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ADAPTIVE FUZZY SLIDING-MODE CONTROL SYSTEM DESIGN FOR BRUSHLESS DC MOTORS

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Abstract. *This paper aims to propose a simple adaptive fuzzy control system to achieve precise trajectory tracking control for brushless DC (BLDC) motors even when the input commands and their frequencies are changed. Moreover, the developed control system is implemented in a field programmable gate array (FPGA) board for the on-line real time control. The precise control of BLDC motors becomes very important because they are applied widely nowadays. Moreover, since the BLDC motors are nonlinear systems, the effective control algorithm is essential. An adaptive fuzzy sliding-mode control (AFSMC) system is proposed. This control system is composed of a fuzzy sliding-mode controller which is utilized to approximate an ideal controller, and a compensator which is used to eliminate the approximation error and guarantee the stability of the system. To eliminate the approximation error properly, two kinds of compensators are designed which are bound estimation compensator and fuzzy compensator. Consequently, satisfactory tracking performance can be achieved and the system's stability can be guaranteed. Then, an FPGA board is used to implement the control system for the on-line real time control. From the practical experimental results, it is shown favorable tracking performance can be achieved by the proposed adaptive fuzzy sliding-mode control system even when the input commands and their frequencies are changed. Moreover, it is also shown the proposed control scheme can achieve better control performance than other control methods.*
Keywords: Fuzzy control, Sliding-mode control, Brushless DC motor, FPGA

1. **Introduction.** Fuzzy logic control has become a powerful tool in control engineering, especially for systems that are structurally difficult in model due to naturally existing nonlinearities and other modeling complexities [1]. Moreover, by equipping with adaptive law to tune the parameters of fuzzy systems, a lot of adaptive fuzzy control systems have been proposed [2]. Sliding-mode control (SMC) is a powerful control system and famous for its capability to eliminate the uncertainty and noise [3]. According to the definition of sliding surface, tracking error of control system can approach to zero along the surface. In conventional design, an equivalent controller containing system dynamic information is designed to force system to maintain on the sliding surface. If the system dynamic equations are unknown, it will become difficult to design the controller [4,5]. In order to combine the model free advantage of fuzzy control and uncertainty removable advantage of SMC, some fuzzy sliding-mode controllers have been introduced by replacing equivalent controller with fuzzy controller [6,7]. However, there is a drawback that fuzzy controller lacks a systematic design technique. To ensure the performance and stability of

control systems, adaptive fuzzy sliding-mode controller equipped with training algorithm has been proposed by some researchers [8,9]. These adaptive systems are designed to approximate an ideal controller with guaranteed system's stability. In the adaptive fuzzy control system design, adaptive parameter tuning laws are utilized to modify the control parameters. Moreover, the stability of closed-loop control system can be proved in the sense of Lyapunov function. In addition, the adaptive fuzzy controller can be classified into the first type and the second type [2]. The first type adaptive fuzzy controller is based on a linear fuzzy logic system which has fixed membership functions, so the adjustable parameters are limited to be linear; whereas the second type adaptive fuzzy controller is based on a nonlinear fuzzy logic system which has adjustable membership functions, so its optimal fuzzy logic system can fit into nonlinear system better than the first type.

The adaptive systems described in previous paragraph utilize the fuzzy system and fuzzy sliding-mode system to approximate the ideal controller. Since the membership functions of the fuzzy system and fuzzy sliding-mode system are finite, the approximation error is inevitable when they are used to approximate the ideal controller. To eliminate this approximation error, a compensator plays an important role in the adaptive fuzzy systems. Firstly, the commonly used compensator is a sliding-mode type controller, but this kind of controller requires the information of approximation error bound. Because the approximation error is usually unknown, the bound of approximation error needs to be estimated. However, if the estimated bound is too small, the compensator cannot eliminate the approximation error well; on the otherhand, if the estimated bound is too large, it will result in control chattering.

Brushless DC (BLDC) motor utilizes an electronically controlled commutation system to replace mechanical commutation system which is used in conventional brush motors, and it has the advantages such as simple to construct, high torque capability, small inertia, low noise and long life operation [10]. Because of these merits, BLDC motor has been applied on home appliances, electrical vehicles, advanced manufacturing system and consumer appliances [11]. The difficulty in BLDC motor control is that it is a nonlinear system. The internal parameters of BLDC motor would change slightly with different input commands and environments; thus the design of controller becomes complicated. Recently, some researchers have brought up various control approaches to tackle this difficulty [12-14]. Liu et al. proposed a PI controller to achieve instantaneous torque control with reduced torque ripple [12]. However, the selection of PI gains is a trade-off between robustness and fast transient response. Rubaai et al. proposed an adaptive fuzzy-neural-network controller to achieve satisfactory tracking performance [13], but it suffered the heavy computational loading. Rubaai et al. also proposed a robust adaptive fuzzy controller [14]. Although the experimental results show that robust tracking performance can be achieved, the control effort may lead to a large control signal as the specified robustness is increased. In this paper, an adaptive fuzzy sliding-mode control system is proposed to achieve the trajectory tracking control of BLDC motors.

In recent years, practical applications are implemented under the requirement of small scale. Thus, the PC-based system becomes unsuitable for many applications. To satisfy the requirement, field programmable gate array (FPGA) chip has been utilized widely with its advantages of low cost, high speed and small volume.

In this study, an adaptive fuzzy sliding-mode control (AFSMC) system is proposed to control a BLDC motor. The AFSMC system is associated with different compensators; one is a bound estimation compensator and another one is a fuzzy compensator. The control of BLDC motor is practically implemented by a FPGA experimental board. Finally, different command inputs with different frequencies are tested to illustrate the effectiveness of the proposed design methods.

2. **Problem Formulation.** The system equations of a BLDC motor driver in a d-q model can be expressed as [10,12]

$$\dot{i}_{qs} = -\frac{R_s}{L_q} i_{qs} - \frac{L_d}{L_q} \omega_r i_{ds} + \frac{1}{L_q} V_{qs} - \frac{\lambda_m}{L_q} \omega_r \tag{1}$$

$$\dot{i}_{ds} = -\frac{R_s}{L_d} i_{ds} + \frac{L_q}{L_d} \omega_r i_{qs} + \frac{1}{L_d} V_{ds} \tag{2}$$

$$L_q = L_{is} + L_{mq} \tag{3}$$

$$L_d = L_{is} + L_{md} \tag{4}$$

where i_{ds} and i_{qs} represent the d and q axes stator currents, respectively; v_{ds} and v_{qs} are the d and q axes stator voltage, respectively; L_d and L_q are the d and q axes stator inductances, respectively; R_s is the stator resistance; L_{is} is the stator leakage inductance; L_{md} and L_{mq} are the d and q axes magnetizing inductances, respectively; ω_r is the electrical rotor angular velocity; and λ_m is the flux linkage of permanent magnet. The torque equation is expressed as [13]

$$J\frac{2}{P_m}\dot{\omega}_r + B\frac{2}{P_m}\omega_r = T_e - T_L \tag{5}$$

where P_m is the number of poles; J is the inertia of the rotor; B is the damping coefficient; T_e is the electromagnetic torque and T_L is the load disturbance torque. By using the field-oriented control, the electromagnetic torque of BLDC motor driver can be expressed as

$$T_e = \frac{3}{2}\frac{P_m}{2}\lambda_m i_{qs} = k_t i_{qs} \tag{6}$$

where $k_t = \frac{3}{2}\frac{P_m}{2}\lambda_m$ is the constant gain. From (5) and (6), the system dynamic equation can be obtained as

$$\ddot{\theta}_m = f_m \dot{\theta}_m + g_m u + d \tag{7}$$

where θ_m is the rotor position; $f_m = -\frac{B}{J}$; $d = -\frac{P_m}{2J}T_L$; $g_m = \frac{P_m}{2}\frac{k_t}{J}$ is a positive constant gain; and $u = i_{qs}$ is the control effort. To control the position of motor, the tracking error can be defined as

$$e = \theta_c - \theta_m \tag{8}$$

where θ_c is the rotor position command. If the system parameters in (7) are assumed to be known, there exists an ideal controller [15]

$$u^* = g_m^{-1}(-f_m\dot{\theta}_m - d + \ddot{\theta}_c + k_{m1}\dot{e} + k_{m2}e) \tag{9}$$

where k_{m1} and k_{m2} are nonzero positive constants. Applying this ideal controller (9) into system dynamic Equation (7), yields

$$\ddot{e} + k_{m1}\dot{e} + k_{m2}e = 0 \tag{10}$$

If the parameters k_{m1} and k_{m2} are selected to satisfy that all the roots of (10) lie on the left half side of s-plane, it can be derived that $\lim_{t\to\infty} e = 0$. Unfortunately, because the precise system parameters in (7) cannot be exactly known, the ideal controller in (9) is unobtainable.

3. **Adaptive Fuzzy Sliding-Mode Control System Design.** In this section, an adaptive fuzzy sliding-mode controller is proposed to approximate the ideal controller in (9).

3.1. **Fuzzy sliding-mode controller design.** The output of a j-rules fuzzy control system is defined as [2]

$$u_{fc}(\boldsymbol{z}) = \sum_{i=1}^{j} \boldsymbol{\beta}_i \Theta_i(\boldsymbol{\sigma}_i, (\boldsymbol{z} - \boldsymbol{m}_i)) \tag{11}$$

where $\boldsymbol{z} = [z_1 \; z_2 \; \ldots \; z_n \;]^T$ is the input vector, $\Theta_i(\boldsymbol{\sigma}_i, (\boldsymbol{z} - \boldsymbol{m}_i))$, $i = 1, 2, \ldots, j$ are the Gaussian functions, $\boldsymbol{\sigma}_i = [\sigma_{1i} \; \sigma_{2i} \; \ldots \; \sigma_{ni} \;]^T$ and $\boldsymbol{m}_i = [m_{1i} \; m_{2i} \; \ldots \; m_{ni} \;]^T$ are vectors which denote the inverse of standard deviation and mean of Gaussian membership function, respectively, and β_i is an adjustable parameter. Each Gaussian membership function in the fuzzy logic system can be represented by

$$\Theta_i = \exp\left(-\sum_{k=1}^{n} \sigma_{kj}^2 (z_k - m_{kj})^2\right) \tag{12}$$

To ease the notation, (12) can be expressed in a compact vector form as

$$u_{fc}(\boldsymbol{z}, \boldsymbol{\beta}, \boldsymbol{\sigma}, \boldsymbol{m}) = \boldsymbol{\beta}^T \boldsymbol{\Theta}(\boldsymbol{z}, \boldsymbol{\sigma}, \boldsymbol{m}) \tag{13}$$

where $\boldsymbol{\beta} = [\beta_1 \; \beta_2 \; \ldots \; \beta_j]^T$, $\boldsymbol{\Theta} = [\Theta_1 \; \Theta_2 \; \ldots \; \Theta_j]^T$, $\boldsymbol{\sigma} = [\boldsymbol{\sigma}_1^T \; \boldsymbol{\sigma}_2^T \; \ldots \; \boldsymbol{\sigma}_j^T]^T$ and $\boldsymbol{m} = [\boldsymbol{m}_1^T \; \boldsymbol{m}_2^T \; \ldots \; \boldsymbol{m}_j^T]^T$.

According to the universal approximation theorem [2], there exists an optimal fuzzy system u^* to uniformly approximate the ideal controller, such that

$$u^* = u_{fc}^*(\boldsymbol{z}, \boldsymbol{\beta}^*, \boldsymbol{\sigma}^*, \boldsymbol{m}^*) + \Delta = \boldsymbol{\beta}^{*T} \boldsymbol{\Theta}(\boldsymbol{z}, \boldsymbol{\sigma}^*, \boldsymbol{m}^*) + \Delta = \boldsymbol{\beta}^{*T} \boldsymbol{\Theta}^* + \Delta \tag{14}$$

where Δ denotes the approximation error and $\boldsymbol{\sigma}^*$, $\boldsymbol{m}^*$ and $\boldsymbol{\beta}^*$ are the optimal parameter vectors of $\boldsymbol{\sigma}$, $\boldsymbol{m}$ and $\boldsymbol{\beta}$, respectively. In fact, the optimal parameter vectors in the optimal fuzzy system to approximate the ideal controller are unobtainable. Thus, an adaptive fuzzy sliding-mode controller is proposed to estimate the optimal parameters, and it is defined as

$$\hat{u}_{fsmc}(\boldsymbol{z}, \hat{\boldsymbol{\beta}}, \hat{\boldsymbol{\sigma}}, \hat{\boldsymbol{m}}) = \hat{\boldsymbol{\beta}}^T \boldsymbol{\Theta}(\boldsymbol{z}, \hat{\boldsymbol{\sigma}}, \hat{\boldsymbol{m}}) = \hat{\boldsymbol{\beta}}^T \hat{\boldsymbol{\Theta}} \tag{15}$$

where $\hat{\boldsymbol{\sigma}}$, $\hat{\boldsymbol{m}}$ and $\hat{\boldsymbol{\beta}}$ are the estimation values of $\boldsymbol{\sigma}^*$, $\boldsymbol{m}^*$ and $\boldsymbol{\beta}^*$, respectively. By using the estimation fuzzy control system to approximate the ideal controller, the estimation error can be written as

$$\tilde{u}_{fsmc} = u^* - \hat{u}_{fsmc} = u_{fc}^* - \hat{u}_{fsmc} + \Delta = \tilde{\boldsymbol{\beta}}^T \tilde{\boldsymbol{\Theta}} + \hat{\boldsymbol{\beta}}^T \tilde{\boldsymbol{\Theta}} + \tilde{\boldsymbol{\beta}}^T \hat{\boldsymbol{\Theta}} + \Delta \tag{16}$$

where $\tilde{\boldsymbol{\beta}} = \boldsymbol{\beta}^* - \hat{\boldsymbol{\beta}}$ and $\tilde{\boldsymbol{\Theta}} = \boldsymbol{\Theta}^* - \hat{\boldsymbol{\Theta}}$. In order to achieve favorable estimation of the ideal controller, the parameters of fuzzy controller will be modified by some adaptive tuning laws. Because the parameters of membership functions are also expected to be modified, the Taylor expansion linearization technique is employed to transform the nonlinear function into a partially linear form [2,16], i.e.,

$$\tilde{\boldsymbol{\Theta}} = \begin{bmatrix} \tilde{\Theta}_1 \\ \tilde{\Theta}_2 \\ \vdots \\ \tilde{\Theta}_j \end{bmatrix} = \begin{bmatrix} \frac{\partial \Theta_1}{\partial \boldsymbol{\sigma}} \\ \frac{\partial \Theta_2}{\partial \boldsymbol{\sigma}} \\ \vdots \\ \frac{\partial \Theta_j}{\partial \boldsymbol{\sigma}} \end{bmatrix} \Bigg|_{\boldsymbol{\sigma} = \hat{\boldsymbol{\sigma}}} \tilde{\boldsymbol{\sigma}} + \begin{bmatrix} \frac{\partial \Theta_1}{\partial \boldsymbol{m}} \\ \frac{\partial \Theta_2}{\partial \boldsymbol{m}} \\ \vdots \\ \frac{\partial \Theta_j}{\partial \boldsymbol{m}} \end{bmatrix} \Bigg|_{\boldsymbol{m} = \hat{\boldsymbol{m}}} \tilde{\boldsymbol{m}} + \boldsymbol{h} \tag{17}$$

or

$$\tilde{\boldsymbol{\Theta}} = \boldsymbol{A}^T \tilde{\boldsymbol{\sigma}} + \boldsymbol{B}^T \tilde{\boldsymbol{m}} + \boldsymbol{h} \tag{18}$$

where $\tilde{\boldsymbol{\sigma}} = \boldsymbol{\sigma}^* - \hat{\boldsymbol{\sigma}}$; $\tilde{\boldsymbol{m}} = \boldsymbol{m}^* - \hat{\boldsymbol{m}}$; $\boldsymbol{h}$ is a vector of higher-order terms; $\boldsymbol{A} = \left[\frac{\partial\Theta_1}{\partial\boldsymbol{\sigma}} \frac{\partial\Theta_2}{\partial\boldsymbol{\sigma}} \cdots \frac{\partial\Theta_j}{\partial\boldsymbol{\sigma}}\right]\Big|_{\boldsymbol{\sigma}=\hat{\boldsymbol{\sigma}}}$; $\boldsymbol{B} = \left[\frac{\partial\Theta_1}{\partial\boldsymbol{m}} \frac{\partial\Theta_2}{\partial\boldsymbol{m}} \cdots \frac{\partial\Theta_j}{\partial\boldsymbol{m}}\right]\Big|_{\boldsymbol{m}=\hat{\boldsymbol{m}}}$; and $\frac{\partial\Theta_i}{\partial\boldsymbol{\sigma}}$ and $\frac{\partial\Theta_i}{\partial\boldsymbol{m}}$ are defined as

$$\left[\frac{\partial\Theta_i}{\partial\boldsymbol{\sigma}}\right]^T = \left[\underset{(i-1)\times n}{0\cdots0} \ \frac{\partial\Theta_i}{\partial\sigma_{1i}} \cdots \frac{\partial\Theta_i}{\partial\sigma_{ni}} \ \underset{(j-i)\times n}{0\cdots0}\right] \tag{19}$$

$$\left[\frac{\partial\Theta_i}{\partial\boldsymbol{m}}\right]^T = \left[\underset{(i-1)\times n}{0\cdots0} \ \frac{\partial\Theta_i}{\partial m_{1i}} \cdots \frac{\partial\Theta_i}{\partial m_{ni}} \ \underset{(j-i)\times n}{0\cdots0}\right] \tag{20}$$

Substituting (18) into (16) yields

$$\tilde{u}_{fsmc} = \tilde{\boldsymbol{\beta}}^T\tilde{\boldsymbol{\Theta}} + \hat{\boldsymbol{\beta}}^T(\boldsymbol{A}^T\tilde{\boldsymbol{\sigma}} + \boldsymbol{B}^T\tilde{\boldsymbol{m}} + \boldsymbol{h}) + \tilde{\boldsymbol{\beta}}^T\hat{\boldsymbol{\Theta}} + \Delta = \tilde{\boldsymbol{\beta}}^T\hat{\boldsymbol{\Theta}} + \tilde{\boldsymbol{\sigma}}^T\boldsymbol{A}\hat{\boldsymbol{\beta}} + \tilde{\boldsymbol{m}}^T\boldsymbol{B}\hat{\boldsymbol{\beta}} + \varepsilon \tag{21}$$

where the uncertain term $\varepsilon = \tilde{\boldsymbol{\beta}}^T\tilde{\boldsymbol{\Theta}} + \hat{\boldsymbol{\beta}}^T\boldsymbol{h} + \Delta$ is assumed to be bounded by $|\varepsilon| \leq E$ and E is a positive constant. If the fuzzy control system can perfectly approximate the ideal controller, that is $\tilde{u}_{fsmc}$ nearly equals zero, then the fuzzy control system can be utilized to control the BLDC motor stably.

3.2. **AFSMC with bound estimation compensator.** In order to control the BLDC motor, the structure of the developed AFSMC system with bound estimation is shown in Figure 1 and the control system is defined as

$$u_{AFSE} = u_{fsmc} + u_{sg} \tag{22}$$

where the adaptive fuzzy sliding-mode controller u_{fsmc} is used to approximate the ideal controller u^*, and the compensator u_{sg} is utilized to compensate the approximation error between the fuzzy sliding-mode controller and the ideal controller. Substituting (22) into (7) yields

$$\ddot{\theta}_m = f_m\dot{\theta}_m + g_m(u_{fsmc} + u_{sg}) + d \tag{23}$$

Define a sliding surface as

$$s = \dot{e} + k_{m1}e + k_{m2}\int edt \tag{24}$$

Then from (9), (23) and (24), the following equation can be obtained

$$\ddot{e} + k_{m1}\dot{e} + k_{m2}e = \dot{s} = g_m(u^* - u_{fsmc} - u_{sg}) \tag{25}$$

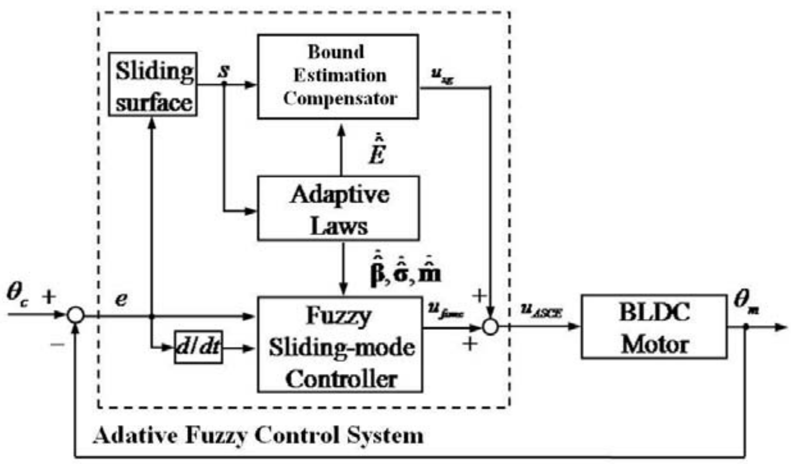


FIGURE 1. The structure of adaptive fuzzy sliding-mode control system with bound estimation

Substituting (21) into (25), yields

$$\dot{s} = g_m(\tilde{\boldsymbol{\beta}}^T\hat{\boldsymbol{\Theta}} + \tilde{\boldsymbol{\sigma}}^T\boldsymbol{A}\hat{\boldsymbol{\beta}} + \tilde{\boldsymbol{m}}^T\boldsymbol{B}\hat{\boldsymbol{\beta}} + \varepsilon - u_{sg}) \tag{26}$$

To prove the stability of the control system, a Lyapunov function is defined as

$$V_{AFSE} = \frac{1}{2}s^2 + \frac{g_m}{2\eta_\beta}\tilde{\boldsymbol{\beta}}^T\tilde{\boldsymbol{\beta}} + \frac{g_m}{2\eta_\sigma}\tilde{\boldsymbol{\sigma}}^T\tilde{\boldsymbol{\sigma}} + \frac{g_m}{2\eta_m}\tilde{\boldsymbol{m}}^T\tilde{\boldsymbol{m}} + \frac{g_m}{2\eta_e}\tilde{E}^2 \tag{27}$$

where η_β, η_σ, η_m, and η_e are the learning-rates with positive constants, and $\tilde{E} = E - \hat{E}$ in which $\hat{E}$ is an estimation value of the approximation error bound E. Differentiating (27) with respect to time and using (26), it can be obtained

$$\begin{aligned} \dot{V}_{AFSE} &= s\dot{s} + \tfrac{g_m}{\eta_\beta}\tilde{\boldsymbol{\beta}}^T\dot{\tilde{\boldsymbol{\beta}}} + \tfrac{g_m}{\eta_\sigma}\tilde{\boldsymbol{\sigma}}^T\dot{\tilde{\boldsymbol{\sigma}}} + \tfrac{g_m}{\eta_m}\tilde{\boldsymbol{m}}^T\dot{\tilde{\boldsymbol{m}}} + \tfrac{g_m}{\eta_e}\tilde{E}\dot{\tilde{E}} \\ &= sg_m(\tilde{\boldsymbol{\beta}}^T\hat{\boldsymbol{\Theta}} + \tilde{\boldsymbol{\sigma}}^T\boldsymbol{A}\hat{\boldsymbol{\beta}} + \tilde{\boldsymbol{m}}^T\boldsymbol{B}\hat{\boldsymbol{\beta}} + \varepsilon - u_{sg}) \\ &\quad + \tfrac{g_m}{\eta_\beta}\tilde{\boldsymbol{\beta}}^T\dot{\tilde{\boldsymbol{\beta}}} + \tfrac{g_m}{\eta_\sigma}\tilde{\boldsymbol{\sigma}}^T\dot{\tilde{\boldsymbol{\sigma}}} + \tfrac{g_m}{\eta_m}\tilde{\boldsymbol{m}}^T\dot{\tilde{\boldsymbol{m}}} + \tfrac{g_m}{\eta_e}\tilde{E}\dot{\tilde{E}} \\ &= g_m\tilde{\boldsymbol{\beta}}^T(s\hat{\boldsymbol{\Theta}} + \tfrac{\dot{\tilde{\boldsymbol{\beta}}}}{\eta_\beta}) + g_m\tilde{\boldsymbol{\sigma}}^T(s\boldsymbol{A}\hat{\boldsymbol{\beta}} + \tfrac{\dot{\tilde{\boldsymbol{\sigma}}}}{\eta_\sigma}) + g_m\tilde{\boldsymbol{m}}^T(s\boldsymbol{B}\hat{\boldsymbol{\beta}} + \tfrac{\dot{\tilde{\boldsymbol{m}}}}{\eta_m}) \\ &\quad + sg_m(\varepsilon - u_{sg}) + \tfrac{g_m}{\eta_e}\tilde{E}\dot{\tilde{E}} \end{aligned} \tag{28}$$

Since $\boldsymbol{\beta}^*$, $\boldsymbol{\sigma}^*$ and $\boldsymbol{m}^*$ are constant vectors, the adaptation laws are selected as

$$\dot{\hat{\boldsymbol{\beta}}} = -\dot{\tilde{\boldsymbol{\beta}}} = \eta_\beta s\hat{\boldsymbol{\Theta}} \tag{29}$$

$$\dot{\hat{\boldsymbol{\sigma}}} = -\dot{\tilde{\boldsymbol{\sigma}}} = \eta_\sigma s\boldsymbol{A}\hat{\boldsymbol{\beta}} \tag{30}$$

$$\dot{\hat{\boldsymbol{m}}} = -\dot{\tilde{\boldsymbol{m}}} = \eta_m s\boldsymbol{B}\hat{\boldsymbol{\beta}} \tag{31}$$

and the compensator is given as

$$u_{sg} = \hat{E}\mathrm{sgn}(s) \tag{32}$$

where $\mathrm{sgn}(\cdot)$ is a signum function, and the bound estimation law is selected as

$$\dot{\hat{E}} = -\dot{\tilde{E}} = \eta_e\left|s\right| \tag{33}$$

Then Equation (28) can be rewritten as

$$\dot{V}_{AFCE} = \varepsilon sg_m - \hat{E}\left|s\right|g_m - (E - \hat{E})\left|s\right|g_m \leq \left|\varepsilon\right|\left|s\right|g_m - E\left|s\right|g_m = -(E - \left|\varepsilon\right|)\left|s\right|g_m \leq 0 \tag{34}$$

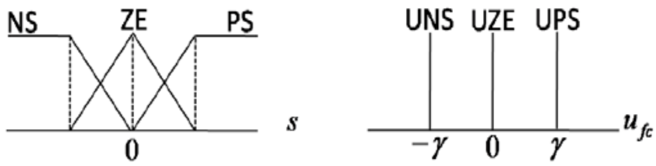
In addition, define

$$\rho_{AFSE} = (E - \varepsilon)sg_m \leq (E - \left|\varepsilon\right|)sg_m \leq -\dot{V}_{AFSE} \tag{35}$$

Then by integrating both sides of (35) with time from 0 to T, it can be obtained

$$\begin{aligned} \textstyle\int_0^T \rho_{AFSE}dt \leq\ & V_{AFSE}(s(0), \tilde{\boldsymbol{\beta}}(0), \tilde{\boldsymbol{\sigma}}(0), \tilde{\boldsymbol{m}}(0), \tilde{E}(0)) \\ & -V_{AFSE}(s(T), \tilde{\boldsymbol{\beta}}(T), \tilde{\boldsymbol{\sigma}}(T), \tilde{\boldsymbol{m}}(T), \tilde{E}(T)) \end{aligned} \tag{36}$$

In (36), because $V_{AFSE}(s(0), \tilde{\boldsymbol{\beta}}(0), \tilde{\boldsymbol{\sigma}}(0), \tilde{\boldsymbol{m}}(0), \tilde{E}(0))$ is bounded and $V_{AFSE}(s(T), \tilde{\boldsymbol{\beta}}(T),$ $\tilde{\boldsymbol{\sigma}}(T), \tilde{\boldsymbol{m}}(T), \tilde{E}(T))$ is not increasing, it implies $\int_0^T \rho_{AFSE}dt < \infty$. Moreover, since $\dot{\rho}_{AFSE} < \infty$, it can be claimed that $\lim_{t\to\infty} \rho_{AFSE} = 0$ by Barbalat's Lemma [17]. Consequently, it also implies $s \to 0$ as $t \to \infty$. This guarantees the stability of the control system.



(a) Input (b) Output

FIGURE 2. Input and output membership functions of fuzzy compensator

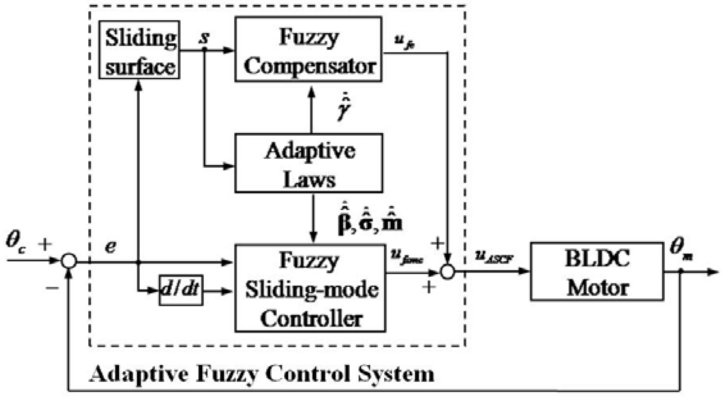


FIGURE 3. The structure of adaptive fuzzy sliding-mode control system with fuzzy compensator

3.3. **AFSMC with fuzzy compensator.** In the previous section, an AFSMC system with bound estimation is proposed. However, the proposed compensator contains a signum function, so that the chattering phenomena in the control effort would occur if the uncertainty bound is overestimated. To overcome this problem, a fuzzy compensator is proposed to replace the bound compensator.

The fuzzy compensator is designed by three simple fuzzy rules to reduce the calculation. The fuzzy rules are defined as

$$\begin{array}{l}\text{Rule 1: IF } s \text{ is } PS \text{ THEN } u_{fc} \text{ is } UPS\\ \text{Rule 2: IF } s \text{ is } ZE \text{ THEN } u_{fc} \text{ is } UZE\\ \text{Rule 3: IF } s \text{ is } NS \text{ THEN } u_{fc} \text{ is } UNS\end{array} \tag{37}$$

where the input and output membership functions are shown in Figure 2.

According to these fuzzy rules, the output of fuzzy compensator is given as [18]

$$u_{fc} = \gamma\varphi_{PS} + 0\varphi_{ZE} - \gamma\varphi_{NS} = \gamma(\varphi_{PS} - \varphi_{NS}) \tag{38}$$

where $-\gamma$, 0 and γ are the centers of the output membership functions, and φ_{PB}, φ_{ZE} and φ_{NB} are the fire strengths of the rules.

Replacing the bound estimation compensator with the fuzzy compensator, the structure of the developed AFSMC system with fuzzy compensator is shown in Figure 3. The control system is defined as

$$u_{AFSF} = u_{fsmc} + u_{fc} \tag{39}$$

Substituting (39) into (7), it can be obtained

$$\ddot{\theta}_m = f_m\dot{\theta}_m + g_m(u_{fsmc} + u_{fc}) + d \tag{40}$$

From (9), (24) and (40), it can be obtained

$$\dot{s} = g_m(u^* - u_{fsmc} - u_{fc}) \tag{41}$$

Substituting (21) into (41), yields

$$\dot{s} = g_m \left(\tilde{\boldsymbol{\beta}}^T \hat{\boldsymbol{\Theta}} + \tilde{\boldsymbol{\sigma}}^T \boldsymbol{A} \hat{\boldsymbol{\beta}} + \tilde{\boldsymbol{m}}^T \boldsymbol{B} \hat{\boldsymbol{\beta}} + \varepsilon - u_{fc} \right) \tag{42}$$

To prove the stability, a Lyapunov function is defined as

$$V_{AFSF1} = \frac{1}{2}s^2 + \frac{g_m}{2\eta_\beta}\tilde{\boldsymbol{\beta}}^T\tilde{\boldsymbol{\beta}} + \frac{g_m}{2\eta_\sigma}\tilde{\boldsymbol{\sigma}}^T\tilde{\boldsymbol{\sigma}} + \frac{g_m}{2\eta_m}\tilde{\boldsymbol{m}}^T\tilde{\boldsymbol{m}} \tag{43}$$

Differentiating (43) with respect to time and using (42), it can be obtained

$$\begin{aligned} \dot{V}_{AFSF1} &= \tfrac{1}{2}s\dot{s} + \tfrac{g_m}{\eta_\beta}\tilde{\boldsymbol{\beta}}^T\dot{\tilde{\boldsymbol{\beta}}} + \tfrac{g_m}{\eta_\sigma}\tilde{\boldsymbol{\sigma}}^T\dot{\tilde{\boldsymbol{\sigma}}} + \tfrac{g_m}{\eta_m}\tilde{\boldsymbol{m}}^T\dot{\tilde{\boldsymbol{m}}} \\ &= g_m\tilde{\boldsymbol{\beta}}^T\left(s\hat{\boldsymbol{\Theta}} + \tfrac{\dot{\tilde{\boldsymbol{\beta}}}}{\eta_\beta}\right) + g_m\tilde{\boldsymbol{\sigma}}^T\left(s\boldsymbol{A}\hat{\boldsymbol{\beta}} + \tfrac{\dot{\tilde{\boldsymbol{\sigma}}}}{\eta_\sigma}\right) + g_m\tilde{\boldsymbol{m}}^T\left(s\boldsymbol{B}\hat{\boldsymbol{\beta}} + \tfrac{\dot{\tilde{\boldsymbol{m}}}}{\eta_m}\right) + sg_m(\varepsilon - u_{fc}) \end{aligned} \tag{44}$$

If the adaptive laws are selected as (29)-(31), then (44) can be rewritten as

$$\dot{V}_{AFSF1} = sg_m(\varepsilon - u_{fc}) = sg_m(\varepsilon - \gamma(\varphi_{PS} - \varphi_{NS})) \tag{45}$$

Because the fire strengths φ_{PS} and φ_{NS} are excited only in the condition of $s > 0$ and $s < 0$, respectively, it implies $s\gamma(\varphi_{PS} - \varphi_{NS}) = |s|\,\gamma\,|\varphi_{PS} - \varphi_{NS}|$. Therefore, (45) can be rewritten as

$$\begin{aligned} \dot{V}_{AFSF1} &= sg_m\varepsilon - |s|\,g_m\gamma\,|\varphi_{PS} - \varphi_{NS}| \\ &\leq |s|\,g_m\,|\varepsilon| - |s|\,g_m\gamma\,|\varphi_{PS} - \varphi_{NS}| \\ &= |s|\,g_m(|\varepsilon| - \gamma\,|\varphi_{PS} - \varphi_{NS}|) \end{aligned} \tag{46}$$

If the condition $\gamma > \frac{|\varepsilon|}{|\varphi_{PS} - \varphi_{NS}|}$ can be satisfied, the differentiation of V_{AFSF1} is less than zero. To guarantee this condition, assume an optimal value exists

$$\gamma^* = \frac{|\varepsilon|}{|\varphi_{PS} - \varphi_{NS}|} + \psi \tag{47}$$

where ψ is a small positive constant. To find out the value of γ^* which includes uncertainty term $|\varepsilon|$, an estimation parameter $\hat{\gamma}$ is utilized to estimate the optimal value γ^*. Therefore, the fuzzy compensator in (38) can be rewritten as

$$u_{fc} = \hat{\gamma}(\varphi_{PS} - \varphi_{NS}) \tag{48}$$

and the Lyapunove function is redefined as

$$V_{AFSF2} = \frac{1}{2}s^2 + \frac{g_m}{2\eta_\beta}\tilde{\boldsymbol{\beta}}^T\tilde{\boldsymbol{\beta}} + \frac{g_m}{2\eta_\sigma}\tilde{\boldsymbol{\sigma}}^T\tilde{\boldsymbol{\sigma}} + \frac{g_m}{2\eta_m}\tilde{\boldsymbol{m}}^T\tilde{\boldsymbol{m}} + \frac{g_m}{2\eta_\gamma}\tilde{\gamma}^2 \tag{49}$$

where η_γ is a positive learning-rate of $\hat{\gamma}$, and $\tilde{\gamma} = \gamma^* - \hat{\gamma}$. Differentiating (49) with respect to time and using (42), it can be obtained

$$\begin{aligned} \dot{V}_{AFSF2} &= s\dot{s} + \tfrac{g_m}{\eta_\beta}\tilde{\boldsymbol{\beta}}^T\dot{\tilde{\boldsymbol{\beta}}} + \tfrac{g_m}{\eta_\sigma}\tilde{\boldsymbol{\sigma}}^T\dot{\tilde{\boldsymbol{\sigma}}} + \tfrac{g_m}{\eta_m}\tilde{\boldsymbol{m}}^T\dot{\tilde{\boldsymbol{m}}} + \tfrac{g_m}{\eta_\gamma}\tilde{\gamma}\dot{\tilde{\gamma}} \\ &= g_m\tilde{\boldsymbol{\beta}}^T\left(s\hat{\boldsymbol{\Theta}} + \tfrac{\dot{\tilde{\boldsymbol{\beta}}}}{\eta_\beta}\right) + g_m\tilde{\boldsymbol{\sigma}}^T\left(s\boldsymbol{A}\hat{\boldsymbol{\beta}} + \tfrac{\dot{\tilde{\boldsymbol{\sigma}}}}{\eta_\sigma}\right) \\ &\quad + g_m\tilde{\boldsymbol{m}}^T\left(s\boldsymbol{B}\hat{\boldsymbol{\beta}} + \tfrac{\dot{\tilde{\boldsymbol{m}}}}{\eta_m}\right) + sg_m(\varepsilon - u_{fc}) + \tfrac{g_m}{\eta_\gamma}\tilde{\gamma}\dot{\tilde{\gamma}} \end{aligned} \tag{50}$$

If the adaptive laws are selected as (29)-(31) and the fuzzy compensator is defined as (48), then (50) can be rewritten as

$$\begin{aligned} \dot{V}_{AFSF2} &= sg_m\left[\varepsilon - \hat{\gamma}(\varphi_{PS} - \varphi_{NS})\right] + \tfrac{g_m}{\eta_\gamma}\tilde{\gamma}\dot{\tilde{\gamma}} \\ &= sg_m\varepsilon + sg_m(\tilde{\gamma} - \gamma^*)(\varphi_{PS} - \varphi_{NS}) + \tfrac{g_m}{\eta_\gamma}\tilde{\gamma}\dot{\tilde{\gamma}} \\ &= g_m\tilde{\gamma}\left[s(\varphi_{PS} - \varphi_{NS}) - \tfrac{1}{\eta_\gamma}\dot{\hat{\gamma}}\right] + sg_m\left[\varepsilon - \gamma^*(\varphi_{PS} - \varphi_{NS})\right] \end{aligned} \tag{51}$$

If the adaptive law of fuzzy compensator is defined as

$$\dot{\hat{\gamma}} = \eta_\gamma s(\varphi_{PS} - \varphi_{NS}) \tag{52}$$

Then (51) can be rewritten as

$$\dot{V}_{AFSF2} = sg_m\left[\varepsilon - \gamma^*(\varphi_{PS} - \varphi_{NS})\right] \le |s|\, g_m\left[|\varepsilon| - \gamma^*\,|\varphi_{PS} - \varphi_{NS}|\right] \tag{53}$$

Substituting (47) into (53), yields

$$\dot{V}_{AFSF2} \le -\,|s|\, g_m\psi_\gamma\,|\varphi_{PS} - \varphi_{NS}| \le 0 \tag{54}$$

In addition, define

$$\rho_{AFSF} = sg_m\psi_\gamma(\varphi_{PS} - \varphi_{NS}) \le -\dot{V}_{AFSF2} \tag{55}$$

Then, by integrating both sides of (55) with time from 0 to T, it can be obtained

$$\begin{aligned}\int_0^T \rho_{AFSF}(t)dt \;\le\; & V_{AFSF2}(s(0), \tilde{\boldsymbol{\beta}}(0), \tilde{\boldsymbol{\sigma}}(0), \tilde{\boldsymbol{m}}(0), \tilde{\gamma}(0)) \\ & -V_{AFSF2}(s(T), \tilde{\boldsymbol{\beta}}(T), \tilde{\boldsymbol{\sigma}}(T), \tilde{\boldsymbol{m}}(T), \tilde{\gamma}(T))\end{aligned} \tag{56}$$

Because V_{AFSF2} is a nonnegative function and $\dot{V}_{AFSF2}$ is less or equal to zero, it implies V_{AFSF2} is not increasing. Moreover, $V_{AFSF2}(s(0), \tilde{\boldsymbol{\beta}}(0), \tilde{\boldsymbol{\sigma}}(0), \tilde{\boldsymbol{m}}(0), \tilde{\gamma}(0))$ is bounded, so it can be shown that $\int_0^T \rho_{AFSF}dt < \infty$. Furthermore, because $\dot{\rho}_{AFSF} < \infty$, by Barbalat's Lemma [16], it implies $\lim_{t\to\infty} \rho_{AFSF2}(t) = 0$ or $s \to 0$ when $t \to \infty$.

4. **Experiment Result.** The proposed AFSMC systems are applied for the trajectory tracking control of a BLDC motor. The control parameters of the proposed control systems are selected as $k_{m1} = 2000$, $k_{m2} = 1000\eta_\sigma = \eta_m = 0.000002$, $\eta_e = 0.00005$, $\eta_\beta = 0.000005$. The selections of these values are to achieve satisfactory control performance considering the requirement of stability and possible operating conditions through some trials. Moreover, the learning-rates are selected by concerning the stability of parameter convergence and learning speed. The experimental environment is shown in Figure 4. In order to test the trajectory tracking performance of the command inputs with different frequencies, the period of command signals would change from $T = 2.25$ to $T = 1.5$ at the 5th second. To compare the control efficiency between the proposed controllers, the experimental results of the BLDC motor to follow (i) a sinusoidal command signal is shown in Figure 5, and (ii) a step command signal is shown in Figure 6. Figures 5(a) and 5(b) illustrate the experimental results of the AFSMC system with the bound estimation compensator and the fuzzy compensator for the sinusoidal command signal, respectively. Figures 6(a) and 6(b) illustrate the experimental results of the AFSMC system with the bound estimation compensator and the fuzzy compensator for the step command signal, respectively. Obviously, bound estimation compensator has the chattering phenomena because of the signum function. On the contrary, fuzzy compensator can eliminate the chattering phenomena effectively. It is easy to find that the tracking responses of the fuzzy compensator are superior to the bound estimation compensator for different command inputs. The comparisons of these experimental results are summarized in Table 1. From these comparisons, it can be seen that the fuzzy compensator can reduce the control chattering and also reduce the mean square error when compared with the bound estimation compensator. Moreover, comparing these experimental results with the PI control [12], adaptive fuzzy-neural-network control [13], and robust adaptive fuzzy control [14] for the BLDC motors, it is shown the proposed AFSMC system can achieve better control performance than these control methods.

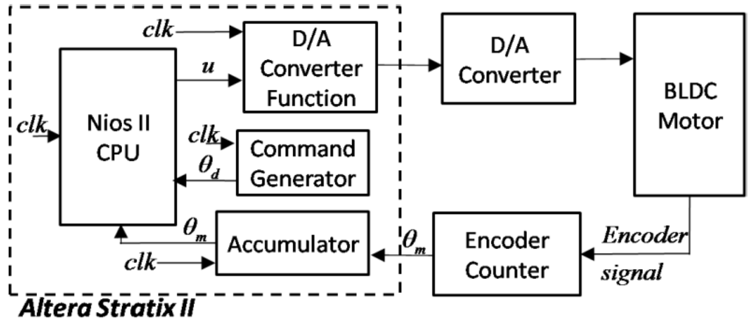


Figure 4. The structure of experimental environment

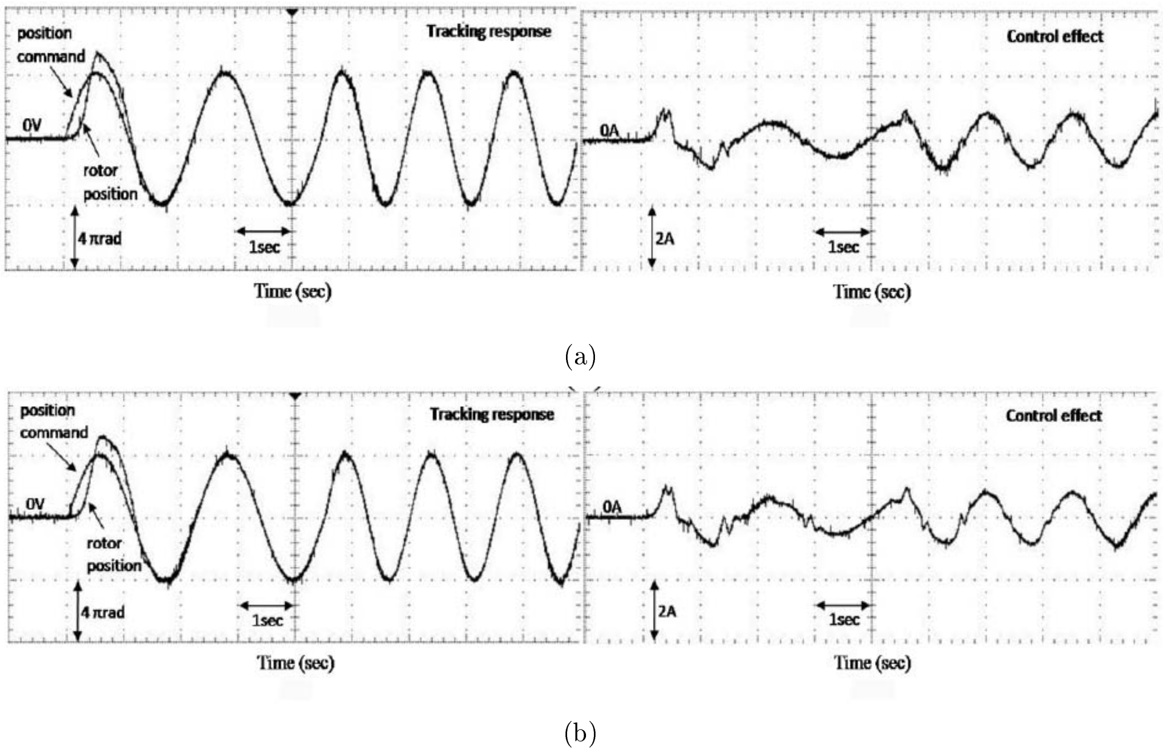
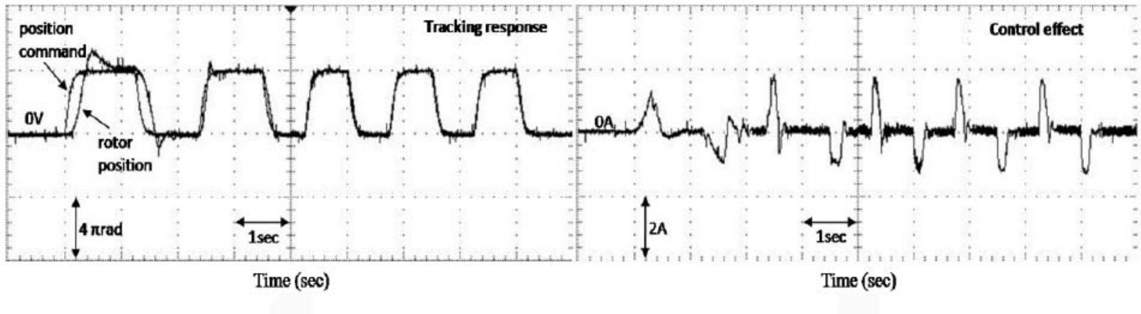


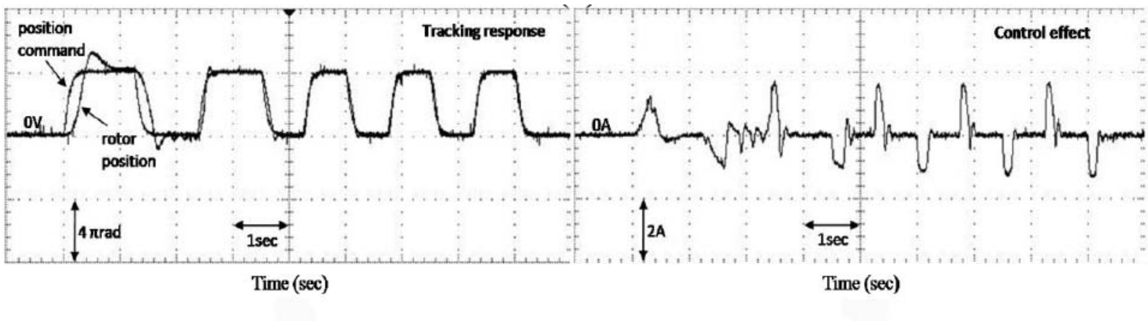
Figure 5. Trajectory tracking responses of BLDC motor for the sinusoidal command

Table 1. Comparison for different compensators

	Control System	MSE (e)
Sinusoidal Command	AFSMC+bound compensator	9.2364°
	AFSMC+fuzzy compensator	7.6161°
Step Command	AFSMC+bound compensator	10.1231°
	AFSMC+fuzzy compensator	9.6196°



(a)



(b)

FIGURE 6. Trajectory tracking responses of BLDC motor for the step command

5. **Conclusion.** In this paper, two adaptive fuzzy sliding-mode control (AFSMC) systems have been designed and successfully implemented by an FPGA board to control a BLDC motor. The proposed AFSMC systems are designed without the need of the information about system dynamic equations. In order to eliminate the approximation error properly, a bound estimation compensator and a fuzzy compensator are designed. According to the adaptive tuning laws, satisfactory control performance can be achieved by modifying the control parameters. The algorithm of bound estimation can overcome the guess of the real approximation error bound. However, it leads to the control chattering because of a signum function. The fuzzy compensator can get rid of chattering phenomena. The experimental results of the trajectory tracking control of a BLDC motor have been provided to verify the efficiency of the proposed control systems. In addition to the BLDC motor control, the developed control algorithm can be also applied to the other servomotor control systems and other automatic control systems such as robotic systems, magnetic levitation systems, power converters and anti-braking systems.

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