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STATE FEEDBACK CONTROL DESIGN FOR A NETWORKED CONTROL MODEL OF SYSTEMS WITH TWO ADDITIVE TIME-VARYING DELAYS

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Abstract. *This paper is centered on state feedback control design for a networked control model of systems with two additive time-varying delays. Firstly, with a Lyapunov functional (LF) defined on some symmetric matrices, the LF approach is employed to study delay-dependent stability for the system. Note in existing papers all the symmetric matrices are required to be positive definite in order to ensure the LF to be positive definite, and this requirement is only sufficient but not necessary for the LF to be positive definite, a new stability result is derived by slackening the requirement. It is shown both theoretically and numerically that the stability result is less conservative than some existing ones. Based on the stability result, a state feedback controller is designed, such that the closed-loop system is asymptotically stable. Finally, examples are given to show the less conservatism of the stability criteria and the effectiveness of the proposed control method.*

Keywords: Networked control systems, Delay systems, State feedback control, Delay-dependent stability, Stabilization

1. **Introduction.** For years systems with time delays have received considerable attention since they are often encountered in various practical systems, such as engineering systems, biology, economics, neural networks, networked control systems and other areas [1-6]. Since time-delay is frequently the main cause of oscillation, divergence or instability, considerable effort has been devoted to stability for systems with time delays. According to whether stability criteria include the information of the delay, they are divided into two classes: delay-independent stability criteria and delay-dependent ones. It is well known that delay-independent stability criteria tend to be more conservative especially for small size delays. More attention has been paid to delay-dependent stability. For delay-dependent stability results, we refer readers to [7-13]. Among these papers, [11-13] were of systems with interval time-varying delay. It should be pointed out that all the stability results mentioned are based on systems with one single delay in the state.

On the other hand, networked control systems have been receiving great attention these years due to their advantages in low cost, reduced weight and power requirements, simple installation and maintenance, and high reliability. It is well known that the transmission delay and the data packet dropout are two fundamental issues in networked control systems. The transmission delay generally includes the sensor-to-control delay and the control-to-actuator delay. In most of existing papers the sensor-to-control delay and the control-to-actuator delay were combined into one state delay, while the data packet dropouts were modeled as delays and absorbed by the state delay, thus formulating networked control systems as systems with one state delay. Among recently reported results based on this modeling idea, to mention a few, H_∞ control co-design problems were addressed for networked control systems in [4], while the event-triggered control were discussed for networked control systems via dynamic output feedback controllers [5]. It is worth noting that there are new research reports [19-22] about networked control systems with random property. For example, in [19] stability was investigated and stabilization considered for networked control systems with random delays.

Note that the sensor-to-control delay and the control-to-actuator delay are different in nature because of the network transmission conditions. The transmission delay and the data packet dropout also have different properties. It is not rational to lump them into one state delay. In this paper, to study networked control systems we adopt the model of systems with multi-additive time-varying delay components. For simplicity, the system with two additive time-varying delay components will be employed to address state feedback control problem for networked control systems. When the physical plant is a linear system and the controller is a linear state-feedback one with a controller gain K, the networked control system takes the following form:

$$\dot{x}(t) = Ax(t) + BKx(t - d_1(t) - d_2(t)) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state; A and B are known real constant matrices; $d_1(t)$, $d_2(t)$ are two time-varying delays. A nature assumption on the two delays is made

$$0 \leq d_1(t) \leq h_1, \ 0 \leq d_2(t) \leq h_2 \tag{2}$$

and

$$\dot{d}_1(t) \leq \mu_1, \ \dot{d}_2(t) \leq \mu_2 \tag{3}$$

Stability analysis for this kind of system was conducted in [14], and a delay-dependent stability criterion was obtained. An improved stability criterion was derived in [15] by constructing a Lyapunov functional to employ the information of the marginally delayed state $x(t-h)$, where h is defined in (6). However, another marginally delayed state $x(t-h_1)$ was not considered, which caused $-\int_{t-h_1}^{t-d_1(t)} \dot{x}(\alpha)^T Z_1 \dot{x}(\alpha) d\alpha$ to be discarded when bounding the derivative of the Lyapunov functional. On the other hand, in the process of the bounding, many free weighting matrices were introduced, making the stability result complicated. Furthermore, in [14,15] as well as in most existing papers, to guarantee the positive definite of a Lyapunov functional, it was required for all the involved matrices to be positive definite.

In this paper we first consider delay-dependent stability for system (1) by constructing a new Lyapunov functional to employ the information of the marginally delayed state $x(t-h_1)$ as well as $x(t-h)$. Based on the observation that the positive definite of a Lyapunov functional does not necessarily imply that of all the symmetric matrices, less conservative conditions are derived for the LF to be positive definite. On the other hand, when bounding the derivative of the LF, we use a novel technique, motivated from [13], to avoid introducing slack variables but produce a fairly tighter upper bound. The upper bound is dependent on the two time-varying delays. To check negative definiteness for the upper bound we propose a so called convex polyhedron method. The resulting stability criteria turn out to be less conservative with fewer matrix variables. Then we apply the stability criteria to control design problem, which is to determine a state feedback controller gain K such that system (1) is asymptotically stable. A delay-dependent condition will be presented for the state feedback controller gain K such that the system is asymptotically stable. When the condition is feasible, the controller gain can be computed.

Throughout this paper the superscript 'T' stands for matrix transposition. I refers to an identity matrix with appropriate dimensions. E_i Stands for the i^{th} column of $diag\{I_n, I_n, I_n, I_n, I_n\}$ $(i = 1, 2, 3, 4, 5)$. For real symmetric matrices X and Y, the notation $X > Y$ means that the matrix $X - Y$ is positive definite. The $X \geq Y$ follows similarly. The symmetric term in a matrix is denoted by $*$.

To end this section, a lemma is given, which will play an important role in deriving our results.

Lemma 1.1. [16] *For any symmetric positive definite matrix $M > 0$, scalar $\gamma > 0$ and vector function $\omega : [0, \gamma] \to \mathbb{R}^n$ such that the integrations concerned are well defined, the following inequality holds*

$$\left(\int_0^\gamma \omega(s)ds\right)^T M \left(\int_0^\gamma \omega(s)ds\right) \leq \gamma \left(\int_0^\gamma \omega(s)^T M \omega(s)ds\right)$$

2. **Stability Analysis.** In the following we consider the stability analysis problem. Specifically, given K we investigate the condition for system (1) to be asymptotically stable. Lump $d_1(t)$ and $d_2(t)$ into one delay

$$d(t) = d_1(t) + d_2(t) \tag{4}$$

and then system (1) is changed into

$$\dot{x}(t) = Ax(t) + BKx(t - d(t)) \tag{5}$$

where $0 \leq d(t) \leq h$, $\dot{d}(t) \leq \mu$ with

$$h = h_1 + h_2 \tag{6}$$

$$\mu = \mu_1 + \mu_2 \tag{7}$$

However, this treatment is not suitable. On the one hand, from an engineering point of view, the two delays may have different properties, and it is not appropriate to lump them together. On the other hand, from a mathematical point of view, it is very conservative to bound $d(t)$ with $h = h_1 + h_2$, since the maximum of $d(t)$ is generally less than $h = h_1 + h_2$. In the following we present a new stability result for system (1) by considering the two delays separately.

Theorem 2.1. *The systems (1)-(3) are asymptotically stable for given K, h_1, h_2, μ_1 and μ_2 if there exist matrices $P = P^T$, $Q_2 = Q_2^T$, $Q_4 = Q_4^T$, $Q_i > 0$ $(i = 1, 3)$, $Z_j > 0$ $(j = 1, 2)$ such that the following LMIs hold*

$$\begin{bmatrix} \frac{h_1}{h}P + Z_1 + Z_2 & -Z_1 - Z_2 \\ * & Z_1 + Z_2 + h_1(Q_2 + Q_4) \end{bmatrix} > 0 \tag{8}$$

$$\begin{bmatrix} P + Z_2 & -Z_2 \\ * & Z_2 + hQ_2 \end{bmatrix} > 0 \tag{9}$$

$$\Phi - E_{13}h_1^{-1}(Z_1 + Z_2)E_{13}^T - E_{23}h_2^{-1}Z_2E_{23}^T < 0 \tag{10}$$

$$\Phi - E_{13}h_1^{-1}(Z_1 + Z_2)E_{13}^T - E_{24}h_2h^{-2}Z_2E_{24}^T < 0 \tag{11}$$

$$\Phi - E_{35}h_1^{-1}Z_1E_{35}^T - E_{24}h_1h^{-2}Z_2E_{24}^T - E_{23}h_2^{-1}Z_2E_{23}^T < 0 \tag{12}$$

$$\Phi - E_{35}h_1^{-1}Z_1E_{35}^T - E_{24}h^{-1}Z_2E_{24}^T < 0 \tag{13}$$

where $E_{ij} = E_i - E_j$, *and*

$$\Phi = \begin{bmatrix} \varphi_1 & PBK & h_1^{-1}(Z_1+Z_2) & 0 & 0 \\ * & \varphi_2 & h_2^{-1}Z_2 & h^{-1}Z_2 & 0 \\ * & * & \varphi_3 & 0 & h_1^{-1}Z_1 \\ * & * & * & -Q_2 - h^{-1}Z_2 & 0 \\ * & * & * & * & -Q_4 - h_1^{-1}Z_1 \end{bmatrix} + \begin{bmatrix} A^T \\ (BK)^T \\ 0 \\ 0 \\ 0 \end{bmatrix} [h_1 Z_1 + h Z_2] \begin{bmatrix} A^T \\ (BK)^T \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \tag{14}$$

with h *and* μ *given in (6) and (7) respectively, and*

$$\begin{aligned} \varphi_1 &= PA + A^T P + \sum\nolimits_{i=1}^4 Q_i - h_1^{-1}(Z_1+Z_2) \\ \varphi_2 &= -(1-\mu)Q_3 - (h_2^{-1} + h^{-1})Z_2 \\ \varphi_3 &= -(1-\mu_1)Q_1 - (h_2^{-1} + h_1^{-1})Z_2 - 2h_1^{-1}Z_1 \end{aligned}$$

Proof: Define a Lyapunov functional as

$$\begin{aligned} V(t) = {} & x(t)^T P x(t) + \int_{t-d_1(t)}^t x(\alpha)^T Q_1 x(\alpha) d\alpha + \int_{t-h}^t x(\alpha)^T Q_2 x(\alpha) d\alpha \\ & + \int_{t-d(t)}^t x(\alpha)^T Q_3 x(\alpha) d\alpha + \int_{t-h_1}^t x(\alpha)^T Q_4 x(\alpha) d\alpha \\ & + \int_{-h_1}^0 \int_{t+s}^t \dot{x}(\alpha)^T (Z_1+Z_2) \dot{x}(\alpha) d\alpha ds + \int_{-h}^{-h_1} \int_{t+s}^t \dot{x}(\alpha)^T Z_2 \dot{x}(\alpha) d\alpha ds \end{aligned} \tag{15}$$

where $d(t)$ is defined in (4). Clearly this Lyapunov functional can employ the information of the marginally delayed state $x(t-h_1)$ as well as $x(t-h)$.

Under the condition of Theorem 2.1, we first show Lyapuonv functional (15) is positive definite. By Lemma 1.1 it follows that

$$\begin{aligned} & \int_{-h_1}^0 \int_{t+s}^t \dot{x}(\alpha)^T (Z_1+Z_2) \dot{x}(\alpha) d\alpha ds \\ & \geq \frac{1}{h_1} \int_{-h_1}^0 [x(t) - x(t+s)]^T (Z_1+Z_2) [x(t) - x(t+s)] ds \end{aligned} \tag{16}$$

and

$$\int_{-h}^{-h_1} \int_{t+s}^t \dot{x}(\alpha)^T Z_2 \dot{x}(\alpha) d\alpha ds \geq \frac{1}{h} \int_{-h}^{-h_1} [x(t) - x(t+s)]^T Z_2 [x(t) - x(t+s)] ds \tag{17}$$

On the other hand,

$$x(t)^T P x(t) = \frac{1}{h} \int_{-h_1}^0 x(t)^T P x(t) ds + \frac{1}{h} \int_{-h}^{-h_1} x(t)^T P x(t) ds \tag{18}$$

Noting $Q_i > 0$ $(i = 1, 3)$, it follows from (15)-(18) that

$$\begin{aligned} V(t) \geq {} & \frac{1}{h} \int_{-h_1}^0 x(t)^T P x(t) ds + \frac{1}{h} \int_{-h}^{-h_1} x(t)^T P x(t) ds \\ & + \int_{-h}^0 x(t+s)^T Q_2 x(t+s) ds + \int_{-h_1}^0 x(t+s)^T Q_4 x(t+s) ds \end{aligned}$$

$$+\frac{1}{h_1}\int_{-h_1}^{0}[x(t)-x(t+s)]^T(Z_1+Z_2)[x(t)-x(t+s)]ds$$
$$+\frac{1}{h}\int_{-h}^{-h_1}[x(t)-x(t+s)]^T Z_2[x(t)-x(t+s)]ds$$
$$=\int_{-h_1}^{0}\begin{bmatrix} x(t) \\ x(t+s)\end{bmatrix}^T\begin{bmatrix}\frac{1}{h}P+\frac{1}{h_1}(Z_1+Z_2) & -\frac{1}{h_1}(Z_1+Z_2) \\ * & \frac{1}{h_1}(Z_1+Z_2)+Q_2+Q_4\end{bmatrix}\begin{bmatrix} x(t) \\ x(t+s)\end{bmatrix}ds$$
$$+\int_{-h}^{-h_1}\begin{bmatrix} x(t) \\ x(t+s)\end{bmatrix}^T\begin{bmatrix}\frac{1}{h}P+\frac{1}{h}Z_2 & -\frac{1}{h}Z_2 \\ * & \frac{1}{h}Z_2+Q_2\end{bmatrix}\begin{bmatrix} x(t) \\ x(t+s)\end{bmatrix}ds$$

This together with (8)-(9) means Lyapuonv functional (15) is positive definite.

Now calculating the time derivative of the Lyapuonv functional along the trajectory of (1) yields

$$\begin{aligned}\dot{V}(t) \le\ & 2x(t)^TP(Ax(t)+BKx(t-d(t))) \\ & +\sum\nolimits_{i=1}^{4}x(t)^TQ_ix(t)-x(t-h_1)^TQ_4x(t-h_1)-x(t-h)^TQ_2x(t-h) \\ & -(1-\mu)x(t-d(t))^TQ_3x(t-d(t))-(1-\mu_1)x(t-d_1(t))^TQ_1x(t-d_1(t)) \\ & +(Ax(t)+BKx(t-d(t)))^T(h_1Z_1+hZ_2)[Ax(t)+BKx(t-d(t))] \\ & -\int_{t-h_1}^{t}\dot{x}(\alpha)^T(Z_1+Z_2)\dot{x}(\alpha)d\alpha-\int_{t-h}^{t-h_1}\dot{x}(\alpha)^TZ_2\dot{x}(\alpha)d\alpha \qquad (19)\end{aligned}$$

Note that

$$\begin{aligned}& -\int_{t-h_1}^{t}\dot{x}(\alpha)^T(Z_1+Z_2)\dot{x}(\alpha)d\alpha-\int_{t-h}^{t-h_1}\dot{x}(\alpha)^TZ_2\dot{x}(\alpha)d\alpha \\ =\ & -\int_{t-h_1}^{t}\dot{x}(\alpha)^TZ_1\dot{x}(\alpha)d\alpha-\int_{t-h}^{t}\dot{x}(\alpha)^TZ_2\dot{x}(\alpha)d\alpha \\ =\ & -\int_{t-d_1(t)}^{t}\dot{x}(s)^TZ_1\dot{x}(s)ds-\int_{t-h_1}^{t-d_1(t)}\dot{x}(s)^TZ_1\dot{x}(s)ds \\ & -\int_{t-d_1(t)}^{t}\dot{x}(s)^TZ_2\dot{x}(s)ds-\int_{t-d(t)}^{t-d_1(t)}\dot{x}(s)^TZ_2\dot{x}(s)ds-\int_{t-h}^{t-d(t)}\dot{x}(s)^TZ_2\dot{x}(s)ds \qquad (20)\end{aligned}$$

Write $\alpha=d_1(t)/h_1$ and $\beta=d_2(t)/h_2$. Then

$$\begin{aligned}-\int_{t-d_1(t)}^{t}\dot{x}(s)^TZ_1\dot{x}(s)ds &= -h_1^{-1}\int_{t-d_1(t)}^{t}h_1\dot{x}(s)^TZ_1\dot{x}(s)ds \\ &= -h_1^{-1}\int_{t-d_1(t)}^{t}d_1(t)\dot{x}(s)^TZ_1\dot{x}(s)ds \\ &\quad -h_1^{-1}\int_{t-d_1(t)}^{t}[h_1-d_1(t)]\dot{x}(s)^TZ_1\dot{x}(s)ds \qquad (21)\end{aligned}$$

It follows from (21) that

$$-\int_{t-d_1(t)}^{t}\dot{x}(s)^TZ_1\dot{x}(s)ds \le -h_1^{-1}\int_{t-d_1(t)}^{t}d_1(t)\dot{x}(s)^TZ_1\dot{x}(s)ds \qquad (22)$$

Using (22) we have that

$$-h_1^{-1}\int_{t-d_1(t)}^{t}[h_1-d_1(t)]\dot{x}(s)^TZ_1\dot{x}(s)ds$$

$$
\begin{aligned}
&= -(1-\alpha)\int_{t-d_1(t)}^{t} \dot{x}(s)^T Z\dot{x}(s)ds \\
&\le -(1-\alpha)h_1^{-1}\int_{t-d_1(t)}^{t} d_1(t)\dot{x}(s)^T Z_1\dot{x}(s)ds
\end{aligned} \tag{23}
$$

By Lemma 1.1 combining (21) and (23) leads to

$$
\begin{aligned}
-\int_{t-d_1(t)}^{t} \dot{x}(s)^T Z_1\dot{x}(s)ds \le\ & -[x(t)-x(t-d_1(t))]^T h_1^{-1} Z_1[x(t) \\
& -x(t-d_1(t))] - (1-\alpha)[x(t)-x(t \\
& -d_1(t))]^T h_1^{-1} Z_1[x(t)-x(t-d_1(t))]
\end{aligned} \tag{24}
$$

Similarly we have

$$
\begin{aligned}
-\int_{t-h_1}^{t-d_1(t)} \dot{x}(s)^T Z_1\dot{x}(s)ds \le\ & -[x(t-d_1(t))-x(t-h_1)]^T h_1^{-1} Z_1[x(t-d_1(t)) \\
& -x(t-h_1)] - \alpha[x(t-d_1(t)) \\
& -x(t-h_1)]^T h_1^{-1} Z_1[x(t-d_1(t))-x(t-h_1)]
\end{aligned} \tag{25}
$$

$$
\begin{aligned}
-\int_{t-d_1(t)}^{t} \dot{x}(s)^T Z_2\dot{x}(s)ds \le\ & -[x(t)-x(t-d_1(t))]^T h_1^{-1} Z_2[x(t) \\
& -x(t-d_1(t))] - (1-\alpha)[x(t) \\
& -x(t-d_1(t))]^T h_1^{-1} Z_2[x(t)-x(t-d_1(t))]
\end{aligned} \tag{26}
$$

$$
\begin{aligned}
-\int_{t-d(t)}^{t-d_1(t)} \dot{x}(s)^T Z_2\dot{x}(s)ds \le\ & -[x(t-d(t))-x(t-d_1(t))]^T h_2^{-1} Z_2[x(t-d(t)) \\
& -x(t-d_1(t))] - (1-\beta)[x(t-d(t)) \\
& -x(t-d_1(t))]^T h_2^{-1} Z_2[x(t-d(t))-x(t-d_1(t))]
\end{aligned} \tag{27}
$$

$$
\begin{aligned}
-\int_{t-h}^{t-d(t)} \dot{x}(s)^T Z_2\dot{x}(s)ds \le\ & -[x(t-d(t))-x(t-h)]^T h^{-1} Z_2[x(t-d(t)) \\
& -x(t-h)] - \alpha[x(t-d(t)) \\
& -x(t-h)]^T h_1 h^{-2} Z_2[x(t-d(t)) \\
& -x(t-h)] - \beta[x(t-d(t)) \\
& -x(t-h)]^T h_2 h^{-2} Z_2[x(t-d(t))-x(t-h)]
\end{aligned} \tag{28}
$$

Define

$$
\zeta(t) = \begin{bmatrix} x(t)^T & x(t-d(t))^T & x(t-d_1(t))^T & x(t-h)^T & x(t-h_1)^T \end{bmatrix}^T \tag{29}
$$

By (19), (20), (24)-(28) and using (29) we have

$$
\begin{aligned}
\dot{V}(t) \le\ & \zeta(t)^T\Phi\zeta(t) - (1-\alpha)[x(t)-x(t-d_1(t))]^T h_1^{-1}(Z_1+Z_2)[x(t)-x(t-d_1(t))] \\
& -\alpha\{[x(t-d_1(t))-x(t-h_1)]^T h_1^{-1} Z_1[x(t-d_1(t))-x(t-h_1)] \\
& +[x(t-d(t))-x(t-h)]^T h_1 h^{-2} Z_2[x(t-d(t))-x(t-h)]\} \\
& -\beta[x(t-d(t))-x(t-h)]^T h_2 h^{-1} Z_2[x(t-d(t))-x(t-h)] \\
& -(1-\beta)[x(t-d(t))-x(t-d_1(t))]^T h_2^{-1} Z_2[x(t-d(t))-x(t-d_1(t))] \\
=\ & \zeta(t)^T M(\alpha,\beta)\zeta(t)
\end{aligned} \tag{30}
$$

where

$$
\begin{aligned}
M(\alpha,\beta) &= \Phi - \alpha(E_{35}h_1^{-1}Z_1E_{35}^T + E_{24}h_1h^{-2}Z_2E_{24}^T) + (1-\alpha)[E_{13}h_1^{-1}(Z_1+Z_2)E_{13}^T] \\
&\quad -\beta E_{24}h_2h^{-2}Z_2E_{24}^T - (1-\beta)E_{23}h_2^{-1}Z_2E_{23}^T \\
&= \alpha[\Phi - (E_{35}h_1^{-1}Z_1E_{35}^T + E_{24}h_1h^{-2}Z_2E_{24}^T)] + (1-\alpha)[\Phi - E_{13}h_1^{-1}(Z_1+Z_2)E_{13}^T] \\
&\quad -\beta E_{24}h_2h^{-2}Z_2E_{24}^T - (1-\beta)E_{23}h_2^{-1}Z_2E_{23}^T \\
&= \alpha[\Phi - (E_{35}h_1^{-1}Z_1E_{35}^T + E_{24}h_1h^{-2}Z_2E_{24}^T) - \beta E_{24}h_2h^{-2}Z_2E_{24}^T \\
&\quad -(1-\beta)E_{23}h_2^{-1}Z_2E_{23}^T] + (1-\alpha)[\Phi - E_{13}h_1^{-1}(Z_1+Z_2)E_{13}^T \\
&\quad -\beta E_{24}h_2h^{-2}Z_2E_{24}^T - (1-\beta)E_{23}h_2^{-1}Z_2E_{23}^T] \\
&= \alpha[\beta(\Phi - E_{35}h_1^{-1}Z_1E_{35}^T - E_{24}h^{-1}Z_2E_{24}^T) \\
&\quad +(1-\beta)(\Phi - E_{35}h_1^{-1}Z_1E_{35}^T - E_{24}h_1h^{-2}Z_2E_{24}^T - E_{23}h_2^{-1}Z_2E_{23}^T)] \\
&\quad +(1-\alpha)[\beta(\Phi - E_{13}h_1^{-1}(Z_1+Z_2)E_{13}^T - E_{24}h_2h^{-2}Z_2E_{24}^T) \\
&\quad +(1-\beta)(\Phi - E_{13}h_1^{-1}(Z_1+Z_2)E_{13}^T - E_{23}h_2^{-1}Z_2E_{23}^T)]
\end{aligned}
$$

By (10)-(13) we have $M(\alpha,\beta) < 0$. From (30) it follows that $\dot{V}(t) < 0$. Therefore, system (1) subject to (2) and (3) is asymptotically stable. This ends the proof.

Remark 2.1. *Recently in [18] the stability for systems (1)-(3) was also investigated using the LF in (15) that involves symmetric matrices P, Q_i $(i = 1, 2, 3, 4)$ and Z_j $(j = 1, 2)$. As seen from the proof, this paper does not require $P > 0$, $Q_i > 0$ $(i = 1, 2, 3, 4)$ and $Z_j > 0$ $(j = 1, 2)$ to guarantee $V(t) > 0$ as [18]. Instead of $P > 0$ and $Q_i > 0$ $(i = 2, 4)$, the conditions (8) and (9) are derived in this paper to ensure $V(t) > 0$. By further requiring $P > 0$ and $Q_i > 0$ $(i = 2, 4)$ to ensure $V(t) > 0$, we can obtain the stability result in [18].*

Corollary 2.1. [18]. *The system (1) subject to (2) and (3) is asymptotically stable for given K, h_1, h_2, μ_1 and μ_2 if there exist matrices $P > 0$, $Q_i > 0$ $(i = 1, 2, 3, 4)$, $Z_j > 0$ $(j = 1, 2)$ such that the following LMIs hold*

$$
\begin{aligned}
&\Phi - E_{13}h_1^{-1}(Z_1+Z_2)E_{13}^T - E_{23}h_2^{-1}Z_2E_{23}^T < 0 \\
&\Phi - E_{13}h_1^{-1}(Z_1+Z_2)E_{13}^T - E_{24}h_2h^{-2}Z_2E_{24}^T < 0 \\
&\Phi - E_{35}h_1^{-1}Z_1E_{35}^T - E_{24}h_1h^{-2}Z_2E_{24}^T - E_{23}h_2^{-1}Z_2E_{23}^T < 0 \\
&\Phi - E_{35}h_1^{-1}Z_1E_{35}^T - E_{24}h^{-1}Z_2E_{24}^T < 0
\end{aligned}
$$

where $E_{ij} = E_i - E_j$, and

$$
\Phi = \begin{bmatrix} \varphi_1 & PBK & h_1^{-1}(Z_1+Z_2) & 0 & 0 \\ * & \varphi_2 & h_2^{-1}Z_2 & h^{-1}Z_2 & 0 \\ * & * & \varphi_3 & 0 & h_1^{-1}Z_1 \\ * & * & * & -Q_2 - h^{-1}Z_2 & 0 \\ * & * & * & * & -Q_4 - h_1^{-1}Z_1 \end{bmatrix}
+ \begin{bmatrix} A^T \\ (BK)^T \\ 0 \\ 0 \\ 0 \end{bmatrix} [h_1Z_1 + hZ_2] \begin{bmatrix} A^T \\ (BK)^T \\ 0 \\ 0 \\ 0 \end{bmatrix}^T
$$

with h and μ given in (6) and (7) respectively, and

$$
\begin{aligned}
\varphi_1 &= PA + A^TP + \sum\nolimits_{i=1}^{4} Q_i - h_1^{-1}(Z_1+Z_2) \\
\varphi_2 &= -(1-\mu)Q_3 - (h_2^{-1} + h^{-1})Z_2
\end{aligned}
$$

$$\varphi_3 = -(1-\mu_1)Q_1 - (h_2^{-1}+h_1^{-1})Z_2 - 2h_1^{-1}Z_1$$

Proof: Note that

$$\begin{bmatrix} \frac{h_1}{h}P + Z_1 + Z_2 & -Z_1 - Z_2 \\ * & Z_1 + Z_2 + h_1(Q_2+Q_4) \end{bmatrix}$$
$$= \begin{bmatrix} Z_1 + Z_2 & -Z_1 - Z_2 \\ * & Z_1 + Z_2 \end{bmatrix} + \begin{bmatrix} \frac{h_1}{h}P & 0 \\ 0 & h_1(Q_2+Q_4) \end{bmatrix}$$

and

$$\begin{bmatrix} P + Z_2 & -Z_2 \\ * & Z_2 + hQ_2 \end{bmatrix} = \begin{bmatrix} P & 0 \\ 0 & hQ_2 \end{bmatrix} + \begin{bmatrix} Z_2 & -Z_2 \\ * & Z_2 \end{bmatrix}$$

Therefore, if there exist $P > 0$, $Q_i > 0$ $(i = 1,2,3,4)$, $Z_j > 0$ $(j = 1,2)$ such that Corollary 2.1 holds, these matrices satisfy (8)-(9) as well as (10)-(13). From this it follows that Corollary 2.1 is covered by Theorem 2.1.

Remark 2.2. *As seen from the proof of Corollary 2.1, to ensure an LF to be positive definite, it is not necessary for all the symmetric matrices involved to be positive definite. Requiring all the symmetric matrices to be positive definite may induce conservatism, while slackening the requirement can reduce the conservatism.*

Remark 2.3. *Note that when estimating the upper bound of $\dot{V}(t)$ we have not introduced a slack variable. The obtained upper bound of $\dot{V}(t)$ is dependent on the two time-varying delays while those in [14,15] are dependent on the upper bounds of the two time-varying delays. That is, the corresponding matrix $M(\alpha,\beta)$ to the upper bound of $\dot{V}(t)$ is a function matrix of the two time-varying delays. To check the negative definiteness for the function matrix, we adopt a new method that is motivated from [13]. The basic idea is that a function matrix is negative definite over a convex polyhedron only if it is negative definite at the vertexes. Note that*

$$\begin{aligned}
M(1,1) &= \Phi - E_{35}h_1^{-1}Z_1E_{35}^T - E_{24}h^{-1}Z_2E_{24}^T \\
M(1,0) &= \Phi - E_{35}h_1^{-1}Z_1E_{35}^T - E_{24}h_1h^{-2}Z_2E_{24}^T - E_{23}h_2^{-1}Z_2E_{23}^T \\
M(0,1) &= \Phi - E_{13}h_1^{-1}(Z_1+Z_2)E_{13}^T - E_{24}h_2h^{-2}Z_2E_{24}^T \\
M(0,0) &= \Phi - E_{13}h_1^{-1}(Z_1+Z_2)E_{13}^T - E_{23}h_2^{-1}Z_2E_{23}^T.
\end{aligned}$$

From this we can see the negative definiteness of $M(\alpha,\beta)$ over the rectangle: $0 \le \alpha \le 1$, $0 \le \beta \le 1$ is actually determined by that of $M(\alpha,\beta)$ at the vertexes. This approach to the negative definiteness of a function matrix is called a convex polyhedron method. Apparently the convex polyhedron method can be extended to more than two time-varying delays.

Gao et al. [15] took advantages of $x(t-h)$ to derive a stability criterion, which improved over that in [14], but another marginally delayed state $x(t-h_1)$ was not employed. In this paper we make use of it to construct the Lyapunov functional $V(t)$ in (15), thus making $-\int_{t-h_1}^{t-d_1(t)} \dot{x}(\alpha)^T Z_1 \dot{x}(\alpha)d\alpha$ retained in the estimate of $\dot{V}(t)$. On the other hand, when estimating the integrals in $\dot{V}(t)$ we do not introduce any free weighting matrix as [14,15], but use a new technique. Take $-\int_{t-h_1}^{t} \dot{x}(\alpha)^T Z_1 \dot{x}(\alpha)d\alpha$ as an example. First divide it into two parts as (20), and then calculate each of them as (21)-(25), both of the two parts employed. In contrast, the second part was ignored in [15]. Note that although both of the two parts were kept in [5,6], either was estimated conservatively. For instance, the first part $-\int_{t-d_1(t)}^{t} \dot{x}(\alpha)^T Z_1 \dot{x}(\alpha)d\alpha$ was simply enlarged as $-h_1^{-1}\int_{t-d_1(t)}^{t} d_1(t)\dot{x}(s)^T Z_1 \dot{x}(s)ds$ in [5,6], and $-h_1^{-1}\int_{t-d_1(t)}^{t} [h_1 - d_1(t)]\dot{x}(s)^T Z_1 \dot{x}(s)ds$ was disregarded. Thanks to the new technique, the upper bound of $\dot{V}(t)$ is estimated tighter without introducing a slack variable, and the *resulting stability criterion Theorem 2.1 is less conservative with fewer matrix variables, as shown in the following example.*

Remark 2.4. *In engineering practice, the information of the delay range is generally available, so Theorem 2.1 is useful in checking stability for the delayed system described by (1)-(3). On the other hand, it is significant to know the maximums of the two delays the system can tolerate. As seen in the following example, by Theorem 2.1 and Matlab LMI Control Toolbox we can compute the admissible upper bounds of the two delays, which guarantee the system to be stable.*

Example 2.1. *Consider the system (1) with*

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \quad BK = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}$$

and

$$\dot{d}_1(t) \leq 0.1, \quad \dot{d}_2(t) \leq 0.8$$

For given upper bound h_1 of $d_1(t)$, we intend to find the admissible upper bound h_2 of $d_2(t)$, which guarantees the system remains asymptotically stable for $0 \leq d_1(t) \leq h_1$, $0 \leq d_2(t) \leq h_2$.

Table 1. Admissible upper bound h_2 for various h_1

Method	h_1	1	1.2	1.5
[14]	h_2	0.415	0.376	0.248
[15]	h_2	0.512	0.406	0.283
Corollary 2.1	h_2	0.5955	0.4632	0.3129
Theorem 2.1	h_2	0.6377	0.4923	0.3283

It is well known that for a system with time-varying delays, it is significant to compute the maximum of the delay the system can tolerate [9,11]. As seen from the above table, for given h_1 the stability result Corollary 2.1 can provide a larger h_2 than those in [14,15] to ensure the system stable for $0 \leq d_1(t) \leq h_1$, $0 \leq d_2(t) \leq h_2$, while in this regard Theorem 2.1 can provide an even larger h_2. In this sense, the stability criterion Corollary 2.1 is less conservative than those in [14,15], while Theorem 2.1 is less conservative than Corollary 2.1.

When h_2 is given, the admissible h_1 can be seen from Table 2.

Table 2. Admissible upper bound h_1 for various h_2

Method	h_2	0.1	0.2	0.3
[14]	h_1	2.263	1.696	1.324
[15]	h_1	2.300	1.779	1.453
Corollary 2.1	h_1	2.3400	1.8337	1.5318
Theorem 2.1	h_1	2.3426	1.8598	1.5660

From Table 2, it is clear that the stability criterion Corollary 2.1 is less conservative than those in [14,15], while Theorem 2.1 is less conservative than Corollary 2.1. It is worth noting that both Corollary 2.1 and Theorem 2.1 have much fewer matrix variables, compared with those in [14,15].

Remark 2.5. *Note that Theorem 2.1 can only provide sufficient stability conditions for systems (1)-(3). Though less conservative than the existing ones [14,15,18], Theorem 2.1 still has some conservatism. To further reduce the conservatism, it is needed to construct a new LF and propose a new scheme bounding the derivative of the LF. This is our future work.*

3. **Control Design.** Theorem 2.1 can be served as a useful tool for the control design problem formulated above.

Theorem 3.1. *Consider systems (1)-(3). Given h_1, h_2, μ_1 and μ_2, there exists a state-feedback controller gain K ensuring that the system is asymptotically stable, if there exist $\hat{K}$, $\bar{P} = \bar{P}^T$, $\tilde{Q}_2 = \tilde{Q}_2^T$, $\tilde{Q}_4 = \tilde{Q}_4^T$, $\tilde{Q}_i > 0$ $(i = 1, 3)$, $\tilde{Z}_j > 0$ $(j = 1, 2)$ such that the following LMIs hold*

$$\begin{bmatrix} \frac{h_1}{h}\bar{P} + \tilde{Z}_1 + \tilde{Z}_2 & -\tilde{Z}_1 - \tilde{Z}_2 \\ * & \tilde{Z}_1 + \tilde{Z}_2 + h_1(\tilde{Q}_2 + \tilde{Q}_4) \end{bmatrix} > 0 \tag{31}$$

$$\begin{bmatrix} \bar{P} + \tilde{Z}_2 & -\tilde{Z}_2 \\ * & \tilde{Z}_2 + h\tilde{Q}_2 \end{bmatrix} > 0 \tag{32}$$

$$\begin{bmatrix} \Omega_i & \Gamma \\ \Gamma^T & \Lambda \end{bmatrix} < 0 \quad (i = 1, 2, 3, 4) \tag{33}$$

where

$$\Omega_1 = \Psi - E_{13}h_1^{-1}(\tilde{Z}_1 + \tilde{Z}_2)E_{13}^T - E_{23}h_2^{-1}\tilde{Z}_2E_{23}^T \tag{34}$$

$$\Omega_2 = \Psi - E_{13}h_1^{-1}(\tilde{Z}_1 + \tilde{Z}_2)E_{13}^T - E_{24}h_2h^{-2}\tilde{Z}_2E_{24}^T \tag{35}$$

$$\Omega_3 = \Psi - E_{35}h_1^{-1}\tilde{Z}_1E_{35}^T - E_{24}h_1h^{-2}\tilde{Z}_2E_{24}^T - E_{23}h_2^{-1}\tilde{Z}_2E_{23}^T \tag{36}$$

$$\Omega_4 = \Psi - E_{35}h_1^{-1}\tilde{Z}_1E_{35}^T - E_{24}h^{-1}\tilde{Z}_2E_{24}^T \tag{37}$$

$$\Lambda = diag\{-h_1^{-1}\bar{P}\tilde{Z}_1^{-1}\bar{P}, -h^{-1}\bar{P}\tilde{Z}_2^{-1}\bar{P}\} \tag{38}$$

$$\Gamma = \begin{bmatrix} \bar{P}A^T & \bar{P}A^T \\ \hat{K}^TB^T & \hat{K}^TB^T \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{39}$$

$$\Psi = \begin{bmatrix} \psi_1 & B\hat{K} & h_1^{-1}(\tilde{Z}_1 + \tilde{Z}_2) & 0 & 0 \\ * & \psi_2 & h_2^{-1}\tilde{Z}_2 & h^{-1}\tilde{Z}_2 & 0 \\ * & * & \psi_3 & 0 & h_1^{-1}\tilde{Z}_1 \\ * & * & * & -\tilde{Q}_2 - h^{-1}\tilde{Z}_2 & 0 \\ * & * & * & * & -\tilde{Q}_4 - h_1^{-1}\tilde{Z}_1 \end{bmatrix} \tag{40}$$

$$\psi_1 = A\bar{P} + (A\bar{P})^T + \sum\nolimits_{i=1}^{4} \tilde{Q}_i - h_1^{-1}(\tilde{Z}_1 + \tilde{Z}_2)$$

$$\psi_2 = -(1-\mu)\tilde{Q}_3 - (h_2^{-1} + h^{-1})\tilde{Z}_2$$

$$\psi_3 = -(1-\mu_1)\tilde{Q}_1 - (h_2^{-1} + h_1^{-1})\tilde{Z}_2 - 2h_1^{-1}\tilde{Z}_1$$

and E_{ij} is given in Theorem 2.1. Moreover, if the foregoing conditions hold, a desired controller gain is given by

$$K = \hat{K}\bar{P}^{-1} \tag{41}$$

Proof: Let us start from Theorem 2.1. If P is singular, obviously there exists a small enough $\varepsilon > 0$ such that $P + \varepsilon I$ is nonsingular and Theorem 2.1 still holds with P replaced by $P + \varepsilon I$. Without loss of generality, we assume there exists a nonsingular P such that

Theorem 2.1 holds. In the following, we write $\tilde{Z}_i = P^{-1}Z_iP^{-1}$ $(i = 1, 2)$, $\tilde{Q}_j = P^{-1}Q_jP^{-1}$, $(j = 1, 2, 3, 4)$, $\bar{P} = P^{-1}$, $J_1 = diag\{P^{-1}, P^{-1}\}$ and $J = diag\{P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}\}$. With these notations and (41) in mind, performing a congruence transformation to (8) and (9) by J_1 we obtain (31) and (32) respectively, while performing a congruence transformation to (10)-(13) by J we can obtain

$$\begin{aligned}
&\Psi - \Gamma\Lambda^{-1}\Gamma^T - E_{13}h_1^{-1}(\tilde{Z}_1 + \tilde{Z}_2)E_{13}^T - E_{23}h_2^{-1}\tilde{Z}_2E_{23}^T < 0\\
&\Psi - \Gamma\Lambda^{-1}\Gamma^T - E_{13}h_1^{-1}(\tilde{Z}_1 + \tilde{Z}_2)E_{13}^T - E_{24}h_2h^{-2}\tilde{Z}_2E_{24}^T < 0\\
&\Psi - \Gamma\Lambda^{-1}\Gamma^T - E_{35}h_1^{-1}\tilde{Z}_1E_{35}^T - E_{24}h_1h^{-2}\tilde{Z}_2E_{24}^T - E_{23}h_2^{-1}\tilde{Z}_2E_{23}^T < 0\\
&\Psi - \Gamma\Lambda^{-1}\Gamma^T - E_{35}h_1^{-1}\tilde{Z}_1E_{35}^T - E_{24}h^{-1}\tilde{Z}_2E_{24}^T < 0
\end{aligned}$$

where Λ, Γ and Ψ are given in (38)-(40), respectively. Using notations (34)-(37) we have

$$\begin{bmatrix} \Omega_i & \Gamma \\ \Gamma^T & \Lambda \end{bmatrix} < 0 \quad (i = 1, 2, 3, 4)$$

That is, (33) holds. It is shown that Theorem 3.1 holds.

Note that the non-linearity of (33) due to Λ in (38). Theorem 3.1 suggests a non-convex feasibility problem for the controller design. Employing CCL method [17] one can change this problem into the following minimization problem:

$$\min \mathrm{tr}(P\bar{P} + Y_1\tilde{Z}_1 + Y_2\tilde{Z}_2 + Z_1\bar{Z}_1 + Z_2\bar{Z}_2)$$

subject to

$$\begin{bmatrix} P & I \\ I & \bar{P} \end{bmatrix} \geq 0, \begin{bmatrix} Z_j & I \\ I & \bar{Z}_j \end{bmatrix} \geq 0, \begin{bmatrix} Y_j & I \\ I & \tilde{Z}_j \end{bmatrix} \geq 0,$$
$$\begin{bmatrix} Y_j & P \\ P & Z_j \end{bmatrix} \geq 0 \ (j = 1, 2), \begin{bmatrix} \Omega_i & \Gamma \\ \Gamma^T & \bar{\Lambda} \end{bmatrix} < 0 \ (i = 1, 2, 3, 4)$$

with $\bar{\Lambda} = diag\{-h_1^{-1}\bar{Z}_1, -h^{-1}\bar{Z}_2\}$, Ω_i and Γ given in Theorem 3.1.

The conditions in Theorem 3.1 are feasible if $\min \mathrm{tr}(P\bar{P}+Y_1\tilde{Z}_1+Y_2\tilde{Z}_2+Z_1\bar{Z}_1+Z_2\bar{Z}_2) = 5n$. To deal with the non-linearity in Theorem 3.1, we have an alternative way. Note that $(\bar{P} - \tilde{Z}_i)\tilde{Z}_i^{-1}(\bar{P} - \tilde{Z}_i) \geq 0$. It follows that $\bar{P}\tilde{Z}_i^{-1}\bar{P} \geq -\tilde{Z}_i + 2\bar{P}$ $(i = 1, 2)$. From this and Theorem 3.1 we can obtain the following result, though it is a little more conservative.

Theorem 3.2. *Consider system (1) with the delays subject to (2) and (3). Given h_1, h_2, μ_1 and μ_2, there exists a state-feedback controller gain K ensuring that the system is asymptotically stable, if there exist matrices $\hat{K}$, $\bar{P} = \bar{P}^T$, $\tilde{Q}_2 = \tilde{Q}_2^T$, $\tilde{Q}_4 = \tilde{Q}_4^T$, $\tilde{Q}_i > 0$ $(i = 1, 3)$, $\tilde{Z}_j > 0$ $(j = 1, 2)$ such that the following LMIs hold*

$$\begin{bmatrix} \frac{h_1}{h}\bar{P} + \tilde{Z}_1 + \tilde{Z}_2 & -\tilde{Z}_1 - \tilde{Z}_2 \\ * & \tilde{Z}_1 + \tilde{Z}_2 + h_1(\tilde{Q}_2 + \tilde{Q}_4) \end{bmatrix} > 0$$
$$\begin{bmatrix} \bar{P} + \tilde{Z}_2 & -\tilde{Z}_2 \\ * & \tilde{Z}_2 + h\tilde{Q}_2 \end{bmatrix} > 0$$
$$\begin{bmatrix} \Omega_i & \Gamma \\ \Gamma^T & \tilde{\Lambda} \end{bmatrix} < 0 \quad (i = 1, 2, 3, 4)$$

where Ω_i and Γ are defined in Theorem 3.1, and $\tilde{\Lambda} = diag\{h_1^{-1}(\tilde{Z}_1 - 2\bar{P}), h^{-1}(\tilde{Z}_2 - 2\bar{P})\}$. Moreover, if the foregoing conditions hold, a desired controller gain matrix is given by $K = \hat{K}\bar{P}^{-1}$.

To illustrate the effectiveness of this control method we provide an example.

Example 3.1. *Consider system (1) with parameters given as follows:*

$$A = \begin{bmatrix} 0.12 & 0 \\ 1 & -0.9 \end{bmatrix}, \quad B = \begin{bmatrix} 0.2 \\ -0.5 \end{bmatrix}$$

It is obvious that when $K = 0$ *the system is unstable. However, for* $h_1 = 1$, $h_2 = 2$, $\mu_1 = 0.1$, $\mu_2 = 0.2$ *we find LMIs in Theorem 3.2 are feasible with*

$$\bar{P} = \begin{bmatrix} 1.1665 & 1.5843 \\ 1.5843 & 17.8683 \end{bmatrix}, \quad \hat{K} = \begin{bmatrix} -0.9667 & -1.6770 \end{bmatrix}$$

By Theorem 3.2, there exists a state feedback controller with

$$K = \hat{K}\bar{P}^{-1} = \begin{bmatrix} -0.8071 & -0.0223 \end{bmatrix}$$

such that the system (1) is asymptotically stable for $0 \le d_1(t) \le 1$, $0 \le d_2(t) \le 2$.

4. **Conclusions.** In this paper, the state feedback control design has been studied for a networked control model of systems with two additive time-varying delays. A new Lyapunov functional approach was employed to investigate the stability for the system. The approach took the Lyapunov functional as a whole to examine its positive definite, rather than restrict each term of it to positive definite as usual. Without introducing a slack variable, the novel technique got a tighter upper bound of the Lyapunov functional's derivative. The resultant stability results, only involving the matrices in the Lyapunov functional, are not only less conservative but also with fewer matrix variables than existing ones. Based on the stability results a state feedback controller was designed, such that the closed-loop system is asymptotically stable. Finally, examples were given to show the less conservatism of the stability results and the effectiveness of the proposed control method.

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