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RESEARCH OF PERIODIC OSCILLATIONS IN CONTROL SYSTEMS WITH FUZZY CONTROLLERS

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Abstract. *In this paper, the problem of the detailed analysis of the nonlinear transformations in fuzzy regulators is solved. As example, the non-linear transformations, using Mamdani model of fuzzy controller are presented. Much attention is paid to the development of a method of harmonious balance for research of periodic oscillations in the intelligent systems of automatic control on the base of fuzzy logic technology. And the examples and simulation results are shown to verify the effectiveness of the method. Furthermore, for many industrial applications it may be acceptable to use simplified versions of the controller, in particular fuzzy P-controller. For this case, a simplified research procedure, based on the method of harmonic balance with simulation result is given.*
Keywords: Controller on the basis of fuzzy logic, Fuzzy controller, Nonlinear transformations in fuzzy controller, Research of periodic oscillation, Method of harmonious balance, Intelligent systems of automatic control

1. Introduction.

With perfection of various types of engineering technologies, both for civilian and military purposes, the requirements to control systems on different levels of leadership, including executive, become more stringent.

It should be emphasized that in many cases of advanced technology, where high speed and running accuracy are required (servo drives, guidance system, items of microsystem technology, etc.), traditional methods can no longer provide control of the highest quality, especially under the influence of different uncertainties acting on a system. These uncertainties include, for example, changing kinematic relations while changing the configuration of various multi-jointed electromechanical devices; temperature changes in the coefficients of viscous friction; size changes in the back play and backlash of the driver transmission elements; mismatch in dynamic characteristics of the power switches for required frequency range; various types of perturbations, which both act upon the shaft of operating motor and its input, etc.

As shown in foreign [1,12] and domestic studies, both theoretical and experimental [2-4], application of intelligent control technologies can provide a significant increase in response time and other characteristics of control process: invariance to the exposure of disturbing factors, improvement of the energy indicators in the system, etc. In cases of such objects, it would be efficient to use control systems with intellectuality in small [4]. This way, formation of the control algorithm can be carried by the intelligent controller (IC), which connects in series with the object in the circuit of the system (controller of sequential operation).

Naturally, the question arises about the impact of such a regulator on the system's dynamics. It should be emphasized, that although the "fuzziness boom" began in the

80's of the last century, more or less successful verification methods of the fuzzy systems' stability were received at the end of the last decade [1]. Moreover, some of them were borrowed from the classical control theory and were finalized thereafter.

Numerous studies are dedicated to understanding of the dynamics of the fuzzy control systems. As these systems represent a specific class of nonlinear systems, it is natural that to research these systems, experts attract methods that are widely spread in the theory of automatic control, such as unconditional stability criterion of V. Popov [6,12], Lyapunov's direct method [7-11]. Sufficiently detailed analyses of the works are shown in [1].

At the same time, it is obvious that the data collection on the features of dynamic processes in the fuzzy control systems is taking place now and researchers should look for different approaches in their studies to create the most complete picture.

The task of this article is to analyze the nature of nonlinear transformations carried by the IC in case, when the method of fuzzy logic technology is used for its construction, and on the basis of conducted analysis, the development of the harmonic balance method is presented for further investigation of the periodic oscillations in the class of intelligent automatic control systems (IACS).

It should be noted that the fuzzy logic technology finds an increasingly wide application in the recent years. Methods of fuzzy inference allow providing the parallel data interpretation through specialized hardware with high operation speed, which makes fuzzy logic extremely promising for the development of intelligent fuzzy controllers (FC) of sequential type. Such regulators, oriented on the processing of logic-linguistic models of data representation, are intended to formalize inaccurate judgments since they classify the input data at the level of fuzzy sets and can be used in high-speed systems operating under conditions of incomplete information.

2. **Nonlinear Transformations in the Fuzzy Controllers.** The development of common approaches to solving research tasks on the IACS, design and setting of intelligent controllers implies the need for a detailed understanding of the specifics of transformations performed while processing the existing data in the system.

It is well-known that with all the differences of the methods and technologies used in data processing, the transformations which are implemented in the intelligent regulators are considered to be nonlinear. However, the most important issue is to identify key factors that determine the actual type of the corresponding non-linear dependency between the input and the output signals of intelligent controller, built on the basis of data processing technology. From this perspective, we can talk about two different aspects of the analysis and synthesis of intelligent controllers:

– Identification of the nature of non-linear transformations in the intelligent controller, whose settings are known;

– Provision of the required nature of nonlinear transformations in intelligent controller by regulating its settings.

Let us take a closer look at the features of nonlinear transformations in intelligent controllers, which are based on the fuzzy logic technology. Such transformations are gaining popularity and are becoming more wide-spread. Considering the last circumstance, today researchers allow to disregard the well-known facts, related to the fuzzy inference models, and point out that the analysis of the most common schemes for constructing logical-linguistic models and the organization of their subsequent processing has been carried out: Mamdani, Larsen, Tsukamoto and Sugeno. And the main results of this analysis, related to the research of the design principles and specifics of different types of fuzzy systems, on the one hand, and comprehension of experience of their practical application in control tasks on the other, lead to a number of fundamentally important generalizations:

– The use of tools and methods adopted in fuzzy logic for representation and data processing provides easy formation, understanding, debugging and modification of created management models;

– The all transformations in fuzzy controllers including the differences of accepted modes of representation and knowledge processing methods are inherently non-linear;

– The character of different types of nonlinear transformations, implemented in a fuzzy controller, essentially depends on its settings, including the number of input and output terms, as well as the shape and relative placement of membership functions;

– Sugeno model is an effective tool for describing systems with a priori known or suspected character of nonlinear transformations between the input and output signals;

– Models Mamdani, Larsen and Tsukamoto are an effective tool for the formation of non-linear transformations between the input and output signals of the system, based on the description of desired laws of its functioning in the form of rules of behavior in the whole range of possible situations; at the same time, the possibilities of Tsukamoto models in the formation nonlinear transformations are largely limited by the necessity of applying monotone membership functions to describe the output terms.

The question of the analysis of nonlinear transformations is the sufficient detail considered in [3,4].

Some examples of non-linear transformations, using Mamdani models, are shown in Table 1.

As could be noted, even a primary analysis of the nonlinearities (primarily it is 1.1-1.4 items from Table 1) explains the fundamental possibility to improve the transient performance in systems FC compared with PID-controller.

At the same time, the addition of nonlinearities to the ACS circuit leads to various types of problems related to the dynamics of the system and, in particular, with the possibility of the emergence of stable periodic oscillations.

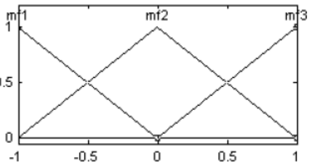
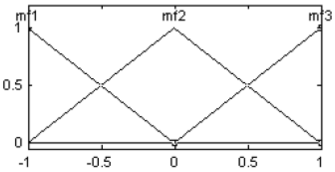
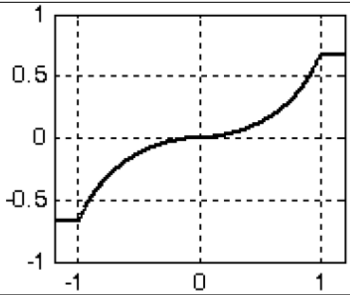
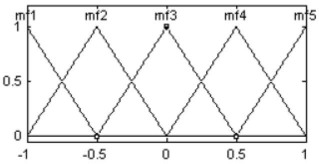
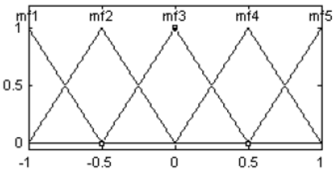
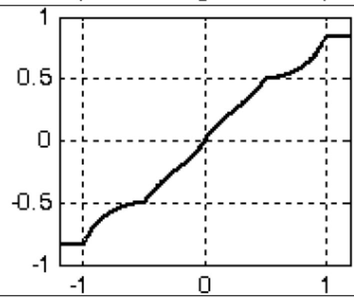
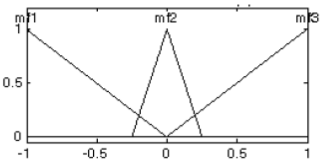
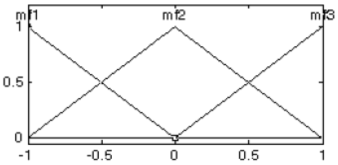
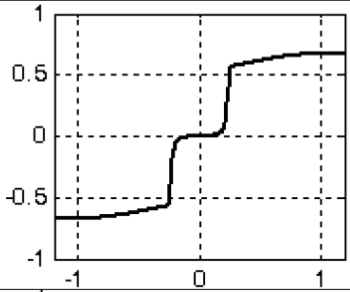
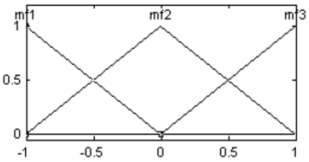
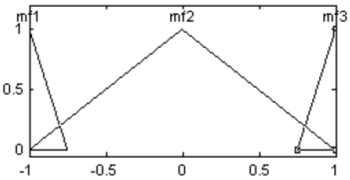
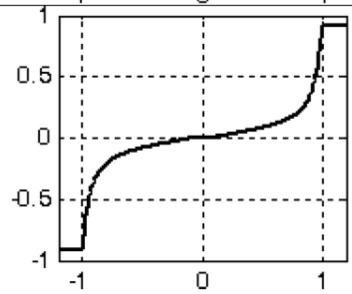
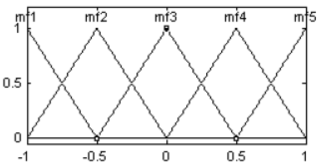
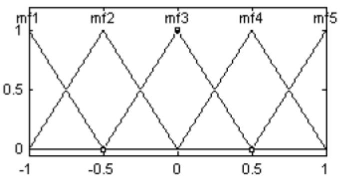
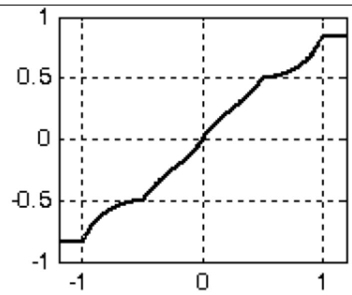
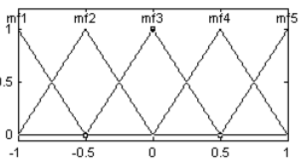
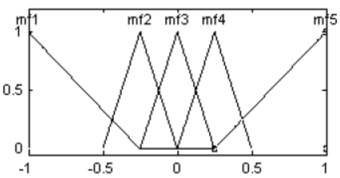
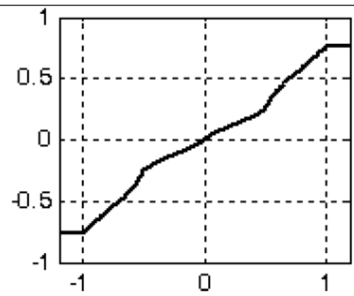
3. **Research of Periodic Oscillations by Harmonic Balance Method.** Structural diagram of intellectual automatic control system with fuzzy controller (FC) is represented as a series connection of fuzzy calculations unit (FCU), having h – inputs with connected to them linear dynamical elements, and one output, and a control object (CO) with the transfer function $W_{CO}(s)$ (Figure 1), wherein $g(t)$ – is reference-input signal, $u(t)$ – is the control signal, $y(t)$ – is output signal, $e(t)$ – is error signal of control, and s – is Laplace operator.

Fuzzy controller can be based on two types of structures: the first type is the fuzzy controller with parallel one-dimensional fuzzy calculations units FCU_i and the second type is the fuzzy calculations unit with multidimensional input (in Figures 2 and 3 are examples of a fuzzy structural scheme of PID controllers of the first and second types).

Consider the fuzzy controller (Figure 1) as a nonlinear frequency-dependent element with one input and one output and study the periodic oscillations based on the harmonic balance method on it.

The program module for construct on the complex plane the characteristic of equivalent complex gain of fuzzy controller (ECG FC), was created in the MatLab Simulink, and its structural diagram is shown in Figure 4. The values of the real and imaginary parts for ECG FC when fixed frequency and different amplitudes are recorded in the data array, then the real and imaginary parts of the inverse complex gain – ICG FC are calculated and displayed on the complex plane.

Table 1. Non-linear transformations, using Mamdani models

№	Input	Output	non-linear transformations
1.1			
1.2			
1.3			
1.4			
1.5			
1.6			

By the result of calculation in MatLab Simulink, a family of curves ICG $-J^{-1}(A,\omega)$ and hodograph of control object $W_{CO}(j\omega)$ are displayed on the complex plane. The solution of harmonic balance equation on condition of equal frequencies on intersecting graphs $\omega_{CO} = \omega_{FC}$ is the point belonging both to the inverse complex gain FC, and to hodograph of object control. The amplitude of the oscillations is determined at the point of intersection along the curve $-J^{-1}(A,\omega_{FC})$ at the frequency of $\omega_{CO} = \omega_{FC}$.

Thus, the task of research of periodic oscillations in IACS with fuzzy controller reduces to the well-known and widely accepted in engineering practice; it is the method of graphic-analytical calculation of oscillations in nonlinear ACS.

4. **Control Design.** In this section, we present specific examples of research of IACS with fuzzy controllers, the scheme shown in Figure 1.

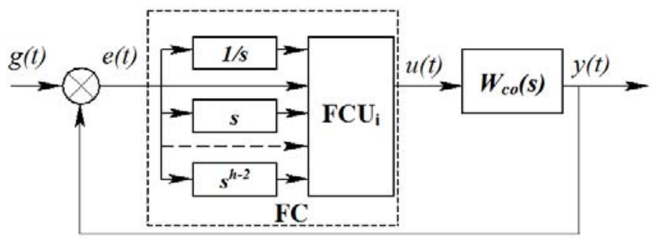


Figure 1. Generalized structural diagram of IACS with fuzzy controller

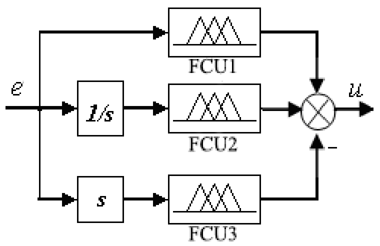


Figure 2. Fuzzy PID-controller of the first type

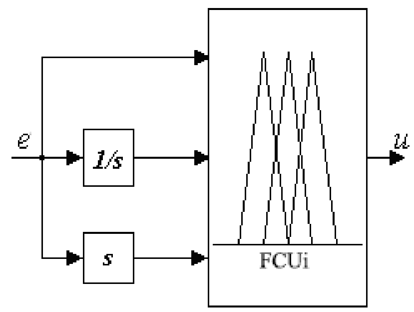


Figure 3. Fuzzy PID-controller of the second type

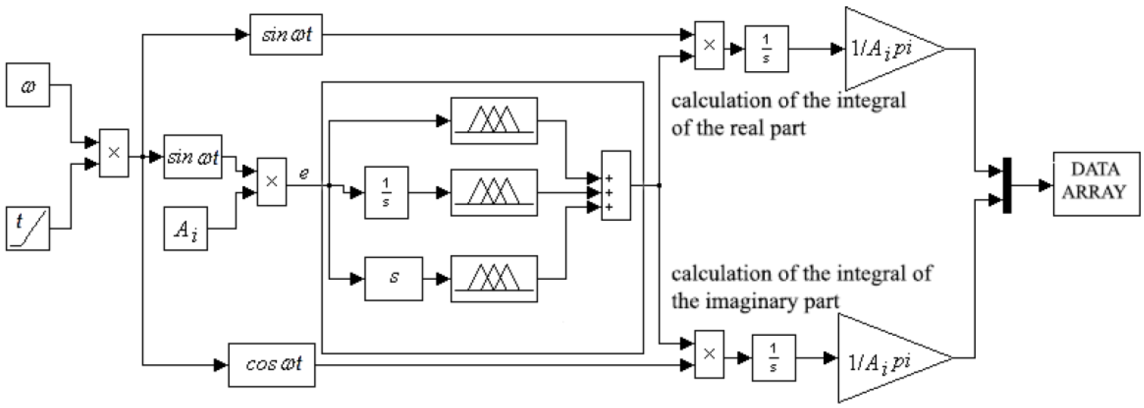


Figure 4. The program module in MatLab Simulink to calculate the real and imaginary parts of ECG FC

Example 4.1. *Type of the controller: fuzzy PID of the first type, Model FC – Mamdani.*

$$W_{CO}(s) = \frac{K_{CO}}{s(0.1s+1)(0.4s+1)(0.6s+1)}, \quad K_{CO} = 4$$

Membership functions of the input (a) and output linguistic variables FC (b) and the production rules base (c) in the integral, differential and proportional channels are presented in Figure 5 A, B, C, and non-linear transformations are shown in Figures 6(a), 6(b), 6(c).

At the previously given parameters of the system, the solution of harmonic balance equation is presented in Figure 7(a).

As it can be seen from Figure 7(a), in the researched system there are periodic oscillations with frequency $\omega \approx 3$ rad/s and amplitude $A = 0.66$. Figure 7(b) shows the results of direct modeling, which coincide with the calculation results with sufficient accuracy.

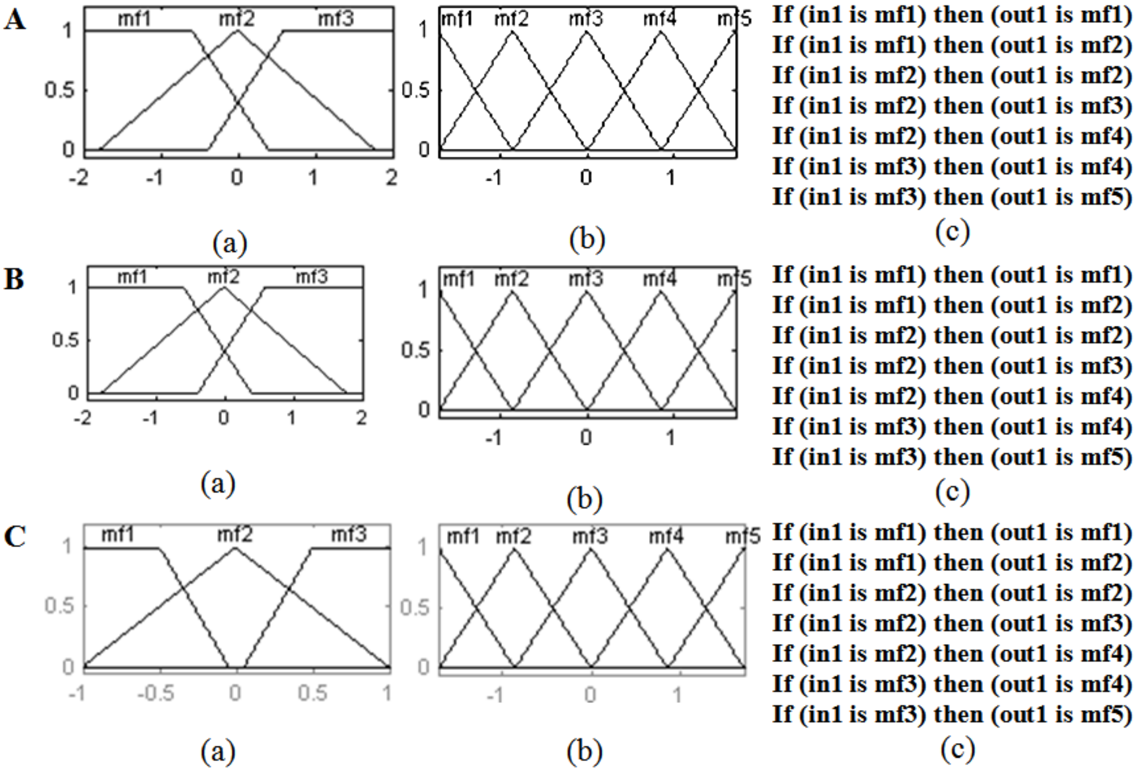


FIGURE 5. Membership functions of the input (a) and output linguistic variables FC (b) and the production rules base (c) in the integral, differential and proportional channels

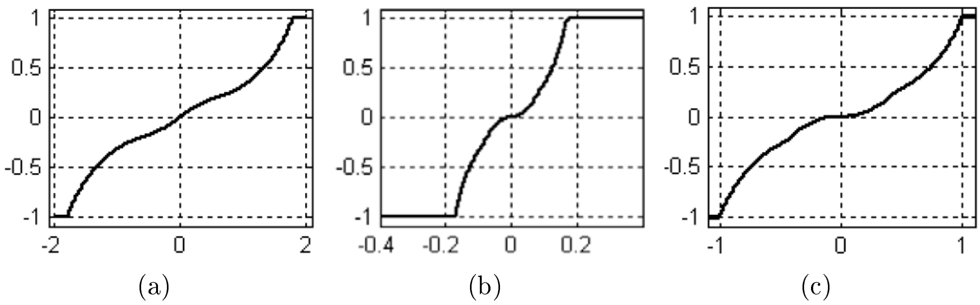
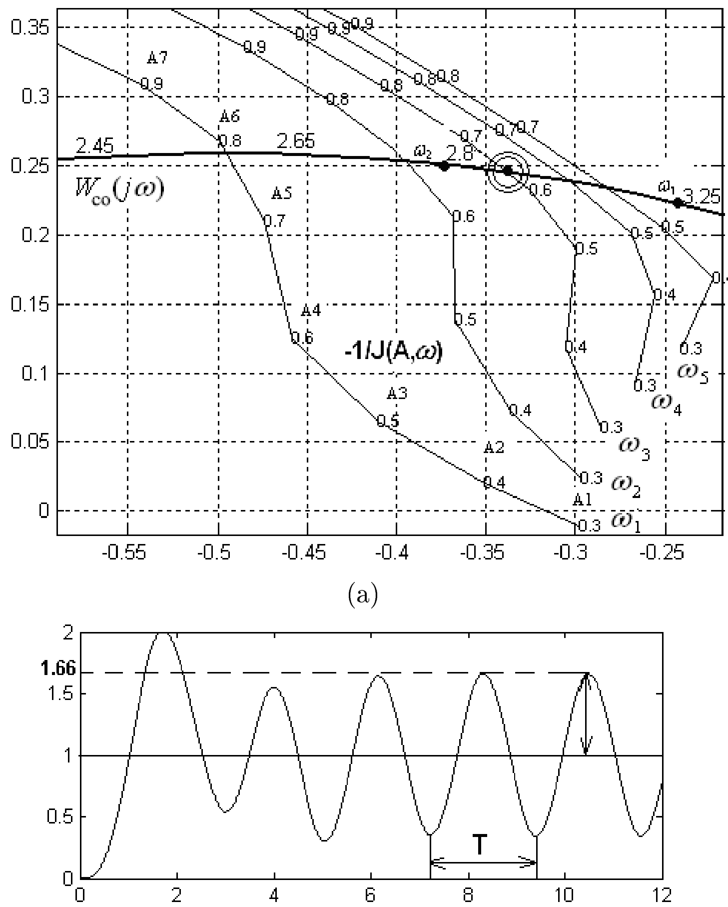


FIGURE 6. Non-linear transformations for Example 4.1



(a)

(b)

FIGURE 7. (a) The solution of harmonic balance equation for Example 4.1, (b) simulation results for Example 4.1

Figure 8 shows the results of a research of the same system with a decrease in the gain, K_{CO} =0,5.

As it can be seen, the characteristics $-J^{-1}(A,\omega)$ and $W_{CO}\,(j\omega)$ do not intersect (Figure 8(b)), and hence, there are no oscillations in the system; these calculations are confirmed by the transition of the system (Figure 8(a)).

Example 4.2. *Type of the controller: fuzzy PID of the second type. Model FC – Mamdani.*

$$W_{CO}(s) = \frac{K_{CO}}{s(0.1s+1)(0.4s+1)(0.6s+1)}, \quad K_{CO} = 0,8.$$

Membership functions of the input linguistic variables of proportional, integral and differential channels, and output variable are presented in Figures 9(a)-9(d) and nonlinear transformations are shown in Figure 10.

Base of production rules has the form:

If (in1 is mf1) and (in2 is mf1) and (in3 is mf1) then (out1 is mf1)
If (in1 is mf1) and (in2 is mf1) and (in3 is mf2) then (out1 is mf1)
If (in1 is mf1) and (in2 is mf1) and (in3 is mf3) then (out1 is mf2)
If (in1 is mf1) and (in2 is mf2) and (in3 is mf1) then (out1 is mf2)
If (in1 is mf1) and (in2 is mf2) and (in3 is mf2) then (out1 is mf2)

If (in1 is mf1) and (in2 is mf2) and (in3 is mf3) then (out1 is mf3)
If (in1 is mf1) and (in2 is mf3) and (in3 is mf1) then (out1 is mf3)
If (in1 is mf1) and (in2 is mf3) and (in3 is mf2) then (out1 is mf3)
If (in1 is mf1) and (in2 is mf3) and (in3 is mf3) then (out1 is mf4)
If (in1 is mf1) and (in2 is mf4) and (in3 is mf1) then (out1 is mf4)
If (in1 is mf1) and (in2 is mf4) and (in3 is mf2) then (out1 is mf4)
If (in1 is mf1) and (in2 is mf4) and (in3 is mf3) then (out1 is mf5)
If (in1 is mf3) and (in2 is mf4) and (in3 is mf3) then (out1 is mf6)
If (in1 is mf3) and (in2 is mf4) and (in3 is mf2) then (out1 is mf6)
If (in1 is mf3) and (in2 is mf4) and (in3 is mf1) then (out1 is mf5)
If (in1 is mf3) and (in2 is mf3) and (in3 is mf3) then (out1 is mf5)
If (in1 is mf3) and (in2 is mf3) and (in3 is mf2) then (out1 is mf5)
If (in1 is mf3) and (in2 is mf3) and (in3 is mf1) then (out1 is mf4)
If (in1 is mf3) and (in2 is mf2) and (in3 is mf3) then (out1 is mf4)
If (in1 is mf3) and (in2 is mf2) and (in3 is mf2) then (out1 is mf4)
If (in1 is mf3) and (in2 is mf2) and (in3 is mf1) then (out1 is mf3)
If (in1 is mf3) and (in2 is mf1) and (in3 is mf3) then (out1 is mf3)
If (in1 is mf3) and (in2 is mf1) and (in3 is mf2) then (out1 is mf3)
If (in1 is mf3) and (in2 is mf1) and (in3 is mf1) then (out1 is mf2)
If (in1 is mf2) and (in2 is mf1) and (in3 is mf1) then (out1 is mf2)
If (in1 is mf2) and (in2 is mf1) and (in3 is mf2) then (out1 is mf2)
If (in1 is mf2) and (in2 is mf1) and (in3 is mf3) then (out1 is mf3)
If (in1 is mf2) and (in2 is mf4) and (in3 is mf3) then (out1 is mf5)
If (in1 is mf2) and (in2 is mf4) and (in3 is mf2) then (out1 is mf5)
If (in1 is mf2) and (in2 is mf4) and (in3 is mf1) then (out1 is mf4)
If (in1 is mf2) and (in2 is mf2) and (in3 is mf1) then (out1 is mf3)
If (in1 is mf2) and (in2 is mf2) and (in3 is mf2) then (out1 is mf3)
If (in1 is mf2) and (in2 is mf2) and (in3 is mf3) then (out1 is mf4)
If (in1 is mf2) and (in2 is mf3) and (in3 is mf3) then (out1 is mf4)
If (in1 is mf2) and (in2 is mf3) and (in3 is mf2) then (out1 is mf4)
If (in1 is mf2) and (in2 is mf3) and (in3 is mf1) then (out1 is mf3)

Graph-analytical solutions of the harmonic balance equation and the results of simulation for a given system, as well as for a similar system with a gain of 0.2 are shown in Figure 11 and Figure 12. As seen at $K_{CO} = 0,8$ the system has periodic oscillations with frequency $\omega = 1,75$ rad/s and $A = 0,18$ (Figures 11(a) and 11(b)); at $K_{CO} = 0,2$ there is no periodic oscillations (Figures 12(a) and 12(b)).

Example 4.3. *The results obtained above relate to systems with fuzzy PID-controller, and this regulator has a rather general form. At the same time, it is obvious that for many industrial applications it may be acceptable to use simplified versions of the controller, in particular fuzzy P-controller [13,14]. In this case a research procedure based on the method of harmonic balance is significantly simplified. If the fuzzy P-controller constructed under the scheme:*

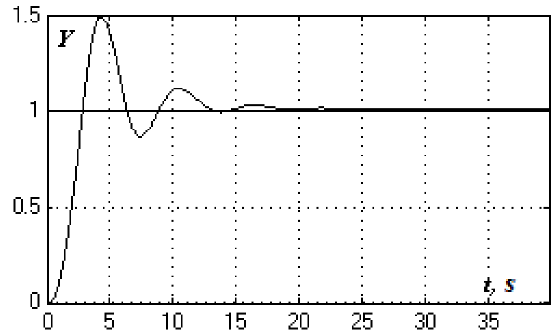
- *Membership function – triangular;*
- *The number of membership functions – 7;*

· *Terms of proportional controller, according to the example shown in Figure 6(b);*
· *Defuzzification by Mamdani;*
then, the nonlinear character of the transformed fuzzy P-controller has the form shown in Figure 13.

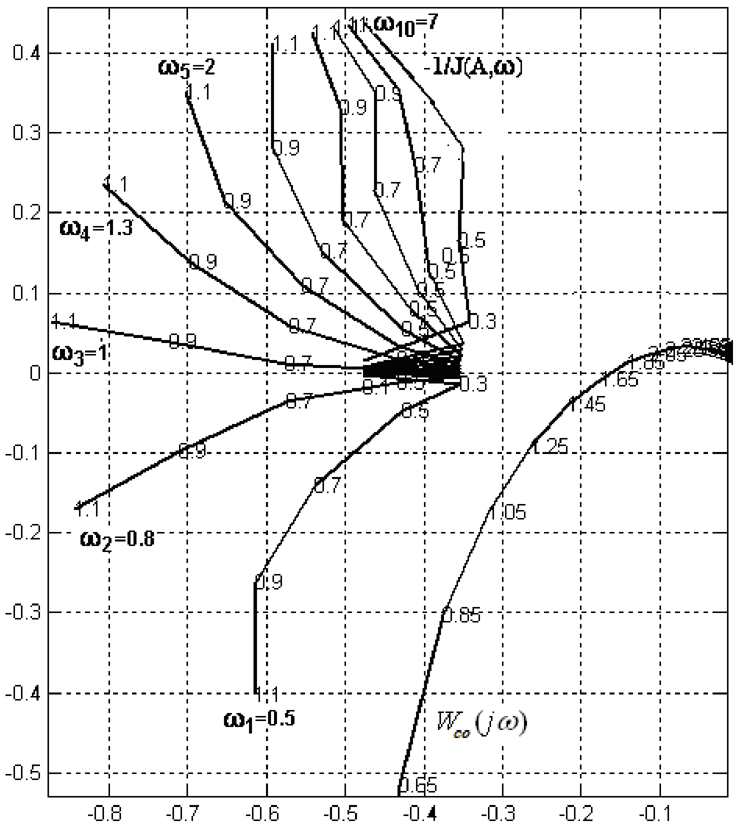
Its equivalent complex gain is defined by

$$J(A) = \frac{2}{\pi}\left[(m-k)\arcsin\frac{a}{A} + k\arcsin\frac{b}{A} + \frac{2ka+2c-kb}{A}\sqrt{1-\left(\frac{b}{A}\right)^2} - \frac{ka+ma}{A}\sqrt{1-\left(\frac{a}{A}\right)^2}\right],$$
$$m = \frac{d}{a},\ k = \frac{c-d}{b-a}$$

and has a real character.



(a)



(b)

FIGURE 8. (a) Simulation results for Example 4.1 with $K_{CO} = 0,5$, (b) the solution of harmonic balance equation for Example 4.1 with $K_{CO} = 0,5$

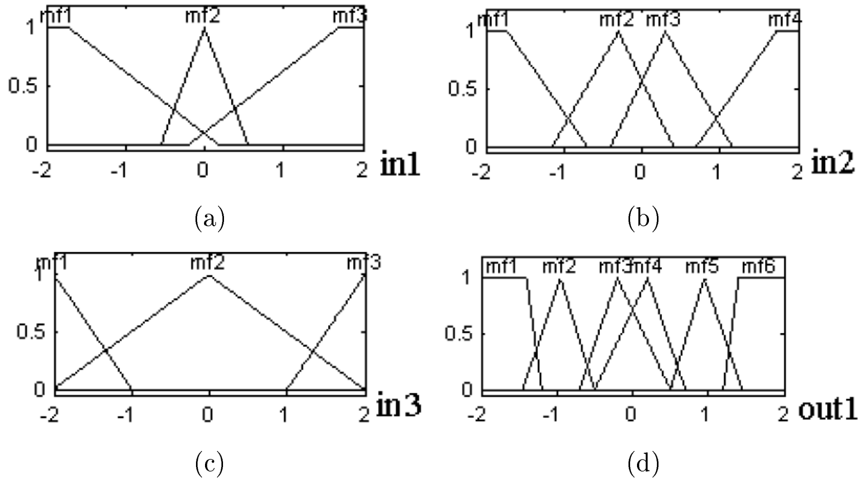


FIGURE 9. Membership functions of the input linguistic variables of proportional (a), integral (b) and differential (c) channels, and output variable (d) for Example 4.2

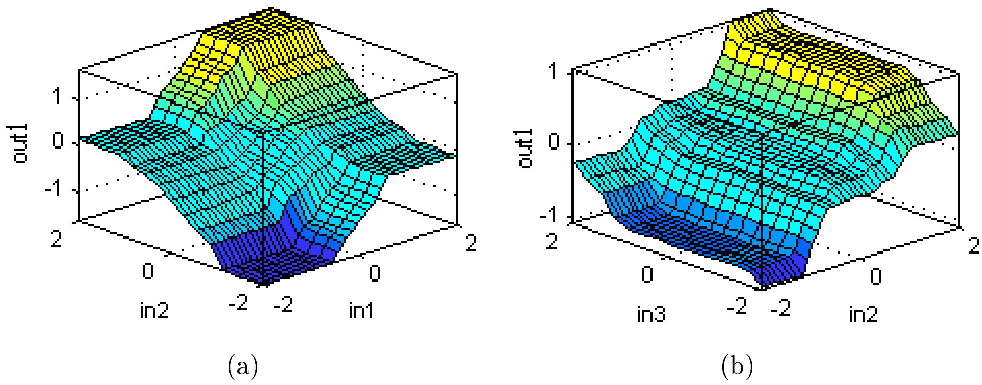


FIGURE 10. Non-linear transformations for Example 4.2

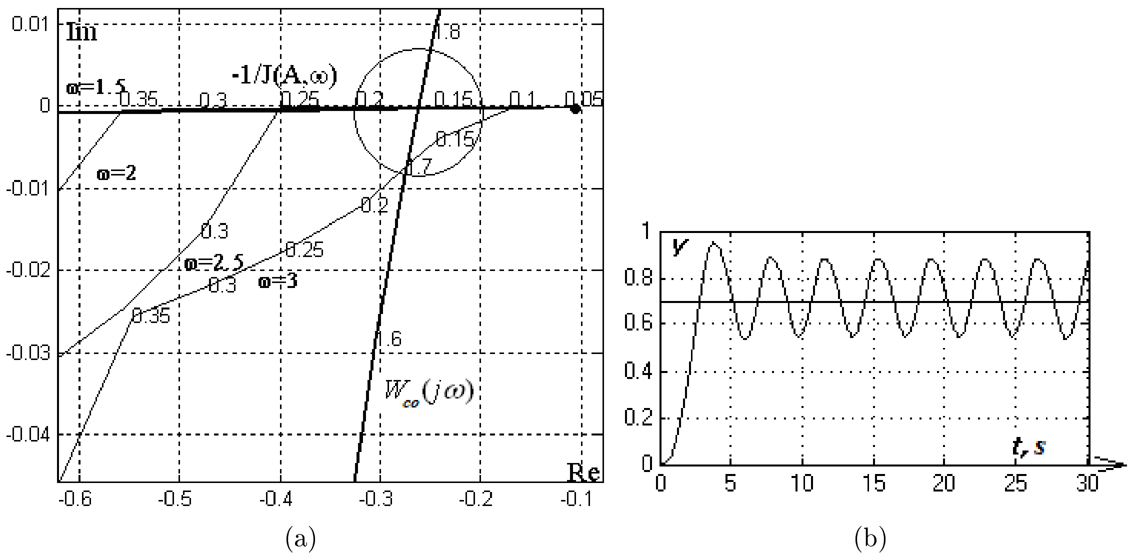
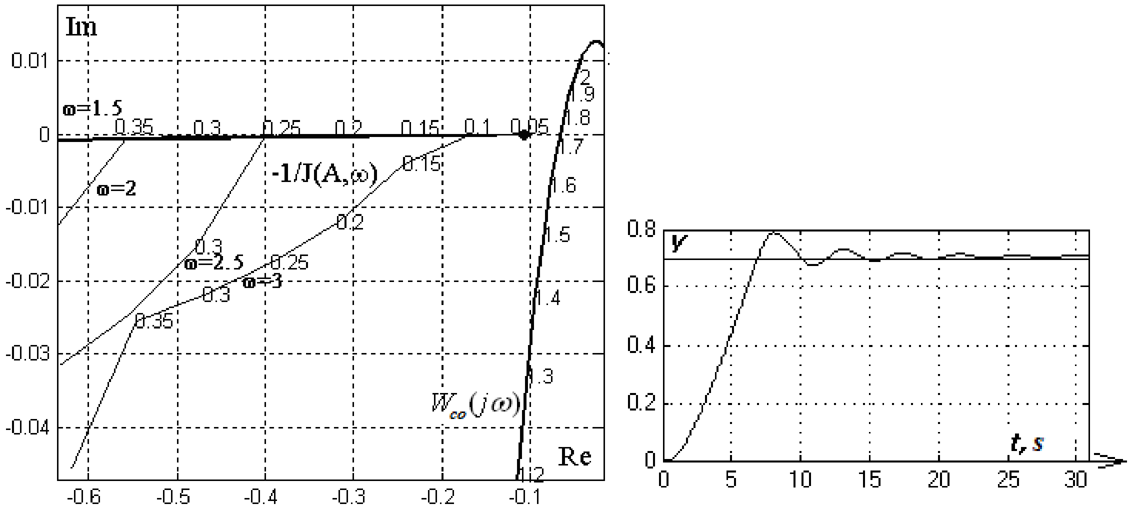


FIGURE 11. Graph-analytical solutions of the harmonic balance equation for Example 4.2 (a) and result of simulation (b)



(a) (b)

FIGURE 12. Graph-analytical solutions of the harmonic balance equation for Example 4.2 with $K_{CO} = 0,2$ (a) and result of simulation (b)

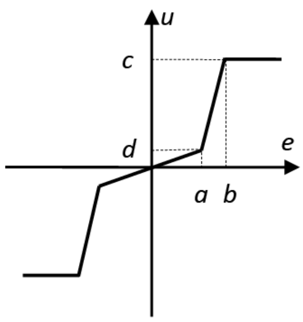


FIGURE 13. Nonlinear character of the transformed fuzzy P-controller

Thus, the procedure of the graph-analytical solution of the harmonic balance equation reduces to the determination of the intersection points of the curve $-J^{-1}(A,\omega)$ and the hodograph of the linear part $W_{CO}(j\omega)$.

Figure 14 shows an example of a system where $W_{CO}(j\omega) = k[p(1+pT_1)(1+pT_2)]^{-1}$, $k = 10$, $T_1 = 1$, $T_2 = 0,1$.

The simulation results of the system are shown in Figure 15.

Similarly it is possible to build researches with several of simplified modifications of fuzzy controllers that implement the various types of nonlinear transformations.

5. **Conclusions.** Although in this article we solve the problem of the research of the periodic oscillations in IACS with fuzzy controllers, the results of this are more general. And if we use the well-known conversion of ANFYS type, then it is possible to convert the fuzzy controller into neuro-fuzzy and thus extend the results to a new class of IACS.

It is important to emphasize that the more difficult the object control and the higher the order of models while describing it are the more accurate the results obtained by the method of harmonic balance will be.

And finally, with development of well-known techniques from the theory of automatic control, which give indirect assessment of the quality of non-linear systems, and using the results of this article, it is possible to develop a simple estimation technique of the

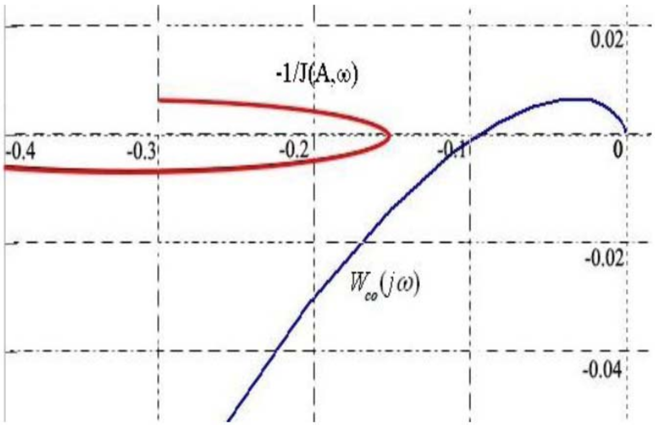


FIGURE 14. The graph-analytical solution of the harmonic balance equation for Example 4.3

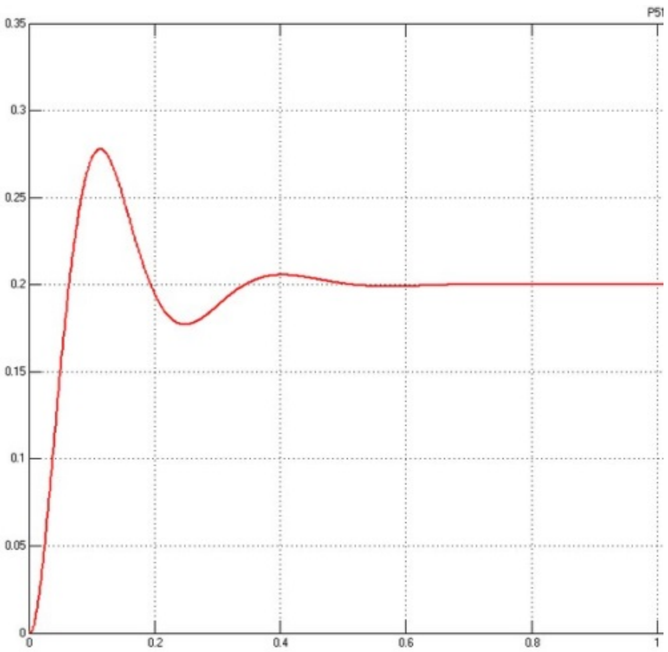


FIGURE 15. Result of simulation for Example 4.3

overshoot in ISAU with the fuzzy controller in terms of the oscillation index of a nonlinear system.

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