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DESIGN OF ROBUST SELF-TUNING CONTROL SCHEMES FOR STOCHASTIC SYSTEMS DESCRIBED BY INPUT-OUTPUT MATHEMATICAL MODELS

Nabiha Touijer and Samira Kamoun

Laboratory of Sciences and Techniques of Automatic Control and Computer Engineering
National School of Engineering of Sfax
University of Sfax
BP 1173, 3038 Sfax, Tunisia
{ nabiha_ettouijer; kamounsamira }@yahoo.fr

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Abstract. *This paper deals with the robust self-tuning control schemes for stochastic systems, which can be described by the input-output Auto-Regressive Auto-Regressive Moving Average with eXogenous (ARARMAX) mathematical model with unknown parameters in the presence of unmodelled dynamics. This explicit self-tuning control scheme is based on the proposed modified filtering recursive least squares algorithm with dead zone (m-F-RLS) in the step of estimation. We have applied the developed generalized minimum variance self-tuning regulator to a numerical simulation of a climate control in building, in order to test its performances. The obtained numerical simulation results are satisfactory.*

Keywords: Stochastic systems, ARARMAX mathematical models, Recursive parametric estimation algorithms, Robust explicit self-tuning control schemes

1. **Introduction.** The description of a stochastic system by an ARARMAX mathematical model was studied in [1]. Wang and Ding [2] have proposed a recursive least squares parametric estimation algorithm based on the filter input and system output (F-RLS), in order to estimate the parameters of the stochastic system, which can be described by an ARARMAX mathematical model. The F-RLS algorithm has been applied to the Box-Jenkins system non-uniformly sampled [3].

A modified recursive least squares parametric estimation algorithm based on the filter input and system output with dead zone approach (m-F-RLS) was proposed in [4], in order to estimate the parameters of the stochastic systems, which can be described by the ARARMAX mathematical models in the presence of unmodelled dynamics. Furthermore, the robust regulation problem for the considered systems was solved on the basis of m-F-RLS, where the minimum phase system was considered. In this case, an explicit scheme of minimum variance self-tuning regulation was developed.

The minimum variance regulator was been developed in [5], which can be applied only for minimum phase system. To overcome this problem, a generalized minimum variance self-tuning control was developed in [6] in order to control the non-minimum systems. The strategies of self-tuning control have been developed and applied to several systems (see [5,7-12]). Chen et al. [13] have proposed robust adaptive inverse dynamics control schemes for the trajectory tracking control of robot manipulator with uncertain dynamics. Wang [14] has treated the adaptive tracking control problem for a class of uncertain MIMO switched nonlinear systems.

In this paper, we will focus on the study of the self-tuning control problems for dynamical systems (minimum phase systems and non-minimum phase systems) in the presence of unmodelled dynamics. The developed explicit schemes of self-tuning control can be applied to the linear stochastic systems described by the ARARMAX mathematical models in the presence of unmodelled dynamics. The proposed recursive parametric estimation algorithm m-F-RLS is used in the step of the system parameters estimation.

This paper is organized as follows. The second section is devoted to the description of dynamical systems by a mathematical discrete input-output ARARMAX in the presence of unmodelled dynamics. Furthermore, a modified recursive parametric estimation algorithm m-F-RLS with a dead zone will be studied. In the third section, an explicit scheme of generalized minimum variance self-tuning regulation is analyzed and developed based on the proposed parametric estimation algorithm to solve the problem of regulation. Moreover, an explicit scheme of self-tuning control is analyzed and developed based on the proposed parametric estimation algorithm to solve the problem of regulation-tracking. In the fourth section we are going to use the simplified model of developed Mi2 building model from physic law, which is available from CLIM2000, to show the performance of the self-tuning regulation on the basis of the proposed recursive parametric estimation algorithm m-F-RLS with dead zone. And we conclude in the last section.

2. **Parametric Estimation.**

Let us consider a linear time-varying system, which can be described by the following discrete-time ARARMAX mathematical model:

$$A_c(q^{-1},k)y(k) = q^{-d}B_c(q^{-1},k)u(k) + \frac{D(q^{-1})}{C(q^{-1})}e(k) \tag{1}$$

where $u(k)$ and $y(k)$ represent the input and the output of the system at the discrete-time k, respectively, $e(k)$ is a white noise with zero mean and constant variance acting on the system, d is the dead-time (is an integral number of sample intervals), and $A_c(q^{-1},k)$, $B_c(q^{-1},k)$, $C(q^{-1})$ and $D(q^{-1})$ are polynomials of degree n_{Ac}, n_{Bc}, n_C and n_D, respectively, which are defined by:

$$A_c(q^{-1},k) = 1 + a_1(k)q^{-1} + \cdots + a_{n_{Ac}}(k)q^{-n_{Ac}} \tag{2}$$

$$B_c(q^{-1},k) = b_1(k)q^{-1} + \cdots + b_{n_{Bc}}(k)q^{-n_{Bc}} \tag{3}$$

$$C(q^{-1}) = 1 + c_1q^{-1} + \cdots + c_{n_C}q^{-n_C} \tag{4}$$

$$D(q^{-1}) = 1 + d_1q^{-1} + \cdots + d_{n_D}q^{-n_D} \tag{5}$$

where, for $i = 1, \cdots, n_{Ac}$, and $j = 1, \cdots, n_{Bc}$, we can write:

$$a_i(k) = a_i + \varepsilon_{a_i}(k) \tag{6}$$

$$b_j(k) = b_j + \varepsilon_{b_j}(k) \tag{7}$$

where $\varepsilon_{a_i}(k)$ and $\varepsilon_{b_j}(k)$ represent the unmodelled dynamics into parameters.

2.1. **Formulation of the parametric estimation problem.**

The filtered mathematical model of the system is given by:

$$y_f(k) = \theta_s^T\varphi_f(k) + e_d(k) \tag{8}$$

where

$$\theta_s^T = [a_1 \dots a_{n_{Ac}} \quad b_1 \dots b_{n_{Bc}}] \tag{9}$$

$$\varphi_f^T(k) = [-y_f(k-1) \dots - y_f(k-n_{Ac}) \quad u_f(k-d-1) \dots u_f(k-d-n_{Bc})] \tag{10}$$

$$y_f(k) = \frac{C(q^{-1})}{D(q^{-1})}y(k) \tag{11}$$

$$u_f(k) = \frac{C(q^{-1})}{D(q^{-1})} u(k) \tag{12}$$

The noise $e_d(k)$ is defined as follows:

$$e_d(k) = \varepsilon_{AB}^T(k)\varphi_f(k) + e(k) \tag{13}$$

where

$$\varepsilon_{AB}^T(k) = [\varepsilon_{a_1}(k) \dots \varepsilon_{a_{Ac}}(k) \quad \varepsilon_{b_1}(k) \dots \varepsilon_{b_{Bc}}(k)] \tag{14}$$

The output $y(k)$ of the considered system can be defined as follows:

$$y(k) = \theta_s^T \varphi_s(k) + v(k) \tag{15}$$

where

$$\varphi_s^T(k) = [-y(k-1) \dots - y(k - n_{Ac}) \quad u(k-d-1) \dots u(k-d-n_{Bc})] \tag{16}$$

$$v(k) = \frac{D(q^{-1})}{C(q^{-1})} e_d(k) \tag{17}$$

We can write (17) as follows:

$$v(k) = \theta_n^T \varphi_n(k) + e_d(k) \tag{18}$$

where

$$\theta_n^T = [c_1 \dots c_{n_C} \quad d_1 \dots d_{n_D}] \tag{19}$$

$$\varphi_n^T(k) = [-v(k-1) \dots - v(k-n_C) \quad e_d(k-1) \dots e_d(k-n_D)] \tag{20}$$

The output $y(k)$ of the system defined in (1) can be given by:

$$y(k) = \theta_s^T \varphi_s(k) + v(k) \tag{21}$$

We consider in our work the two hypotheses presented in [15], which are: an upper bound ρ of $\varepsilon_{AB}(k)$ is known; the noise $e(k)$ is bounded and the upper bound m_0 of $e(k)$ is known. Based on these hypotheses and the expression (13), the upper bound $d(k)$ of $e_d(k)$ is defined as follows:

$$d(k) = \rho \left\|\varphi_f(k)\right\| + m_0 \tag{22}$$

2.2. **Recursive parametric estimation algorithm with dead zone.** The proposed parametric estimation algorithm m-F-RLS with dead zone is defined as:
Step 1: Estimation of the parameter vector θ_s given by (9) using the following recursive parametric estimation algorithm RLS with dead zone:

$$\begin{aligned} &\hat{\theta}_s(k) = \hat{\theta}_s(k-1) + \delta(k) L_f(k) \xi(k) \\ &L_f(k) = \frac{P_f(k-1)\widehat{\varphi}_f(k)}{o(k) + \widehat{\varphi}_f^T(k) P_f(k-1)\widehat{\varphi}_f(k)} \\ &P_f(k) = \left[I - \delta(k) L_f(k) \widehat{\varphi}_f(k)\right] P_f(k-1) \\ &\xi(k) = \widehat{y}_f(k) - \hat{\theta}_s^T(k-1)\widehat{\varphi}_f(k) \end{aligned} \tag{23}$$

where

$$\delta(k) = \begin{cases} 0, & \text{if } |\xi(k)| \le \beta d(k) \\ \gamma, & \text{otherwise, where } \gamma \in [\sigma, 1/\beta - \sigma] \end{cases} \tag{24}$$

$$d(k) = \rho \left\|\widehat{\varphi}_f(k)\right\| + m_0 \tag{25}$$

such as

$$\widehat{y}_f(k) = \frac{\hat{C}(k-1)}{\hat{D}(k-1)} y(k) \tag{26}$$

$$\widehat{u}_f(k) = \frac{\hat{C}(k-1)}{\hat{D}(k-1)} u(k) \tag{27}$$

$$\widehat{\varphi}_f^T(k) = [-\widehat{y}_f(k-1) \ldots - \widehat{y}_f(k-n_{Ac}) \quad \widehat{u}_f(k-d-1) \ldots \widehat{u}_f(k-d-n_{Bc})] \tag{28}$$

Step 2: Estimation of the parameter vector θ_n given by (19) using the following recursive parametric estimation algorithm RELS:

$$\begin{array}{l} \hat{\theta}_n(k) = \hat{\theta}_n(k-1) + L_n(k)[\mu(k) - \hat{\theta}_n^T(k-1)\widehat{\varphi}_n(k)] \\ L_n(k) = \frac{P_n(k-1)\widehat{\varphi}_n(k)}{1+\widehat{\varphi}_n^T(k)P_n(k-1)\widehat{\varphi}_n(k)} \\ P_n(k) = \left[I - L_n(k)\widehat{\varphi}_n(k)\right] P_n(k-1) \\ \mu(k) = y(k) - \hat{\theta}_s^T(k)\varphi_s(k) \\ \varepsilon(k) = \mu(k) - \hat{\theta}_n^T(k-1)\widehat{\varphi}_n(k) \end{array} \tag{29}$$

such as:

$$\widehat{\varphi}_n^T(k) = [-\mu(k-1) \ldots - \mu(k-n_C) \quad \varepsilon(k-1) \ldots \varepsilon(k-n_D)] \tag{30}$$

3. **Robust Self-Tuning Control.** In this section, we are going to develop the robust generalized minimum variance self-tuning regulation and the robust self-tuning control for the stochastic systems, which can be described by the mathematical model (1).

3.1. **Robust explicit generalized minimum variance self-tuning regulation scheme.** The regulation problem for the non-minimum phase system described by (1) is discussed. In this case, an explicit scheme of generalized minimum variance self-tuning regulation was developed.

Therefore, we introduce the following criterion $J(k+d+1)$:

$$J(k+d+1) = E\left[(y(k+d+1))^2 + \alpha(u(k))^2\right] \tag{31}$$

where E denotes the expectation and α is a weighting coefficient.

Based on the mathematical model (1) of the considered system, the output $y(k+d+1)$ can be defined as follows:

$$y(k+d+1) = q\frac{B_c(q^{-1},k)}{A_c(q^{-1},k)}u(k) + \frac{D(q^{-1})}{A_c(q^{-1},k)C(q^{-1})}e(k+d+1) \tag{32}$$

We can write:

$$y(k+d+1) = q\frac{B_c(q^{-1},k)}{A_c(q^{-1},k)}u(k) + \frac{G(q^{-1},k)}{A_c(q^{-1},k)C(q^{-1})}e(k) + F(q^{-1},k)e(k+d+1) \tag{33}$$

where $F(q^{-1},k)$ and $G(q^{-1},k)$ are solutions of the following polynomial equation:

$$D(q^{-1}) = A_c(q^{-1},k)C(q^{-1})F(q^{-1},k) + q^{-d-1}G(q^{-1},k) \tag{34}$$

with

$$F(q^{-1},k) = 1 + f_1(k)q^{-1} + \ldots + f_d(k)q^{-d} \tag{35}$$

$$G(q^{-1},k) = g_0(k) + g_1(k)q^{-1} + \ldots + g_{n_{Ac}+n_C-1}(k)q^{1-n_{Ac}-n_C} \tag{36}$$

Using (1), we can write $e(k)$ as follows:

$$e(k) = -\frac{q^{-d}B_c(q^{-1},k)C(q^{-1})}{D(q^{-1})}u(k) + \frac{A_c(q^{-1},k)C(q^{-1})}{D(q^{-1})}y(k) \tag{37}$$

and using (37), the output system $y(k+d+1)$ given by (33) can be rewritten as follows:

$$y(k+d+1) = \frac{H(q^{-1},k)}{D(q^{-1})}u(k) + \frac{G(q^{-1},k)}{D(q^{-1})}y(k) + F(q^{-1},k)e(k+d+1) \tag{38}$$

where

$$H(q^{-1},k) = qB_c(q^{-1},k)C(q^{-1})F(q^{-1},k) \tag{39}$$

Computing (38) in (31), the criterion $J(k+d+1)$ is defined as follows:

$$J(k+d+1) = \left[\frac{H(q^{-1},k)}{D(q^{-1})}u(k) + \frac{G(q^{-1},k)}{D(q^{-1})}y(k)\right]^2 + \left[1 + f_1^2(k) + \cdots + f_d^2(k)\right]\sigma^2 + \alpha[u(k)]^2 \tag{40}$$

Then, the searched control law is determined by minimizing the criterion (40), by using the following derive of the criterion $J(k+d+1)$:

$$\frac{\partial J(k+d+1)}{\partial u(k)} = \frac{\partial}{\partial u(k)}\left(\left[\frac{H(q^{-1},k)}{D(q^{-1})}u(k) + \frac{G(q^{-1},k)}{D(q^{-1})}y(k)\right]^2 + \left[1 + f_1^2(k) + \cdots + f_d^2(k)\right]\sigma^2 + \alpha[u(k)]^2\right) \tag{41}$$

Then, (41) is written as:

$$\frac{\partial J(k+d+1)}{\partial u(k)} = 2h_1(k)(H(q^{-1})u(k) + G(q^{-1})y(k)) + 2\alpha D(q^{-1})u(k) \tag{42}$$

So, the control law is calculated by minimizing (42), which is given by:

$$u(k) = -\frac{G(q^{-1},k)y(k)}{Z(q^{-1},k)} \tag{43}$$

such as:

$$Z(q^{-1},k) = H(q^{-1},k) + \frac{\alpha}{h_1(k)}D(q^{-1}) \tag{44}$$

The recursive algorithm of the explicit scheme of self-tuning control is formulated by the following steps:
Step 1: Estimation of the parameters intervening of the ARARMAX mathematical model (1) using the modified recursive parametric estimation algorithm m-F-RLS with dead zone (23)-(29);
Step 2: Calculation of the parameters intervening in polynomials $F(q^{-1},k)$ and $G(q^{-1},k)$ by solving the following polynomial equation:

$$\hat{D}(q^{-1},k) = \hat{A}(q^{-1},k)\hat{C}(q^{-1},k)F(q^{-1},k) + q^{-d-1}G(q^{-1},k) \tag{45}$$

Polynomials $H(q^{-1},k)$ and $Z(q^{-1},k)$ are defined by, respectively:

$$H(q^{-1},k) = q\hat{B}(q^{-1},k)\hat{C}(q^{-1},k)F(q^{-1},k) \tag{46}$$

$$Z(q^{-1},k) = H(q^{-1},k) + \frac{\alpha}{h_1(k)}\hat{D}(q^{-1},k) \tag{47}$$

where: $h_1(k) = \hat{b}_1(k)$ and $z_1(k) = h_1(k) + \alpha/h_1(k)$;
Step 3: Calculation of the control law $u(k)$ by (43).

Noting that, if we have: $\hat{b}_1(k) = 0$, then we can choose, for example, $h_1(k) = 0.1$.

3.2. **Robust explicit self-tuning control scheme.** The regulation-tracking problem for the considered system is discussed. In this case, an explicit scheme of self-tuning control was developed.

Therefore, we introduce the following criterion $J(k+d+1)$:

$$J(k+d+1) = E([S(q^{-1})(y(k+d+1) - y_r(k+d+1))]^2 + [Q(q^{-1})(u(k) - \bar{u}(k))]^2) \tag{48}$$

where E denotes the expectation, $y_r(k+d+1)$ is a bounded and desired output signal, $\bar{u}(k)$ is the average value of the control signal $u(k)$ and $S(q^{-1})$ and $Q(q^{-1})$ are polynomials of known order ns and nq, respectively, such as:

$$\bar{u}(k) = \bar{u}(k-1) + \frac{1}{k}[u(k-1) - \bar{u}(k-1)] \tag{49}$$

$$S(q^{-1}) = 1 + s_1 q^{-1} + \cdots + s_{ns} q^{-ns} \tag{50}$$

$$Q(q^{-1}) = q_0 + q_1 q^{-1} + \cdots + q_{nq} q^{-nq} \tag{51}$$

Based on the mathematical model (1) of the considered system, $S(q^{-1})y(k+d+1)$ can be defined as follows:

$$S(q^{-1})y(k+d+1) = q\frac{S(q^{-1})B_c(q^{-1},k)}{A_c(q^{-1},k)}u(k) + \frac{S(q^{-1})D(q^{-1})}{A_c(q^{-1},k)C(q^{-1})}e(k+d+1) \tag{52}$$

We can write:

$$\begin{aligned} S(q^{-1})y(k+d+1) = {} & q\frac{B_c(q^{-1},k)S(q^{-1})}{A_c(q^{-1},k)}u(k) \\ & + \frac{G(q^{-1},k)}{A_c(q^{-1},k)C(q^{-1})}e(k) + F(q^{-1},k)e(k+d+1) \end{aligned} \tag{53}$$

where the polynomials $F(q^{-1},k)$ and $G(q^{-1},k)$ are solutions of the following polynomial equation:

$$S(q^{-1})D(q^{-1}) = A_c(q^{-1},k)C(q^{-1})F(q^{-1},k) + q^{-d-1}G(q^{-1},k) \tag{54}$$

Using (37), the output system $S(q^{-1})y(k+d+1)$ given by (53) can be rewritten as follows:

$$S(q^{-1})y(k+d+1) = \frac{H(q^{-1},k)}{D(q^{-1})}u(k) + \frac{G(q^{-1},k)}{D(q^{-1})}y(k) + F(q^{-1},k)e(k+d+1) \tag{55}$$

where $H(q^{-1},k)$ is given by (39).

Computing (55) in (48), the criterion $J(k+d+1)$ can be defined as follows:

$$\begin{aligned} J(k+d+1) = {} & \left[\frac{H(q^{-1},k)}{D(q^{-1})}u(k) + \frac{G(q^{-1},k)}{D(q^{-1})}y(k) - S(q^{-1})y_r(k+d+1)\right]^2 \\ & + \left[1 + f_1^2(k) + \cdots + f_d^2(k)\right]\sigma^2 + [Q(q^{-1})(u(k) - \bar{u}(k))]^2 \end{aligned} \tag{56}$$

Then, the searched control law is determined by minimizing the criterion (56), by using the following derive of the criterion $J(k+d+1)$:

$$\begin{aligned} \frac{\partial J(k+d+1)}{\partial u(k)} = {} & \frac{\partial}{\partial u(k)}\left(\left[\frac{H(q^{-1})}{D(q^{-1})}u(k) + \frac{G(q^{-1})}{D(q^{-1})}y(k) - S(q^{-1})yr(k+d+1)\right]^2\right. \\ & \left. + [Q(q^{-1})(u(k) - \bar{u}(k))]^2 + [1 + f_1^2(k) + \cdots + f_d^2(k)]\sigma^2\right) \end{aligned} \tag{57}$$

Then, (57) is written as:

$$\begin{aligned} \frac{\partial J(k+d+1)}{\partial u(k)} = {} & h_1(k)[H(q^{-1})u(k) + G(q^{-1})y(k) \\ & - D(q^{-1})S(q^{-1})yr(k+d+1)] + q_0 D(q^{-1})Q(q^{-1})[u(k) - \bar{u}(k)] \end{aligned} \tag{58}$$

So, the control law, which is calculated by minimizing (58), is given by:

$$\begin{aligned} u(k) = \frac{1}{Z(q^{-1},k)} \Big[& D(q^{-1})S(q^{-1})yr(k+d+1) \\ & + \frac{q_0}{h_1(k)} D(q^{-1})Q(q^{-1})\bar{u}(k) - G(q^{-1},k)y(k) \Big] \end{aligned} \tag{59}$$

such as:

$$Z(q^{-1},k) = H(q^{-1},k) + \frac{q_0}{h_1(k)} D(q^{-1})Q(q^{-1}) \tag{60}$$

The recursive algorithm of the explicit scheme of minimum variance self-tuning regulator is formulated by the following steps:
Step 1: Estimation of the parameters intervening of the ARARMAX mathematical model (1) using the modified recursive parametric estimation algorithm m-F-RLS with dead zone (23)-(29);
Step 2: Calculation of the parameters intervening in the polynomials $F(q^{-1},k)$ and $G(q^{-1},k)$ by solving the following polynomial equation:

$$S(q^{-1})\hat{D}(q^{-1},k) = \hat{A}(q^{-1},k)\hat{C}(q^{-1},k)F(q^{-1},k) + q^{-d-1}G(q^{-1},k) \tag{61}$$

Polynomial $H(q^{-1},k)$ is given by (46) and polynomial $Z(q^{-1},k)$ is defined by:

$$Z(q^{-1},k) = H(q^{-1},k) + \frac{q_0}{h_1(k)} \hat{D}(q^{-1},k)Q(q^{-1}) \tag{62}$$

where $h_1(k) = \hat{b}_1(k)$ and $z_1(k) = h_1(k) + q_0^2/h_1(k)$.
Step 3: Calculation of the control law $u(k)$, such as:

$$\begin{aligned} u(k) = \frac{1}{Z(q^{-1},k)} \Big[& \hat{D}(q^{-1},k)S(q^{-1})yr(k+d+1) \\ & + \frac{q_0}{h_1(k)} \hat{D}(q^{-1},k)Q(q^{-1})\bar{u}(k) - G(q^{-1},k)y(k) \Big] \end{aligned} \tag{63}$$

Note that, if we have: $\hat{b}_1(k) = 0$, then we can choose, for example, $h_1(k) = 0.1$ and $z_1(k) = 0.1$.

4. **Numerical Simulation Example.** This section presents an application of robust self-tuning regulator to the air conditioning system in order to control the building climate. The design of the self-tuning regulator described in last section is now applied to the developed Mi2 building model from physical laws which are available from CLIM2000's component library (see [16,17]).

A simple synthesis of control model is given by:

$$G_s(s) = \frac{1.025 + 1.195s}{1 + 5.71s} \exp(-s/6) \tag{64}$$

The discrete-time form of (64) can be defined as follows:

$$G_s(q^{-1}) = q^{-1}\frac{B(q^{-1})}{A(q^{-1})} = \frac{0.2094q^{-1} - 0.1799q^{-2}}{1 - 0.9712q^{-1}} \tag{65}$$

The used sampling period T_e is chosen, such that: $T_e = 600\,\text{sec}$.

The posed regulation problem is the application of the self-tuning regulation of the temperature of room at reference temperature equal to 26°C. This room is affected by a set of external noises such as the outside air inlets caused by the opening of doors and windows, variation of the external temperature, the solar flux, the quality of the wall and the energy delivered by the occupants of the building (called internal contribution).

The considered air conditioning system, which operates in the stochastic environment and in the presence of unmodelled dynamics, can be described by the following ARARMAX mathematical model:

$$\begin{aligned} y(k) = {} & 0.9712y(k-1) + 0.2094u(k-2) - 0.1799u(k-3) \\ & - \varepsilon_{a1}(k)y(k-1) + \varepsilon_{b1}(k)u(k-2) + \varepsilon_{b2}(k)u(k-3) + v(k) \end{aligned} \tag{66}$$

where

$$v(k) = -0.1v(k-1) - 0.15e(k-1) + e(k) \tag{67}$$

The external disturbance $e(k)$ is a white noise sequence with zero mean and constant variance $\sigma^2 = 0.5^2$.

The unmodelled dynamics $\varepsilon_{a1}(k)$, $\varepsilon_{b1}(k)$ and $\varepsilon_{b2}(k)$ are given, respectively, as follows:

$$\varepsilon_{a1}(k) = 0.01\sin(0.15k) \tag{68}$$

$$\varepsilon_{b1}(k) = 0.01\cos(0.15k) \tag{69}$$

$$\varepsilon_{b2}(k) = 0.01\cos(0.2k) \tag{70}$$

Using the simulation results of the self tuning regulator ($\alpha = 0$) on the basis of the proposed recursive parametric estimation algorithm m-F-RLS and on the basis of the parametric estimation algorithm F-RLS, we are going to give the calculations of the statistical average values $\bar{m}_y$, $\bar{m}_\mu$, $\bar{m}_{te}$ and $\bar{m}_T$, respectively, of the output $y(k)$, of the prediction error $\mu(k)$, of the regulation error $te(k)$ and of the temperature $T(k)$ in Table 1, and the calculations of the variance of the prediction error σ_μ^2 and of the regulation error σ_{te}^2 in Table 2.

Table 1. Statistical average values

	$\bar{m}_y$	$\bar{m}_\mu$	$\bar{m}_{te}$	$\bar{m}_T$
F-RLS	3.0113	−0.0091	−0.0304	28.1343
m-F-RLS	2.9941	−0.0024	−0.0065	25.9415
True values	2.99			26°C

Table 2. Variance values

	σ_μ^2	σ_{te}^2
F-RLS	0.0118	0.0253
m-F-RLS	0.0102	0.0117

From Tables 1 and 2, we conclude that the explicit scheme of self-tuning regulator on the basis of the proposed recursive parametric estimation algorithm m-F-RLS is more robust than the explicit scheme of self-tuning regulator on the basis of the recursive parametric estimation algorithm F-RLS.

We are going to present the results of numerical simulation for the explicit generalized minimum variance self-tuning regulation scheme with $\alpha = 0.25$, on the basis of the proposed recursive parametric estimation algorithm, by the solid line (control scheme (1)), and the results of simulation for the explicit scheme of minimum variance self-tuning regulation ($\alpha = 0$), on the basis of the proposed recursive parametric estimation algorithm, by the dashed line (control scheme (2)). Using a linear converter temperature to tension (the temperature sensor used is of the type LM335), the reference output signal is: $y_r = 2.99v$. We will take $\hat{\theta}_s(0) = 0$, $\hat{\theta}_n(0) = 0$, $Pf(0) = 1000I$, $Pn(0) = I$ (where I is an identity matrix), $\beta = 1$ and $\gamma = 0.77$.

Figures 1 and 2 show, respectively, the evolution curve of the output $y(k)$ in control scheme (1) and in control scheme (2). Figures 3 and 4 show, respectively, the evolution curves of the control law $u(k)$ in control scheme (1) and in control scheme (2). Figure 5 shows the evolution curve of the output variance. Figure 6 shows the evolution curve of the control law variance.

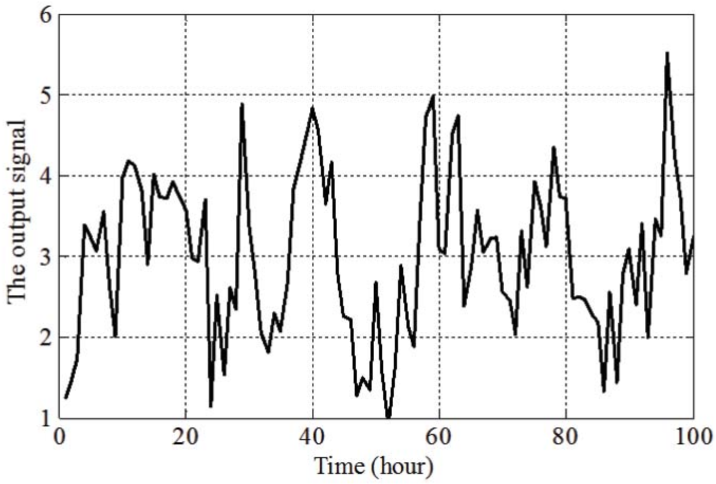

FIGURE 1. Evolution curve of the output $y(k)$ in control scheme (1)

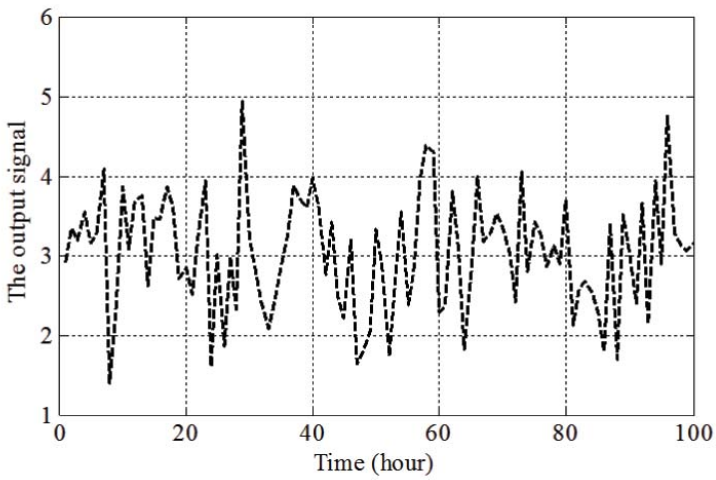

FIGURE 2. Evolution curve of the output $y(k)$ in control scheme (2)

The statistical average values of the output $\bar{m}_y$, of the prediction error $\bar{m}_\mu$, of the regulation error $\bar{m}_{te}$ and of the temperature are given in Table 3. The variances of the input σ_u^2, of the output σ_y^2, of the prediction error σ_μ^2 and of the regulation error σ_{te}^2 are given in Table 4.

From Figures 1-6, and Tables 1 and 2, we can get the following conclusions:

i. The control scheme (1) can overcome the effect of transitional regime and the control law can be applied in real time;
ii. The average of the temperature reaches the desired temperature in the control scheme (1) contrary to the average of the temperature in the control scheme (2);

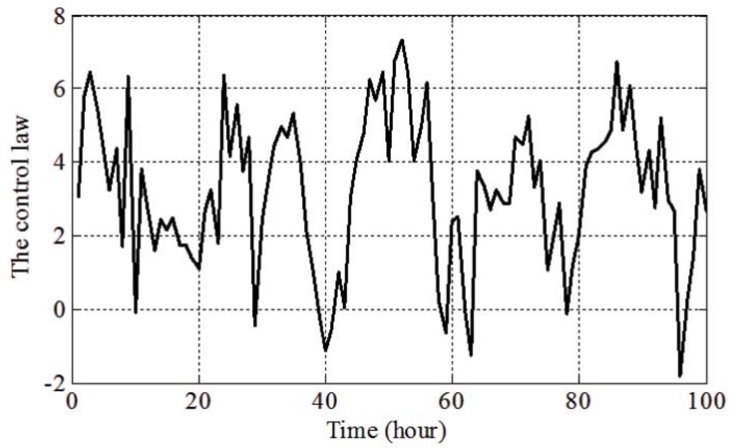

FIGURE 3. Evolution curve of the control law $u(k)$ in control scheme (1)

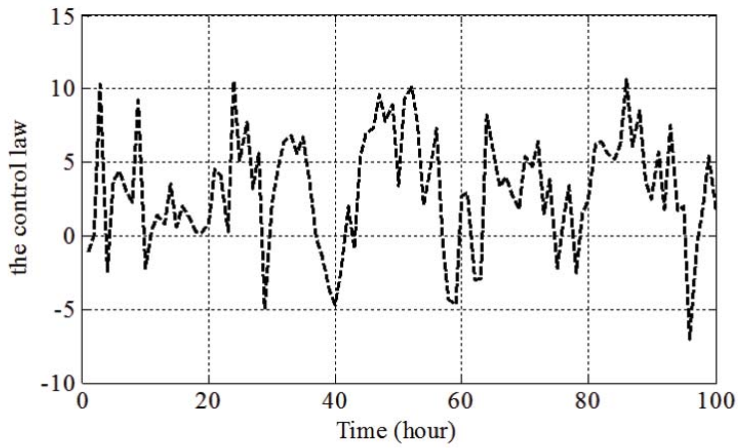

FIGURE 4. Evolution curve of the control law $u(k)$ in control scheme (2)

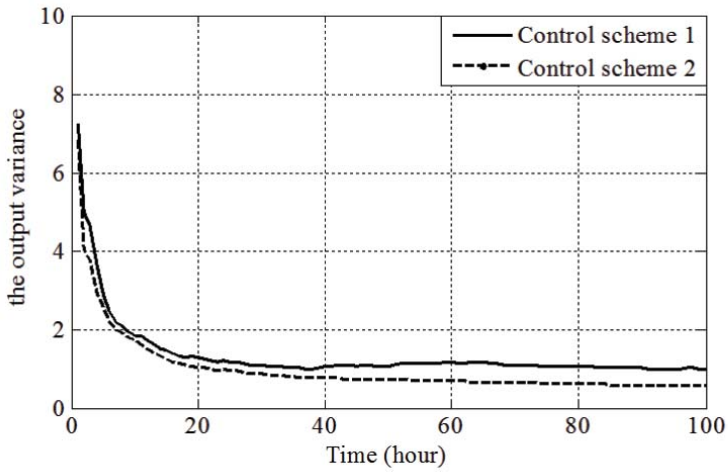

FIGURE 5. Evolution curve of the output variance

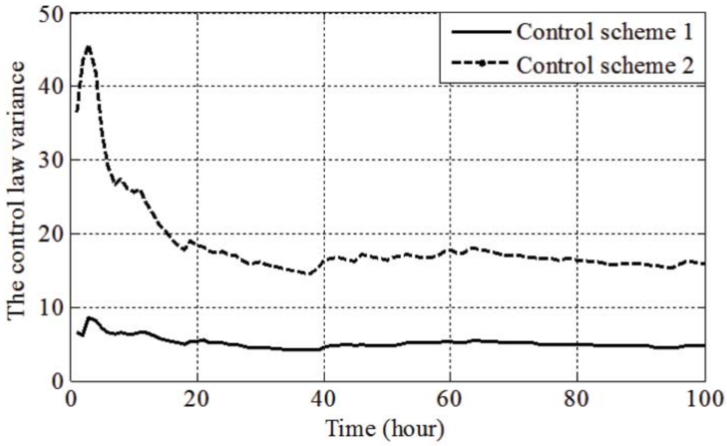

FIGURE 6. Evolution curve of control law variance

TABLE 3. Statistical average values

	$\bar{m}_y$	$\bar{m}_\mu$	$\bar{m}_{te}$	$\bar{m}_T$
Control scheme 1	2.9894	0.0448	0.0432	25.9384
Control scheme 2	3.0286	0.0185	0.0201	28.3636
Desired values	2.99v			26°C

TABLE 4. Variance values

	σ_u^2	σ_y^2	σ_μ^2	σ_{te}^2
Control scheme 1	3.7507	0.8220	0.2434	0.6129
Control scheme 2	15.0619	0.4343	0.2479	0.2321

iii. The inconvenience of the control scheme (1) is observed in the evolution curve of the output signal and the evolution curve of the output variance. This inconvenience has effect on the statistical average and on the variance of the regulation error.

5. **Conclusion.** This paper has presented the robust generalized minimum variance self-tuning control schemes for stochastic systems in the presence of unmodelled dynamics, which are described by the ARARMAX mathematical models. The modified recursive parametric estimation algorithm m-F-RLS with dead zone has been proposed to be used in the step of the determination of the parameter estimation. The simulation results of the control scheme applied to the simplified model of Mi2 building show the effectiveness of the explicit scheme of generalized minimum variance of self-tuning regulator based on the proposed modified recursive parametric estimation algorithm m-F-RLS with dead zone.

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