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H_∞ CONTROLLER DESIGN FOR UNCERTAIN NETWORKED CONTROL SYSTEMS WITH SCHEDULING STRATEGY BASED ON PREDICTED ERROR

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Abstract. *For network environment, scheduling policy based on model prediction error and accomplished by the switch matrix is presented for networked control systems (NCS), in which the characters of NCS are considered, such as limited network bandwidth, limited node energy and high collision probability. Data is transmitted and computed only if the absolute value of prediction error is larger than the threshold value. And the model of NCS based on prediction error scheduling with uncertain parameters and network-induced delay is established. To make the model closer to real system, a constant time-delay model is also introduced in the NCS model. The H_∞ controller of such NCS is designed by using a Lyapunov-Krasovskii function and inequality theory. Finally, simulations are included to demonstrate the theoretical results.*

Keywords: Networked control systems, Model prediction error, Scheduling, Uncertain, Time delay, H_∞ controller

1. Introduction.

Feedback control systems wherein the control loops are closed through a real-time network are called networked control systems [1]. Due to their suitable and flexible structure, NCS is frequently encountered in practice for such fields as information technology, life science and aeronautical and space technologies. Meanwhile, the characteristics of NCS, such as authorization of the spectrum, dynamic mobile, limited channels and broadcast transmission, make itself inevitably have transmission delay and data packet loss, which could cause adverse effect to system, and even lead to instability. How to reduce the negative influence on the system control performance and energy consumption of nodes has been becoming one of the popular issues in the control field.

Literature in the aspects of NCS has gotten plenty of achievements on stability analysis and controller design with considering uncertain parameters, time delay and other factors [2-6,12-14,16], in which the scheduling problem is not included. However, when a large number of data share the limited bandwidth, it is difficult to improve the control performance of system effectively only by relying on the controller design. Reasonable network scheduling strategies to reduce the conflict and the energy consumption of controller nodes are introduced in [7-11]. The relation between dead-zone threshold and control performance is analyzed in literature [7]. Transmission dead-zone and state estimator are set at the sensors side in [8], and the state of system is transmitted if and only if the absolute value of the difference between state value and its estimated value is larger than the dead-zone threshold. The stability of NCS with dead-zone scheduling strategy is analyzed in [8], but it has not referred to network-induced delay. Y. B. Zhao et al. [9] proposed a predictive control and scheduling co-design approach to deal with the controller and scheduler design for a set of networked control systems which are connected to a shared communication network. In [10], the scheduling of sensor information towards the controller is ruled by the classical Round-Robin protocol and the induced L_2-gain of NCS is analyzed, but it does not consider the effect of outside disturbance.

With the rapid development of computer technology, sampling frequency is being improved continually. Network conflict is becoming more and more serious at sensors and actuators nodes because of the limited channels of network during the transmission of information. So, it is important to explore a reasonable scheduling policy to reduce the network conflict at nodes and to avoid the loss of important information. This motivates us to conduct the research work.

In this paper, scheduling strategy based on predicted error is proposed to reduce the energy consumption for a class of time continuous NCS with uncertain parameters and outside disturbance. A certain model is set at the sensors side to predict the state of system, and a transmission threshold is set at the sensor node. Sampling data is transmitted and calculated if and only if the absolute value of prediction error is larger than the threshold value. By using Lyapunov-Krasovskii functional and linear matrix inequality method, H_∞ controller is designed to render the NCS with scheduling strategy based on predicted error asymptotically stable.

Notation: R^n denotes the n-dimensional Euclidean space. The superscript 'T' stands for matrix transposition. The notation $X > 0$ means that the matrix X is a real positive definite matrix. I is the identity matrix of appropriate dimensions. $\begin{bmatrix} X & Z \\ * & Y \end{bmatrix}$ denotes a symmetric matrix, where $*$ denotes the entries implied by symmetry.

2. Modeling for NCS with Scheduling Strategy Based on Predicted Error.

The structure of NCS with scheduling strategy based on predicted error is shown as Figure 1. $x(i_kh)$, $\bar{x}(i_kh)$, $\tilde{x}(i_kh)$ separately represent the sampling data, predicted value and

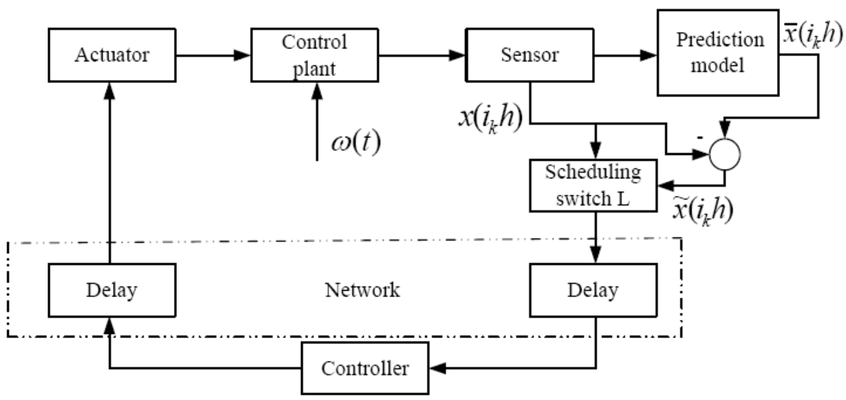

FIGURE 1. The structure of NCS with scheduling strategy based on predicted error

predicted error at time $i_k h$, where h is sampling period, i_k is sampling sequence, and $i_k = 1, 2, 3 \cdots$.

Uncertain linear time continuous networked control system can be described as follows.

$$\begin{cases} \dot{x}(t) = \vec{A}x(t) + \vec{D}x(t-d) + \vec{B}K\Gamma(x(i_k h)) + Hw(t) \\ y(t) = Cx(t) \\ t \in [i_k h + \tau_k, i_{k+1}h + \tau_{k+1}] \end{cases} \tag{1}$$

where $x \in R^n$, $u \in R^m$, $y \in R^r$ and $w \in R^p$ represent state value, input, output and outside disturbance separately; τ_k is the total time that data reaches actuator from sensor; $\vec{A} = A + \Delta A$, $\vec{B} = B + \Delta B$, $\vec{D} = D + \Delta D$; A, B, D, H are matrices with appropriate dimensions; d is the constant delay; ΔA, ΔB, ΔD are matrices with uncertain time-varying parameters, satisfying $[\Delta A \ \Delta D \ \Delta B] = \Theta F(t)[E_1 \ E_2 \ E_3]$; $F(t)$ is an unknown matrix function with Legesgue measurable properties, satisfying $F^T F \leq I$; Θ, E_1, E_2, E_3 are constant matrices with appropriate dimensions.

To facilitate discussion, some assumptions are employed as follows.

A1. *Sensor is time driven while the controller and actuator are event driven.*

A2. *Before the first controlled input reaches the actuator, controlled input always maintain $u(t) = 0$, and zero-order holder is used at actuator node.*

2.1. **The description about prediction error.** The k^{th} sampling data is used to predict the state value at next time. After comparing with the real value at next time, the predicted error can be obtained. And then the state value is updated spontaneously. The prediction model can be described as follows.

$$\bar{x}(i_{k+1}h) = A'x(i_k h) + B'u(i_k h) \tag{2}$$

where $A' = e^{Ah}$, $B' = e^{Ah}B$.

The predicted error produced by the model is shown as follows.

$$\tilde{x}(i_k h) = x(i_k h) - \bar{x}(i_k h) \tag{3}$$

2.2. **The description about scheduling strategy.** Considering the restrained condition of transmission as: $|\tilde{x}_j(i_k h)| \leq r_j$ (r_j transmission threshold, $j = [1, 2, \cdots, n]$), when the restrained condition of transmission meets, state $x_j(i_k h)$ will not be transmitted and keeps $x_j(i_k h) = 0$.

According to the description above, piecewise function is introduced as follows.

$$l_j = \begin{cases} 0 & |\tilde{x}_j(i_k h)| \leq r_j \\ 1 & |\tilde{x}_j(i_k h)| > r_j \end{cases} \tag{4}$$

when $l_j = 1$, state $x_j(i_k h)$ will be transmitted to controller and calculated; when $l_j = 0$, the data $x_j(i_k h)$ will be discarded. We define the switch matrix as $L = diag(l_1, l_2, \cdots, l_n)$.

$$\Gamma(x(i_k h)) = \begin{cases} Lx(i_k h) & i_k > 1 \\ x(i_k h) & i_k = 1 \end{cases} \tag{5}$$

Based on Equation (5), we know the first sampling data cannot be discarded. We assume the state of NCS is completely measurable, and state feedback can be introduced as follows.

$$u(t) = K\Gamma(x(i_k h)) \tag{6}$$

2.3. **The whole NCS model.** Submitting (6) into (1), closed loop model of NCS can be obtained

$$\begin{cases} \dot{x}(t) = (A + \Delta A)x(t) + (D + \Delta D)x(t-d) + (B + \Delta B)K\Gamma(x(i_k h)) + Hw(t) \\ \Gamma(x(i_k h)) = Lx(i_k h) \\ y(t) = Cx(t) \\ t \in [i_k h + \tau_k, i_{k+1}h + \tau_{k+1}] \end{cases} \tag{7}$$

We define

$$(i_{k+1} - i_k)h + \tau_{k+1} \le \eta \tag{8}$$

where η is the maximum value of delay affected by transmission delay and data packets dropouts.

3. H_∞ Controller Design for NCS with Scheduling Strategy Based on Predicted Error.

Definition 3.1. [14]. *It is called that system (7) is asymptotically stable with H_∞ norm bound γ, if it satisfies that*

(1) The closed loop system is asymptotically stable when $w(t) = 0$.

(2) In any zero initial condition, given $\gamma > 0$, for any nonzero vector $w(t) \in L_2[0, \infty)$, the output $y(k)$ satisfies $\|y(t)\|_2 \le \gamma \|w(t)\|_2$. It is called that system (7) is asymptotically stable with H_∞ norm bound γ.

Lemma 3.1 (Newton-Leibniz). *For any variable vector $x \in R^n$, the following equation holds.*

$$x(t) - x(i_k h) - \int_{i_k h}^{t} \dot{x}(\partial) d_\partial = 0 \tag{9}$$

Lemma 3.2. [4]. *For any matrices N, L with appropriate dimensions, as well as variable vectors $\varepsilon(t)$, $\dot{x}(t)$ and constants a, b, the following equation exists.*

$$\begin{aligned} &\textstyle\int_a^b [\varepsilon^T(t)N + \dot{x}^T(v)L]L^{-1}[N^T\varepsilon(t) + L\dot{x}(v)]d_v \\ &= (b-a)\varepsilon^T(t)NL^{-1}N^T\varepsilon(t) + 2\varepsilon^T(t)N \textstyle\int_a^b \dot{x}(v)d_v + \int_a^b \dot{x}^T(v)L\dot{x}(v)d_v \end{aligned} \tag{10}$$

Corollary 3.1. *Given a symmetric positive definite matrix L and a set of constants a, b satisfying $b > a$, for any matrix N with appropriate dimension, the following inequality is established.*

$$-2\varepsilon^T(t)N \int_a^b \dot{x}(v)d_v \le (b-a)\varepsilon^T(t)NL^{-1}N^T\varepsilon(t) + \int_a^b \dot{x}^T(v)L\dot{x}(v)d_v \tag{11}$$

Proof: Because $L > 0$, we have $L^{-1} > 0$.

We have

$$\begin{aligned} &\int_a^b [\varepsilon^T(t)N + \dot{x}^T(v)L]L^{-1}[N^T\varepsilon(t) + L\dot{x}(v)] \\ &= \int_a^b [N^T\varepsilon(t) + L\dot{x}(v)]^T L^{-1}[N^T\varepsilon(t) + L\dot{x}(v)] \ge 0 \end{aligned}$$

Based on Equation (10) in Lemma 3.2, we have

$$(b-a)\varepsilon^T(t)NL^{-1}N^T\varepsilon(t) + 2\varepsilon^T(t)N \int_a^b \dot{x}(v)d_v + \int_a^b \dot{x}^T(v)L\dot{x}(v)d_v \ge 0$$

This is equivalent to inequality (11).

Lemma 3.3. [15]. *For any matrices W, M, N, $F(t)$ with $F^TF \le I$, and any scalar $\varepsilon > 0$, the inequality holds as*

$$W + MF(t)N + N^TF^T(t)M^T \le W + \varepsilon MM^T + \varepsilon^{-1}N^TN \tag{12}$$

3.1. **Stability analysis.**

Theorem 3.1. *Given a set of constants $\gamma > 0$, $d > 0$, $\eta > 0$, if there exist matrices M_i, N_i, G_i ($i = 1, 2, 3, 4, 5, 6$), K and symmetric matrices $P > 0$, $T_j > 0$, $R_j > 0$ ($j = 1, 2$), as well as a constant $\varepsilon > 0$, satisfying*

$$\begin{bmatrix} \Phi & \bar{\Theta}^T & \bar{E} \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon^{-1}I \end{bmatrix} < 0 \tag{13}$$

where

$$\Phi = \begin{bmatrix} \bar{\Omega} & dN & \eta M \\ * & -dR_2 & 0 \\ * & * & -\eta T_2 \end{bmatrix}, \ \bar{\Omega} = \begin{bmatrix} \bar{\Omega}_{11} & \bar{\Omega}_{12} & \bar{\Omega}_{13} & \bar{\Omega}_{14} & \bar{\Omega}_{15} & \bar{\Omega}_{16} \\ * & \bar{\Omega}_{22} & \bar{\Omega}_{23} & \bar{\Omega}_{24} & \bar{\Omega}_{25} & \bar{\Omega}_{26} \\ * & * & \bar{\Omega}_{33} & \bar{\Omega}_{34} & \bar{\Omega}_{35} & \bar{\Omega}_{36} \\ * & * & * & \bar{\Omega}_{44} & \bar{\Omega}_{45} & \bar{\Omega}_{46} \\ * & * & * & * & \bar{\Omega}_{55} & \bar{\Omega}_{56} \\ * & * & * & * & * & \bar{\Omega}_{66} \end{bmatrix},$$

$\bar{\Omega}_{11} = T_1 + R_1 + M_1 + M_1^T + N_1 + N_1^T + C^TC + G_1A + A^TG_1^T$,
$\bar{\Omega}_{12} = M_2^T + N_2^T - N_1 + A^TG_2^T + G_1D$,
$\bar{\Omega}_{13} = M_3^T - M_1 + N_3^T + A^TG_3^T$, $\bar{\Omega}_{14} = M_4^T + N_4^T + A^TG_4^T + G_1BK$,
$\bar{\Omega}_{15} = P + M_5^T + N_5^T - G_1 + A^TG_5^T$, $\bar{\Omega}_{16} = M_6^T + N_6^T + G_1H + A^TG_6^T$,
$\bar{\Omega}_{22} = -R_1 - N_2 - N_2^T + G_2D + D^TG_2^T$, $\bar{\Omega}_{23} = -M_2 - N_3^T + D^TG_3^T$,
$\bar{\Omega}_{24} = -N_4^T + D^TG_4^T + G_2BK$, $\bar{\Omega}_{25} = -N_5^T - G_2 + D^TG_5^T$,
$\bar{\Omega}_{26} = -N_6^T + G_2H + D^TG_6^T$, $\bar{\Omega}_{33} = -T_1 - M_3 - M_3^T$, $\bar{\Omega}_{34} = -M_4^T + G_3BK$,
$\bar{\Omega}_{35} = -M_5^T - G_3$, $\bar{\Omega}_{36} = -M_6^T + G_3H$, $\bar{\Omega}_{44} = G_4BK + K^TB^TG_4^T$,
$\bar{\Omega}_{45} = -G_4 + K^TB^TG_5^T$, $\bar{\Omega}_{46} = G_4H + K^TB^TG_6^T$, $\bar{\Omega}_{55} = \eta T_2 + dR_2 - G_5 - G_5^T$,
$\bar{\Omega}_{56} = G_5H - G_6^T$, $\bar{\Omega}_{66} = G_6H + H^TG_6^T - \gamma^2 I$, $M^T = \left[M_1^T, M_2^T, M_3^T, M_4^T, M_5^T, M_6^T\right]$,
$N^T = \left[N_1^T, N_2^T, N_3^T, N_4^T, N_5^T, N_6^T\right]$, $\bar{E} = [E_1, E_2, 0, E_3K, 0, 0, 0, 0]^T$,
$\bar{\Theta} = [\Theta^TG_1^T, \Theta^TG_2^T, \Theta^TG_3^T, \Theta^TG_4^T, \Theta^TG_5^T, \Theta^TG_6^T, 0, 0]^T$,

then the system (7) is asymptotically stable with H_∞ norm bound γ.

Proof: We consider the Lyapunov-Krasovskii function as follows.

$$\begin{aligned} V(t) &= x^T(t)Px(t) + \int_{i_kh}^{t} x^T(\alpha)T_1x(\alpha)d\alpha + \int_{t-d}^{t} x^T(\beta)R_1x(\beta) \\ &\quad + \int_{t-\eta}^{t}\int_{\theta}^{t} x^T(\delta)T_2x(\delta)d_\theta d_\delta + \int_{t-d}^{t}\int_{\tau}^{t} x^T(\omega)R_2x(\omega)d_\tau d_\omega \end{aligned} \tag{14}$$

Calculating the derivative of Lyapunov-Krasovskii function, we have

$$\begin{aligned} \dot{V}(t) &= 2x^T(t)P\dot{x}(t) + x^T(t)(T_1 + R_1)x(t) - x^T(i_kh)T_1x(i_kh) \\ &\quad -x^T(t-d)R_1x(t-d) + \eta\dot{x}^T(t)T_2\dot{x}(t) + d\dot{x}^T(t)R_2\dot{x}(t) \\ &\quad - \int_{t-\eta}^{t} \dot{x}^T(\delta)T_2\dot{x}(\delta)d_\delta - \int_{t-d}^{t} \dot{x}^T(\omega)R_2\dot{x}(\omega)d_\omega \end{aligned} \tag{15}$$

Using weighted technology based on Lemma 3.1 and model (7), we have

$$\begin{aligned}\dot{V}(t) = {}& 2x^T(t)P\dot{x}(t) + x^T(t)(T_1+R_1)x(t) - x^T(i_kh)T_1x(i_kh) \\ & -x^T(t-d)R_1x(t-d) + \eta\dot{x}^T(t)T_2\dot{x}(t) + d\dot{x}^T(t)R_2\dot{x}(t) - \int_{t-\eta}^{t}\dot{x}^T(\delta)T_2\dot{x}(\delta)d_\delta \\ & -\int_{t-d}^{t}\dot{x}^T(\omega)R_2\dot{x}(\omega)d_\omega + 2\xi^T(t)M\left[x(t) - x(i_kh) - \int_{i_kh}^{t}\dot{x}(\beta)d_\beta\right] \\ & -\int_{i_kh}^{t}\dot{x}(\beta)d_\beta + 2\xi^T(t)N\left[x(t) - x(t-d) - \int_{t-d}^{t}\dot{x}(\lambda)d_\lambda\right] \\ & +2\xi^T(t)G[(A+\Delta A)x(t) + (D+\Delta D)x(t-d) \\ & +(B+\Delta B)K\Gamma(x(i_kh)) + Hw(t) - \dot{x}(t)]\end{aligned} \tag{16}$$

where

$$\begin{aligned}\xi^T(t) &= \left[x^T(t), x^T(t-d), x^T(i_kh), \Gamma^T(x(i_kh)), \dot{x}^T(t), w^T(t)\right], \\ M^T &= \left[M_1^T, M_2^T, M_3^T, M_4^T, M_5^T, M_6^T\right], \\ N^T &= \left[N_1^T, N_2^T, N_3^T, N_4^T, N_5^T, N_6^T\right], \\ G^T &= \left[G_1^T, G_2^T, G_3^T, G_4^T, G_5^T, G_6^T\right].\end{aligned}$$

Based on (8), we have

$$-\int_{t-\eta}^{t}\dot{x}^T(\delta)T_2\dot{x}(\delta)d_\delta \le -\int_{i_kh}^{t}\dot{x}^T(\delta)T_2\dot{x}(\delta)d_\delta \tag{17}$$

Based on Corollary 3.1, we have

$$\begin{aligned}-2\xi^T(t)M\int_{i_kh}^{t}\dot{x}(\beta)d_\beta &\le (t-i_kh)\varepsilon^T(t)MT_2^{-1}M^T\varepsilon(t) + \int_{i_kh}^{t}\dot{x}^T(\beta)T_2\dot{x}(\beta)d_\beta \\ &\le \eta\varepsilon^T(t)MT_2^{-1}M^T\varepsilon(t) + \int_{i_kh}^{t}\dot{x}^T(\beta)T_2\dot{x}(\beta)d_\beta\end{aligned} \tag{18}$$

and

$$-2\xi^T(t)N\int_{i_kh}^{t}\dot{x}(\beta)d_\beta \le d\varepsilon^T(t)NR_2^{-1}N^T\varepsilon(t) + \int_{i_kh}^{t}\dot{x}^T(\beta)R_2\dot{x}(\beta)d_\beta \tag{19}$$

Submitting (17)-(19) to (16), we have

$$\begin{aligned}\dot{V}(t) \le {}& 2x^T(t)P\dot{x}(t) + x^T(t)(T_1+R_1)x(t) - x^T(i_kh)T_1x(i_kh) - x^T(t-d)R_1x(t-d) \\ & +\eta\dot{x}^T(t)T_2\dot{x}(t) + d\dot{x}^T(t)R_2\dot{x}(t) + \eta\xi^T(t)MT_2^{-1}M^T\xi(t) + d\xi^T(t)NR_2^{-1}N^T\xi(t) \\ & +2\xi^T(t)M[x(t) - x(i_kh)] + 2\xi^T(t)N[x(t) - x(t-d)] + 2\xi^T(t)G[(A+\Delta A)x(t) \\ & +(D+\Delta D)x(t-d) + (B+\Delta B)K\Gamma(x(i_kh)) + Hw(t) - \dot{x}(t)]\end{aligned} \tag{20}$$

Inequality (20) can be rewritten as

$$\begin{aligned}\dot{V}(t) \le {}& [x^T(t)P\dot{x}(t) + \dot{x}^T(t)P^Tx(t)] + x^T(t)(T_1+R_1)x(t) - x^T(i_kh)T_1x(i_kh) \\ & -x^T(t-d)R_1x(t-d) + \eta\dot{x}^T(t)T_2\dot{x}(t) + d\dot{x}^T(t)R_2\dot{x}(t) + \eta\xi^T(t)MT_2^{-1}M^T\xi(t) \\ & +d\xi^T(t)NR_2^{-1}N^T\xi(t) + \xi^T(t)M[x(t) - x(i_kh)] + [x(t) - x(i_kh)]^TM^T\xi(t) \\ & +\xi^T(t)N[x(t) - x(t-d)] + [x(t) - x(t-d)]^TN^T\xi(t) + \xi^T(t)G[(A+\Delta A)x(t) \\ & +(D+\Delta D)x(t-d) + (B+\Delta B)K\Gamma(x(i_kh)) + Hw(t) - \dot{x}(t)] + [(A+\Delta A)x(t) \\ & +(D+\Delta D)x(t-d) + (B+\Delta B)K\Gamma(x(i_kh)) + Hw(t) - \dot{x}(t)]^TG^T\xi(t)\end{aligned} \tag{21}$$

Plus $y^T(t)y(t) - \gamma^2 w^T(t)w(t)$ in (21) at both sides, it can be obtained

$$\begin{aligned}
&\dot V(t) + y^T(t)y(t) - \gamma^2 w^T(t)w(t) \\
\le{}& [x^T(t)P\dot x(t) + \dot x^T(t)P^T x(t)] + x^T(t)(T_1 + R_1)x(t) - x^T(i_k h)T_1 x(i_k h) \\
&-x^T(t-d)R_1 x(t-d) + \eta \dot x^T(t)T_2\dot x(t) + d\dot x^T(t)R_2\dot x(t) + \eta\xi^T(t)MT_2^{-1}M^T\xi(t) \\
&+d\xi^T(t)NR_2^{-1}N^T\xi(t) + \xi^T(t)M[x(t) - x(i_k h)] + [x(t) - x(i_k h)]^T M^T\xi(t) \\
&+\xi^T(t)N[x(t) - x(t-d)] + [x(t) - x(t-d)]^T N^T\xi(t) + \xi^T(t)G[(A + \Delta A)x(t) \\
&+(D + \Delta D)x(t-d) + (B + \Delta B)K\Gamma(x(i_k h)) + Hw(t) - \dot x(t)] \\
&+[(A + \Delta A)x(t) + (D + \Delta D)x(t-d) + (B + \Delta B)K\Gamma(x(i_k h)) \\
&+Hw(t) - \dot x(t)]^T G^T\xi(t) + x^T(t)C^T Cx(t) \\
={}& \eta\xi^T(t)MT_2^{-1}M^T\xi(t) + d\xi^T(t)NR_2^{-1}N^T\xi(t) + \xi^T(t)\Omega\xi(t)
\end{aligned} \tag{22}$$

where

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} & \Omega_{16} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & \Omega_{25} & \Omega_{26} \\ * & * & \Omega_{33} & \Omega_{34} & \Omega_{35} & \Omega_{36} \\ * & * & * & \Omega_{44} & \Omega_{45} & \Omega_{46} \\ * & * & * & * & \Omega_{55} & \Omega_{56} \\ * & * & * & * & * & \Omega_{66} \end{bmatrix},$$

$$\begin{aligned}
&\Omega_{11} = T_1 + R_1 + M_1 + M_1^T + N_1 + N_1^T + G_1(A + \Delta A) + (A + \Delta A)^T G_1^T + C^T C, \\
&\Omega_{12} = M_2^T + N_2^T - N_1 + (A + \Delta A)^T G_2^T + G_1(D + \Delta D), \\
&\Omega_{13} = M_3^T - M_1 + N_3^T + (A + \Delta A)^T G_3^T, \\
&\Omega_{14} = M_4^T + N_4^T + (A + \Delta A)^T G_4^T + G_1(B + \Delta B)K, \\
&\Omega_{15} = P + M_5^T + N_5^T - G_1 + (A^T + \Delta A)^T G_5^T, \\
&\Omega_{16} = M_6^T + N_6^T + G_1 H + (A^T + \Delta A)^T G_6^T, \\
&\Omega_{22} = -R_1 - N_2 - N_2^T + G_2(D + \Delta D) + (D + \Delta D)^T G_2^T, \\
&\Omega_{23} = -M_2 - N_3^T + (D + \Delta D)^T G_3^T, \\
&\Omega_{24} = -N_4^T + (D + \Delta D)^T G_4^T + G_2(B + \Delta B)K, \\
&\Omega_{25} = -N_5^T + (D + \Delta D)^T G_5^T - G_2, \\
&\Omega_{26} = -N_6^T + (D + \Delta D)^T G_6^T + G_2 H,\ \Omega_{33} = -T_1 - M_3 - M_3^T, \\
&\Omega_{34} = -M_4^T + G_3(B + \Delta B)K,\ \Omega_{35} = -M_5^T - G_3,\ \Omega_{36} = -M_6^T + G_3 H, \\
&\Omega_{44} = K^T(B + \Delta B)^T G_4^T + G_4(B + \Delta B)K,\ \Omega_{45} = K^T(B + \Delta B)^T G_5^T - G_4, \\
&\Omega_{46} = K^T(B + \Delta B)^T G_6^T + G_4 H,\ \Omega_{55} = \eta T_2 + dR_2 - G_5 - G_5^T, \\
&\Omega_{56} = G_5 H - G_6^T,\ \Omega_{66} = G_6 H + H^T G_6^T - \gamma^2 I.
\end{aligned}$$

Now we consider

$$\eta MT_2^{-1}M^T + dNR_2^{-1}N^T + \Omega < 0 \tag{23}$$

From (23), we know

$$\dot V(t) + y^T(t)y(t) - \gamma^2 w^T(t)w(t) < 0 \tag{24}$$

1) If $w(t) \equiv 0$, we have $V(\infty) > 0$, $\dot V(t) < 0$;

2) For (24), we have $V(\infty) - V(t_0) + \int_{t_0}^{\infty} y^T(v)y(v)d_v - \gamma^2\int_{t_0}^{\infty} w^T(v)w(v)d_v \le 0$.

If $V(t_0) = 0$, we must have $\int_{t_0}^{\infty} y^T(v)y(v)d_v \le \gamma^2\int_{t_0}^{\infty} w^T(v)w(v)d_v$. Therefore, $\|y(t)\|_2 \le \gamma\|w(t)\|_2$. From Definition 3.1, we know the system (7) is asymptotically stable with H_∞ norm bound γ.

Applying Schur complement to (23), pre- and post-multiplying with $diag(I, dI)$ at both sides, we have

$$\begin{bmatrix} \eta M T_2^{-1} M^T + \Omega & dN \\ * & -dR_2 \end{bmatrix} < 0 \tag{25}$$

Inequality (25) can be rewritten as

$$\begin{bmatrix} \Omega & dN \\ * & -dR_2 \end{bmatrix} + \eta \begin{bmatrix} M \\ 0 \end{bmatrix} T_2^{-1} \begin{bmatrix} M \\ 0 \end{bmatrix}^T < 0$$

Applying Schur complement again, we have

$$\begin{bmatrix} \Omega & dN & \eta M \\ * & -dR_2 & 0 \\ * & * & -\eta T_2 \end{bmatrix} < 0 \tag{26}$$

Because $[\Delta A \ \Delta D \ \Delta B] = \Theta F(t)[E_1 \ E_2 \ E_3]$, inequality (26) can be rewritten as

$$\Phi + \bar{\Theta} F \bar{E} + \bar{E}^T F^T \bar{\Theta}^T < 0 \tag{27}$$

Based on Lemma 3.3, the uncertain matrix F can be eliminated and a sufficient condition of (27) is obtained.

$$\Phi + \varepsilon^{-1} \bar{\Theta}^T \bar{\Theta} + \varepsilon \bar{E} \bar{E}^T < 0 \tag{28}$$

Based on Schur complement, we know inequality (28) is equivalent to inequality (13). Therefore, this completes the proof.

Remark 3.1. *Because the items such as G_1BK exist in inequality (13), it is not a linear matrix inequality, which cannot be solved by using the LMI Tool-box. It will be reformulated into LMI via a change of variables in our next work.*

3.2. **Controller design for NCS.**

Theorem 3.2. *Given a set of constants μ_i ($i = 1, 2, 3, 4, 5$), $d > 0$, $\eta > 0$, if there exist matrices Y, $\bar{M}_i$ and $\bar{N}_i$ ($i = 1, 2, 3, 4, 5, 6$), invertible matrix X, symmetric matrices $\bar{P} > 0$, $\bar{T}_j > 0$, $\bar{R}_j > 0$ ($j = 1, 2$), as well as a set of constants $\varepsilon > 0$, $\sigma > 0$ satisfying the following LMIs.*

$$X - I > 0 \tag{29}$$

$$\begin{bmatrix} \Psi & 0 & 0 & 0 \\ * & \Pi_1 & \Pi_2 & \Pi_3 \\ * & * & -\varepsilon I & 0 \\ * & * & * & -\varepsilon I \end{bmatrix} < 0 \tag{30}$$

where

$$\Psi = [-I, C, 0, 0, 0, 0], \ \Pi_1 = \begin{bmatrix} \Delta & d\bar{N} & \eta \bar{M} \\ * & -d\bar{R}_2 & 0 \\ * & * & -\eta \bar{T}_2 \end{bmatrix},$$

$$\Delta = \begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & \Delta_{14} & \Delta_{15} & \Delta_{16} \\ * & \Delta_{22} & \Delta_{23} & \Delta_{24} & \Delta_{25} & \Delta_{26} \\ * & * & \Delta_{33} & \Delta_{34} & \Delta_{35} & \Delta_{36} \\ * & * & * & \Delta_{44} & \Delta_{45} & \Delta_{46} \\ * & * & * & * & \Delta_{55} & \Delta_{56} \\ * & * & * & * & * & \Delta_{66} \end{bmatrix},$$

$$\Delta_{11} = \bar{T}_1 + \bar{R}_1 + \bar{M}_1 + \bar{M}_1^T + \bar{N}_1 + \bar{N}_1^T + AX^T + XA^T,$$
$$\Delta_{12} = \bar{M}_2^T + \bar{N}_2^T - \bar{N}_1 + \mu_1 XA^T + DX^T, \ \Delta_{13} = \bar{M}_3^T - \bar{M}_1 + \bar{N}_3^T + \mu_2 XA^T,$$
$$\Delta_{14} = \bar{M}_4^T + \bar{N}_4^T + \mu_3 XA^T + BY, \ \Delta_{15} = \bar{P} + \bar{M}_5^T + \bar{N}_5^T - X^T + \mu_4 XA^T,$$

$\Delta_{16} = \bar{M}_6^T + \bar{N}_6^T + HX^T + \mu_5 XA^T,\ \Delta_{22} = -\bar{R}_1 - \bar{N}_2 - \bar{N}_2^T + \mu_1 DX^T + \mu_1 XD^T,$
$\Delta_{23} = -\bar{M}_2 - \bar{N}_3^T + \mu_2 XD^T,\ \Delta_{24} = -\bar{N}_4^T + \mu_3 XD^T + \mu_1 BY,$
$\Delta_{25} = -\bar{N}_5^T - \mu_1 X^T + \mu_4 XD^T,\ \Delta_{26} = -\bar{N}_6^T + \mu_1 HX^T + \mu_5 XD^T,$
$\Delta_{33} = -\bar{T}_1 - \bar{M}_3 - \bar{M}_3^T,\ \Delta_{34} = -\bar{M}_4^T + \mu_2 BY,\ \Delta_{35} = -\bar{M}_5^T - \mu_2 X^T,$
$\Delta_{36} = -\bar{M}_6^T + \mu_2 HX^T,\ \Delta_{44} = \mu_3 BY + \mu_3 Y^T B^T,\ \Delta_{45} = -\mu_3 X^T + \mu_3 Y^T B^T,$
$\Delta_{46} = \mu_3 HX^T + \mu_5 Y^T B^T,\ \Delta_{55} = \eta\bar{T}_2 + d\bar{R}_2 - \mu_4 X^T - \mu_4 X,\ \Delta_{56} = \mu_4 HX^T - \mu_5 X,$
$\Delta_{66} = \mu_5 HX^T + \mu_5 XH^T - \sigma I,\ \bar{M}^T = \left[\bar{M}_1^T, \bar{M}_2^T, \bar{M}_3^T, \bar{M}_4^T, \bar{M}_5^T, \bar{M}_6^T\right],$
$\bar{N}^T = \left[\bar{N}_1^T, \bar{N}_2^T, \bar{N}_3^T, \bar{N}_4^T, \bar{N}_5^T, \bar{N}_6^T\right],$
$\Pi_2 = [\Theta^T, \mu_1\Theta^T, \mu_2\Theta^T, \mu_3\Theta^T, \mu_4\Theta^T, \mu_5\Theta^T, 0, 0]^T,$
$\Pi_3 = \left[E_1 X^T, E_2 X^T, 0, E_3 Y, 0, 0, 0, 0\right]^T,$

then the system (7) is asymptotically stable with H_∞ norm bound $\gamma = \sqrt{\sigma}$, and the gain matrix is $K = YX^{-T}$.

Proof: The proof is based on a suitable congruence transformation and a change of variables allowing us to obtain inequality (13) in Theorem 3.1. Based on Schur complement, inequality (13) is equivalent to

$$\begin{bmatrix} \Psi & 0 & 0 & 0 \\ * & \Phi' & \bar{\Theta}^T & \bar{E} \\ * & * & -\varepsilon I & 0 \\ * & * & * & -\varepsilon^{-1} I \end{bmatrix} < 0 \tag{31}$$

where

$$\Psi = [-I, C, 0, 0, 0, 0],\ \Phi' = \begin{bmatrix} \bar{\Omega}' & dN & \eta M \\ * & -dR_2 & 0 \\ * & * & -\eta T_2 \end{bmatrix},$$

$$\bar{\Omega}' = \begin{bmatrix} \bar{\Omega}'_{11} & \bar{\Omega}_{12} & \bar{\Omega}_{13} & \bar{\Omega}_{14} & \bar{\Omega}_{15} & \bar{\Omega}_{16} \\ * & \bar{\Omega}_{22} & \bar{\Omega}_{23} & \bar{\Omega}_{24} & \bar{\Omega}_{25} & \bar{\Omega}_{26} \\ * & * & \bar{\Omega}_{33} & \bar{\Omega}_{34} & \bar{\Omega}_{35} & \bar{\Omega}_{36} \\ * & * & * & \bar{\Omega}_{44} & \bar{\Omega}_{45} & \bar{\Omega}_{46} \\ * & * & * & * & \bar{\Omega}_{55} & \bar{\Omega}_{56} \\ * & * & * & * & * & \bar{\Omega}_{66} \end{bmatrix},$$

$$\bar{\Omega}'_{11} = T_1 + R_1 + M_1 + M_1^T + N_1 + N_1^T + G_1 A + A^T G_1^T.$$

We define $X = G_1^{-1}$; $G_i = \mu_{i-1} G_1$ $(i = 2, 3, 4, 5, 6)$; $\bar{T}_i = XT_iX^T$; $\bar{R}_i = XR_iX^T$ $(i = 1, 2)$; $\bar{P} = XPX^T$; $\bar{M}_i = XM_iX^T$; $\bar{N}_i = XN_iX^T$ $(i = 1, 2, 3, 4, 5, 6)$; $Y = KX^T$. Pre-multiplying with $diag(I, X, X, X, X, X, X, X, X, I, \varepsilon I)$ and post-multiplying $diag(I, X^T, X^T, X^T, X^T, X^T, X^T, X^T, X^T, I, \varepsilon I)$, we have

$$\begin{bmatrix} \Psi & 0 & 0 & 0 \\ * & \Pi'_1 & \Pi_2 & \Pi_3 \\ * & * & -\varepsilon I & 0 \\ * & * & * & -\varepsilon I \end{bmatrix} < 0 \tag{32}$$

where

$$\Pi_1' = \begin{bmatrix} \Delta' & dN & \eta M \\ * & -dR_2 & 0 \\ * & * & -\eta T_2 \end{bmatrix},\ \Delta' = \begin{bmatrix} \Delta_{11} & \Delta_{12} & \Delta_{13} & \Delta_{14} & \Delta_{15} & \Delta_{16} \\ * & \Delta_{22} & \Delta_{23} & \Delta_{24} & \Delta_{25} & \Delta_{26} \\ * & * & \Delta_{33} & \Delta_{34} & \Delta_{35} & \Delta_{36} \\ * & * & * & \Delta_{44} & \Delta_{45} & \Delta_{46} \\ * & * & * & * & \Delta_{55} & \Delta_{56} \\ * & * & * & * & * & \Delta_{66}' \end{bmatrix},$$

$$\Delta_{66}' = \mu_5 HX^T + \mu_5 XH^T - \gamma^2 XX^T.$$

From (29), we have $XX^T - I > 0$. It can be obtained that $\Delta_{66}' = \mu_5 HX^T + \mu_5 XH^T - \gamma^2 XX^T < \Delta_{66}$. With the definition $\sigma = \gamma^2$, we know (30) is a sufficient condition of inequality (32). Based on $Y = KX^T$, we have $K = YX^{-T}$. Therefore, this completes the proof.

Remark 3.2. *Obviously, inequality (29) and inequality (30) are linear matrix inequalities. So we can find the control parameters by solving the feasibilities with the help of LMI Toolbox.*

$$\Omega_{26} = -N_6^T + (D+\Delta D)^T G_6^T + G_2 H,\ \Omega_{33} = -T_1 - M_3 - M_3^T,$$
$$\Omega_{34} = -M_4^T + G_3(B+\Delta B)K,\ \Omega_{35} = -M_5^T - G_3,\ \Omega_{36} = -M_6^T + G_3 H.$$

4. **Simulations.** Consider the parameters of an inverted pendulum model with delays as follows.

$$A = \begin{bmatrix} 0.07 & -0.023 \\ -0.15 & 0.2 \end{bmatrix},\ B = \begin{bmatrix} -0.4 & 0 \\ -0.5 & 1.08 \end{bmatrix},\ D = \begin{bmatrix} 0.4 & -0.5 \\ 0.15 & 0.45 \end{bmatrix},\ \Theta = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix},$$
$$C = \begin{bmatrix} -1.2 & 0.45 \\ 0 & -0.25 \end{bmatrix},\ H = \begin{bmatrix} -0.89 & 1 \\ 0.54 & 0.8 \end{bmatrix},\ \Delta A = \begin{bmatrix} -0.03\sin t & 0 \\ 0 & -0.012\sin t \end{bmatrix},$$
$$\Delta B = \begin{bmatrix} -0.05\sin t & 0 \\ 0 & -0.04\sin t \end{bmatrix},\ \Delta D = \begin{bmatrix} -0.1\sin t & 0 \\ 0 & -0.03\sin t \end{bmatrix},$$
$$w(t) = \begin{cases} [0.6, -1.4]^T & 55 \le t \le 85 \\ 0 & \text{others.} \end{cases}$$

We consider $F = \begin{bmatrix} -0.1\sin t & 0 \\ 0 & -0.1\sin t \end{bmatrix}$; therefore, $E_1 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.12 \end{bmatrix}$, $E_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0.3 \end{bmatrix}$, $E_3 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.4 \end{bmatrix}$.

With the development of computer technology, the calculating speed of computer is increasingly developed. Here we select the sampling period $h = 1\times 10^{-3}s$. Other parameters are assumed as

$$\mu_1 = -0.029,\ \mu_2 = -2.27,\ \mu_3 = -0.1,\ \mu_4 = 2.51,\ \mu_5 = -0.007,\ \eta = 0.004,\ d = 0.002.$$

By taking advantage of LMI tool-box and submitting these parameters above into inequalities (29) and (30), it can obtained

$$K = YX^{-T} = \begin{bmatrix} 1.0797 & -1.5653 \\ -1.1534 & -2.1894 \end{bmatrix} \text{ with } \gamma = \sqrt{1.6013\times 10^6} = 1.2654\times 10^3.$$

We consider the transmission threshold as $r = \begin{bmatrix} 0.02 & 0.12 \end{bmatrix}$, namely, if the predicted error produced by model (2) satisfies

1) $|\tilde{x}_1(i_k h)| < 0.02$, sensor will not send the data $x_1(i_k h)$ to the controller;
2) $|\tilde{x}_2(i_k h)| < 0.12$, sensor will not send the data $x_2(i_k h)$ to the controller.

We assume the initial state as $x(0) = \begin{bmatrix} 9.8 \\ -5 \end{bmatrix}$, $x(t) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $(t \in [-\eta, 0])$. When the uncertain delay exists and satisfies $(i_{k+1} - i_k)h + \tau_{k+1} \leq \eta$, the response of state and the prediction error curve are separately shown in Figure 2 and Figure 3. And the data packets dropouts in the whole control loops are shown in Figure 4. From Figure 2 and Figure 3, it is clear that the NCS with scheduling strategy based on predicted error is asymptotically stable. After the system reaches the steady state, the data transmission stops. Due to the outside disturbance, the predicted error could exceed the transmission threshold, which should force the senor to transmit the data again. In order to facilitate comparison, the number of data packets dropouts at time $i_k h$ is obtained by stacking number of data packets dropouts nearby 20 data. Obviously, number of data packets dropouts is rare. As time goes on, data transmission gradually stops. After calculated, the average rate of data packets dropouts in the whole simulation is 66.33%.

In addition, we apply the method proposed by S. Longo et al. [17] into the same problem. The average data dropout rate of NCS is just 5.62%. And the design of controller fails with $K = \begin{bmatrix} 0.10912 & 0.1108 \\ -0.5208 & 0.9083 \end{bmatrix}$, and the response of system state is shown as Figure 5. Thus, it sufficiently demonstrates the effectiveness and feasibility of this paper.

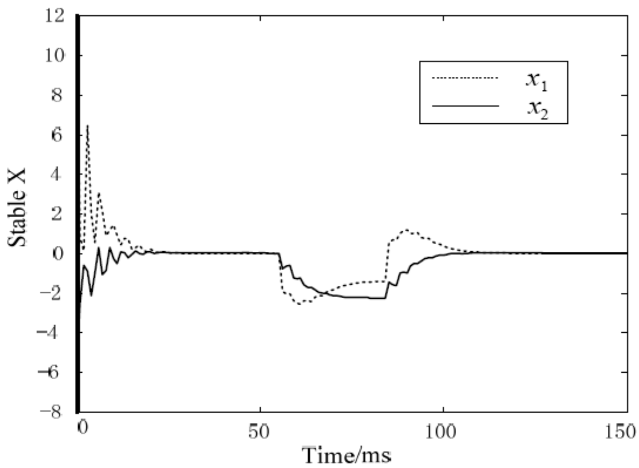

FIGURE 2. The state response curve of NCS

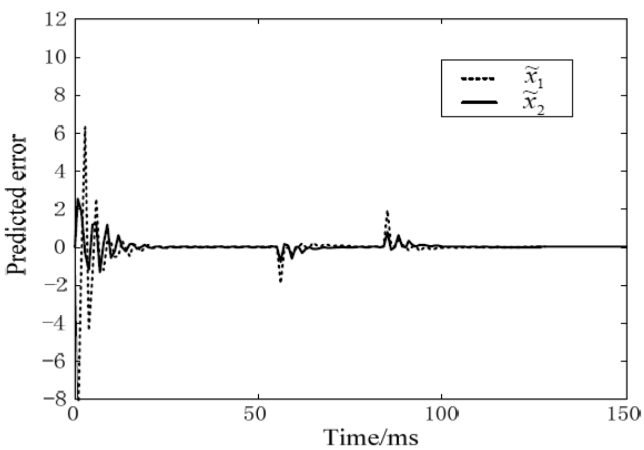

FIGURE 3. The curve of prediction error in NCS

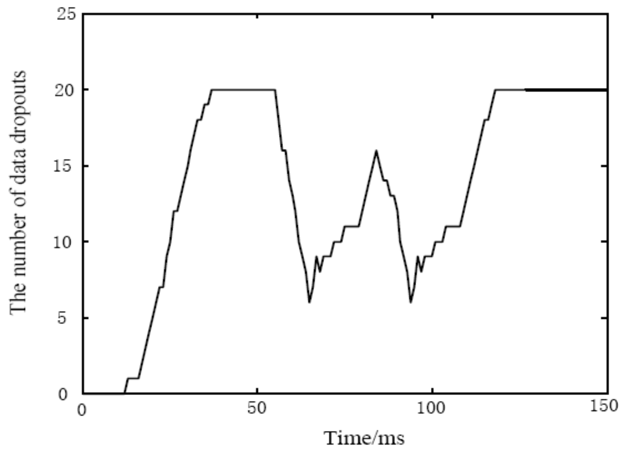

FIGURE 4. The curve of data packets dropouts in NCS

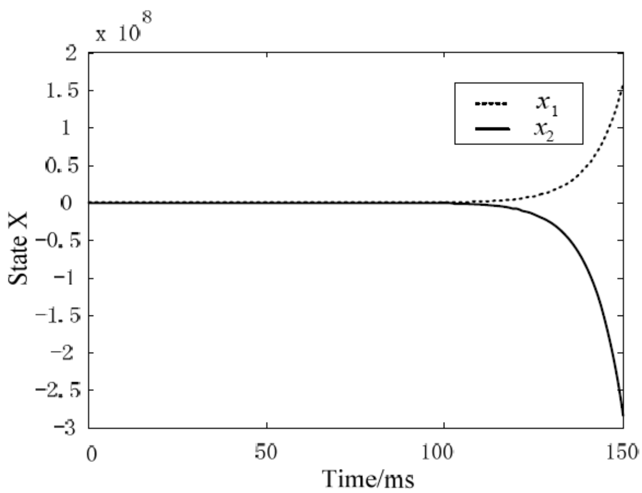

FIGURE 5. The state response curve of NCS

5. **Conclusions.** For network environment, scheduling policy based on model prediction error and accomplished by the switch matrix is presented for networked control systems (NCS). Data is transmitted and computed only if the absolute value of prediction error is larger than the threshold value. And the model of NCS based on prediction error scheduling is established with uncertain parameters and network-induced delay considered. By using Lyapunov-Krasovskii functional and linear matrix inequality method, the H_∞ controller is designed to render the NCS with scheduling strategy based on predicted error asymptotically stable. And the result shows this method can effectively decrease the transmissions of data, which can reduce the network conflict and energy consumption of nodes. Our next research task will be choosing more reasonable values of parameters μ_i $(i = 1, 2, 3, 4, 5)$ to reduce the conservatism.

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