However, the production structure itself is expressed as continuous rather than time independent. Because make-to-order production is stochastic, the production structure is a stochastic process; so too is the rate-of-return over time, which is calculated on a monthly or term basis. By analyzing the monthly rate-of-return deviation as a stochastic process, we found that the cumulative distribution had a characteristic power-law distribution [4].

This suggests that the coupling elements inherent in each rate-of-return are continuous in time. Therefore, we first define the kind of coupling and then analyze the time-series data for the rate-of-return deviation. This allows us to identify the kinds of coupling that give rise to on-off intermittency. In addition, self-similar characteristics were also found.

Figure 4 shows monthly rate-of-return deviations over a two-year period. The behavior in Figures 8 to 14 can be predicted to some extent. In particular, on-off intermittent behavior is observed in the rate-of-return deviation. By introducing a self-similarity, integrated, infinite linear Langevin equation that describes the local throughput, a power-law distribution can be assumed. Figures 5 and 6 show the rate-of-return deviations and the probability density function, respectively, from our earlier study [4]. It is well known that the cumulative distribution $P(>|\Delta D|)$ obeys the power-law distribution as in (2), which indicates variations in the model such as stock prices.

$$P(>|\Delta D|) \propto |\Delta D|^{-\beta} \tag{2}$$

Thus, by fitting (2) to a power-law distribution using the observed data $|\Delta D| > 0.1$, we obtain (see Figure 6, $\mathrm{R}^2 = 0.926$).

$$P(>|\Delta D|) = 0.0008|\Delta D|^{-2.63} \tag{3}$$

In this manner, a "fluctuation model of the rate-of-return deviation" exhibits self-similar characteristics, and hence, fractal features appear in the results [8]. Such power-law characteristics are also related to the "fluctuations" that accompany phase transitions in physical systems [8].

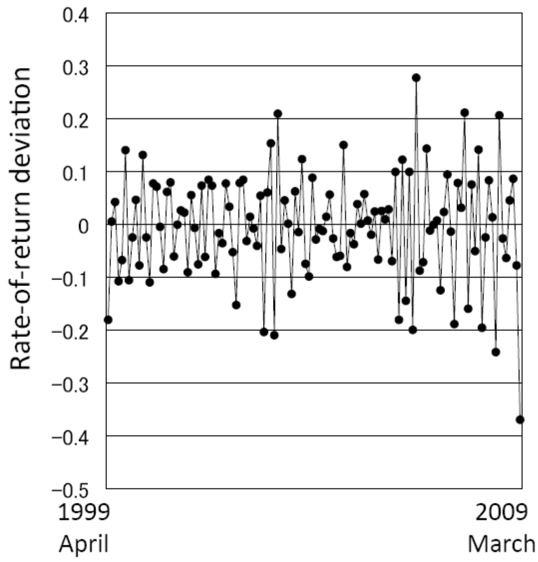

FIGURE 4. Monthly rates of return deviation

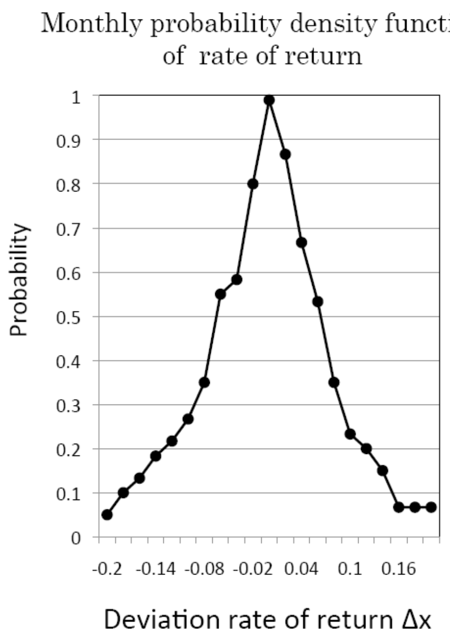

FIGURE 5. Probability density function

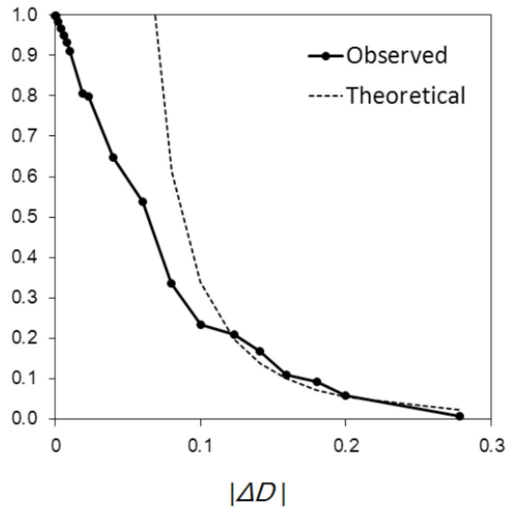

FIGURE 6. Cumulative distribution function

Figure 3 shows our model again, in which $d_i(t)$ represents the rate of return for term i.

Definition 3.2. *Rate-of-return deviation $h_i(t)$*

$$h_i(t) = d_i(t) - d_{i-1}(t) \tag{4}$$

Equation (4) represents the rate-of-return deviation for term i.

Assumption 1. *Mathematical model of the lead time $d_i(t)$*

$$\dot{d}_i(t) = \mathcal{F}(d_i(t)) + C_i(d_i(t) - d_{i+1}(t)) \tag{5}$$

where, $\mathcal{F}$ is an analytical function, $C_i(> 0)$ is a coupling coefficient. For example, using a coupling function, this coupling coefficient C_i is assumed as follows.

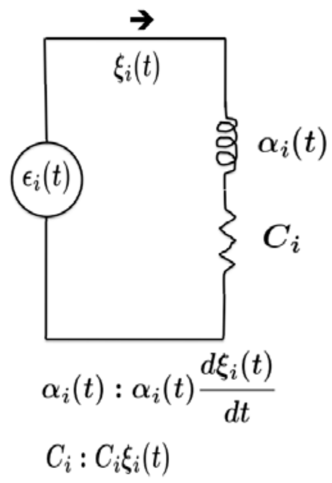

Figure 7. Equivalent circuit of the coupling function

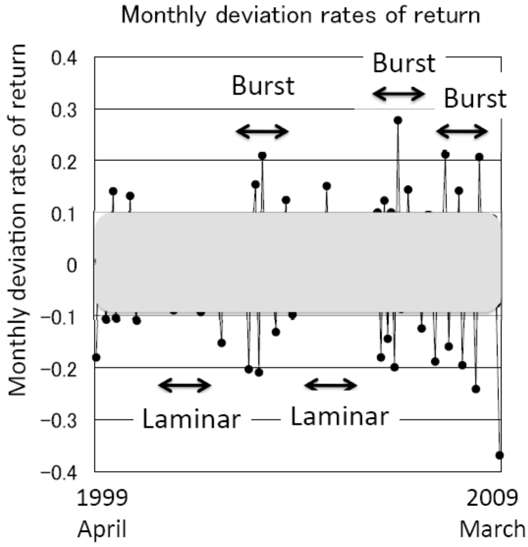

Figure 8. On-off intermittency during the laminar and burst states

Assumption 2. *Mathematical model of the coupling function* $\xi_i(t)$

$$\alpha_i \frac{d\xi_i(t)}{dt} + C_i\xi_i(t) = \epsilon_i \tag{6}$$

where, the coupling function $\xi_i(t)$ *is coupled by the differential form (6), and it is assumed that* $h_i(t)$ *is mapped by (7).*

Assumption 3. *Mapping form of lead time deviation* $h_i(t)$

$$\tilde{h}_i(t) = f\left(\tilde{d}_i(t)\right) + \xi_i(t)\left[\tilde{d}_i(t) - \tilde{d}_{i+1}(t)\right] \tag{7}$$

That is, Figure 7 indicates the coupling form. $\xi_i(t)$ is represented by the coupling parameter that is calculated with Equation (8).

$$\xi_i(t)\Big|_{t=T} = \frac{1}{C_i}\left(\exp\left(-\frac{C_i}{\alpha_i}T\right) + \epsilon_i\right) \tag{8}$$

Then, coupled chaotic mappings are obtained, such as Poincare mappings [10, 14].

Therefore, the lead time per month or per term has differential elements as the coupling parameters.

It can be seen that the rate-of-return deviation $h_i(t)$ is described by

$$h_i(t) \sim \exp(n\zeta_n)h_i(0) \tag{9}$$

where, ζ_n indicates strong fluctuations around the average value of $h_i(t)$.

For this reason, when $\zeta_n > 0$, the fluctuations burst, and then $h_i(t)$ grows large. However, when $\zeta_n < 0$, the fluctuations stop [10, 11].

Here, $h_i(t)$ is [10, 16]:

$$h_i(t) = h_{i_0}(t) + \delta h_i(t) \tag{10}$$

That is, to examine the continuous transitions in on-off intermittency, fluctuations in the local transverse Lyapunov exponents which set $\Lambda_\perp$ are required [10]. Then, Equation (11) is:

$$\frac{dh_i(t)}{dt} = \Lambda_\perp h_i(t) - \beta h_i^2(t) \tag{11}$$

Here, $\Lambda_\perp$ is:

$$\Lambda_\perp = \lambda_\perp + f(t) \tag{12}$$

where $\lambda_\perp$ is a cross Lyapunov exponent [10, 11, 16] and $f(t)$ is assumed as follows.

Assumption 4. *Assume that $f(t)$ is Gaussian white noise.*

$$< f(t) > = 0 \tag{13}$$

$$< f(t) \cdot f^{'}(t) > = 2D\delta\left(t - t^{'}\right) \tag{14}$$

where the Fokker-Planck equation for Equation (11) is represented by the following equation [10, 16].

$$\frac{\partial P(h,t)}{\partial t} = -\frac{\partial}{\partial h}[(\lambda_\perp - \beta h + D)hP(h,t)] + \frac{\partial^2}{\partial h^2}(h^2 P(h,t)) \tag{15}$$

The steady-state solution to Equation (15) is represented by the following equation when $\lambda_\perp > 0$.

$$P(h) \cong h^{-1+\eta} \exp\left[-\frac{\beta h}{D}\right] \tag{16}$$

When h is small value, we obtain

$$P(h) \sim h^{-1+\eta}, \quad \eta = \frac{\lambda_\perp}{D} \tag{17}$$

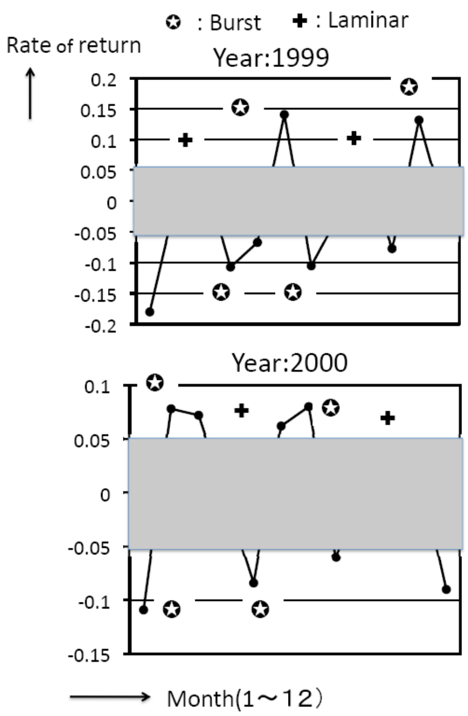

Figure 9. Self-similarity during the laminar and burst states (1999-2000)

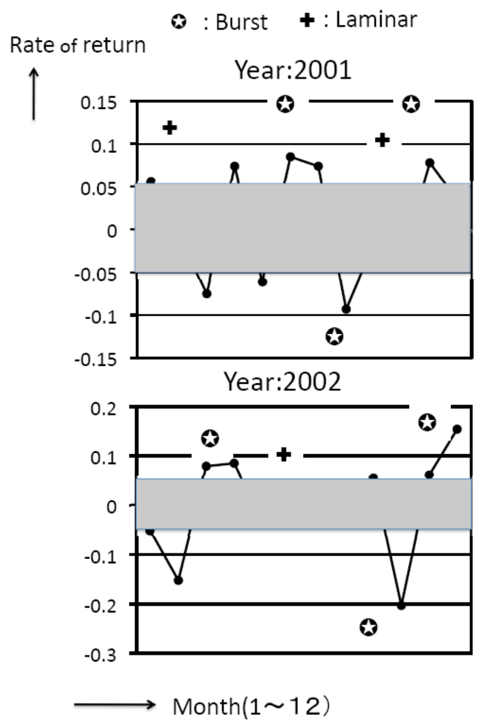

Figure 10. Self-similarity during the laminar and burst states (2001-2002)

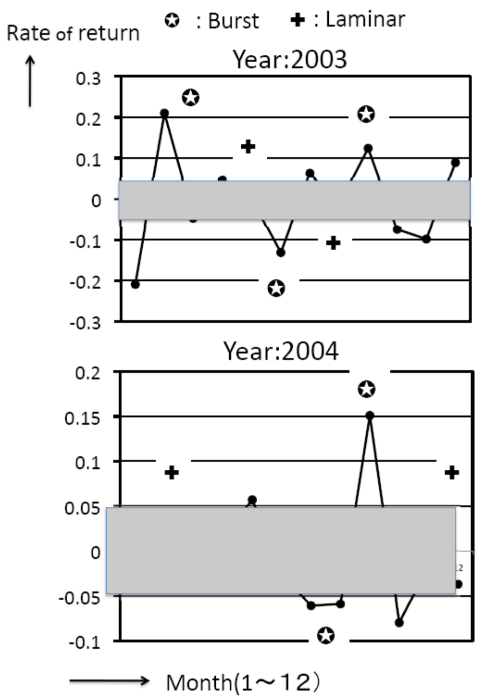

FIGURE 11. Self-similarity during the laminar and burst states (2003-2004)

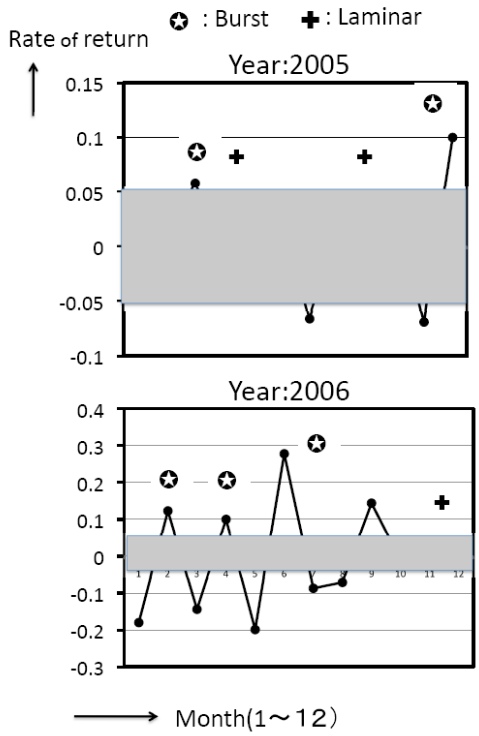

FIGURE 12. Self-similarity during the laminar and burst states (2005-2006)

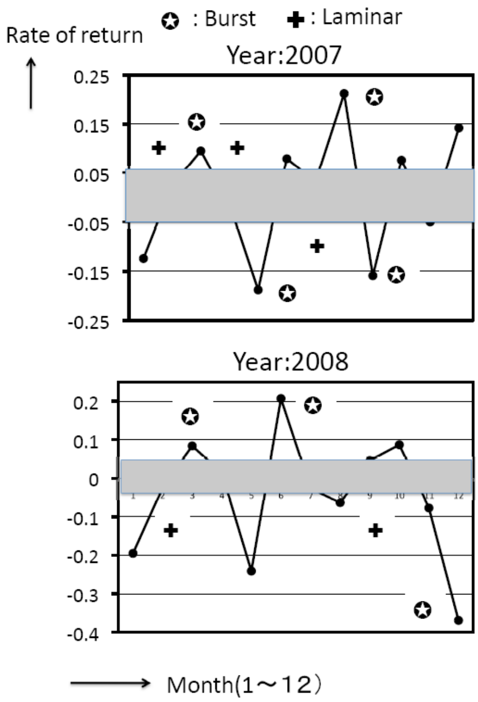

FIGURE 13. Self-similarity during the laminar and burst states (2007-2008)

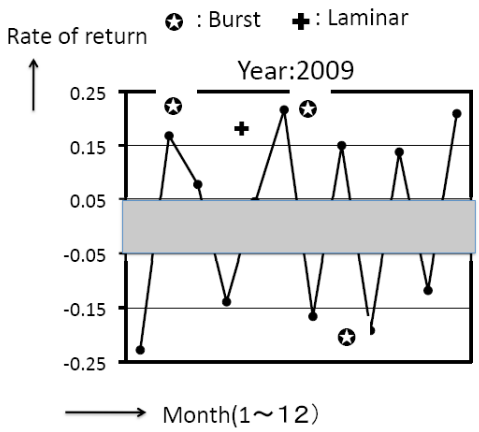

FIGURE 14. Self-similarity during the laminar and burst states (2009)

Here, according to previous results, the mathematical model for the rate-of-return deviation is represented by

$$\frac{dh_i(t)}{dt} + r_i(t)h_i(t) = \tilde{f}(t) \tag{18}$$

Now, the rate-of-return deviation depending on period k is set to h_{i_k}. The power-law characteristics for the moment of h_{i_k} are described by

$$S_q^{h_{i_k}}(t) =< h_{i_k}^q >\sim k^{\tau(q)} \tag{19}$$

where $< h_{i_k}^q >$ represents the "fluctuation" in the rate-of-return deviation per period and $\tau(q)$ is a characteristic function. Here, Equation (19) represents the periodic existence of self-similarity in the lead time deviation. Then, $S_q^{h_i}(k)$ is the function that characterizes the statistical properties of self-similar processes for revenue. In other word, the behavior of revenue processes suggests the existence of some kind of revenue potential cascade. It is to be understood by considering the coupling coefficient as described above.

It is known that the relationship between $S_q^{h_{i_k}}(t)$ and $\tau(q)$ in Equation (19) becomes a Legendre transformation. $S_q^{h_{i_k}}(t)$ is approximated by a parabolic function, such as $S(z)$ in following equation [8].

$$S(z) = \frac{1}{2\mu}\left(z + \frac{\mu}{2}\right)^2 \tag{20}$$

where μ is called as the intermittency index [8].

Then, the Legendre transformation of $S(z)$ is defined as follows.

Definition 3.3. *Legendre transformation of $S(z)$*

$$\tau(q) = \min_z\{S(z) - q_z\} \tag{21}$$

where as shown in Figure 9-Figure 14, the self-similarity representing the on-off intermittency is described as follows.

$$t_n = Te^{-n} \ \forall n = 0, 1, 2, \cdots, \quad N\left(\equiv \ln\left(\frac{A_{n+1}}{A_n}\right)\right) \tag{22}$$

Here $n = 1, 2, \cdots$, T is the correlation time related to generating on-off intermittency. In addition, the following equation is introduced for the fluctuation index.

$$e^{-z_n} = \left(\frac{A_{t_{n+1}}}{A_{t_n}}\right) \tag{23}$$

The statistical properties of index z_n assume a stationary state that does not depend on the coarse-grained scale step n. It assumes that the correlation step of z_j is sufficiently short compared to the N, where A_{t_n} and the average of $\overline{z}_n$ are as follows.

On solving Equation (22) and Equation (23), the following equations are obtained.

$$At_n = A_Te^{-n\overline{z}_n} \tag{24}$$

$$\overline{z}_n = \frac{1}{n}\sum_{j=0}^{n-1} z_j \tag{25}$$

Now, the following equation for the logarithmic lead time is defined as follows.

Definition 3.4. *Logarithmic lead time*

$$h_t(s) = \ln\left(\frac{P(s)}{P(s-t)}\right) \tag{26}$$

where $P(s)$ represents a lead time at time s.

Then, the logarithmic lead time deviation is defined as follows.

Definition 3.5. *Logarithmic lead time deviation*

$$\eta_k(s) \equiv \left| \sum_{k=0}^{t/\Delta t-1} h_t(s - k\Delta t) \right| = |h_t(s)| \tag{27}$$

where, to formulate the similarity of the fluctuation, z_n is introduced as follows.

Definition 3.6. *Definition of z_n*

$$z_n = \ln\left(\frac{\eta_{n+1}}{\eta_n}\right) \tag{28}$$

$$\frac{\eta_{n+1}}{\eta_n} = e^{z_n} \tag{29}$$

Assuming that z_n is stationary with respect to n, and that it has a short correlation step, the following equation is obtained.

$$\eta_t \sim t^{-\overline{z}(t)} \tag{30}$$

Considering the index z_j as stationary, and taking advantage of the large deviation statistics, we obtain the following equation [10].

$$z_t(\eta) = \frac{\ln(\eta/\eta_T)}{\ln(T/t)} \tag{31}$$

The asymptotic distribution of $z_t(\eta)$ assumes the convex function with beneath, and then, it is proportional to $(T/t)^{-S(z)}$.

Here, the probability density function is

$$P_t \cong a^{-1}\left(\frac{T}{t}\right)^{-S(z_t(a))} \tag{32}$$

$$z_t(a) = \frac{\ln(a/A_t)}{\ln(T/t)} \tag{33}$$

where A_t is the burst strength of intermittency. We obtain

$$A_t = \frac{1}{t}\int_0^t h_k(s)ds \tag{34}$$

where, when $n \gg 0$, the distribution of $\overline{z}_n$ is obtained as follows with respect to $S(z)$.

$$Q(z) \cong e^{-S(z)n} \tag{35}$$

where, because the distribution of ξ_k is the normal distribution of log-normal type, we obtain

$$S_q^h =< h_k >\sim k^{\tau(q)} \tag{36}$$

$$P(h_k) = \frac{1}{\sqrt{2\pi}\sigma_k h_k} \exp\left\{-\frac{(\ln h_k - m_k)^2}{2\sigma_k^2}\right\} \tag{37}$$

where, the average m_k and volatility σ_k are as follows.

$$m_k = \ln h_k - \frac{1}{2}\sigma_k^2$$

$$\sigma_k^2 = A + \mu \ln\frac{L}{k} \tag{38}$$

where, L represents the end of project evaluation.

Therefore, we obtain

$$\begin{aligned} < h_k^q > &= \int_0^\infty h_k^q \cdot P(h_k) dh_k \\ &= \exp\left\{ q \cdot m_k + \frac{q^2\sigma_k^2}{2} \right\} \\ &\approx h_k^{-q} \cdot \left(\frac{L}{k}\right)^{\mu q(q-1)/2} \\ &\equiv k^{\tau(q)} \end{aligned} \tag{39}$$

From this, we obtain

$$\tau(q) = \frac{q}{3} + \frac{\mu}{2} q(q-1) \tag{40}$$

On the other hand, when the Legendre transformation is performed, we obtain

$$\tau(q) = \max_q (qz - S(z)) \tag{41}$$

From this, we obtain

$$q = \frac{dS(z)}{dz} \tag{42}$$

In addition, we obtain

$$\frac{d\tau(q)}{dq} = z(q) \tag{43}$$

From this, $S(z)$ is described as follows.

$$S(z) = \int_0^z q dz \tag{44}$$

Therefore, because h_k follows the log-normal distribution, we obtain

$$S(z) = \frac{1}{2\mu}\left(z + \frac{\mu}{2}\right)^2 \tag{45}$$

Therefore, calculating $\tau(q)$ and $S(z)$ under appropriate parameters, $S(z)$ looks like Figure 15.

Now, the model equation of the growth rate of $h(t)$ is defined as follows.

Definition 3.7.

$$\frac{dh(t)}{dt} = \{r + \xi(t)\} h(t) \tag{46}$$

$$< \xi(t) \cdot \xi(t') > = 2s\delta\left(t - t'\right) \tag{47}$$

where r represents a trend coefficient, and $\xi(t)$ represents a stochastic variable.

At this time, the distribution of h satisfies the Fokker-Planck equation as follows.

$$\frac{\partial P(h,t)}{\partial t} = -\frac{\partial}{\partial h}(rhP(h,t)) + \frac{\partial^2}{\partial h^2}(sh^2 P(h,t)) \tag{48}$$

This is called as the Stochastic Autocatalytic Process with Diffusion, or "SAPD" [15]. Thus, to characterize the intermittent field $h(t)$, we introduce kth order moments as the follows.

$$\begin{aligned} \frac{d}{dt} < h(t)^k > &= \frac{d}{dt} \int dhh^k P(h,t) \\ &= [kr + (k^2 - k)s] < h(t)^k > \end{aligned} \tag{49}$$

k-th order moment

$$M_k(t) =< h(t)^k >\sim \exp[kht + (k^2 - k)st] \tag{50}$$

Thus, the first order moment is

$$M_1(t) =< h(t) >\sim \exp(kht) \tag{51}$$

Therefore, the intermittent index and the index of the lead time are as follows.

$$\lambda_k = kh - (k^2 - k)s \tag{52}$$

$$\Lambda_k = (k^2 - k)s \tag{53}$$

As described above, when $\Lambda_k \neq 0$, the time evolution of the distribution h becomes intermittent. In this case, Λ_k is defined as follows.

Definition 3.8. *Index of intermittency*

$$\frac{M_k(t)}{M_1(t)} \sim \exp[\Lambda_k(t)] \tag{54}$$

$$\Lambda_k = \lim_{t\to\infty} \frac{1}{t} \ln \left[\frac{M_k(t)}{M_1(t)^k}\right] \tag{55}$$

Therefore, using the statistic weight ratio $P(z,t)$ for the appropriate coefficient $C(z,t)$, we obtain

$$P(z,t) = \int_0^\infty C(z,t) \exp[-tS(z)] \approx C_0(z,t) \exp\Big[-tS(z)\Big] \tag{56}$$

Here, $C_0(z,t)$ is an initial value, $S(z)$ is a convex function in which the fluctuations z in finite time t become zero at the point where it matches the true expected value.

Alternatively, $M_k(t)$ can be approximated by

$$M_k(t) \sim \exp(\lambda_k t) \tag{57}$$

In accordance with (41), the Legendre transformation of λ_k is

$$\lambda_k = \max_z[kz - S(z)] \tag{58}$$

Therefore, the inverse Legendre transformation is

$$S(z) \sim \frac{(z+s)^2}{4s} \approx \exp\left[-S(\tilde{z}) \cdot t\right] \tag{59}$$

From this result, Equation (59) matches Equation (45). Figure 15 shows Equation (59). Considering the graph of $S(z)$ in Figure 15, we see that the fluctuations z in finite time t are converging to the vicinity (0.366) of the true expected value. Figure 16 shows the normalized lead time by the period batch production process on time series. The delay of lead time is conspicuous by the value in the time series outset 10, 34, 37, 47 in Figure 16. The standardized lead time represents (Lead time/25). Figure 17 shows the normalized lead time deviation adjacent on time series outset. The fluctuation average data exists at around 20 of time series outset in Figure 18 and Figure 19.

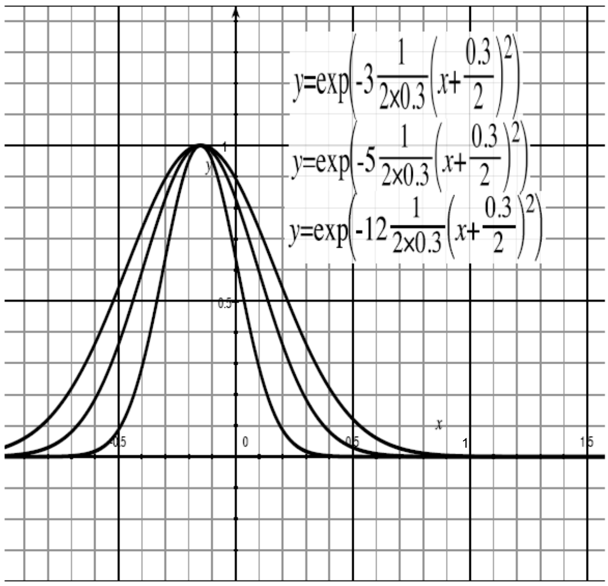

FIGURE 15. Fluctuation spectrum on-off intermittency

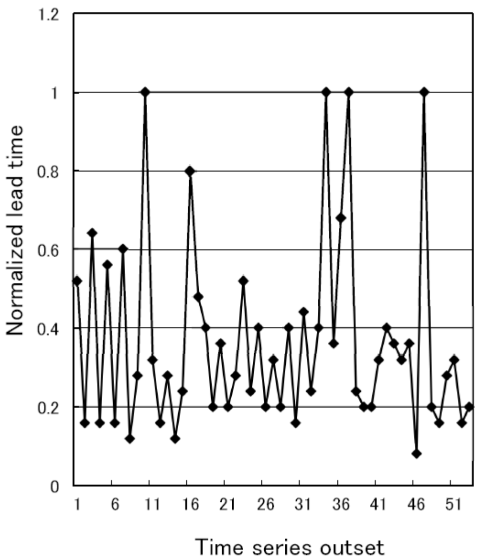

FIGURE 16. Normalized lead time by the period batch production process

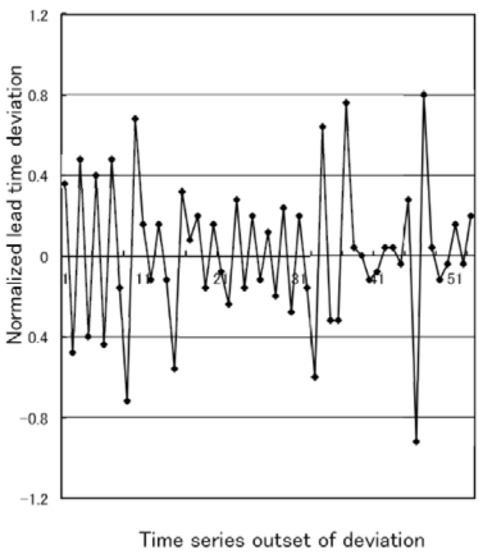

FIGURE 17. Normalized lead time deviation by the period batch production process

TABLE 1. Standardization lead time/variance before reclassification of production process

Lead time	Variance
9.67	0.236

4. **On-Off Intermittency Verification by Actual Data.** The current business style is a complete make-to-order production system and the production process is a batch process. Figure 18 and Figure 19 show the deviation of the lead time of a batch production standardization. C1, C2, D1, D2 in Figure 20 become C1', C2', D1', D2' by moving the

Table 2. Standardization lead time/variance after reclassification of production process

Lead time	Variance
9.67	0.165

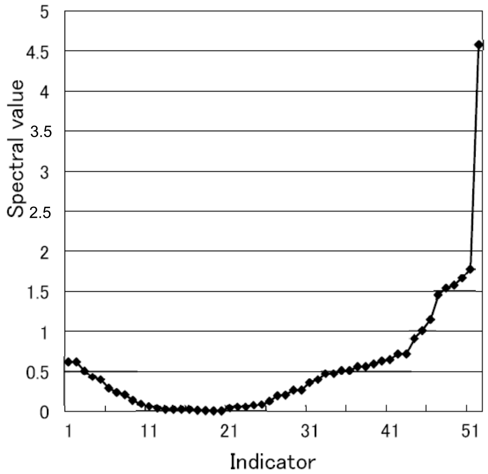

Figure 18. Fluctuation spectrum on the batch production normalized lead time

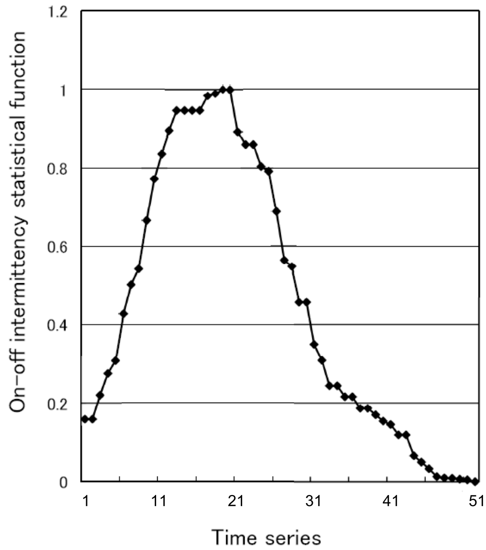

Figure 19. On-off intermittency statistical function $\exp(-t \cdot S(\Phi))$ on the batch production normalized lead time

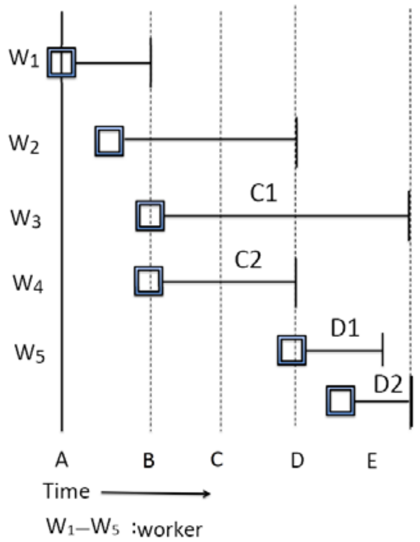

Figure 20. Process before managing of on-off intermittency

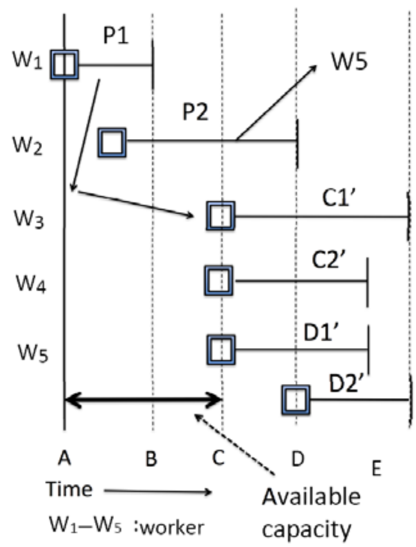

Figure 21. Process after managing of on-off intermittency

work start time respectively. P1 in Figure 21 represents the movement of the working power to W3. P2 in Figure 21 represents the movement of the working power to W5.

Therefore, Figure 20 and Figure 21 show the production processes before and after on-off intermittency, respectively. Our strategy was to change the start time of the production and reduce the worker's variance, as shown in Figure 21. After reconstructing the process shown in Figure 21, we could handle sudden orders by appropriately managing the available manpower and prevented opportunity loss. As a result, we increased monthly shipments.

From Table 2, no change was observed in the actual lead times. However, the variance is reduced. In the process diagram, the time allocated for the operator is the first half of the entire process. We could offer a sudden customer support for the production. As a result, the production throughput was also improved.

5. **Calculation of the Spectral Density and the Weight Rate of Fluctuations for the Normalized Lead Time.** The actual manufacturing is across the two periods in Figure 22. In Figure 22, there is about 24 days which is the longest lead time in one period. In case of production in uncertainty situation, it is an important issue to decide the production capacity by any method. Therefore, we propose a method such as the following.

In accordance with Equations (28) and (29), we obtain:

$$z_n = \ln\left[\frac{h_T^{n+1}}{h_T^n}\right] \tag{60}$$

$$\exp(z_n) = \left[\frac{h_T^{n+1}}{h_T^n}\right] \tag{61}$$

From Equations (60) and (61),

Definition 5.1. *On-off intermittency indicator*

$$h_T^n = \frac{\tilde{h}_T^n}{h_{LT}} \tag{62}$$

where h_{LT} is a longest lead time, and $\tilde{h}_T^n$ is the lead time of the nth project.

In accordance with Equation (45), the lead time spectrum is represented by

$$S(\tilde{z}_n) = \frac{1}{2}\left(\tilde{z}_n + \frac{1}{2}\right) \tag{63}$$

where $\mu = 1$, $\tilde{z}_n$ is the ordinal data of z_n, and represents a relative indicator (See Equations (24) and (25)). Moreover, in accordance with Equation (56),

$$P(\tilde{z}_n, t) \approx \exp\left[-t \cdot S(\tilde{z}_n)\right] \tag{64}$$

We obtained the spectral density and weight rate using the annual project data in Figures 16-19. From Figure 18 and Figure 19, we can see that the spectral fluctuation value at

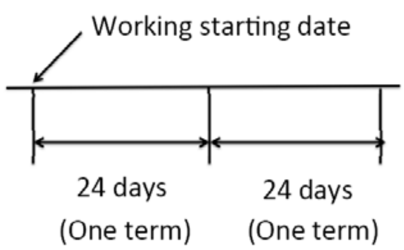

FIGURE 22. Actual work across the two periods

around time $t = 20$ is converging to the true average value. In other word, the lead time fluctuates around the average value.

Therefore, from the lead time average value $\tilde{z}_n$ on fluctuations, $\tilde{z}_n = 0.366$ is derived in Figure 18 and Figure 19. Moreover, using this $\tilde{z}_n = 0.366$, a production capacity in consideration of the fluctuation is derived approximately:

$$S \simeq \frac{\sum_i h_T^i}{\left[h_T^n\right] \cdot h_{LT}} = N \cdot \left[h_T^k\right] + \alpha \equiv kN \cdot \left[h_T^k\right] \tag{65}$$

where h_T^i is the actual lead time, $[h_T^n]$ is the average of fluctuations and h_{LT} is the longest lead time.

The total lead time of annual data h_T^N is 485. From $z_n = \ln\left[h_T^{k+1}/h_T^k\right]$, $h_T^k = h_{LT} \cdot \exp(\bar{z})$. $h_T^k = h_{LT} \cdot \exp(\bar{z}_n) = 60 \times 1.442 = 86.52$, because $h_T^k = \exp(\bar{z})$.

1. Long term manufacturing order

$$S \simeq k \cdot \frac{h_T^N}{h_T^k} = k \cdot \frac{485}{86.52} \approx k \times 5.6 \simeq [5 \sim 6]/month, \text{ where } k = 1$$

2. Short term manufacturing order

$$S \simeq k \cdot \frac{h_T^N}{h_T^k} = k \cdot \frac{61}{1.442 \times 20} = k \times 2.12 \simeq [2 \sim 3]/month, \text{ where } k = 1$$

From above data, the calculation method of production capacity can handle both of No.1 and No.2. We can provide the number of product and the number of worker in advance.

In the production business with an uncertain element, uncertainty risk depends on the ambiguity of the design information which orders from customer. Therefore, the production starting date is not fixed. Manufacturers had to allow it until now. However, from the extremely strict requirements of the market, secure profits are becoming difficult at present. Therefore, we think the simple calculation of production capacity is a useful technique based on mathematical approach greatly.

6. **Conclusion.** On-off intermittency exists in the rate-of-return and lead time deviations of production processes. In physics, an on-off intermittency is present in case of power-law distributions, phase transitions, and self-similar phenomena. In the production process described in this study, we observed on-off intermittency on a lead time data with respect to time series outset.

To change the production process schedule, the lead time volatility was reduced. With respect to the lead time spectrum, we calculated the production capacity in consideration of lead time fluctuation.

Thus, in case of having strong stochastic elements, we can reduce the uncertainty risks by analyzing the stochastic process and can construct a system that can reduce the uncertainty.

In this study, we analyzed the actual lead time data and found that the on-off intermittency existed in the lead time on time series outset. To explore the production process improvement from this phenomenon is very significant and our study has originality.

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