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H_∞ FAULT-TOLERANT CONTROLLER DESIGN FOR NETWORKED CONTROL SYSTEMS WITH TIME-VARYING ACTUATOR FAULTS

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Abstract. *For the large scale and complicated structure of networked control system, time-varying actuator faults could inevitably occur when it works in a poor environment. H_∞ fault-tolerant controller for a new networked control system with time-varying actuator faults occurring is designed in this paper. Based on the network transmission environment, the networked control system is firstly modeled as a closed loop discrete-time system with time varying actuator faults and outside disturbance considered. And the time varying property of actuator faults is reflected by a time-varying parameter. Moreover, using Lyapunov stability theory and linear matrix inequality (LMI) approach, the H_∞ fault-tolerant controller is proposed to guarantee such faulty networked control system asymptotically stable. Finally, simulations are included to demonstrate the theoretical results.*

Keywords: Networked control system, Actuator faults, H_∞ fault-tolerant controller, Lyapunov stability theory, LMI

1. **Introduction.** Feedback control system wherein the control loops are closed through a real-time network is called networked control system (NCS). Because of its suitable and flexible structure, NCS is widely used in the fields such as information technology, life science and aeronautical and space technologies. However, there not only exists induced delay, data packet loss and sequence disordering in NCS [1-3], but also faults usually occur when it works [11], which could cause negative impact on the performance of the system, even leading to system instability. Recently, the fault-tolerant control of NCS with delays has become a new popular issue in the control field [4-12,15-17].

A robust fault-tolerant control based on the integrity control theory when actuator faults occur is discussed by Y. N. Guo et al. [4]. The faults of each sensor or actuator were taken as occurring randomly by E. Tian et al. [5], and their failure rates are governed by two sets of unrelated random variables satisfying certain probabilistic distribution. A methodology for the design of fault-tolerant control systems for chemical plants with distributed interconnected processing units is presented based on Lyapunov stability theorem by N. H. El-Farra et al. [6]. X. Y. Luo et al. [7] propose the guaranteed cost active fault-tolerant controller (AFTC) strategy that the fault detection and isolation unit sends out the information to the controller choosing strategy when actuator failures appear, and then the optimal stabilizing controller with the smallest guaranteed cost value is chosen. A switched model based on probability is proposed to research problems of fault-tolerant control when actuators become aging or partially disabled in [8], but not considering the outside disturbance. X. Li and X. B. Wu [17] investigate the problem of integrity against actuator faults for NCS under variable-period sampling, in which the existence conditions of guaranteed cost faults-tolerant control law are testified in terms of Lyapunov stability theory, but not referring to the effects of uncertain parameters and outside disturbance.

Almost all literature above considers the faults in some special cases, and no design considers the time-varying faults. However, in practical application, because of large scale and complicated structure of NCS, the faults could vary from time to time when it works in a poor environment. It is significant and necessary to explore a reasonable control method to improve the performance of NCS when time-varying faults occur, which motivates us to conduct this research.

In this paper, H_∞ fault-tolerant control problem of networked control system with time varying actuator faults is studied. On the basis of the network transmission environment, the networked control system is modeled as a closed loop discrete-time system with outside disturbance considered. And the model of networked control system is related to the boundary values of the actuator faults and the time varying property of actuator faults is reflected by a time-varying parameter. Using Lyapunov stability theory and linear matrix inequality (LMI) approach, the H_∞ fault-tolerant controller is proposed to guarantee such faulty networked control system asymptotically stable.

2. Modeling of Networked Control Systems with Actuator Faults.

The structure of NCS is shown as follows.

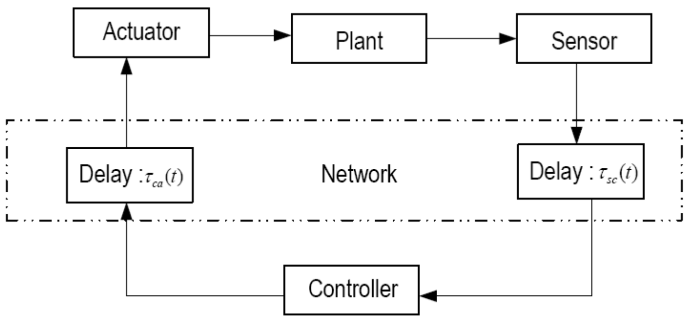

FIGURE 1. The structure of networked control system

In Figure 1, τ_{sc} represents the transmission delay from sensor to the controller, while τ_{ca} represents that from controller to the actuator. Induced delay of system can be calculated as $\tau = \tau_{sc} + \tau_{ca}$.

A linear control plant is described by state equation as follows.

$$\begin{cases} \dot{x}(t) = A_o x(t) + B_o u(t) + H_o \omega(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $x \in R^n$, $u \in R^m$, $y \in R^r$ and $\omega(t) \in L_2[0, \infty) \in R^p$ represent state, input, output vectors and outside disturbance separately, while A_o, B_o, C and H_o are matrices with appropriate dimensions.

To facilitate the model, some rational assumptions are introduced as follows.

A1. *Single data package is transmitted. The packet loss and sequence disorder are not taken into consideration during the transmission process.*

A2. *Time-varying time delay exists during the data transmission process, but it is bounded, and the maximum time delay does not exceed one sampling period, namely* $\tau \in [0, T]$*, where* T *is the sampling period.*

A3. *Sensor is clock driving; controller and actuator are all event driving.*

The timing diagram of signals in NCS is shown in Figure 2. Due to the structure of NCS and the above assumptions, within a sampling period, the system input is not a

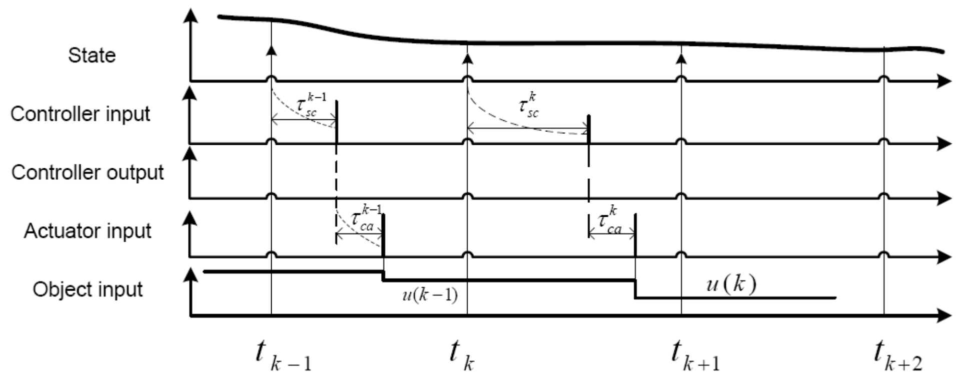

FIGURE 2. Timing diagram of signals transmitting in NCS

constant value, but is a piecewise constant. In a cycle, input objects can be described as

$$u(t) = \begin{cases} u(k-1), & t_k < t \le t_k + \tau_k \\ u(k), & t_k + \tau_k < t \le t_k + T \end{cases} \tag{2}$$

where t_k is the k^{th} cycle sampling time, and τ_k is the corresponding delay.

The discrete-time model of system (1) can be obtained as follows.

$$\begin{cases} x(k+1) = Ax(k) + B_1u(k) + B_2u(k-1) + H\omega(k) \\ y = Cx(k) \end{cases} \tag{3}$$

where $A = e^{A_oT}$, $B_1 = \int_0^{T-\tau_k} e^{A_ot}B_odt$, $B_2 = \int_{T-\tau_k}^{T} e^{A_ot}B_odt$, $B = \int_0^T e^{A_ot}B_odt$, $H = \int_0^T e^{A_ot}H_odt$.

As τ_k is uncertain, B_1 and B_2 are also time-varying, so the model of NCS is converted to a discrete system with uncertain parameters shown as Equation (3). The equivalent model of system (3) is given by C. Xie et al. [10], also used by Q. Zhu and K. Lu [11].

$$\begin{cases} x(k+1) = Ax(k) + DFEu(k) + (B - DFE)u(k-1) + H\omega(k) \\ y = Cx(k) \end{cases} \tag{4}$$

where $B = B_1 + B_2 = \int_0^T e^{A_ot}B_odt$, D and E represent constant matrices with appropriate dimension. F is an uncertain component matrix satisfying $F^TF \le I$. Detailed calculating method of D, E and F is proposed in [10].

Assuming that the system is fully measurable, state feedback is introduced as follows.

$$u(k) = Kx(k) \tag{5}$$

Considering the actuator faults may occur, the controller is expanded as

$$u^F(k) = \Phi(k)Kx(k) \tag{6}$$

where $u^F = [u_1^F, u_2^F, \cdots, u_m^F]^T$ represents faulty signal. $\Phi(k) = diag(\phi_1(k), \phi_2(k), \cdots, \phi_m(k))$; $\phi_i = 0$ represents that actuator i faults occur; $\phi_i = 1$ represents that actuator i is normal; $0 \le \phi_i \le 1$ represents partial faults occur at actuator i. When $\Phi = I$, it represents all actuators are normal. The condition that all actuators failure occurs at the same time is not taken into consideration here.

Moreover, the faults are usually bounded in the practical work and they are assumed to be measurable. And the upper bound of fault matrices is defined as $\Phi_u = diag(\phi_{u1}, \phi_{u2}, \cdots, \phi_{un})$, $1 \ge \phi_{ui} > 0$; while the lower bound of fault matrices is defined as $\Phi_l = diag(\phi_{l1}, \phi_{l2}, \cdots, \phi_{ln})$, $1 > \phi_{li} \ge 0$. That is to say $\Phi(k) \in [\Phi_l, \Phi_u]$, which is time-varying.

The mean value of matrices Φ_l and Φ_u can also be expressed as

$$\Phi_0 = diag(\phi_{01}, \phi_{02}, \cdots, \phi_{0n}), \quad \phi_{0i} = \frac{\phi_{ui} + \phi_{li}}{2} \tag{7}$$

Furthermore, following matrices are introduced.

$$L(k) = diag(l_1(k), l_2(k), \cdots, l_n(k)), \quad l_i(k) = \frac{\phi_i(k) - \phi_{0i}}{\phi_{0i}} \tag{8}$$

Obviously, we have

$$-1 \leq \frac{\phi_{li} - \phi_{0i}}{\phi_{0i}} \leq l_i(k) = \frac{\phi_i(k) - \phi_{0i}}{\phi_{0i}} \leq \frac{\phi_{ui} - \phi_{0i}}{\phi_{0i}} = \frac{\phi_{ui} - \phi_{li}}{\phi_{ui} + \phi_{li}} \leq 1 \tag{9}$$

Based on (9), we have $-I_{n\times n} \leq L(k) \leq I_{n\times n}$. From (8), it can be obtained

$$\phi_i = \phi_{0i}(1 + l_i), \quad i = 1, 2, \cdots, n$$

Naturally, it is denoted by $\Phi(k) = \Phi_0(I + L(k))$, and the closed-loop systems model with actuator faults can be obtained as:

$$\begin{cases} x(k+1) = Ax(k) + DFE\Phi_0(I + L(k))Kx(k) \\ \qquad\qquad +(B - DFE)\Phi_0(I + L(k-1))Kx(k-1) + H\omega(k) \\ y = Cx(k) \end{cases} \tag{10}$$

Remark 2.1. *Unlike the previous models as [6,11], the NCS with time-varying actuator faults considered is modeled as a closed-loop system (10) with time-varying parameter $L(k)$ which reflects the time varying property of actuator faults, and this model is related to the boundary values of the faults Φ_u and Φ_l.*

3. H_∞ Fault-Tolerant Controller Design.

To analyze the stability of the system expediently, following definition and lemma are introduced.

Definition 3.1. [13, 14]. *For system (10), if it satisfies that: 1. The closed-loop system is asymptotically stable if $\omega(k) = 0$; 2. Under any zero initial condition, given $\gamma > 0$, for any nonzero vector $\omega(k) \in L_2[0, \infty)$, the output $y(k)$ satisfies $\|y(k)\|_2 \leq \gamma \|\omega(k)\|_2$. It is called that system (10) is asymptotically stable with H_∞ norm bound γ.*

Lemma 3.1. [14]. *For any matrices W, M, N, $F(t)$ with $F^TF \leq I$, and any scalar $\varepsilon > 0$, the inequality holds as follows.*

$$W + MF(t)N + N^TF^T(t)M^T \leq W + \varepsilon MM^T + \varepsilon^{-1}N^TN \tag{11}$$

Theorem 3.1. *Given gain matrix K, if there exists symmetric positive definite matrices P and Q, as well as a set of constants $\varepsilon > 0$ and $\gamma > 0$, satisfying*

$$\begin{bmatrix} -\varepsilon I & 0 & 0 & E\Phi_0(I+L)K & -E\Phi_0(I+L)K & 0 \\ * & -\varepsilon^{-1}I & D^T & 0 & 0 & 0 \\ * & * & -P^{-1} & A & B\Phi_0(I+L)K & H \\ * & * & * & -P+Q+C^TC & 0 & 0 \\ * & * & * & * & -Q & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{12}$$

then system (10) is asymptotically stable with H_∞ norm bound γ. $*$ represents the symmetry blocks of matrix.

Proof: Consider the following Lyapunov function

$$v(k) = x^T(k)Px(k) + x^T(k-1)Qx(k-1).$$

For the convenience of writing, we denote $L = L(k)$ in the following expressions. Conducting subtract operating along the arbitrary trajectory of system (10) is given by

$$\begin{aligned}\Delta v(k) &= v(k+1) - v(k) \\ &= x^T(k+1)Px(k+1) + x^T(k)(Q-P)x(k) - x^T(k-1)Qx(k-1) \\ &= [Ax(k) + DFE\Phi_0(I+L)Kx(k) + (B-DFE)\Phi_0(I+L)Kx(k-1) \\ &\quad +H\omega(k)]^T P[Ax(k) + DFE\Phi_0(I+L)Kx(k) + (B-DFE)\Phi_0K(I+L)x(k-1) \\ &\quad +H\omega(k)] + x^T(k)(Q-P)x(k) - x^T(k-1)Qx(k-1) \\ &= \begin{bmatrix} x(k) \\ x(k-1) \\ \omega(k) \end{bmatrix}^T \begin{bmatrix} \Gamma^T P\Gamma - P + Q & \Gamma^T P\Omega & \Gamma^T PH \\ * & \Omega^T P\Omega - Q & \Omega^T PH \\ * & * & H^T PH \end{bmatrix} \begin{bmatrix} x(k) \\ x(k-1) \\ \omega(k) \end{bmatrix} \end{aligned} \tag{13}$$

where $\Gamma = A + DFE\Phi_0(I+L)K$, $\Omega = B\Phi_0(I+L)K - DFE\Phi_0(I+L)K$.

Plus $y^T(k)y(k) - \gamma^2\omega^T(k)\omega(k)$ at two ends of Equation (13), we have

$$\begin{aligned}&\Delta v(k) + y^T(k)y(k) - \gamma^2\omega^T(k)\omega(k) \\ =& \begin{bmatrix} x(k) \\ x(k-1) \\ \omega(k) \end{bmatrix}^T \begin{bmatrix} \Gamma^T P\Gamma - P + Q + C^TC & \Gamma^T P\Omega & \Gamma^T PH \\ * & \Omega^T P\Omega - Q & \Omega^T PH \\ * & * & H^T PH - \gamma^2 I \end{bmatrix} \begin{bmatrix} x(k) \\ x(k-1) \\ \omega(k) \end{bmatrix}\end{aligned} \tag{14}$$

If

$$\begin{bmatrix} \Gamma^T P\Gamma - P + Q + C^TC & \Gamma^T P\Omega & \Gamma^T PH \\ * & \Omega^T P\Omega - Q & \Omega^T PH \\ * & * & H^T PH - \gamma^2 I \end{bmatrix} < 0 \tag{15}$$

from Equation (14), it is known that,

$$\Delta v(k) + y^T(k)y(k) - \gamma^2\omega^T(k)\omega(k) < 0 \tag{16}$$

If $\omega(k) \equiv 0$, obviously, there is $\Delta v(k) < 0$.

From zero initial condition, we know $v(0) = 0$. And it can be obtained that $v(\infty) \geq 0$. Therefore,

$$\begin{aligned}&\sum_{k=0}^{\infty}[\Delta v(k) + y^T(k)y(k) - \gamma^2\omega^T(k)\omega(k)] \\ &= v(\infty) - v(0) + \sum_{k=0}^{\infty}[y^T(k)y(k) - \gamma^2\omega^T(k)\omega(k)] \\ &< 0\end{aligned} \tag{17}$$

Moreover,

$$\sum_{k=0}^{\infty}[y^T(k)y(k) - \gamma^2\omega^T(k)\omega(k)] < -v(\infty) \leq 0 \tag{18}$$

Therefore, we have

$$\sum_{k=0}^{\infty} y^T(k)y(k) < \sum_{k=0}^{\infty} \gamma^2\omega^T(k)\omega(k) \tag{19}$$

Namely, $\|y(k)\|_2 < \gamma \|\omega(k)\|_2$. Based on Definition 3.1, it knows that system (10) is asymptotically stable with H_∞ norm bound γ. Based on Schur complement theory, inequality (15) is equivalent to

$$\begin{bmatrix} -P^{-1} & \Gamma & \Omega & H \\ * & -P+Q+C^TC & 0 & 0 \\ * & * & -Q & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{20}$$

Submitting the expressions of Γ and Ω, inequality (20) can be rewritten as

$$\begin{aligned} &\begin{bmatrix} -P^{-1} & A & B\Phi_0(I+L)K & H \\ * & -P+Q+C^TC & 0 & 0 \\ * & * & -Q & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} \\ &+ \begin{bmatrix} D \\ 0 \\ 0 \\ 0 \end{bmatrix} F \begin{bmatrix} 0 \\ (E\Phi_0(I+L)K)^T \\ -(E\Phi_0(I+L)K)^T \\ 0 \end{bmatrix}^T \\ &+ \begin{bmatrix} 0 \\ (E\Phi_0(I+L)K)^T \\ -(E\Phi_0(I+L)K)^T \\ 0 \end{bmatrix} F^T \begin{bmatrix} D \\ 0 \\ 0 \\ 0 \end{bmatrix}^T < 0 \end{aligned} \tag{21}$$

From Lemma 3.1, the sufficient condition of inequality (21) follows that

$$\begin{aligned} &\begin{bmatrix} -P^{-1} & A & B\Phi_0(I+L)K & H \\ * & -P+Q+C^TC & 0 & 0 \\ * & * & -Q & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} \\ &+\varepsilon \begin{bmatrix} D \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} D \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \varepsilon^{-1} \begin{bmatrix} 0 \\ (E\Phi_0(I+L)K)^T \\ -(E\Phi_0(I+L)K)^T \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ (E\Phi_0(I+L)K)^T \\ -(E\Phi_0(I+L)K)^T \\ 0 \end{bmatrix}^T < 0 \end{aligned} \tag{22}$$

Using Schur complement theory again, the inequality (22) is equivalent to inequality (12). This completes the proof.

Now, we reformulate inequality (12) into LMI via a change of variables.

Theorem 3.2. *For NCS (10), if there exists symmetric positive definite matrices X, S and matrix W, as well as a set of constants $\lambda_i (i = 1, 2) > 0$, $\varepsilon > 0$ satisfying the following*

LMI:

$$\begin{bmatrix} -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & CX & 0 & 0 \\ * & \lambda_2 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & W & 0 \\ * & * & \lambda_2 I & 0 & 0 & 0 & 0 & \lambda_2(B\Phi_0)^T & 0 & 0 & 0 \\ * & * & * & \lambda_1 I & 0 & 0 & 0 & 0 & W & -W & 0 \\ * & * & * & * & \lambda_1 I & \lambda_1(E\Phi_0)^T & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon I & 0 & 0 & E\Phi_0 W & -E\Phi_0 W & 0 \\ * & * & * & * & * & * & -\varepsilon I & \varepsilon D^T & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -P^{-1} & A & B\Phi_0 W & H \\ * & * & * & * & * & * & * & * & -P^{-1}+S & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -S & 0 \\ * & * & * & * & * & * & * & * & * & * & -\mu I \end{bmatrix} < 0 \quad (23)$$

then the NCS (10) is asymptotically stable with H_∞ norm bound $\gamma = \sqrt{\mu}$ and the control gain is $K = WX^{-1}$.

Proof: Inequality (12) can be rewritten as

$$\begin{bmatrix} -\varepsilon I & 0 & 0 & E\Phi_0 K & -E\Phi_0 K & 0 \\ * & -\varepsilon^{-1} I & D^T & 0 & 0 & 0 \\ * & * & -P^{-1} & A & B\Phi_0 K & H \\ * & * & * & -P+Q+C^TC & 0 & 0 \\ * & * & * & * & -Q & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} E\Phi_0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} L \begin{bmatrix} 0 \\ 0 \\ 0 \\ K^T \\ -K^T \\ 0 \end{bmatrix}^T$$

$$+ \begin{bmatrix} 0 \\ 0 \\ 0 \\ K^T \\ -K^T \\ 0 \end{bmatrix} L^T \begin{bmatrix} E\Phi_0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \begin{bmatrix} 0 \\ 0 \\ B\Phi_0 \\ 0 \\ 0 \\ 0 \end{bmatrix} L \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ K^T \\ 0 \end{bmatrix}^T + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ K^T \\ 0 \end{bmatrix} L^T \begin{bmatrix} 0 \\ 0 \\ B\Phi \\ 0 \\ 0 \\ 0 \end{bmatrix}^T < 0 \quad (24)$$

Based on Lemma 3.1, a sufficient condition not containing the time varying parameter matrix can be given by

$$\begin{bmatrix} -\varepsilon I & 0 & 0 & E\Phi_0 K & -E\Phi_0 K & 0 \\ * & -\varepsilon^{-1} I & D^T & 0 & 0 & 0 \\ * & * & -P^{-1} & A & B\Phi_0 K & H \\ * & * & * & -P+Q+C^TC & 0 & 0 \\ * & * & * & * & -Q & 0 \\ * & * & * & * & * & -\gamma^2 I \end{bmatrix} + \lambda_1 \begin{bmatrix} E\Phi_0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} E\Phi_0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T$$

$$+\lambda_1^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ K^T \\ -K^T \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ K^T \\ -K^T \\ 0 \end{bmatrix}^T + \lambda_2 \begin{bmatrix} 0 \\ 0 \\ B\Phi_0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ B\Phi_0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T + \lambda_2^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ K^T \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ K^T \\ 0 \end{bmatrix}^T < 0 \quad (25)$$

Using Schur complement theory repeatedly, it can be equivalent to

$$\begin{bmatrix} -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & C & 0 & 0 \\ * & \lambda_2 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & K & 0 \\ * & * & \lambda_2^{-1} I & 0 & 0 & 0 & 0 & (B\Phi_0)^T & 0 & 0 & 0 \\ * & * & * & \lambda_1 I & 0 & 0 & 0 & 0 & K & -K & 0 \\ * & * & * & * & \lambda_1^{-1} I & (E\Phi_0)^T & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon I & 0 & 0 & E\Phi_0 K & -E\Phi_0 K & 0 \\ * & * & * & * & * & * & -\varepsilon^{-1} I & D^T & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -P^{-1} & A & B\Phi_0 K & H \\ * & * & * & * & * & * & * & * & -P+Q & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -Q & 0 \\ * & * & * & * & * & * & * & * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad (26)$$

Pre- and post-multiplying (26) by block-$diag(I, I, \lambda_2 I, I, \lambda_1 I, I, \varepsilon I, I, P^{-1}, P^{-1}, I)$, and then leting $X = P^{-1}$, $W = KP^{-1} = KX$, $S = P^{-1}QP^{-1}$, $\gamma^2 = \mu$, inequality (23) can be obtained. K can be calculated as $K = WX^{-1}$. This completes the proof.

Remark 3.1. *Obviously, in the result above, the time-varying variable L representing the change of actuator faults is not contained, so this result not only can effectively be applied to NCS with some special and constant forms of faults, but also fits to NCS with the faults varying from time to time. Compared with the traditional methods such as [11] in which the result is related to the faults matrix, the method proposed here is more universal.*

4. **Simulation.** Consider the model of an inverted pendulum as follows.

$$\dot{x}(t) = \begin{bmatrix} -1 & -0.36 & 0 & 0.12 \\ 0.1 & -1.1 & 0 & 0.01 \\ 0.16 & 0.19 & -0.42 & 0 \\ 0 & 0 & -0.02 & 0.23 \end{bmatrix} x(t) + \begin{bmatrix} 1.32 \\ -1.1 \\ 1.25 \\ -0.75 \end{bmatrix} u(t) + \begin{bmatrix} 0.31 \\ -0.52 \\ 0.29 \\ 1.01 \end{bmatrix}.$$

We choose the sampling period $T = 0.1s$. It assumes the parameters as $\alpha_1 = 1.53$, $\alpha_2 = -1.68$, $\alpha_3 = -2.1$, $\alpha_4 = 1.83$. Computed as [10,11], it follows

$$D = \begin{bmatrix} -1.3025 & 1.4302 & 0.0117 & 0.17 \\ -0.1798 + 0.6623i & 0.1975 + 0.7272i & 0.0027 & 0.0265 \\ 0.4084 - 0.081i & -0.4484 - 0.089i & 2.099 & 0.0496 \\ 0.0064 - 0.0003i & -0.0071 - 0.0004i & 0.0647 & 1.8212 \end{bmatrix},$$

$$H = \begin{bmatrix} 0.031 \\ -0.0489 \\ 0.0281 \\ 0.1021 \end{bmatrix}, \ E = \begin{bmatrix} -0.8136 + 1.4805i \\ -0.8136 - 1.4805i \\ -1.5499 \\ -0.7954 \end{bmatrix},$$

$$F = diag\left(\frac{e^{(-1.0503+0.1831i)(0.1-\tau_k)} - 1}{-1.6070 + 0.2801i}, \frac{e^{(-1.0503-0.1831i)(0.1-\tau_k)} - 1}{1.7645 + 0.3076i}, \right.$$
$$\left. \frac{e^{-0.4189(0.1-\tau_k)} - 1}{0.8796}, \frac{e^{0.2295(0.1-\tau_k)} - 1}{0.4199} \right).$$

Obviously, $F^T F < I$ is satisfied.

Considering the sensor faults may occur, it assumes the upper bound value is 0.95 and the lower bound value is 0.3, namely $0.3 \le \Phi \le 0.95$. By making use of LMI toolbox in MATLAB to solve the linear matrix inequality (23), H_∞ fault-tolerant controller parameters of NCS can be obtained

$$K = WX^{-1} = \begin{bmatrix} -1.1365 & 1.4213 & 2.1465 & 2.6178 \end{bmatrix}.$$

We choose the initial state of the system as $x(0) = x(-1) = \begin{bmatrix} 1 & 0.2 & 0.1 & -1.1 \end{bmatrix}^T$, the outside disturbance as $\omega(t) = \begin{cases} 0.06 & 55 \le t \le 65 \\ 0 & \text{others} \end{cases}$. When actuator is normal, the state responses of NCS are shown in Figure 3, and the system gets preliminary steadiness at $38s$. When time varying actuator faults occur, the state responses of NCS are shown in Figure 4, from which we can see the transition time of state response obviously becomes longer than that in Figure 3 for the introduction of time-varying faults, but the system is still asymptotically stable and gets preliminary steadiness at $45s$. The corresponding distribution of time varying actuator faults is shown in Figure 5. Moreover, from Figure 3 and Figure 4, we know the state can return to the equilibrium position in a certain period of time when the NCS is affected by outside disturbance. So, the performance of NCS can be well maintained by H_∞ fault-tolerant controller.

Moreover, we apply the method proposed in [11] into the time-varying actuator faults problem. And the design of controller fails with $K = \begin{bmatrix} -1.5359 & -0.3211 & 1.2108 & -0.5793 \end{bmatrix}$, and the response of system state is shown as Figure 6, from which we know the traditional design cannot stabilize the system. Thus, it sufficiently demonstrates the effectiveness and feasibility of this paper.

5. **Conclusions.** When time varying actuator faults occur, H_∞ fault-tolerant control problem of networked control system is studied in this paper. Based on the network transmission environment, the networked control system is modeled as a closed loop discrete-time system with time varying actuator faults and outside disturbance considered. And the time varying property of actuator faults is reflected by a time-varying parameter.

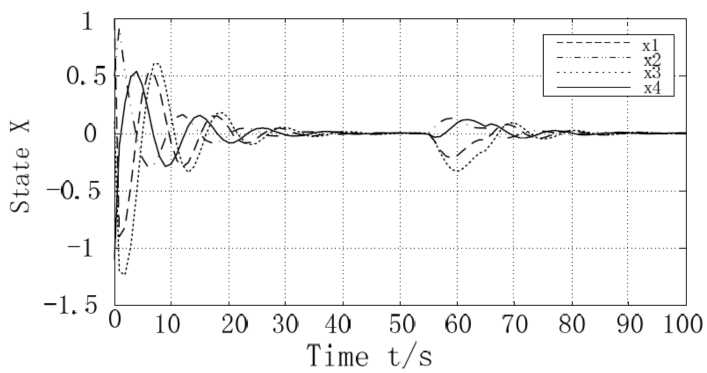

FIGURE 3. State response of NCS without actuator faults

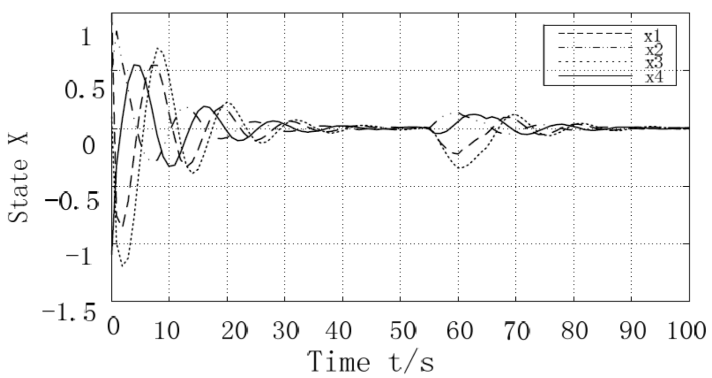

FIGURE 4. State response of NCS with time varying actuator faults

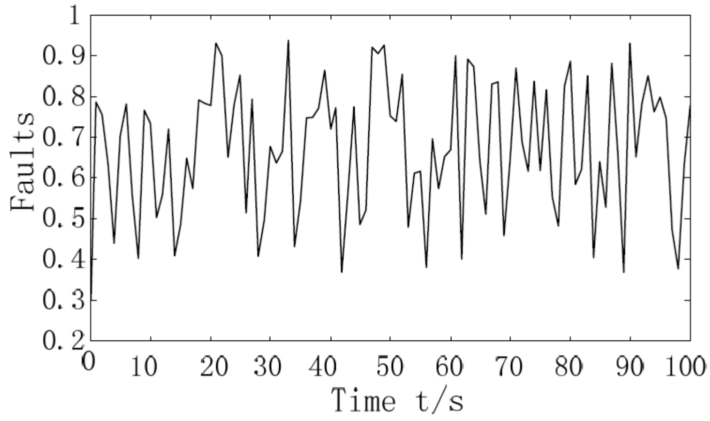

FIGURE 5. The time varying actuator faults in NCS

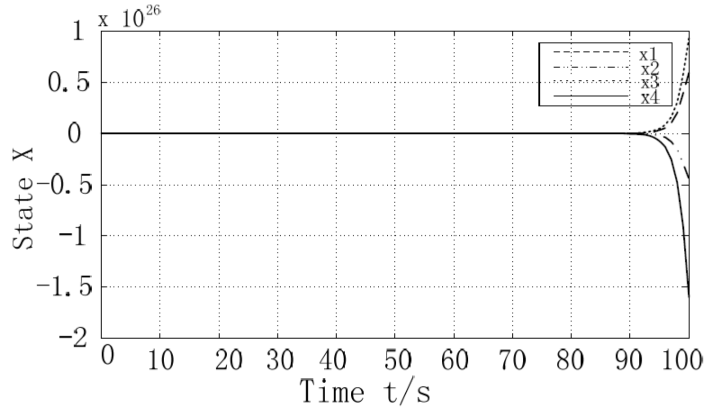

FIGURE 6. State response of NCS with time varying actuator faults

Moreover, using Lyapunov stability theory and linear matrix inequality (LMI) approach, the H_∞ fault-tolerant controller is proposed to guarantee such faulty networked control system asymptotically stable. Finally, the feasibility and effectiveness of this method have been demonstrated by a simulation example.

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