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PROJECTION OF SOCIAL INTEREST HOUSING THROUGH THE HUMAN MIGRATION MODEL

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Abstract. *A human migration model is applied to the study of population growth in the three agglomerated cities in the Comarca Lagunera, México. A utility function for different population groups is introduced, and a balance with conjectural variations is investigated. Numerical experiments based on the data of the three cities in the period 1990-2010 for a projection of social interest housing are obtained to provide the construction of housing in the future.*
Keywords: Projection of social interest housing, Human migration model, Utility function, Conjectural variations

1. Introduction.

Social changes are the product of very different factors. One of the most noticeable changes is population growth. It should be noted, for example, that each year the population grows, and it allows us to easily perceive new situations, such as the saturation of transport, health, electricity, education, and drinking water services, as well as sources of employment, spaces for recreation and housing saturation, among others [1].

The same situation is presented in both developing and most developed countries, since they cannot supply adequate resources for systematic housing, employment sources, quality of life, etc., due to the increased population. It is difficult to have a controlled growth of the population and to have preventive measures to provide solutions to the problems mentioned above.

The following research addresses the main topic of projection of social interest housing by applying the human migration model. This is a phenomenon directly related to population growth. The next research was carried out with the aim that, through a mathematical model and experimenting with the elementary function of the straight line defined by [2], the human migration flow between the three cities that comprise the Comarca Lagunera could be predicted. Those cities are Torreón, Gómez Palacio, and Lerdo. The first one belongs to the State of Coahuila while the last two belong to the State of Durango; both entities are located in northern México.

The human migration equilibrium models have been in the centre of research activity since the early nineties of the last century. In almost all relevant papers and books, a network of locations is considered, and conditions guaranteeing the existence and uniqueness of equilibrium in the proposed models are elaborated. For example, the works by the group of Anna Nagurney examined various forms of the Nash equilibrium under an assumption of perfect competition, that is, each population group neglected the possible influence of the migration flows on the living standards at the destination [3-5].

A new array of conjectural variations equilibrium (CVE) was introduced and investigated, in which the influence coefficients of each agent affected the structure of the Nash equilibrium. In particular, the constant conjectural influence factors were used in the human migration model. More precisely, the potential migration groups were taking account of not only the current difference between the utility function values at the destination and original locations, but also the possible variations in the utility values implied by the change of population volume due to the migration flow. In other words, we considered not a perfect competition but a generalized Cournot-type model with influence coefficients in general different from 1 [6-10].

Also, new interesting results concerning conjectural variation equilibrium a points and bifurcation control were published by Xiang [11]. Jiménez-Lizarrage et al. [12] made further advance in numerical algorithms, which allow finding equilibrium in linear-quadratic stochastic games with unknown parameters. Dynamic multi-objective optimization problem (DMOP) was considered by Liu and Wang [13], who noticed that these often involve incommensurable, competing and varying objectives with time (environment), and the number of their Pareto optimal solutions is usually infinite. The latter thus implies that ways to find a sufficient number of uniformly distributed and representative Pareto optimal solutions at any environment are very important for the decision maker. In their paper [13], Liu and Wang divided the time period of DMOP into several equal sub-periods. In each sub-period, the DMOP is approximated by a static multi-objective optimization problem (SMOP). Furthermore, the static rank variance and the static density variance of the population are defined. Making use of the static rank variance and the static density variance of the population as two objectives, each SMOP is further transformed into a bi-objective optimization problem. As a result, the original DMOP is approximately transformed into several static multi-objective optimization problems. Thereafter, an environment changing feedback operator, which can automatically check out the environment variation, is proposed, and an improved non-uniform mutation operator with quantization is designed. Based on these, a new dynamic multi-objective optimization evolutionary algorithm (denoted by DMEA) is proposed. The comparative study shows that DMEA is more effective and can find better solution set in environment-varying than the compared algorithms can in terms of convergence, diversity and the distribution of the obtained Pareto optimal solutions.

Recently published papers are: *Demand and Equilibrium in A Network of Oligopolistic Markets* [14]; *Simulation of A Conjectural Variations Equilibrium in A Human Migration Model* [15]; *Numerical Experimentation with A Human Migration Model* [16]; *Consistent Conjectures in A Human Migration Model: Definition, Existence and Computation* [17].

This paper presents a human migration model applied to the study of population growth in the three agglomerated cities in the Comarca Lagunera, México. A utility function for different population groups is introduced, and a balance with conjectural variations is investigated. Numerical experiments based on the data of the three cities in the period 1990-2010 for a projection of social interest housing are obtained.

The paper is organized as follows. Section 2 describes the examined human migration model and introduces the appropriate notation. Section 3 is dedicated to the definition of equilibrium with conjectural variations in the particular model. In Section 4, we mention some results concerning the existence and uniqueness of the concerned equilibrium. Section 5 shows the numerical examples of the human migration model with Torreón, Gómez

Palacio and Lerdo, the application for the projection of social interest housing. Section 6 presents the conclusions.

2. **Model Description.** Similar to [15-17], consider a closed economy with: n locations, denoted by i; K population classes, denoted by k; $\overline{Q}_i^k$ initial fixed population of class k in the location i; Q_i^k population of class k in location i at equilibrium; c_{ij} migration cost from location i to location j; s_{ij}^k migration flow of class k from origin i to destination j.

Assume that the migration cost not only reflects the cost of physical movement but also the personal and psychological costs as perceived by a class that moves between locations.

Unlike the model of human migration described in [10], the utility u_i^k (attractiveness of location i as perceived by class k), depends on the population at destination Q_i^k, that is, $u = u(Q)$. This assumption is quite natural: indeed, in many cases, the cities with higher population provide much more possibilities to find a job, better medical service and household facilities, a developed infrastructure, etc. On the other hand, when the infrastructure development lags behind the modern city demands, the higher population may lead to certain decrease in the living standards, and hence, of the utility values.

The conservation of flow equations given for each social class k and each location i, is as follows:

$$Q_i^k = \overline{Q}_i^k + \sum_{j \neq i} s_{ji}^k - \sum_{j \neq i} s_{ij}^k, \ i = 1, \ldots, n; \ j = 1, \ldots, n, \tag{1}$$

and the assumption that there is no repeated migration is written as inequalities:

$$\sum_{j \neq i} s_{ij}^k \leq \overline{Q}_i^k, \ i = 1, \ldots, n, \tag{2}$$

with $s_{ij}^k \geq 0$, $\forall k = 1, \ldots, K$; $j \neq i$. Denote the problem's feasible set by

$$M = \{(Q, s) | s \geq 0, \ \text{and } (Q, s) \text{ satisfies (1) and (2)}\}. \tag{3}$$

Equation (1) says that the population at location i of class k is determined by the initial population of class k in the location i plus the migration flow in i of that class minus the migration flow out of i for that class. Equation (2) states that the flow out of i for class k cannot exceed the initial population of class k to i, since no repeated migration is allowed.

Assume that migrants are rational, and that migration continues until no individual has any incentive to move, since a unilateral decision will no longer yield a positive net gain (the gain in the expected utility minus the migration cost).

In order to extend the human migration model from [16], here, we introduce the following concepts.

Let $w_{ij}^{k+} \geq 0$ be an influence coefficient taken into account by an individual of class k moving from i to j. This coefficient is defined by her assumption that after the movement of s_{ij}^k individuals of class k from i to j the total population of class k at j will become equal to $\overline{Q}_j^k + w_{ij}^{k+} s_{ij}^k$.

On the other hand, let $w_{ij}^{k-} \geq 0$ be an influence coefficient conjectured by an individual of class k moving from i to j, determined by the assumption that after the movement of s_{ij}^k individuals, the total population of class k in i will remain equal to $\overline{Q}_i^k - w_{ij}^{k-} s_{ij}^k$.

We accept the following assumptions concerning the utility functions and expected variations of the utility values.

A1. The utility $u_i^k = u_i^k(Q_i)$ is a monotone decreasing and continuously differentiable function.

A2. Each person of class k, when considering her possibility of moving from location i to location j, takes account of not only the difference in the utility values at the initial location and the destination, but also both the expected (negative) increment of the utility function value at j which is: $s_{ij}^k w_{ij}^{k+} \dfrac{\partial u_j^k}{\partial Q_j^k}$; and the expected (positive) utility value increment in location i which is: $-s_{ij}^k w_{ij}^{k-} \dfrac{\partial u_i^k}{\partial Q_i^k}$.

3. **Defining Equilibrium.** A multi-class population and flow pattern $(Q^*, s^*) \in M$ is an equilibrium, if for each class $k = 1, \ldots, K$, and for each pair of locations $i, j = 1, \ldots, n$; $i \neq j$, the following relationship holds:

$$u_i^k - s_{ij}^{k*} w_{ij}^{k-} \frac{\partial u_i^k}{\partial Q_i^k} \left(Q_i^{k*}\right) + c_{ij}^k \begin{cases} = u_j^k + s_{ij}^{k*} w_{ij}^{k+} \dfrac{\partial u_j^k}{\partial Q_j^k} \left(Q_j^{k*}\right) - \lambda_i^k, & \text{if } s_{ij}^{k*} > 0; \\ \geq u_j^k + s_{ij}^{k*} w_{ij}^{k+} \dfrac{\partial u_j^k}{\partial Q_j^k} \left(Q_j^{k*}\right) - \lambda_i^k, & \text{if } s_{ij}^{k*} = 0; \end{cases} \tag{4}$$

and

$$\lambda_i^k \begin{cases} \geq 0, & \text{if } \sum\limits_{j \neq i} s_{ij}^{k*} = \overline{Q}_i^k; \\ = 0, & \text{if } \sum\limits_{j \neq i} s_{ij}^{k*} < \overline{Q}_i^k. \end{cases} \tag{5}$$

A3. We assume that the influence coefficients are functions depending upon the current population at the location in question (the destination for the coefficients w_{ij}^{k+} and the initial locality for the coefficients w_{ij}^{k-}) and the migration flow from location i to location j, satisfying the following conditions:

$$s_{ij}^k w_{ij}^{k+} \left(Q_j^k, s_{ij}^k\right) = \alpha_{ij}^{k+} s_{ij}^k + \sigma_{ij}^{k+} Q_j^k, \tag{6}$$

and

$$s_{ij}^k w_{ij}^{k-} \left(Q_j^k, s_{ij}^k\right) = \alpha_{ij}^{k-} s_{ij}^k - \sigma_{ij}^{k-} Q_j^k, \tag{7}$$

where

$$\alpha_{ij}^{k\pm} \geq 0,\ \sigma_{ij}^{k\pm} \geq 0,\ k = 1, \ldots, K;\ i, j = 1, \ldots, n;\ i \neq j. \tag{8}$$

Taking A3 into account and omitting for shortness the argument Q^* in the utility functions, we come from (4) to:

$$u_i^k - s_{ij}^{k*} \alpha_{ij}^{k-} \frac{\partial u_i^k}{\partial Q_i^k} + \sigma_{ij}^{k-} Q_i^{k*} \frac{\partial u_i^k}{\partial Q_i^k} + c_{ij}^k \begin{cases} = u_j^k + s_{ij}^{k*} \alpha_{ij}^{k+} \dfrac{\partial u_j^k}{\partial Q_j^k} + \sigma_{ij}^{k+} Q_j^{k*} \dfrac{\partial u_j^k}{\partial Q_j^k} - \lambda_i^k, & \text{if } s_{ij}^{k*} > 0; \\ \geq u_j^k + s_{ij}^{k*} \alpha_{ij}^{k+} \dfrac{\partial u_j^k}{\partial Q_j^k} + \sigma_{ij}^{k+} Q_j^{k*} \dfrac{\partial u_j^k}{\partial Q_j^k} - \lambda_i^k, & \text{if } s_{ij}^{k*} = 0. \end{cases} \tag{9}$$

Now assume that the utility function associated with a particular location and a single class can depend upon the population associated with every class and each location, that is, compose a vector-function $= u(Q)$. Assume also that the cost associated with migration between two locations as perceived by a particular class can depend, in general, upon the flow of each class between every pair of locations, i.e., compose an aggregate vector-function $= c(s)$. Finally, let us compose an auxiliary vector of the appropriate size as follows:

$$d^k(Q, s) = d_{ij}^k(Q, s), \tag{10}$$

where:

$$d_{ij}^k(Q,s) = s_{ij}^k \alpha_{ij}^{k-} \frac{\partial u_i^k}{\partial Q_i^k} - \sigma_{ij}^{k-} Q_i^k \frac{\partial u_i^k}{\partial Q_i^k} + s_{ij}^k \alpha_{ij}^{k+} \frac{\partial u_j^k}{\partial Q_j^k} + \sigma_{ij}^{k+} Q_j^k \frac{\partial u_j^k}{\partial Q_j^k}. \tag{11}$$

4. **The Existence and Uniqueness of Equilibrium.** Now, we are in a position to formulate the following result, established in the previous paper [16].

Theorem 4.1. *A population and migration flow pattern $(Q^*, s^*) \in M$ satisfies the equilibrium conditions (4) and (5) if, and only if it solves the variational inequality problem:*

$$\langle -u(Q^*), Q - Q^* \rangle + \langle c(s^*) - d(Q^*, s^*), s - s^* \rangle \geq 0,\ \forall (Q,s) \in M. \tag{12}$$

The existence of at least one solution to the variational inequality (15) follows from the general theory of variational inequalities, under the sole assumption of continuous differentiability of the utility functions u and the continuity of migration cost functions c, since the feasible convex set M is compact; see [16].

From now on, we omit the superscript k for simplicity purpose. The uniqueness of the equilibrium population and migration flow pattern (Q^*, s^*) follows under the assumption that the compound operator:

$$\begin{pmatrix} -u(Q) \\ c(s) - d(Q,s) \end{pmatrix} : R^{K\times n} \times R^{K\times n\times(n-1)} \to R^{K\times n} \times R^{K\times n\times(n-1)}, \tag{13}$$

involving the utility and migration cost functions, is strictly monotone over the feasible set M:

$$\begin{gathered} \left\langle \begin{pmatrix} -u\left(Q^1\right) \\ c\left(s^1\right) - d\left(Q^1, s^1\right) \end{pmatrix} - \begin{pmatrix} -u(Q^2) \\ c(s^2) - d(Q^2, s^2) \end{pmatrix}, \begin{pmatrix} Q^1 - Q^2 \\ s^1 - s^2 \end{pmatrix} \right\rangle > 0, \\ \forall \begin{pmatrix} Q^1 \\ s^1 \end{pmatrix} \neq \begin{pmatrix} Q^2 \\ s^2 \end{pmatrix}, \end{gathered} \tag{14}$$

that is,

$$\begin{aligned} - \left\langle u\left(Q^1\right) - u\left(Q^2\right), Q^1 - Q^2 \right\rangle + \left\langle c\left(s^1\right) - c\left(s^2\right), s^1 - s^2 \right\rangle \\ - \left\langle d\left(Q^1, s^1\right) - d\left(Q^2, s^2\right), s^1 - s^2 \right\rangle > 0. \end{aligned} \tag{15}$$

The latter is a consequence of the classical result of the Theory of Variational Inequality Problems; see [16].

Theorem 4.2. *Consider the variational inequality: find $y^* \in M \in R^n$ such that,*

$$\left\langle F\left(y^*\right), y - y^* \right\rangle \geq 0 \quad \forall y \in M. \tag{16}$$

If the operator $F : R^n \to R^n$ is strictly monotone over M, i.e.,

$$\left\langle F\left(y^1\right) - F\left(y^2\right), y^1 - y^2 \right\rangle > 0 \quad \forall y^1, y^2 \in M,\ y^1 \neq y^2, \tag{17}$$

then the variational inequality (16) has at most one solution.

Consider now an additional assumption.

A4. The coefficients $\sigma_{ij}^{k\pm}$ satisfy the following conditions:

$$\sigma_{ij}^{k-} = \sigma_{ji}^{k+} = \sigma_i^k,\ i = 1, \ldots, n;\ j \neq i. \tag{18}$$

It is worthwhile to note that under assumptions A3 and A4, the strict monotony condition (15) can be relaxed and replaced with a bit more general condition of strict monotony of two functions:

$$T(Q) = -u(Q) - d_Q(Q) \text{ with respect to } Q, \tag{19}$$

and

$$G(Q,s) = c(s) - d_s(Q,s) \text{ with respect to } s, \text{ for any fixed value of } Q; \tag{20}$$

here,

$$\left(d_Q^k\right)_i = \sigma_i Q_i^k \frac{\partial u_i^k}{\partial Q_i^k},\ i = 1, \ldots, n; \tag{21}$$

and

$$\left(d_s^k\right)_{ij} = s_{ij}^k \alpha_{ij}^{k-} \frac{\partial u_i^k}{\partial Q_i^k} + s_{ij}^k \alpha_{ij}^{k+} \frac{\partial u_j^k}{\partial Q_j^k},\ i \neq j. \tag{22}$$

In the mathematical form, these two conditions are represented as follows:

$$\begin{aligned} &\left\langle T\left(Q^1\right) - T\left(Q^2\right), Q^1 - Q^2 \right\rangle \\ &= -\left\langle u\left(Q^1\right) - u\left(Q^2\right), Q^1 - Q^2 \right\rangle - \left\langle d_Q\left(Q^1\right) - d_Q\left(Q^2\right), Q^1 - Q^2 \right\rangle \quad \forall Q^1 \neq Q^2; \end{aligned} \tag{23}$$

$$\begin{aligned} &\left\langle G\left(Q, s^1\right) - G\left(Q, s^2\right), s^1 - s^2 \right\rangle \\ &= \left\langle c\left(s^1\right) - d_s\left(Q, s^1\right) - c\left(s^2\right) + d_s\left(Q, s^2\right), s^1 - s^2 \right\rangle \quad \forall G\left(Q, s^1\right), G\left(Q, s^2\right) \in M,\ s^1 \neq s^2. \end{aligned} \tag{24}$$

First, we establish the following equivalency result.

Theorem 4.3. *Under assumptions A1-A4, a population and migration flow pattern* $(Q^*, s^*) \in M$ *satisfies the equilibrium conditions (5) and (9) if, and only if it solves the variational inequality problem*

$$\left\langle T\left(Q^*\right), Q - Q^* \right\rangle + \left\langle G\left(Q^*, s^*\right), s - s^* \right\rangle \geq 0 \quad \forall (Q, s) \in M. \tag{25}$$

The proof can be seen in [16].

Uniqueness conditions are given in the following theorem [16].

Theorem 4.4. *If conditions (23) and (24) are valid and the utility functions* $u_i^k = u_i^k(Q)$ *are linear, i.e.,*

$$u_i^k(Q) = b_i^k + \sum_{j=1}^{n} a_j^k Q_j,\ i = 1, \ldots, n;\ k = 1, \ldots, K, \tag{26}$$

then the variational inequality problem (25) has at most one solution.

The proof can be seen in [16].

In case of general (not necessarily linear) utility functions, one can guarantee that if the equilibrium population distribution Q^* is determined uniquely, so is the equilibrium flow pattern s^*.

5. **Numerical Examples of the Human Migration Model with Torreón, Gómez Palacio and Lerdo.** In order to realize numerical examples with the human migration model, we consider three distinct locations with population quantities Q_i, $i = 1$, 2 and 3, for a unique class k. Each inhabitant perceives a utility u_i in each location i, and the cost of being transferred from i to j, denoted by c_{ij}. For a base of our research, three real cities have been selected: Torreón, Coah. ($i = 1$), Gómez Palacio, Dgo. ($i = 2$), and Lerdo, Dgo. ($i = 3$). These three cities form an agglomeration with a well-developed transportation and communication network. To introduce utility functions for each city, we make use of the following scheme.

Assuming that $du(Q(t))/dt = a$, we come to the formula:

$$\frac{du}{dQ} = \frac{a}{dQ/dt}. \tag{27}$$

The function $Q(t)$ reflects the population growth in time, and it is assumed to be approximated more population growth.

Assuming that $Q(t) = A + Bt$ (linear function) and making use of (27) with $a = -1$, we come to the following differential equation:

$$\frac{du}{dQ} = -\frac{1}{B}. \tag{28}$$

Its general solution $u(Q) = C - \dfrac{Q}{B}$ allows one to accept linear utility functions for each of the three locations mentioned above. As a value of the parameter C, we select an average cost (in tens of thousands of Mexican pesos) of a two-bedroom house at the corresponding location, whereas the value of parameter B was determined for each city by the least squares approximation procedure applied to the population data for the time period 1990-2010.

Table 1 shows the number of persons for each city according to INEGI 2010 (National Institute of Statistics, Geography and Informatics).

Table 1. INEGI 2010 (Persons census in the populations)

City	Employed population (Persons)	Total population (Persons)	Group
Torreón, Coahuila, México	242269	639629	1
Gómez Palacio, Durango, México	114643	327985	2
Lerdo, Durango, México	47391	141043	3

Based upon these calculations, we accept the following (linear) utility function for the numerical examples [16]:

$$\begin{cases} u_1(Q) = 25.0 - \dfrac{Q_1}{8635.4337}; \\ u_2(Q) = 22.5 - \dfrac{Q_2}{4795.51}; \\ u_3(Q) = 16.0 - \dfrac{Q_3}{2015.295}. \end{cases} \tag{29}$$

Now assume that the initial population of total employed workers together with their families at each location is: $\overline{Q}_1 = 242269$, $\overline{Q}_2 = 114643$, $\overline{Q}_3 = 47391$; the costs to be transferred from a location to another (in thousands of Mexican pesos) are as follows: $c_{12} = 1.6$; $c_{13} = 1.6$; $c_{21} = 1.6$; $c_{23} = 1.0$; $c_{31} = 1.6$; $c_{32} = 1.0$.

Inequalities (4) and (5) can be re-written as the following complementarity problem:

$$\begin{gathered} \Psi_{ij}^k \equiv u_i^k + c_{ij}^k - u_j^k - s_{ij}^{k*} w_{ij}^{k-} \frac{\partial u_i^k}{\partial Q_i^k}\left(Q_i^{k*}\right) - s_{ij}^{k*} w_{ij}^{k+} \frac{\partial u_j^k}{\partial Q_j^k}\left(Q_j^{k*}\right) + \lambda_i^k \geq 0, \\ s_{ij}^k \geq 0, \text{ and } \Psi_{ij}^k s_{ij}^k \geq 0; \end{gathered} \tag{30}$$

$$\xi_i^k \equiv \overline{Q}_i^k - \sum_{j \neq i} s_{ij}^{k*} \geq 0, \lambda_i^k \geq 0, \text{ and } \xi_i^k \lambda_i^k \geq 0. \tag{31}$$

Considering $w_{ij}^- = 0$ and $w_{ij}^+ \geq 0$, we rewrite (30) and (31) in the following complementarity problem form:

$$\begin{cases} \Psi_{12} \equiv 2.66 - \dfrac{Q_1}{8635.4337} + \dfrac{Q_2 + s_{12}w_{12}^+}{4795.51} + \lambda_1 \geq 0, \quad s_{12} \geq 0, \quad \Psi_{12}s_{12} = 0; \\ \Psi_{13} \equiv 9.16 - \dfrac{Q_1}{8635.4337} + \dfrac{Q_3 + s_{13}w_{13}^+}{2015.295} + \lambda_1 \geq 0, \quad s_{13} \geq 0, \quad \Psi_{13}s_{13} = 0; \\ \Psi_{21} \equiv -2.34 - \dfrac{Q_2}{4795.51} + \dfrac{Q_1 + s_{21}w_{21}^+}{8635.4337} + \lambda_2 \geq 0, \quad s_{21} \geq 0, \quad \Psi_{21}s_{21} = 0; \\ \Psi_{23} \equiv 6.66 - \dfrac{Q_2}{4795.51} + \dfrac{Q_3 + s_{23}w_{23}^+}{2015.295} + \lambda_2 \geq 0, \quad s_{23} \geq 0, \quad \Psi_{23}s_{23} = 0; \\ \Psi_{31} \equiv -8.6 - \dfrac{Q_3}{2015.295} + \dfrac{Q_1 + s_{31}w_{31}^+}{8635.4337} + \lambda_3 \geq 0, \quad s_{31} \geq 0, \quad \Psi_{31}s_{31} = 0; \\ \Psi_{32} \equiv -6.4 - \dfrac{Q_3}{2015.295} + \dfrac{Q_2 + s_{32}w_{32}^+}{4795.51} + \lambda_3 \geq 0, \quad s_{32} \geq 0, \quad \Psi_{32}s_{32} = 0; \end{cases} \tag{32}$$

$$\begin{aligned} &\lambda_1 \geq 0, \quad \xi_1 \equiv 242269 - s_{12} - s_{13} \geq 0, \quad \xi_1\lambda_1 = 0; \\ &\lambda_2 \geq 0, \quad \xi_2 \equiv 114643 - s_{21} - s_{23} \geq 0, \quad \xi_2\lambda_2 = 0; \\ &\lambda_3 \geq 0, \quad \xi_3 \equiv 47391 - s_{31} - s_{32} \geq 0, \quad \xi_3\lambda_3 = 0. \end{aligned} \tag{33}$$

In order to solve problems (32) and (33), we recall that the final population at each location, when reaching the equilibrium, is given by the initial total population at location i minus the migration flow out of i, plus the migration flow into i:

$$\begin{aligned} Q_1 &= 639629 - s_{12} - s_{13} + s_{21} + s_{31}; \\ Q_2 &= 327985 - s_{21} - s_{23} + s_{12} + s_{32}; \\ Q_3 &= 141043 - s_{31} - s_{32} + s_{13} + s_{23}. \end{aligned} \tag{34}$$

Assuming that

$$w_{ij}^+(Q, s) = \alpha_{ij}^+ + \sigma_{ij}^+ \frac{Q_j}{s_{ij}}, \tag{35}$$

we put $\sigma_{ij}^k = 0$, $w^+ = w_{12}^+ = w_{13}^+ = w_{21}^+ = w_{23}^+ = w_{31}^+ = w_{32}^+$. Then solving the above complementarity problem for various values of $\alpha_{ij}^+ = w^+ = 0$; $\alpha_{ij}^+ = w^+ = 0.5$; $\alpha_{ij}^+ = w^+ = 0.75$; $\alpha_{ij}^+ = w^+ = 1.0$; $\alpha_{ij}^+ = w^+ = 1.5$, and $\alpha_{ij}^+ = w^+ = 2.0$, we obtain different results resumed in the following Table 2.

For example, if we consider the Cournot equilibrium case, that is, with $w_{ij}^- = 0$, $w_{ij}^+ \equiv 2.0$, and make use of the software Maple 14, we find the solution given in the last column of Table 2:

$$s_{12} = 2932; \; s_{13} = s_{21} = s_{23} = 0; \; s_{31} = 2548; \; s_{32} = 5452; \; \lambda_1 = 0; \; \lambda_2 = 0; \; \lambda_3 = 0.$$

The insignificant flow $s_{12} = 2932$ demonstrates that for the majority of persons inhabiting in Torreón, the gain in the utility value does not compensate the migration costs: only 2932 migrants move from Torreón to Gómez Palacio; however, almost one-sixth of all the employed workers living in Lerdo should move to Gómez Palacio and Torreón, having enhanced the group's population in Gómez Palacio up to 123027 persons and decreasing the group's population in Lerdo down to 39391 persons, i.e.,

$$\overline{Q}_1^* = 242269 - 2932 + 2548 = 241885; \; \overline{Q}_2^* = 114643 + 2932 + 5452 = 123027;$$
$$\overline{Q}_3^* = 47391 - 2548 - 5452 = 39391,$$

with the total population in Torreón becoming equal to $Q_1^* = 639629 - 2932 = 636697$, in Gómez Palacio $Q_2^* = 327985 + 2932 + 5452 = 3363$, and in Lerdo $Q_3^* = 141043 - 2548 - 5452 = 133043$.

TABLE 2. Migration flows with various influence coefficients $w_{ij}^{\pm}$

$\overline{Q}_1 = 242269$ $\overline{Q}_2 = 114643$ $\overline{Q}_3 = 47391$	$Q_1 = 639629$ $Q_2 = 327985$ $Q_3 = 141043$	$c_{12} = 1.6$ $c_{13} = 1.6$ $c_{21} = 1.6$		$c_{23} = 1.0$ $c_{31} = 1.0$ $c_{32} = 1.0$		
	$w^+ = 0.00$ $w^- = 0.00$	$w^+ = 0.50$ $w^- = 0.00$	$w^+ = 0.75$ $w^- = 0.00$	$w^+ = 1.00$ $w^- = 0.00$	$w^+ = 1.50$ $w^- = 0.00$	$w^+ = 2.00$ $w^- = 0.00$
s_{12}	1835	2082	2520	2877	2977	2932
s_{13}	0	0	0	0	0	0
s_{21}	0	0	0	0	0	0
s_{23}	0	0	0	0	0	0
s_{31}	0	0	1272	1868	2164	2548
s_{32}	9129	6665	5839	5055	4584	5452
λ_1	0	0	0	0	0	0
λ_2	0	0	0	0	0	0
λ_3	0	0	0	0	0	0
$\overline{Q}_1^*$	240434	240187	241026	241260	241456	241885
$\overline{Q}_2^*$	125607	123390	123002	122575	122204	123027
$\overline{Q}_3^*$	38262	40726	40280	40468	40643	39391
Q_1^*	637794	637547	637109	636752	636652	636697
Q_2^*	338949	336732	336344	335917	335546	336369
Q_3^*	131914	134378	133932	134120	134295	133043

TABLE 3. Social interest housing to be built in the next years

Uninhabited housing / Housing construction	Torreón	Gómez Palacio	Lerdo	Total
Torreón	0	838	0	838
Gómez Palacio	0	0	0	0
Lerdo	728	1558	0	2286
Total	728	2396	0	3124

If we consider that there are 3.5 persons living in each social interest house, the information was provided by the INEGI 2010 population and housing census. Then, we can make the conversion of inhabitants to obtain the quantity of social interest houses. If we consider the Cournot equilibrium case, that is, with $w_{ij}^- = 0$, $w_{ij}^+ \equiv 2.0$. This is presented in Table 3.

The number of housing to be built for each city in the next years is observed clearly, for example, in Torreón city 728 housing and Gómez Palacio 2396 housing, with a total of 3124 housing. It also can be noted that in the city of Lerdo, there is no need to build housing. On the contrary, there will be vacant housing that might be rented or abandoned.

6. **Conclusions.**

This paper presents a human migration model involving conjectures of the migration groups concerning the variations of utility function values both in the abandoned location and in the destination site. Also, we assume that the utility function is linear. To formulate equilibrium conditions in this model, we use the concept of conjectural variations equilibrium (CVE). We establish the existence and uniqueness results for the equilibrium in question, and do a series of numerical examples based upon the population data and utility functions for the conglomerate of three cities in the Laguna region of México (Torreón, Gómez Palacio and Lerdo). The results of the examples show a strong dependence of the migration flows upon the conjectures used by the potential migrants groups.

This mathematical model proposes to use as a solid alternative to determine the migration flow of the cities and thus make a population projection for each city, and also to determine the number of housing that must be built in the next years.

We also notice that the human migration model with conjectural variations can be further extended and examined for abandoned location "n" and destination site "m", both for cities, states and countries.

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