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ADAPTIVE FUZZY SLIDING MODE CONTROL FOR SEMI ACTIVE VEHICLE SUSPENSION SYSTEM

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Abstract. *In this paper, a semi-active suspension system using adaptive fuzzy sliding mode control is developed to control the suspension system. The adaptive sliding mode controller is designed so that the practical system can track the state of the reference model. The asymptotical stability of the adaptive sliding mode control system is proved based on the Lyapunov theory. The control force includes the equivalent control and the switching control. The uncertain switching gain in the switching force could be derived by the adaptive fuzzy strategy. Numerical simulations demonstrate the effectiveness of the proposed adaptive fuzzy sliding mode control for semi-active vehicle suspension, showing that the system can be improved in the presence of parameter uncertainties and external disturbances.*

Keywords: Semi-active suspension system, Fuzzy adaptive control, Sliding mode control

1. Introduction.

The vehicle development comes with the people's higher demand. Thus, the good performance of the vehicle suspension system related to the stability of the driving process and the effective control of the vehicle has attracted much attention. The shaking system of the running vehicle has lots of complexities, including the complexity of subsystems, the variances of the system parameters, the uncertain external disturbances. The good performance of the vehicle suspension system plays an important role in driving flexibilities and running stabilities. The semi-active suspension system does not have to be equipped with a complex control system as the active suspension does while having a good control influence on the suspension system. Therefore, the semi-active suspension system is an ideal method to control the suspension system. The current suspension system control strategy includes active suspension strategy, passive suspension strategy and semi-active suspension strategy. Among these strategies, the semi-active suspension system has the advantage of avoiding the complex control law and the high expense of the active suspension system while taking the suspension system under changeable control that the passive suspension system could not realize.

Several control strategies have been applied to the semi-active vehicle suspension system such as skyhook control, linear-quadratic-Gaussian (LQG), sliding mode control and fuzzy control strategies. Hrovat [1] reviewed advanced vehicle suspension and optimal control applications. Karnopp [2] designed a skyhook control strategy and demonstrated that it was effective in controlling the passive suspension system with one-degree-of-freedom (DOF). Dyke [3] proposed a clipped-optimal control strategy combined with an acceleration feedback. Yet, LQG control strategy may have a high demand for the system and bring robustness problem [4]. Fang et al. [5] designed a fuzzy control strategy based on a four-DOF vehicle model. Dan [6] applied the sliding mode control strategy to a suspension system experiment. Sam et al. [7] came up with a kind of proportional and integral sliding mode control with application to active suspension system. Yagiz and Hacioglu [8] presented a feedback controller for the vehicle suspension system. Some research works such as finite frequency H_∞ control [9], active suspension control with frequency band constraints and actuator input delay [10] and saturated adaptive robust control [11] have been investigated for active suspension system. Priyandoko et al. [12] developed a force controller for the vehicle active suspension of a quarter car model with skyhook and adaptive neuro active force control. Yagiz et al. [13] designed robust fuzzy sliding-mode controller for active suspensions of a nonlinear half-car model. Huang and Chen [14] applied an adaptive sliding controller with self-tuning fuzzy compensation to vehicle suspension control. Cao et al. [15] looked back about intelligent control approaches in vehicle active suspension adaptive control systems. Adaptive laws to estimate the upper bound of uncertainties are designed in [16,17]. Robust adaptive sliding mode controller was developed for semi-active vehicle suspension system in [18].

During the process of designing the control law, the appropriate model should be neither too complex nor too simple. The fuzzy control strategy does not rely heavily on the complex model and has the good control performance. In this paper, fuzzy control and sliding mode control strategies are combined together with self-adaptive control law. There are two models in this system, such as semi-active 1/4 vehicle of two-DOF simulation model and the 1/4 vehicle of two-DOF skyhook ideal reference model. The input of the active vehicle suspension system is the condition of the road surface set by human. The human-set disturbances would be added to the simulation model. The external disturbances and unknown factors are incorporated into the simulation model in order to simulate the real vehicle system. Thus, the two models would have two different outputs which will come into the controller as the component of the whole control force. In the end, the simulation model controlled by the proposed adaptive fuzzy sliding mode controller would be as same as the reference model and realize the purpose of controlling the vehicle suspension system.

During the past few years, many researchers have paid highly attention to the adaptive fuzzy sliding mode control method [19-23] which combines the advantages of both sliding mode and adaptive fuzzy control, and overcoming their shortcomings, the usage of sliding mode can easily solve impacts brought by the inaccurate model and the disturbance. In this paper, adaptive fuzzy sliding mode control is proposed to suppress the vibration of the vehicle system. Therefore, the motivation and contributions of the proposed study could be emphasized as follows.

(1) The whole control force includes the equivalent control force and the switching control force. The equivalent control force comes from the equivalent controller applied with the sliding mode control strategy. The switching control force comes from sliding mode controller with adaptive fuzzy control strategy. Thus, the whole control force could be self-adaptive to the change of parameters.

(2) The fuzzy control method combined with the adaptive sliding mode control not only does not require accurate system model, but also obtains the self-learning ability and adjusts the fuzzy parameters. The method designs the adaptive fuzzy controller to adjust the switching controller's parameters, and compensate the model uncertainties, thus greatly improving the robustness of the control system and control accuracy. Therefore, the proposed approach attenuates the model uncertainties and external disturbances.

(3) The proposed adaptive fuzzy sliding mode control adds additional adaptive fuzzy estimator to estimate the unknown upper bound of model uncertainties and external disturbances for achieving and improving the system stability, hence obtaining desired system

behavior and performance. Thus, the entire closed-loop system meets the expectations indicators of dynamic and static performance and achieves better performance.

2. **Semi-Active Vehicle Suspension Model.** Making the appropriate model is the first step in designing the control strategy. Neither the too complex model nor the too simple model would be appropriate for the control law. Therefore, the 1/4 vehicle with two-DOF suspension system is considered as the appropriate choice.

The simulation model is shown in Figure 1.

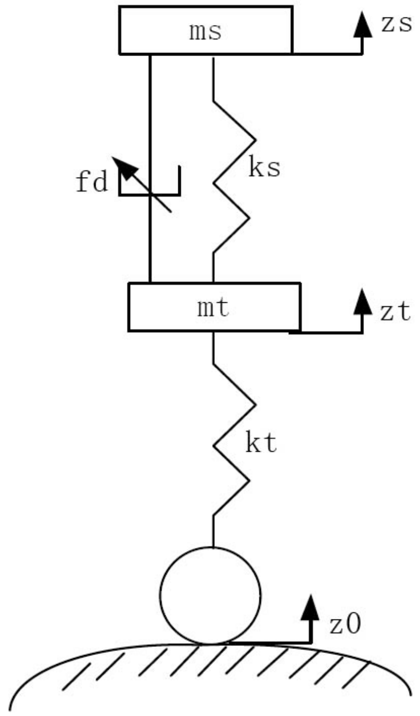

FIGURE 1. 1/4 vehicle model with two-DOF suspension system

In Figure 1, m_s is the sprung mass, m_t is the unsprung mass, k_s is the suspension rigidity, k_t is the tyre rigidity, z_s is the displacements of the sprung mass and z_t is the displacements of the unsprung mass, z_0 is the road displacements, and f_d is the whole control force came from the designed controller.

The dynamic differential equations are set up as follows:

$$\begin{cases} m_s\ddot{z}_s = -k_s(z_s - z_t) - f_d \\ m_t\ddot{z}_t = k_s(z_s - z_t) - k_t(z_t - z_0) + f_d \end{cases} \tag{1}$$

The state space equation could be written as:

$$\dot{X} = AX + B\mu + C\omega \tag{2}$$

where

$$A = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ -\frac{k_s}{m_s} & 0 & 0 & 0 \\ \frac{k_s}{m_t} & -\frac{k_t}{m_t} & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & \frac{-1}{m_s} & \frac{1}{m_t} \end{bmatrix}^T,$$

$$C = \begin{bmatrix} 0 & -1 & 0 & 0 \end{bmatrix}^T, \quad \omega = \dot{z}_0.$$

The 1/4 vehicle of two-DOF skyhook ideal reference model is taken as the reference model shown in Figure 2, where m_s is the sprung mass, m_t is the unsprung mass, k_s is

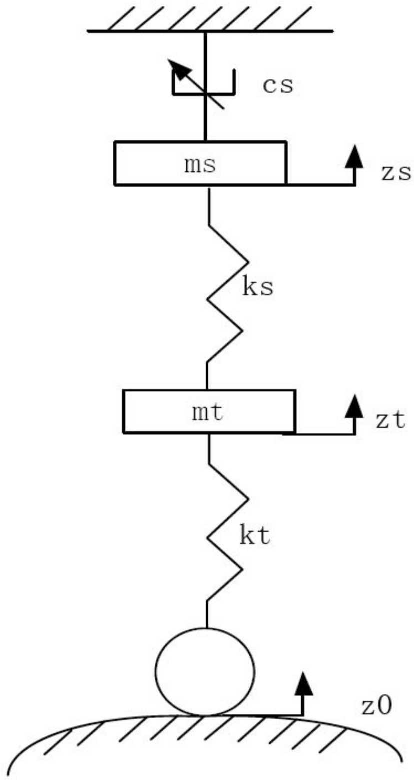

FIGURE 2. Schematic diagram of reference model

the suspension rigidity, k_t is the tyre rigidity, z_s is the displacements of the sprung mass and z_t is the displacements of the unsprung mass, z_0 is the road displacements, and c_s is the parameter of the damper.

The dynamic differential equations are set up as follows:

$$\begin{cases} m_s\ddot{z}_s = -k_s(z_s - z_t) - f_{dr} \\ m_t\ddot{z}_t = k_s(z_s - z_t) - k_t(z_t - z_0) + f_{dr} \end{cases} \qquad f_{dr} = \begin{cases} c_s\dot{z}_s & \dot{z}_s(\dot{z}_s - \dot{z}_t) > 0 \\ 0 & \dot{z}_s(\dot{z}_s - \dot{z}_t) \le 0 \end{cases} \tag{3}$$

The state space equation could be written as:

$$\dot{X}_m = A_m X_m + B_m f_{dr} + C_m \omega \tag{4}$$

where

$$A_m = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ -\frac{k_s}{m_s} & 0 & 0 & 0 \\ \frac{k_s}{m_t} & -\frac{k_t}{m_t} & 0 & 0 \end{bmatrix}, \quad B_m = \begin{bmatrix} 0 & 0 & \frac{-1}{m_s} & \frac{1}{m_t} \end{bmatrix}^T,$$

$$C_m = \begin{bmatrix} 0 & -1 & 0 & 0 \end{bmatrix}^T, \quad \omega = \dot{z}_0.$$

3. **Adaptive Fuzzy Sliding Mode Control.** The block diagram of an adaptive fuzzy sliding mode control for vehicle suspension is shown in Figure 3, and the tracking error comes to the sliding mode controller. The proposed adaptive fuzzy estimator can on-line update the estimates of unknown system nonlinearities and improve the tracking performance of vehicle suspension in the presence of model uncertainties and external disturbances.

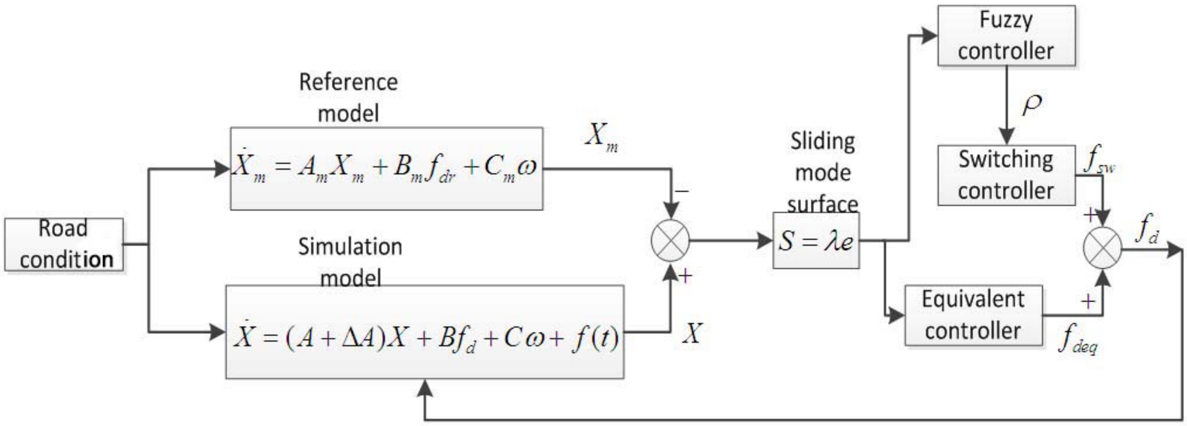

FIGURE 3. Block diagram of adaptive fuzzy sliding mode control for vehicle suspension

Considering the parameter uncertainties and external disturbances, the simulation model could be rewritten as:

$$\dot{X} = (A + \Delta A)X + Bf_d + C\omega + f(t) \tag{5}$$

where ΔA is unknown parameter uncertainties of the matrix A, and $f(t)$ is uncertain external disturbances or unknown non-linearity of the system.

In order to guarantee that the simulation model could be as much objective as possible, we should make the following assumptions.

Assumption 1: The matching condition: there exist $G(t)$ and $D(t)$ which are known matrix functions of appropriate dimensions as:

$$f(t) = BG(t); \quad \Delta A(t) = BD(t) \tag{6}$$

According to Assumption 1, (5) can be rewritten as:

$$\begin{aligned} \dot{X} &= AX + \Delta AX + Bf_d + C\omega + f(t) \\ &= AX + Bf_d + C\omega + BDX + BG \\ &= AX + Bf_d + C\omega + Bf_m \end{aligned} \tag{7}$$

where $f_m = DX + G$, and f_m includes the parameter uncertainties and external disturbances of the system.

Assumption 2: The bounded condition: there exists positive constant ρ which could meet the condition is the parameter uncertainties and external disturbances of the system.

Define the tracking error between the outputs of the simulation model and reference model as $\|f_m\| \le \rho$:

$$e = X - X_m \tag{8}$$

Then, its derivative is:

$$\dot{e} = A_m e + (A - A_m)X + Bf_d + B_m f_{dr} + Bf_m \tag{9}$$

Define the sliding surface as:

$$S = \lambda e \tag{10}$$

Then, the derivative of sliding surface (10) is:

$$\dot{S} = \lambda A_m e + \lambda(A - A_m)X + \lambda Bf_d + \lambda B_m f_{dr} + \lambda Bf_m \tag{11}$$

By setting $\dot{S} = 0$, we could solve the equivalent control force without considering the parameter uncertainties and external disturbances f_m:

$$f_{deq} = -(\lambda B)^{-1}\lambda A_m e - (\lambda B)^{-1}\lambda B_m f_{dr} \tag{12}$$

In order to decrease the effects of parameter uncertainties and external disturbances of the system, we should add the switching controller, so the whole control force contains the equivalent control force and the switching control force:

$$f_d = f_{deq} + f_{sw} = -(\lambda B)^{-1}\lambda A_m e - (\lambda B)^{-1}\lambda B_m f_{dr} - \rho\frac{B^T\lambda^T S}{\|B^T\lambda^T S\|} \tag{13}$$

The switching term $f_{sw} = -\rho\frac{B^T\lambda^T S}{\|B^T\lambda^T S\|}$ is the output of the switching controller which could compensate the effect of the parameter uncertainties and the external disturbances of the system, where ρ is sliding gain.

Substituting (13) with the sliding term $f_{sw} = -\rho\frac{B^T\lambda^T S}{\|B^T\lambda^T S\|}$ into (11), we could get

$$\dot{S} = \lambda\dot{e} = \lambda B\left(f_m - \rho\frac{B^T\lambda^T S}{\|B^T\lambda^T S\|}\right) \tag{14}$$

Define Lyapunov function candidate as:

$$V = \frac{1}{2}S^T S \tag{15}$$

Then, the derivative of (15) is

$$\begin{aligned}\dot{V} &= S^T\lambda B\left(f_m - \rho\frac{B^T\lambda^T S}{\|B^T\lambda^T S\|}\right) = S^T\lambda B f_m - \rho\frac{S^T\lambda B B^T\lambda^T S}{\|B^T\lambda^T S\|} = S^T\lambda B f_m - \rho\left\|B^T\lambda^T S\right\| \\ &\le \left\|B^T\lambda^T S\right\|\|f_m\| - \rho\left\|B^T\lambda^T S\right\| = -\left\|B^T\lambda^T S\right\|(\rho - \|f_m\|)\end{aligned} \tag{16}$$

Assuming d_B is known constant, if we choose sliding gain $\rho > d_B$, then $\dot{V} < 0$, Because $V(0)$ is limited and $V(t)$ is limited and non-increasing. According to the Barbalat lemma, $\lim_{t\to\infty} S(t) = 0$ and $\lim_{t\to\infty} e(t) = 0$.

However, on the condition that f_m is unknown, it is hard to determine the value of the upper bound of $\|f_m\|$, and then we use the fuzzy system $\hat{\rho}$ to approximate ρ.

Substituting ρ with $\hat{\rho}$ in (13), we could get the new adaptive law as follows:

$$f_d = f_{deq} + f_{sw} = -(\lambda B)^{-1}\lambda A_m e - (\lambda B)^{-1}\lambda B_m f_{dr} - \hat{\rho}\frac{B^T\lambda^T S}{\|B^T\lambda^T S\|} \tag{17}$$

Then, (14) could be rewritten as follows:

$$\dot{S} = \lambda B\left(f_m - \hat{\rho}\frac{B^T\lambda^T S}{\|B^T\lambda^T S\|}\right) \tag{18}$$

Using the production inference engine, single value fuzzy controller and the central average defuzzifier to defuzzy the fuzzy control, we could get the output of the fuzzy controller as:

$$\hat{\rho} = \frac{\sum_{j=1}^{m}\alpha_j\mu_A(S_j)}{\sum_{j=1}^{m}\mu_A(S_j)} = \hat{\alpha}^T\xi \tag{19}$$

where adaptive parameter $\hat{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_m]^T$, $\xi = [\xi_1, \xi_2, \ldots, \xi_m]^T$ is fuzzy basis function vector and ξ_j is jth column vector of fuzzy basis function vector ξ. $\mu_A(S_j)$ is the membership function value of the fuzzy variable S_j, $\xi_j(q) = \frac{\mu_A(S_j)}{\sum_{j=1}^{m}\mu_A(S_j)} = \hat{\alpha}^T\xi$.

According to the approximation theory, there exists an optimal fuzzy control system ρ^* which could minimize the difference between ρ^* and ρ, and ρ^* could be written as follows:

$$\rho^* = \rho + \varepsilon = \alpha^{*T}\xi \tag{20}$$

where ε is the error between ρ^* and ρ, and α^* is the optimal parameter.

The optimal parameter is:

$$\alpha^* = \arg \min_{\hat{\alpha} \in \Omega_\alpha} [\sup |\rho - \rho^*|] \tag{21}$$

where Ω_α is the convergence of $\hat{\alpha}$.

Define the Lyapunov function as follows:

$$V = \frac{1}{2} S^T S + \frac{1}{2\eta} \tilde{\alpha}^T \tilde{\alpha} \tag{22}$$

where $\tilde{\alpha} = \hat{\alpha} - \alpha^*$ is estimated error, and η is a positive constant.

Then, the derivative of (22) becomes

$$\begin{aligned}
\dot{V} &= S^T \lambda B f_m - S^T \lambda B \hat{\rho} \frac{B^T \lambda^T S}{\|B^T \lambda^T S\|} + \frac{1}{\eta} \tilde{\alpha}^T \dot{\tilde{\alpha}} \\
&= S^T \lambda B f_m - S^T \lambda B \hat{\rho} \frac{B^T \lambda^T S}{\|B^T \lambda^T S\|} + S^T \lambda B \rho^* \frac{B^T \lambda^T S}{\|B^T \lambda^T S\|} \\
&\quad - S^T \lambda B \rho^* \frac{B^T \lambda^T S}{\|B^T \lambda^T S\|} + \frac{1}{\eta} \tilde{\alpha}^T \dot{\tilde{\alpha}} \\
&= - \left[S^T \lambda B \rho^* \frac{B^T \lambda^T S}{\|B^T \lambda^T S\|} - S^T \lambda B f_m \right] \\
&\quad - \left[S^T \lambda B \hat{\rho} \frac{B^T \lambda^T S}{\|B^T \lambda^T S\|} - S^T \lambda B \rho^* \frac{B^T \lambda^T S}{\|B^T \lambda^T S\|} \right] + \frac{1}{\eta} \tilde{\alpha}^T \dot{\tilde{\alpha}} \\
&= - \left[S^T \lambda B (\rho + \varepsilon) \frac{B^T \lambda^T S}{\|B^T \lambda^T S\|} - S^T \lambda B f_m \right] \\
&\quad - \left[S^T \lambda B \hat{\rho} \frac{B^T \lambda^T S}{\|B^T \lambda^T S\|} - S^T \lambda B \rho^* \frac{B^T \lambda^T S}{\|B^T \lambda^T S\|} \right] + \frac{1}{\eta} \tilde{\alpha}^T \dot{\tilde{\alpha}} \\
&= - \left[S^T \lambda B (\rho + \varepsilon) \frac{B^T \lambda^T S}{\|B^T \lambda^T S\|} - S^T \lambda B f_m \right] - \tilde{\alpha}^T \xi S^T \lambda B \frac{B^T \lambda^T S}{\|B^T \lambda^T S\|} + \frac{1}{\eta} \tilde{\alpha}^T \dot{\tilde{\alpha}}
\end{aligned} \tag{23}$$

In order to make $\dot{V} \le 0$, and make the sum of last two terms equal to zero, we could get the following adaptive law:

$$\dot{\tilde{\alpha}} = \dot{\hat{\alpha}} = \eta \left\| B^T \lambda^T S \right\| \xi \tag{24}$$

Substituting the adaptive law (24) into (23), we could get:

$$\begin{aligned}
\dot{V} &= - \left[S^T \lambda B (\rho + \varepsilon) \frac{B^T \lambda^T S}{\|B^T \lambda^T S\|} - S^T \lambda B f_m \right] \\
&\le - \left\| B^T \lambda^T S \right\| (\rho + \varepsilon - \|f_m\|) \le 0
\end{aligned} \tag{25}$$

$\dot{V} \le 0$ implies that $\dot{V}$ is a negative semi-definite function. Furthermore, we have $\int_0^t \dot{V}(\tau) d\tau = V(t) - V(0) \le - \int_0^t \left\| B^T \lambda^T S \right\| (\rho + \varepsilon - \|f_m\|) d\tau$. Since $V(0)$ is bounded and $V(t)$ is non-increasing and bounded, $\lim_{t \to \infty} \int_0^t \int_0^t \left\| B^T \lambda^T S \right\| (\rho + \varepsilon - \|f_m\|) d\tau < \infty$. According to Barbalat lemma, $\lim_{t \to \infty} S(t) = 0$ and $\lim_{t \to \infty} e(t) = 0$.

4. **Simulation Study.** Numerical simulations will demonstrate the effectiveness of the proposed adaptive fuzzy sliding mode control for semi-active vehicle suspension, showing that the influence of parameter uncertainties and external disturbances can be reduced and system robustness can be improved.

We choose the following parameters: $m_s = 500$kg, $m_t = 50$kg, $k_s = 16800$N/m, $k_t = 168000$N/m, $c_s = 3550$Ns/m; $\lambda = [-10\ 10\ \ -10\ 1]$; three membership functions are: $\mu_{NM} = 1/(1 + \exp(5(s + 3)))$, $\mu_{zo} = \exp(-s^2)$, $\mu_{NM} = 1/\left(1 + \exp(5(s - 3))\right)$.

In the simulation figures, real_yes is the simulation model controlled by the proposed adaptive fuzzy sliding mode controller, real_no is also simulation model without controller. In the beginning, the same road surface condition acts as the input of the two models in the same time. Considering the road condition is $10/3.14 \sin(3.14t)$, the parameter uncertainties and the external disturbances of the vehicle system are considered as $10 \sin t$.

Figure 4 shows the road condition which means the input of the system. It is a kind of simple ideal road condition; yet we believe it could be used to testify the affections of the control system. Figure 5 shows the whole control force f_d which is calculated by the control system in the presence of acting road condition.

Figure 6 and Figure 7 compare the velocities of the sprung mass in vehicle body z_s between real vehicle system and reference model without controller and with controller respectively. Figure 8 and Figure 9 compare the velocities of the sprung mass in vehicle body z_t between real vehicle system and reference model without controller and with controller respectively. It could be easily noted that suspension system's dz_s is much more like reference model's dz_s in Figure 7 than Figure 6, and suspension system's dz_t is much closer to reference model's dz_t in Figure 9 than Figure 8, showing that the proposed controller has good effect on the control of sprung mass.

Figure 10 and Figure 11 compare the difference of the sprung mass between the displacements of the sprung mass z_s and unsprung mass z_t without controller and with controller respectively. Figure 12 and Figure 13 compare the displacements of the sprung mass between the displacements of the sprung mass z_s and the road displacements z_0 without controller and with controller. It could be easily noted that suspension system's z_s is much closer to reference model's z_t, showing that the proposed controller has good

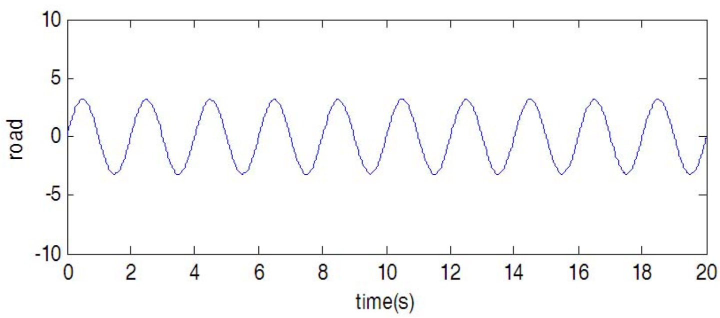

Figure 4. Road condition as the input of the system

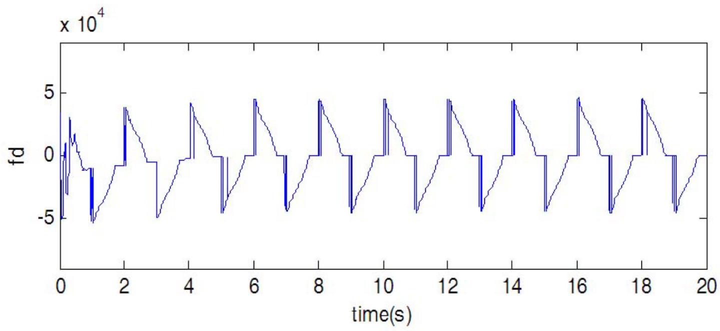

Figure 5. Whole control force f_d

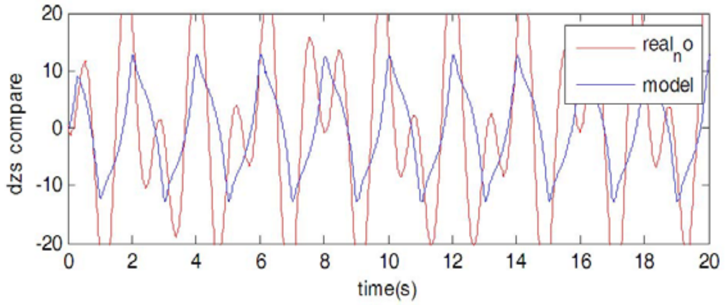

Figure 6. Comparison of the velocities of the sprung mass in vehicle body z_s between real vehicle system and reference model without controller

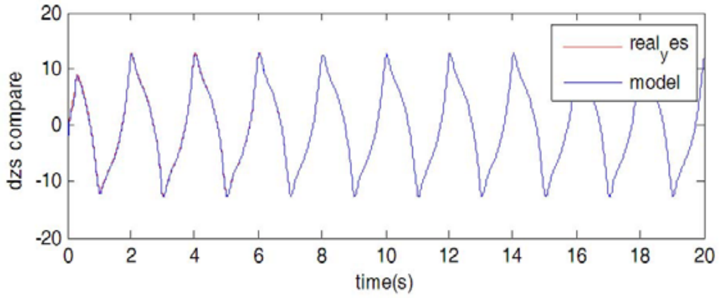

Figure 7. Comparison of the velocities of the sprung mass in vehicle body z_s between real vehicle system and reference model with controller

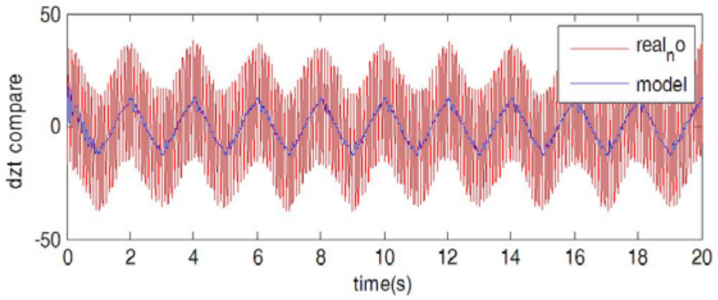

Figure 8. Comparison of the velocities of the sprung mass in vehicle wheel z_t between real vehicle system and reference model

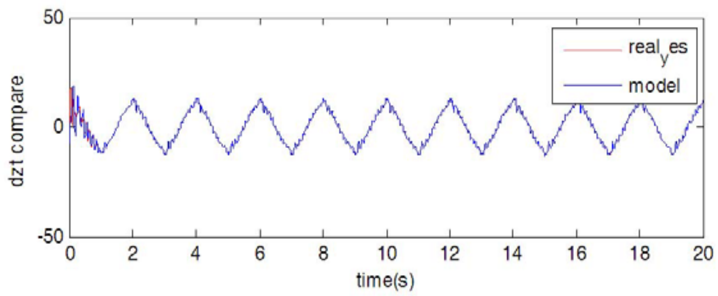

Figure 9. Comparison of the velocities of the sprung mass in vehicle wheel z_t between real vehicle system and reference model

effect on the control of sprung mass. Adaptive parameters of α are drawn in Figure 14, showing that the adaptive parameters in the fuzzy system are stable. The property of sliding surface s can be observed from Figure 15 that it converges to zero.

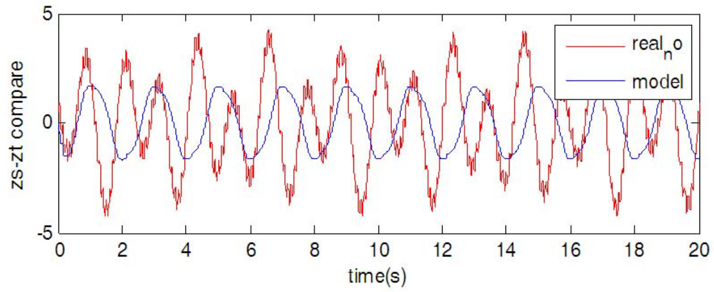

FIGURE 10. Comparison of the displacements of the sprung mass between real vehicle system and reference model

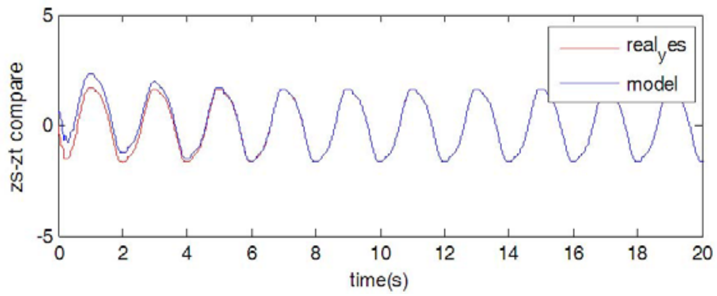

FIGURE 11. Comparison of the displacements of the sprung mass between real vehicle system and reference model

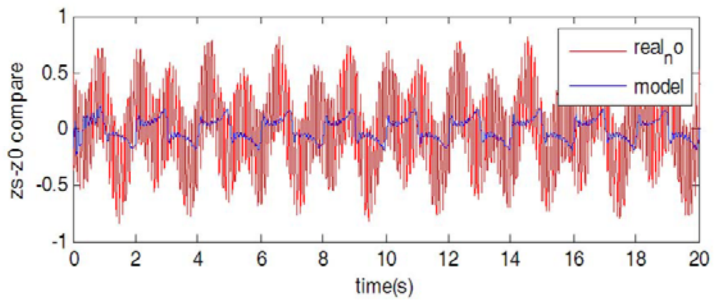

FIGURE 12. Comparison of the displacements of the sprung mass between real vehicle system and reference model

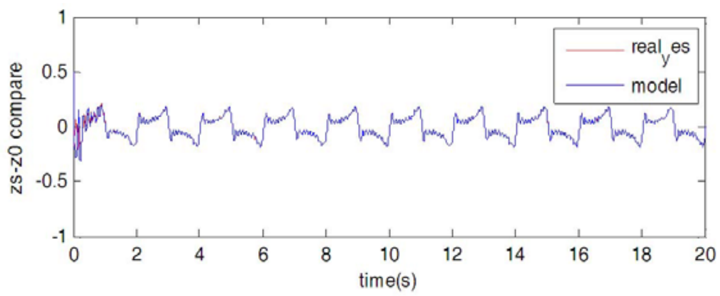

FIGURE 13. Comparison of the displacements of the sprung mass between real vehicle system and reference model

5. **Conclusion.** In this paper, the proposed semi-active control suspension system could have an effective impact on controlling the system with a suitable cost including the complexity of the hardware and the software. The designed control strategy is based on the

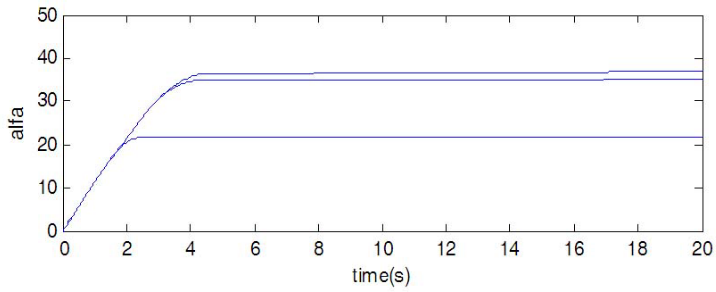

FIGURE 14. Adaptive parameters of α

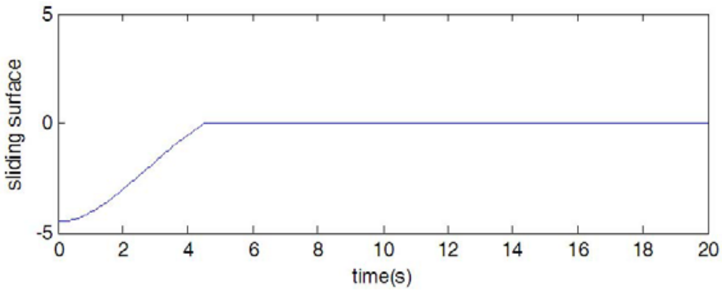

FIGURE 15. Property of sliding surface s

semi-active 1/4 vehicle of two-DOF model and the 1/4 vehicle of two-DOF skyhook ideal reference model. The adaptive fuzzy sliding mode control employs additional adaptive fuzzy estimator to estimate the unknown upper bound of model uncertainties and external disturbances for achieving and improving the system stability. The simulation model under control successfully realizes the goal of tracking the reference model. This strategy has an obvious control effect when it is applied in the vehicle suspension system with the proposed adaptive fuzzy sliding mode control strategy. However, the experimental implementation should be investigated to demonstrate the effectiveness of the proposed controller.

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REFERENCES

[1] D. Hrovat, Survey of advanced suspension development and related optimal control applications, *Antomatics*, vol.33, no.10, pp.1781-1817, 1997.
[2] D. Karnopp, Vibration control using semi-active force generator, *ASMEJ Engineering for Industry*, vol.96, no.2, pp.619-626, 1974.
[3] S. Dyke, *Acceleration Feedback Control Strategies for Active and Semi-Active Control Systems Modeling, Algorithm Development and Experimental Verification*, Ph.D. Thesis, University of NotreDame, 1996.
[4] G. Ulsoy, D. Hrobal and T. Tseng, Stability robustness of LQ and LQG active suspensions, *ASME Journal Dynamic Systems, Measurement and Control*, vol.116, no.1, pp.123-131, 1994.
[5] X. Fang, W. Chen, L. Wu, Q. Wang, D. Fan and Z. Li, Fuzzy control technology and the application to vehicle semi-active suspension, *Chinese Journal of Mechanical Engineering*, vol.35, no.3, pp.98-100, 1999.
[6] C. Dan, Experimental results on sliding mode control of an electromagnetic suspension, *Mechanical Systems and Signal Processing*, vol.7, no.4, pp.283-292, 1983.

[7] Y. Sam, J. H. Osman and M. R. Ghani, A class of proportional-integral sliding mode control with application to active suspension system, *Systems & Control Letters*, vol.51, nos.3-4, pp.217-224, 2004.

[8] N. Yagiz and Y. Hacioglu, Backstepping control of a vehicle with active suspensions, *Control Engineering Practice*, vol.16, no.12, pp.1457-1467, 2008.

[9] W. Sun, H. Gao and O. Kaynak, Finite frequency H_∞ control for vehicle active suspension systems, *IEEE Trans. Control System Technology*, vol.19, no.2, pp.416-422, 2011.

[10] W. Sun, Y. Zhao and J. Li, Active suspension control with frequency band constraints and actuator input delay, *IEEE Trans. Industrial Electronics*, vol.59, no.1, pp.530-537, 2012.

[11] W. Sun, Z. Zhao and H. Gao, Saturated adaptive robust control for active suspension systems, *IEEE Trans. Industrial Electronics*, vol.60, no.9, pp.3889-3896, 2013.

[12] G. Priyandoko, M. Mailah and H. Jamaluddin, Vehicle active suspension system using skyhook adaptive neuro active force control, *Mechanical Systems and Signal Processing*, vol.23, no.3, pp.855-868, 2009.

[13] N. Yagiz, Y. Hacioglu and Y. Taskin, Fuzzy sliding-mode control of active suspensions, *IEEE Trans. Industrial Electronics*, vol.55, no.11, pp.3883-3890, 2008.

[14] S. Huang and H. Chen, Adaptive sliding controller with self-tuning fuzzy compensation for vehicle suspension control, *Mechatronics*, vol.16, no.10, pp.607-622, 2006.

[15] J. Cao, H. Liu, P. Li and D. J. Brown, State of the art in vehicle active suspension adaptive control systems based on intelligent methodologies, *IEEE Trans. Intelligent Transportation Systems*, vol.9, no.3, pp.392-405, 2008.

[16] C. Chou and C. Cheng, A decentralized model reference adaptive variable structure controller for large-scale time-varying delay systems, *IEEE Trans. Automatic Control*, vol.48, no.7, pp.1213-1217, 2003.

[17] J. Fei, Robust adaptive vibration tracking control for a MEMS vibratory gyroscope with bound estimation, *IET Control Theory and Application*, vol.4, no.6, pp.1019-1026, 2010.

[18] J. Fei and M. Xin, Robust adaptive sliding mode controller for semi-active vehicle suspension system, *International Journal of Innovative Computing, Information and Control*, vol.8, no.1(B), pp.691-700, 2012.

[19] B. Wang, P. Shi and H. Karimi, Fuzzy sliding mode control design for a class of disturbed systems, *J. of Franklin Institute*, vol.351, no.7, pp.3593-3609, 2014.

[20] P. Shi, Y. Xia, G. Liu and D. Rees, On designing of sliding mode control for stochastic jump systems, *IEEE Trans. Automatic Control*, vol.51, no.1, pp.97-103, 2006.

[21] M. Liu, P. Shi, L. Zhang and X. Zhao, Fault tolerant control for nonlinear Markovian jump systems via proportional and derivative sliding mode observer, *IEEE Trans. Circuits and Systems I: Regular Papers*, vol.58, no.11, pp.2755-2764, 2011.

[22] L. Wu, P. Shi and H. Gao, State estimation and sliding mode control of Markovian jump singular systems, *IEEE Trans. Automatic Control*, vol.55, no.5, pp.1213-1219, 2010.

[23] J. Zhang, P. Shi and Y. Xia, Robust adaptive sliding mode control for fuzzy systems with mismatched uncertainties, *IEEE Trans. Fuzzy Systems*, vol.18, no.4, pp.700-711, 2010.