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POWER-LAW DISTRIBUTION OF RATE-OF-RETURN DEVIATION AND EVALUATION OF CASH FLOW IN A CONTROL EQUIPMENT MANUFACTURING COMPANY

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Abstract. *Although it is well known that power-law distribution appears in stock price fluctuation models and the like and theoretical analytic methods by mathematical finance has made progress, there are not many examples where such theoretical analysis has been performed for manufacturing industries. Therefore, we collected data about a rate of return and its deviation in a certain control equipment manufacturing company for about 10 years, and analyzed the both by applying methods of mathematical finance. Regarding rate-of-return deviation, it was found that it conforms to log-normal distribution. From analysis of mathematical models about rate-of-return deviation, we obtained the following conclusion. If an amount of money of order entries and an amount of money of production are stochastic, accumulated excessive order entries become of Brownian motion, and thus a random "fluctuation" occurs in hour to hour order entries and production even though it might be of a small degree. In comparison with a case where production is made to conform to the average order entry, profit can be increased in a case where strategy to purposefully lead to excessive production or excessive order entries state is adopted. Regarding rate of return, analysis of characteristics of power-law distribution was made. As a result of analysis of mathematical models for a rate of return, the followings are reported. A cash flow of a target company is log-normally distributed. With regard to equipment manufacturing, whether a value after a repayment of a loan is changed or not in the case where a guaranty by a company president is required is reported. In addition, how a value of manufacturing equipment (remaining value) changes after a repayment of a loan relative to a repayment period is reported. Finally, a degree (sensitivity) of influence of parameters, an initial plan money amount, a repaid money amount and a repayment period, on a remaining value and a result of risk analysis is also reported.*
Keywords: Rate-of-return deviation, Power-law distribution, Log-normal stochastic differential equation, Cash flow evaluation

1. Introduction.

Regarding research related to rate of return, there is a report in which, in order to compare a rate of return and variance of short-term investment, rate of return and variance at the time of long-term investment was researched [1]. In this research, in order to simulate a rate of return, Monte Carlo method was utilized. Also, there is a report that says that, from a result of investigation of long-term return on investment and characteristics of its variance, a geometric Brownian motion model describing a price of risk assets differs substantially from actual phenomena [2]. Monte Carlo method was used also in this research in order to simulate a rate of return.

On the other hand, there is research related to a cost required for fund raising in some company (interest expense, depreciation allowance and the like) and modeling of investments [3]. The feature of this research is that it uses a stochastic Euler-Lagrange Equation in order to specify things, and it is a very interesting research.

Conventionally, in physics, it is known that, assuming that the scaling law holds as is the case with a phase transition phenomenon, the probability density function becomes power-law distribution [4]. Due to occurrence of an unforeseen situation in an economic phenomenon such as a stock price fluctuation and a yen-dollar exchange fluctuation, fast and furious volatility of stock prices or rapid fluctuation of yen-dollar exchange is caused. Further, also in the information communication network field, it has been reported that, as a result of similar data analysis about first and furious traffic fluctuations, it becomes power-law distribution [5-8]. It is widely known that a field of econophysics as we know it today has been established.

A motive that the present writers and the like started to promote such kind of research during many years of experience of manufacturing operations of control equipment for general industrial machines is as follows. With respect to Japan after Lehman Shock, Japan's economy has been in a slump, and production bases of manufacturing industries keep moving overseas. Business environments of equipment manufacturing companies in Japanese are extremely severe. In Japan, the situation here is that thorough cost reduction is required. Therefore, we thought that, by finding relation between a company size and a production size of a company, and management parameters mathematically, cost reduction becomes possible.

Further, as a result of analysis based on data of a rate of return of companies and its deviation collected over 10 years or more from the above-mentioned motive, we have noticed that such data has random variation. By performing the data analysis, relation between a value of rate-of-return deviation and a production throughput became clear to some extent. For example, occurrence of fluctuation and the like were made clear through recognition as a phase transition point. This will be reported some other time.

As a consequence of having performed such data analysis, resultant advantage is that, because an almost rational width of rate-of-return deviation can be set, it has become possible to establish delivery time strategy in which production throughput, that is, a period from manufacturing equipment, which has been requested from an orderer, following the specification to delivering it, is flexible so that a rate-of-return may stay within the width.

First of all, analysis was made focusing attention on business rate-of-return deviation (hereinafter, referred to as rate-of-return deviation). As a result, it is reported that rate-of-return deviation has power-law characteristics. Generally, disnormality of rate-of-return deviation in business is well known about a stock price fluctuation model, although with conditions. For example, there exists widely-known Levy process [9].

However, almost all of the reported actual data was entirely limited to stock price data.

As another example, also in applying Real Option, many of the return fluctuation models are of a log-normal stochastic differential equation, and there is also one that handles a jump process [10].

However, we think that, as far as the present writers and the like know, there has been no report that handles power-law distribution focusing attention on rate-of-return deviation of a privately-owned company of equipment manufacturing business. Further, regarding a make-to-order production department (production-number based manufacturing system), in relation between rate-of-return deviation and a sales amount in the case of recent production departments, a model of rate-of-return deviation becomes Langevin type.

Next, if an amount of money of order entries and an amount of money of production are stochastic, accumulated excessive order entries are of Brownian motion, and a random "fluctuation" occurs in hour to hour order entries and production even though it might be of a small degree. However, in reality, when a "fluctuation" becomes large, force to adjust expected values of them will be added. For example, force to adjust expected values by suppressing an order entry volume, or by making a production amount increase (or decrease) transiently will be added.

It is reported that, in general, profit can be improved in a case where strategy to lead to an excessive production or an excessive order entry state is taken rather than a case where production is made to match the average order entry.

In addition, because, also in the case of the manufacturing business that is the subject of the present research, a rate of return is distributed log-normally, a cash flow of a target company proportional to a rate of return will be also distributed log-normally, naturally. With regard to equipment manufacturing, a small-to-midsize firm is required company president's guarantee for a borrowing inevitably. Therefore, whether a value of manufacturing equipment after a repayment of a loan varies relative to a repayment period or not is reported.

Also, how a value of manufacturing equipment after a repayment of a loan (hereinafter, referred to as a remaining value) varies relative to a repayment period is reported. Finally, a degree of influence (sensitivity) of parameters of an initial plan money amount, a repaid money amount and a repayment period on a remaining value, and also a risk analysis result is reported.

2. **Analysis Result and Discussion about Rate of Return and Its Deviation.** Production framework in equipment manufacturing business that is the subject of the present paper will be cited. A framework that we carry out is not a special framework, but is a manufacturing system called an "order entry type production framework by a production number management method".

Order entry type manufacturing system is a manufacturing system that starts necessary production activities after an order is received. In this system, "variation" is caused due to a delivery time and a lead time. In addition, due to occurrence of an idle in logistics or a lead time, "variation" also occurs in throughput. In addition, also due to an order-accepted product (equipment to be manufactured), "variation" of a lead time might be caused.

However, although a certain level of "variation" can be suppressed by effectively leveraging production forecast information related to order entries, it is difficult to suppress it completely. That is, in such company, a "variation" arises in cash flow for each month, and, naturally, a rate of return is also affected. In such order entry type manufacturing system, production management method which has been suited for a job-order production form in particular in which management is performed by, each time an order is accepted, assigning a production number (keyder) that is a key for management for an individual order entry is called a "production number management method", and it is widely used.

All production management is performed in a form that a production number is added, and directions for fabrication are issued for each production number.

In this way, for most of units to be built in an end product except for strategic stock parts, designing, logistics and ordering to suppliers is carried out for each order-received production number. Because of this, even when careful lead time management or fabrication start time management is performed, there is a case where "variation" in manufacturing (production) cannot be suppressed.

Such "variation" is a cause that a production factor or a production variable will be a random variable, and thus a cash flow (rate of return) in a company will also be a random variable.

2.1. **Rate-of-return deviation.**

2.1.1. *Power-law distribution characteristics of rate-of-return deviation.* Here, it is shown that data of rate-of-return deviation of a certain control equipment manufacturing company collected by us conforms to power-law distribution. About that company, we calculated the return of 10 years from Apr., 1999 to Mar., 2008 on a month-by-month basis to calculate rate-of-return deviation. The result is shown in Figure 1. Here, given that the return of nth month is S_n, a rate of return was defined by the following formula.

$$D_{n+1} = \frac{S_{n+1} - S_n}{S_n} \tag{1}$$

where S_n, $n = 1, 2, \ldots$, indicates monthly return, and D_n indicates a monthly rate of return.

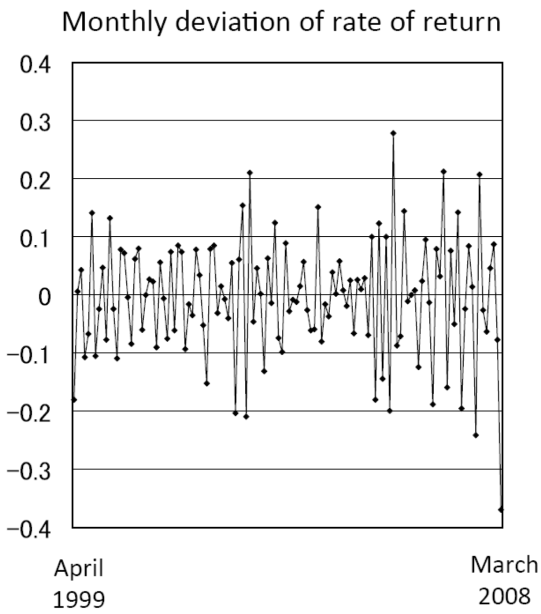

Figure 1. Rate-of-return deviation data

Using this rate of return, the following rate-of-return deviation is defined.

Definition 2.1. *Definition of rate-of-return deviation in Equation (2)*

$$\Delta D(n) \equiv D_{n+1} - D_n \tag{2}$$

Rate-of-return deviation ΔD can be considered as a random variable fluctuating on a monthly basis. As a result of examining distribution of an appearance frequency of this rate-of-return deviation ΔD, the probability density function was as shown in Figure 2 and the distribution was as shown in Figure 3.

Here, the distribution was obtained as follows. A range within which ΔD varies is divided into a plurality of zones, and let $n(\Delta D, \Delta D + \delta D)$ be the number of pieces of data included in a zone of width $\delta D[\Delta D, \Delta D + \delta D]$. Assuming that the number of pieces

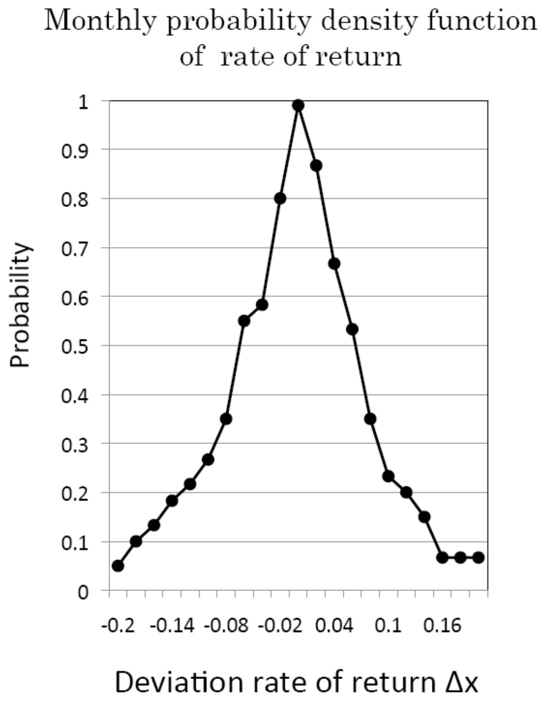

FIGURE 2. Probability density function of rate of return ΔD

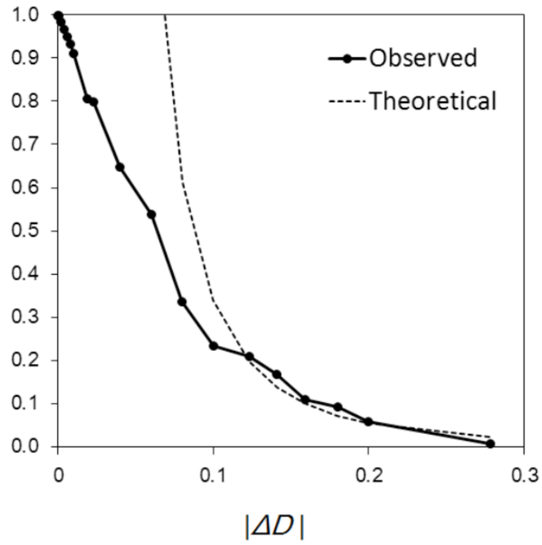

FIGURE 3. Cumulative distribution about ΔD

of data is M, the probability density function $P(\Delta D)$ can be defined by the following formula.

$$P(\Delta D) = \frac{n(\Delta D, \Delta D + \delta D)}{M \cdot \delta D} \tag{3}$$

In this paper, distribution $P(> \Delta D)$ is defined by the following formula.

$$P(> \Delta D) = \frac{N(\Delta D)}{M} = \int_{\Delta D}^{\infty} d\Delta D P(\Delta D) \tag{4}$$

where $N(\Delta D)$ is the rank order of ΔD. In stock price fluctuation models and the like, it is known that a skirt of distribution of this cumulative distribution $P(> |\Delta D|)$ follows the following power-law distribution [7, 8].

$$P(>|\Delta D|) \propto |\Delta D|^{-\beta} \tag{5}$$

Therefore, as a result of performing fitting to the formula of power-law distribution using the observed data of $|\Delta D| > 0.1$ corresponding to a skirt of distribution by MS-Excel,

$$P(>|\Delta D|) = 0.0008|\Delta D|^{-2.63} \tag{6}$$

was obtained (Figure 3, theoretical, $R^2 = 0.926$). As above, a "fluctuation model of rate-of-return deviation" is of self-similarity, and it will show fractal nature [8, 11]. Also, this power-law distribution characteristic has "fluctuation" nature in phase transition [8, 11].

Generally, in phase transition, phase transition from [A] phase to [B] phase occurs taking a balance point as a critical point (Figure 4). In the case of markets prices, a point at which supply and demand balance, that is, a border point between two different states of the excess demand phase and the excess supply phase, is a phase transition point [11]. When this theory of markets prices is applied to the present research, a model of fluctuation of rate-of-return deviation becomes as follows [11].

$$\Delta D(t + \Delta t) = c(t)\Delta D(t) + f(t) \tag{7}$$

where $\Delta D(t)$ is rate-of-return deviation, $c(t)$ is a parameter, $f(t)$ is random external force.

Because cumulative distribution function of the graph of rate-of-return deviation (Figure 1) indicates power-law characteristics (Figure 3), it can be thought that there is a possibility of phase transition in the vicinity of equilibrium point. This suggests a possibility of phase transition in the production departments of this company, and in the equipment manufacturing departments, in particular.

In equipment manufacturing departments, in particular in make-to-order manufacturing departments, a production risk exists between an order entry volume and a production volume, and this will cause fluctuation between equipment production processes and affect their throughputs and eventually the overall throughput.

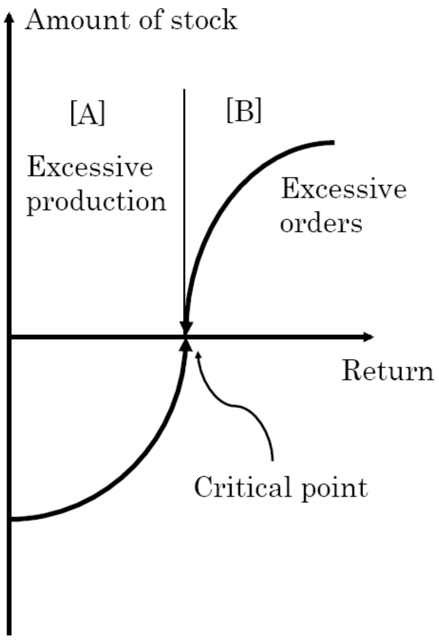

Figure 4. Critical point between excessive production and excessive order entries

Regarding Formula (7), the following holds [12].

$$\langle c(t)^{\beta} \rangle > 1 \tag{8}$$

where β is a positive constant. In Formula (7), subtract $\Delta D(t)$ from both sides, and consider the limit of $\Delta t \to 0$. On this occasion, the following formula is obtained [12].

$$\frac{d\Delta D(t)}{dt} = -r\Delta D(t) + \tilde{f}(t) \tag{9}$$

where r is a viscosity coefficient, and $\tilde{f}(t)$ is random external force and satisfies the following formula.

$$r = \lim_{\Delta t \to 0} \frac{1 - c(t)}{\Delta t} \tag{10}$$

$$\tilde{f}(t) = \lim_{\Delta t \to 0} \frac{f(t)}{\Delta t} \tag{11}$$

Here, Formula (8) includes the following condition.

$$\langle c(t)^2 \rangle > 1 \tag{12}$$

Formula (12) indicates that $c(t) > 1$ holds at some degree of probability, and represents that, if $t \to 0$ in Formula (10), viscosity coefficient r becomes negative. In a situation in which a viscosity coefficient becomes negative, there occurs an unstable state where order entries are amplified increasingly, and thus fluctuation of order entries is also amplified and it will deviate from the normal distribution. That is, in Formula (9) that satisfies the condition Formula (12), because viscosity coefficient can stochastically become not only positive but also negative, fluctuation is amplified.

Figure 2 indicates the probability density function value of rate-of-return deviation, and Figure 3 indicates the cumulative distribution of the absolute values of rate-of-return deviation and shows power-law characteristics. Here, $f(t)$ is random force, and if it is indicated as $\Delta D(t) \equiv h(t)$, the following formula is obtained.

$$\frac{dh(t)}{dt} = -r(t)h(t) + \tilde{f}(t) \tag{13}$$

As mentioned above, in a make-to-order manufacturing department (production number based manufacturing system), regarding relation between rate-of-return deviation and a sales amount, a model of rate-of-return deviation becomes Langevin type in a case of recent production departments [13].

2.1.2. *Sensitivity of order entries against rate-of-return deviation.* Here, an analysis result of mathematical models with respect to rate-of-return deviation is described. First, a result of mathematical modeling of this control equipment manufacturing company is described. Then, description is made with respect to sensitivity of order entries to return, and, in addition, based on Formula (13), a situation where phase transition between an order entry phase and a production phase occurs is analyzed, and relation between an amount of money of order entries and rate-of-return deviation will be cited.

In Figure 5(A), a product manufacturing maker that is the orderer of equipment to be manufactured provides an ordering price considering the market. Now, let this ordering price $V(t)$ be defined by the following formula.

Definition 2.2. *Definition of ordering price* $V(t)$

$$dV(t) = \gamma(t)dW(t) \tag{14}$$

where let $\gamma(t)$ be called a price variability coefficient. It is known nonnegative deterministic function which is a function that is sufficiently smooth and possible to be differentiated. $W(t)$ represents a standard Brownian motion.

Referring to Figure 5(B), in a customer development department (customer's factory), assuming that customer's amount of money of orders entries through sales contracts based on transaction strategy is $X(t)$, $X(t)$ is defined by the following formula.

Definition 2.3. *Definition of customer's amount of money of order entries $X(t)$*

$$dX(t) = \beta(t)\left[V(t) - \hat{P}(t)\right]dt \tag{15}$$

where let $\beta(t)$ be called a strategy factor in customer's market development department. $\left\{\hat{P}(t) : t \geq 0\right\}$ means that a factory price in the customer's factory.

In Figure 5(C), $\hat{P}(t)$ is an ordered amount to the factory within the customer, and order reception will be made with this price. Let $\hat{P}(t)$ be defined by the following formula.

Definition 2.4. *Definition of ordered amount $\hat{P}(t)$ within a customer*

$$d\hat{P}(t) = \lambda_P(t)dY(t) \tag{16}$$

where λ_P is a coefficient of variance of an ordered amount, and $Y(t) = X^Y(t) + Z(t)$, and $Z(t)$ is a strategic amount of money of production ($dZ(t) = \sigma_Z(t)dU(t)$), and $X^Y(t)$ is an amount of money of production to which a production number is assigned. Further, $\sigma_Z(t)$ represents standard deviation of $Z(t)$, and $U(t)$ a standard Brownian motion.

In Figure 5(D), assuming that order entry price for a production number assigned from a customer's factory is $\hat{X}(t)$, let accounting sales amount $T(t)$ due to throughput for each month be defined by the following formula.

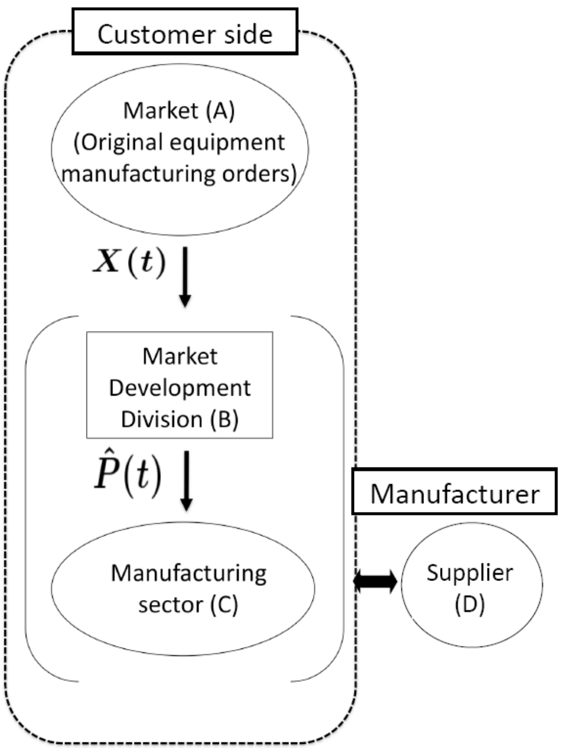

Figure 5. Business association chart of company of research target

Definition 2.5. *Definition of accounting sales amount $T(t)$*

$$dT(t) = \xi(t)dB(t)^{\forall}B(t) = \hat{X}(t) + Q(t) \tag{17}$$

where $Q(t)$ is an expected money amount of production for each month. Also, both $\hat{X}(t)$ and $Q(t)$ will be stochastic due to exogenous and endogenous causes, and are defined by the following formulas.

Definition 2.6. *Definition of throughput for each month $\hat{X}(t)$*

$$d\hat{X}(t) = \sigma_X(t)dW^X(t) \tag{18}$$

where $\sigma_X(t)$ represents standard deviation of an amount of money of order entries, and $W^X(t)$ standard Brownian motion.

Definition 2.7. *Definition of expected production money amount for each month $Q(t)$*

$$dQ(t) = \sigma_Q(t)dW^Q(t) \tag{19}$$

where $\sigma_Q(t)$ indicates standard deviation of an expected production money amount for each month, and $W^Q(t)$ standard Brownian motion.

As above, even if in the case of make-to-order production system, due to exogenous and endogenous causes, it is inevitable for production to be stochastic.

At that time, manufacturing cost of supplier that is the research target is generally indicated as follows.

$$S(t) = S_1(t) + S_2(t) + S_3(B(t), t) \tag{20}$$

where $S_1(t)$ represents cost of raw materials, $S_2(t)$ indirect cost of logistics cost, $B(t)$ constant cost and variable cost, $S_3(B(t), t)$ direct cost related to manufacturing such as constant cost and variable cost.

Generally, power-law distribution phenomenon shares similarity with nature of “fluctuation” observed in phase transition in the matter system dealt with physics.

Accordingly, by stochastic characteristics of rate-of-return deviation becoming power-law distribution, it has the above described nature. That is, it can be said that stability of a rate of return is to remain at “a stationary point of fluctuation” (a state where all processes are synchronizing). To achieve this, it is necessary to control and manage a cause of the instability such as excessive inventory and an opportunity loss.

Also needed is to, in order to achieve optimization of the entire system in a company, understand “manufacturing lead time”, “throughput”, “bottleneck”, “synchronization of systems” and the like deeply and perform operation through measurement.

As described above, assuming that “stationary point of fluctuation” is defined as a state in which all processes are synchronizing, it is considered that staying at this point for a long time leads to stabilization of a system [11].

For this reason, the entire system will be optimized by: finding a bottleneck inherent in a system in order to achieve synchronization of processes; making everything be synchronized there; in addition, improving throughput of the bottleneck; and iterating these [11, 13, 14].

Next, how sensitivity of order entries makes an impact on rate-of-return deviation will be described.

Definition 2.8. *Let $C(t)$ be defined as “sensitivity of order entries to rate-of-return deviation”.*

$$C(t) = \frac{d\,[N(t+\Delta t)/N(t)]}{d[h(t+\Delta t)/h(t)]} \times \frac{[h(t+\Delta t)/h(t)]}{[N(t+\Delta t)/N(t)]} \tag{21}$$

where $h(t)$ is rate-of-return deviation, $N(t)$ is an order entry volume. Let fluctuation of rate of return be defined as random fluctuation of external force and order entries.

Definition 2.9. *Definition of fluctuation of rate-of-return deviation $dh(t)$*

$$\begin{aligned} dh(t) &= d\left(\hat{f}(t), N(t)\right) = \hat{f}(t) \cdot dN(t) + N(t) \cdot d\hat{f}(t) \\ &= \frac{h(t)}{N(t)} \cdot dN(t) + N(t) \cdot d\hat{f}(t) \end{aligned} \tag{22}$$

where let $\hat{f}(t)$ be a return acquisition rate, and it satisfy Formula (23).

Definition 2.10. *Definition of return acquisition rate*

$$\hat{f}(t) = \frac{h(t)}{N(t)} \tag{23}$$

Here, we convert Formula (23) as follows. That is, we analyze, by introducing a return acquisition rate, oppression of rate-of-return deviation due to an increased amount of money of order entries.

$$N(t) \cdot d\hat{f}(t) \equiv dh(t) \left\{ 1 - \frac{dN(t)/N(t)}{dh(t)/h(t)} \right\} \tag{24}$$

From Formula (24), the following formula is obtained.

$$dh(t) = \frac{h(t)}{N(t)} \cdot dN(t) + dh(t) \left\{ 1 - \frac{dN(t)/N(t)}{dh(t)/h(t)} \right\} \tag{25}$$

where $dN(t)/N(t)$ denotes a rate of order entries, $dh(t)/h(t)$ a rate of fluctuation of rate-of-return deviation. Here, let $e_{ng} = \frac{dN(t)/N(t)}{dh(t)/h(t)}$.

Now, if $e_{ng} = 1$, the followings hold,

$$dN(t)/N(t) = \frac{dh(t)}{h(t)} \tag{26}$$

$$dh(t) = \frac{h(t)}{N(t)} \cdot dN(t) \tag{27}$$

In other words, if $e_{ng} = 1$, it denotes that an amount of money of order entries and a fluctuation of rate-of-return deviation are viewed as one. Further, given that $e_{ng} = 0$, $h(t)dN(t)$=0 (no fluctuation of order entries) holds, and assuming that $h(t) > 0$, $dN(t) = 0$, which denotes that a money amount of sales will be constant, holds.

Next, when fluctuation of rate-of-return deviation is indicated as follows, we obtain the following formula.

$$dh(t) = \hat{f}(t)dN(t) - kdh(t), \quad k > 0 \tag{28}$$

When Formula (28) is transformed, we obtain the following formulas.

$$(1 + k)dh(t) = f(t)dN(t), \quad e_{ng} > 1 \tag{29}$$

$$dh(t) = \frac{\hat{f}(t)}{1 + k} dN(t) \tag{30}$$

Also, from $0 \leq k < 1$, the following formula is obtained.

$$\frac{\hat{f}(t)}{1 + k} < \hat{f}(t) \tag{31}$$

Further, from Formula (30), the following formula holds.

$$dh(t) < \hat{f}(t) \cdot dN(t) \tag{32}$$

In other words, from Formula (23) and Formula (32), the following formula is obtained.

$$\frac{dN(t)}{N(t)} > \frac{dh(t)}{h(t)} \tag{33}$$

Formula (33) means that a rate of fluctuation of an amount of money of order entries becomes larger than a rate of fluctuation of rate-of-return deviation, resulting in oppression of rate-of-return deviation (opportunity loss) due to a risk of an amount of money of order entries. Consequently, the following formula holds.

$$\frac{d[N(t+\Delta t)/N(t)]}{[N(t+\Delta t)/N_s(t)]} > \frac{d[h(t+\Delta t)/h(t)]}{[h(t+\Delta t)/h(t)]} \tag{34}$$

That is, from Formula (21) and Formula (34), $C(t) > 1$ is obtained. Consequently, $h(t)$ follows power-law distribution [11].

Therefore, a model of rate-of-return deviation of the relevant company can be prescribed as Formula (13). Now, if $N(0) = 0$, $D(0) = 0$ and $t = T$, accumulated excessive order entries $I(T)$ $(t = T)$ is indicated by the following formula.

$$I(T) = \sum_{t=0}^{T} \{N(t) - D(t)\} \tag{35}$$

where $N(t)$ is an amount of money of order entries, $D(t)$ an amount of money of production, and it is assumed that a price of a product is fixed. Consequently, the followings are understandable.

(1) Excessive order entries phase: an expected value of $\langle N(t)\rangle > \langle D(t)\rangle \to I(T)$ increases monotonically.

(2) Excessive production phase: an expected value of $\langle N(t)\rangle < \langle D(t)\rangle \to I(T)$ decreases monotonically.

(3) Critical point: an expected value of $\langle N(t)\rangle = \langle D(t)\rangle \to I(T)$ is zero.

In the excessive order entries phase, because an order entry volume exceeds a production amount averagely, if this goes on, a cumulative order entry volume keeps increasing. That is, in actual equipment manufacturing business, this state is not stabile, and although it seems as if, in usual economic activities, an effect such as rise of a price or increase of a production amount to a level that accumulated excessive order entries is suppressed works, it is impossible in reality because a physical upper limit of production works (opportunity loss). In excessive production phase, an opposite phenomenon occurs (excessive inventory). In addition, at the critical point, expected values of order entries and production become equal, and, when only the expected values are seen, it seems as if there is equilibrium. However, in reality, looking at "fluctuation" around such expected value, it is unstable state.

That is, considering that amount of money of order entries $N(t)$ and amount of money of production $D(t)$ are stochastic, accumulated excessive order entries $I(T)$ will be of Brownian motion [11]. In other words, when hour to hour order entries and production causes random "fluctuation" even though it might be of a small degree, amount of money of order entries $N(t)$ or, amount of money of production $D(t)$ is accumulated in the critical point and amplified even to infinity.

However, in reality, the larger the "fluctuation" becomes, the more force to make these expected values be adjusted will work. For example, an order entry volume will be suppressed, or, production amount will be transiently increased (or decreased).

Generally, due to occurrence of random "fluctuation" inherent in order entry volumes, a solution to realize a equilibrium point between "order entries and production" that is rational cannot be obtained. There is no way to avoid this under the free economy. When considering on the premise of "fluctuation" of order entries, it is better not to consider making production match the average order entry. In other words, we consider that profit can be increased by employing strategy which purposefully leads to excessive production or excessive order entries rather than adopting the matching strategy.

2.2. **Rate of return.**

2.2.1. *Log-normal distribution characteristics of rate of return.* For a small-to-midsize firm, it is of the upmost importance not to cause default in a cash flow, and it is necessary for business continuity. As is the case with rate-of-return deviation described in the previous half, we also analyzed a return acquisition rate defined by Formula (23). The result is shown in Figure 6.

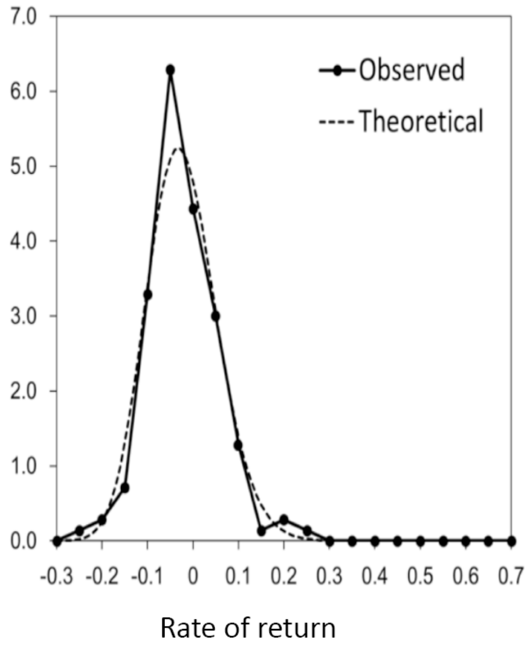

FIGURE 6. Probability density function of rate-of-return deviation: actual data (solid line) and data based on theoretical formula (dotted line)

From the data of monthly rate of return observed, its probability density function was calculated (Figure 6). As a result, it was found that the probability density function conforms to log-normal distribution (Figure 6, theoretical).

Theoretical curve was calculated using EasyFit software (http://www.mathwave.com/), and as a result of Kolmogorov and Smirnov test, the observed values conformed to a log-normal type probability density function. Because, in the goodness-of-fit test of Kolmogorov-Smirnov, a null hypothesis that it is "log-normal" was not rejected with rejection rate 0.2, this data conforms to "log-normal" distribution. P-value was 0.588. The parameters of a theoretical curve were: $\mu_p = -0.134$ (average), $\sigma_p = 0.0873$ (standard deviation), $\gamma_p = -0.900$. The theoretical curve is given by the following formula.

$$f(x) = \frac{1}{\sqrt{2\pi}(x-\gamma_p)\sigma_p} \exp\left\{-\frac{1}{2}\left(\frac{(\ln x - \gamma_p) - \mu}{\sigma_p}\right)^2\right\} \tag{36}$$

2.2.2. *Cash flow evaluation.* In a small-to-midsize firm, because it does not have ample working capital for the company, in order to continue the company operation by any means, it needs to raise working capital from financial institutions. Let this be called a cash flow. In essence, rate of return is at least proportional to a manufacturing cost.

In other words, if a rate of return forms log-normal distribution, it can be said that it is realistic to assume that a cash flow will be also the same log-normal distribution. Therefore, a cash flow model is defined as follows.

Definition 2.11. *Definition of a cash flow model*

$$\frac{dQ(t)}{Q(t)} = \mu dt + \sigma dW^Q(t) \tag{37}$$

where the left-hand side is a monthly rate of return, and a rate of return varies with expected value μ_s and $\sigma^2 t$. Further, σ^2 represents variance, and $W^Q(t)$ standard Brownian motion.

Formulation of repayment guarantee money will be described.

Now, repayment guarantee money for a loan from financial institutions can be defined by the following formula.

Definition 2.12. *Definition of repayment guarantee money for a loan*

$$H = E\left[\max(C - Q(t), 0) \cdot \frac{1}{(1+r)^i}\right] \tag{38}$$

Here, because $1/(1+r)^i$ of Formula (38) means that it is a case where equipment manufacturing is performed within a year, $E[\cdot]$ is an expected value under a risk neutral probability. This expense H can be represented as European Put Option. In other words, regarding repayment guarantee expense, from $Black \cdot Scholes$ model, an appraisal value of guarantee at the time of each repayment can be indicated as follows [15].

$$H = E\left[\max(L - Q(t), 0) \cdot \frac{1}{(1+r)^i}\right] = \frac{L}{(1+r)^i} \cdot N\left(-z + \sigma\sqrt{T-t}\right) - Q_0 \cdot N(-z) \tag{39}$$

where Q_0 is an initial plan expense that is considered to be needed at the time of manufacturing, $T - t = 1$ as it is of single-year, and z is indicated by the following formula.

$$z = \frac{\ln(Q_0/L) + (r + (1/2)\sigma^2(T-t)}{\sigma\sqrt{(T-t)}} \tag{40}$$

Also, $N(\cdot)$ indicates a probability value of standard normal distribution, and is indicated by the following formula.

$$N(z) = \int_{-\infty}^{z} \exp\left(-\frac{1}{2}x^2\right) dx \tag{41}$$

An appraisal value of equipment manufacturing will be described. If financial institutions guarantee repayment L of a small-to-midsize firm with an equipment manufacturing period, the company can avoid a default risk completely if it pays H. At that time, because all money is paid back to the debt guarantor of a company (the company president in the case of a small-to-midsize firm), the appraisal value of such credit obligation is equal to the amount of the loan. On the other hand, because asset value J of manufacturing equipment is a remaining value after subtracting a repaid money amount from a cash flow, it can be represented by the following Formula [15].

$$J = E\left[\max(Q(t) - L, 0) \cdot \frac{1}{(1+r)^i}\right] = Q_0 \cdot N(z) - \frac{L}{(1+r)^i} \cdot N\left(z - \sigma\sqrt{T-t}\right) \tag{42}$$

Numerical simulation will be described.

Figure 7 is results of simulation based on Formula (42). Type 1 to Type 4 in Table 1, representing the possible values when changing the parameters. Referring to Figure 7, Type 1 is a setting that is not problematic to a manufacturing plan at all. In Type 2, although a repaid money amount was set high, because a cash flow initial value is not low, a remaining value becomes high if repayment is performed early. In Type 3, even when variance had been made lowered, no effect was observed. In Type 4, if a cash flow initial value is low, as a matter of course, due to delayed repayment date, a remaining value becomes low.

Figure 8 is a diagram in which Formula (39) is graphed. Type 1 is an example in which a repayment guaranteed amount is low, a risk is relatively low, and profit is not oppressed. Type 2 is an example in which a repaid money amount is set high, and naturally, a repaid money amount is inversely proportional to a repayment period. It can be said that a risk is high. Type 3 is an example where there is a little variation in a cash flow, and because a repayment guaranteed amount is not high, it is an example of the lowest risk. Type 4 has the same tendency with Type 2. However, because a cash flow initial value is the lowest, and thus a repayment guaranteed amount is inversely proportional to repayment date, it can be said that a risk is high.

Sensitivity analysis and risk analysis of remaining value J is described.

Figure 9 and Figure 11 show results of utilizing a simulation tool DECISION SHARE (http=//www.integratto.co.jp/) for quantitative evaluation.

Figure 9 is referred to as a "tornado chart", and on the longitudinal axis, indication is made in order of a degree of influence from highest to lowest starting from the upper side. Horizontal axis shows a value that J, which is a target value, can take. This

Table 1. Set parameter values

	Type 1	Type 2	Type 3	Type 4
Repayment	2.5	5	2.5	2.5
Initial value of cash flow	3	3	3	1
Standard deviation (σ)	0.8	0.8	0.3	0.8
Risk-free rate	0.2	0.2	0.2	0.2

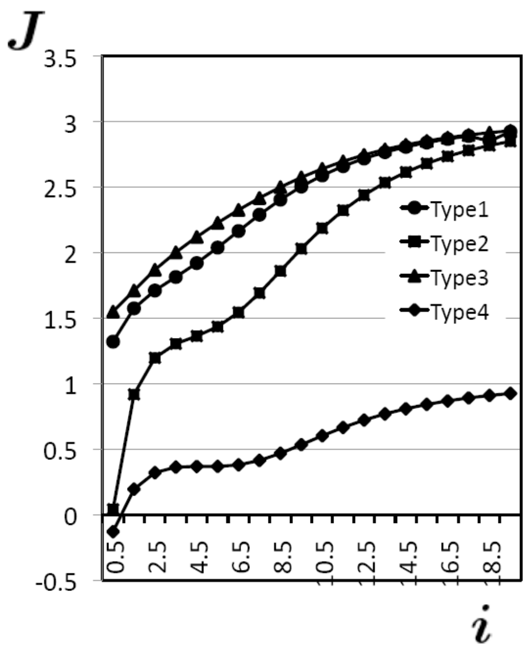

Figure 7. Remaining value obtained by subtracting repaid money from cash flow

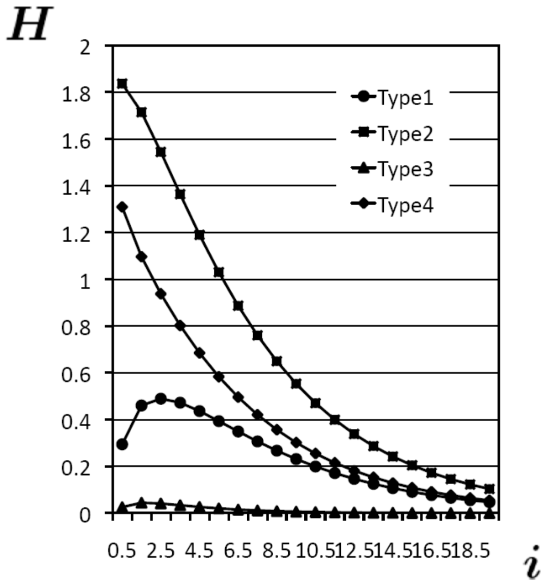

Figure 8. Appraisal value of guarantee at the time of repayment

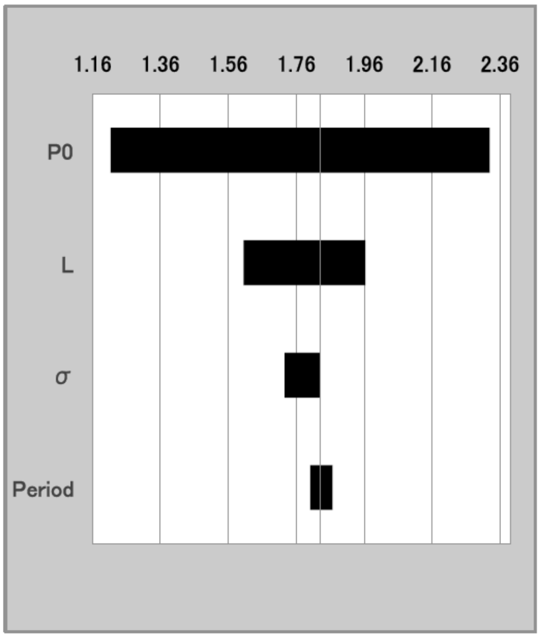

Figure 9. Sensitivity of parameters to target value J (tornado chart)

indicates that, when J is set to a target value, sensitivity of planned value P_0, repaid money amount L, standard deviation of cash flow σ and a repayment period to the target value. Figure 10 is a diagram referred to as a "spider chart", and its longitudinal axis indicates a range that the three parameters can take, and the horizontal axis indicates a value that J can take, and in essence, it represents the same things as Figure 9. The most influential parameter is a planned value, the next is a repaid money amount followed by standard deviation, and, finally, a repayment period. Figure 11 indicates that, when target value $J \approx 2.0$, a risk is about 60. In Figure 11, although simulation was performed with a reference value 3.0, it can be found that this target value itself has a high risk. As a target value recommended by the simulation, in the case of the parameter values set here, $J \approx 2.0$ is recommended. About whether this has a high feasibility as an execution

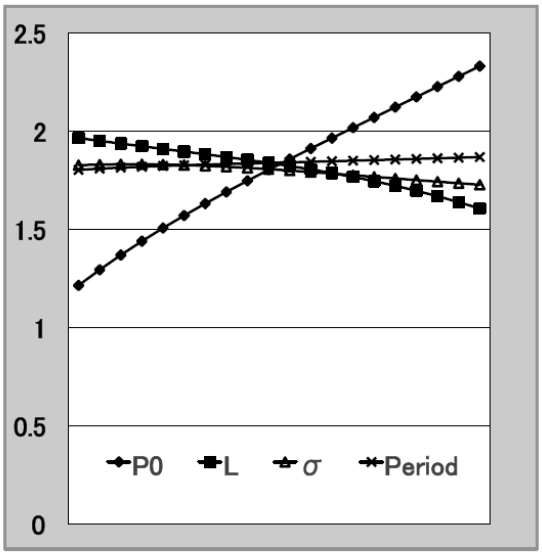

FIGURE 10. Sensitivity of parameters to target value J (spider chart)

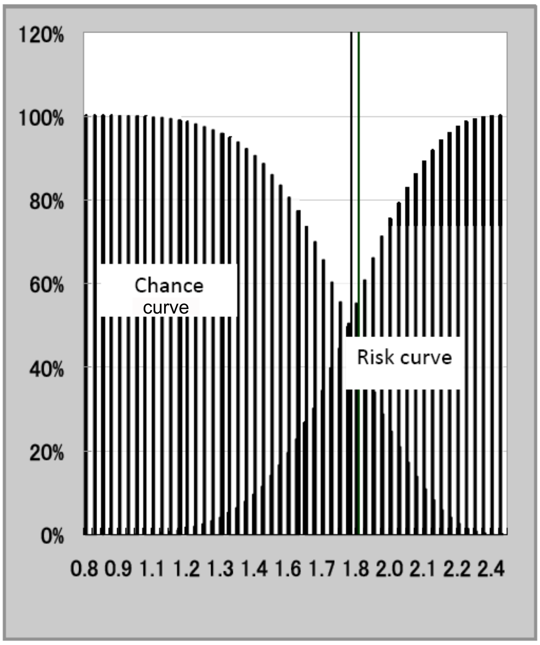

FIGURE 11. Measurement of risk when each parameter is made to vary relative to target value J

plan or not needs to be reviewed once again in the research project. When promoting equipment manufacturing, it will be beneficial information that an uncertainty element has been made clear as above.

Figure 12 is a diagram obtained by making initial plan money amount (P_0), repayment (L), standard deviation of a cash flow (σ) and repayment period ($Period$), which have been adopted as a parameter, vary with the probability distribution. The ranges are $P_0 = 2.4 \to 3.6$, $L = 1.6 \to 2.4$, $\sigma = 0.4 \to 0.9$, $Period = 0.1 \to 0.4$.

3. **Conclusion.** Due to multiple causes that are intricately intertwined with each other, return obtained from a manufacturing process is complicated. From the standpoint of control equipment manufacturing business, the followings are conceivable as this cause.

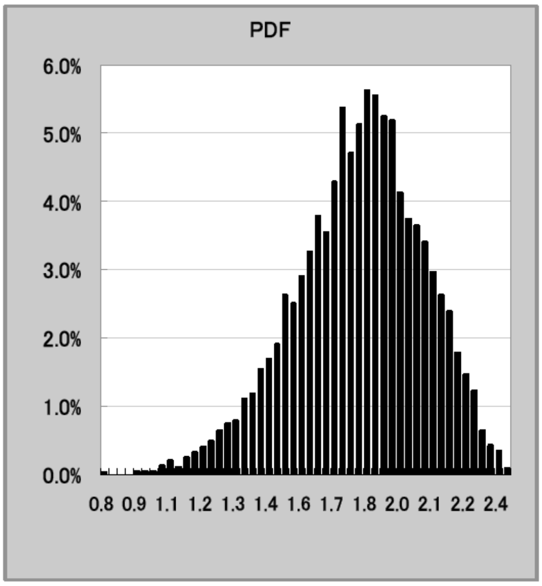

Figure 12. Probability distribution chart when each parameter is made to vary randomly within setting range

That is, transaction prices with customers, impact of the exchange market, variation due to a delivery term, market trends of a specific business community, variation of a processing time of each process within a factory and the like.

We were able to confirm, also in some control equipment manufacturing company, power-law characteristics that are shown in such as stock prices, currency exchanges and other financial prices. Further, we have confirmed that, by expressing a cash flow model by a log-normal stochastic differential equation, a lot of fruits of research carried out in mathematical finance can be also used for evaluation of a manufacturing company, and they are useful as evaluation tools of a business plan.

Company circumstances are becoming increasingly complex and globalized. When considering the financial environments, it is considered that the present research will make a significant contribution also to cash flow management of a small-to-midsize firm.

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