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THE PARAMETERIZATION OF ALL STABILIZING TWO-DEGREES-OF-FREEDOM SIMPLE REPETITIVE CONTROLLERS AND ITS APPLICATION

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Abstract. *The simple repetitive control system proposed by Yamada et al. is a type of servomechanism for periodic reference inputs. This system follows a periodic reference input with a small steady-state error, even if there is periodic disturbance or uncertainty in the plant. In addition, simple repetitive control systems ensure that transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles. Yamada et al. clarified the parameterization of all stabilizing simple repetitive controllers. Recently, Yamada et al. proposed the parameterization of all stabilizing two-degrees-of-freedom simple repetitive controllers that can specify the input-output characteristic and the disturbance attenuation characteristic separately. However, they omitted the complete proof because of space limitations. This paper gives the complete proof and demonstrates the effectiveness of the parameterization. The control characteristics of the system are presented, along with a design procedure for a two-degrees-of-freedom simple repetitive controller. A numerical example and an application for a motor control experiment are presented to illustrate the effectiveness of the proposed method.*
Keywords: Repetitive control, Two-degrees-of-freedom control, Finite number of poles, Parameterization, Motor control experiment

1. Introduction.

A repetitive control system is a servomechanism that can follow a periodic reference input without steady-state error, even if a periodic disturbance or uncertainty exists in the plant [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. It is difficult to design stabilizing controllers for a strictly proper plant, because a repetitive control system that follows any periodic reference input without steady-state error is a neutral type of time-delay control system [11]. To design such repetitive control systems, the plant must be biproper [3, 4, 5, 6, 7, 8, 9, 10, 11]. In practice, however, most plants are strictly proper. Many design methods for repetitive control systems for strictly proper plants have been proposed [3, 4, 5, 6, 7, 8, 9, 10, 11]. These systems can be divided into two types, one that uses a low-pass filter [3, 4, 5, 6, 7, 8, 9, 10] and one that uses an attenuator [11]. The latter system is difficult to design because it uses a state-variable time delay in the repetitive controller [11], whereas the former has a simple structure and is easily designed. The former type of repetitive control system is therefore called the modified repetitive control system [3, 4, 5, 6, 7, 8, 9, 10].

Using modified repetitive controllers, even if the plant does not include time delays, transfer functions from the periodic reference input to the output and from the disturbance to the output have infinite numbers of poles. This makes it difficult to specify the input-output characteristic and the disturbance attenuation characteristic. However, from a practical point of view, it is desirable that these characteristics be easy to specify, which would require these transfer functions to have finite numbers of poles. To overcome this problem, Yamada et al. proposed simple repetitive control systems such that the controller works as a modified repetitive controller, and the transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles [14]. In addition, they clarified the parameterization of all stabilizing simple repetitive controllers. Recently, Yamada et al. clarified the parameterization of all robust stabilizing simple repetitive controllers for time-delay plants with uncertainties [20]. However, using the methods in [14, 20], it is not easy to specify the low-pass filter in the internal model for the periodic reference input that specifies the input-output characteristic, because the low-pass filter is related to more than two free parameters. To make specifying the input-output characteristic easier, Murakami et al. proposed the parameterization of all stabilizing simple repetitive controllers with specified input-output characteristics with the low-pass filter specified beforehand [21]. In [22], Sakanushi et al. proposed a design method for control systems using the parameterization in [21] and demonstrated its application in a motor control experiment. Sakanushi et al. also proposed the parameterization of all robust stabilizing simple repetitive controllers with a specified input-output characteristic for plants with uncertainty [23] and for time-delay plants with uncertainty [24].

However, when employing the methods in [14, 20, 21, 22, 23, 24], we cannot specify the input-output characteristic and the disturbance attenuation characteristic separately, although it is desirable to be able to do so in practice. As the parameterization also is useful in designing stabilizing controllers [15, 16, 17, 18], Yamada et al. examined the parameterization of all stabilizing two-degrees-of-freedom simple repetitive controllers that can specify the input-output characteristic and the disturbance attenuation characteristic separately [19]. Use of that parameterization may enable the easy design of a simple repetitive control system that has the desired input-output and disturbance attenuation characteristics. However, Yamada et al. omitted the complete proof of the parameterization of all stabilizing two-degrees-of-freedom simple repetitive controllers because of space limitations. In addition, the control characteristics of the controllers were not examined, and no design method for stabilizing a two-degrees-of-freedom simple repetitive control system was described. Therefore, we cannot determine the effectiveness of the parameterization of all stabilizing two-degrees-of-freedom simple repetitive controllers in [19].

In this paper, we give the complete proof of this parameterization (omitted from [19]) and demonstrate its effectiveness. First, we give the complete proof. Next, we clarify the control characteristics using the parameterization in [19]. We also present a design procedure using the parameterization. A numerical example is presented to illustrate the effectiveness of the proposed design method. Finally, to demonstrate the effectiveness of the parameterization for real plants, we present an application for the reduction of rotational unevenness in motors.

Notation

R	the set of real numbers
R_+	$R \cup \{\infty\}$
$R(s)$	the set of real rational functions with s
RH_∞	the set of stable proper real rational functions
H_∞	the set of stable causal functions

2. **Two-Degrees-of-Freedom Simple Repetitive Control Systems and Problem Formulation.** Consider the two-degrees-of-freedom control system shown in Figure 1, which can specify the input-output characteristic and the disturbance attenuation characteristic separately. In the figure, $G(s) \in R(s)$ is the strictly proper plant, $C(s)$ is the controller written as

$$C(s) = \left[\begin{array}{cc} C_1(s) & -C_2(s) \end{array} \right], \tag{1}$$

$u(s) \in R(s)$ is the control input written as

$$u(s) = C(s) \left[\begin{array}{c} r(s) \\ z(s) \end{array} \right] = \left[\begin{array}{cc} C_1(s) & -C_2(s) \end{array} \right] \left[\begin{array}{c} r(s) \\ z(s) \end{array} \right], \tag{2}$$

$y(s) \in R(s)$ is the output, $d_1(s) \in R(s)$ and $d_2(s) \in R(s)$ are disturbances, $r(s) \in R(s)$ is the periodic reference input with period $T > 0$ satisfying

$$r(t + T) = r(t) \quad (\forall t \geq 0), \tag{3}$$

and $z(s) = y(s) + d_2(s)$. In the following, we call $C_1(s)$ the feed-forward controller and $C_2(s)$ the feedback controller. From the definition of internal stability [18], when all transfer functions $V_i(s)$ $(i = 1, \ldots, 6)$ written as

$$\left[\begin{array}{c} u(s) \\ y(s) \end{array} \right] = \left[\begin{array}{ccc} V_1(s) & V_2(s) & V_3(s) \\ V_4(s) & V_5(s) & V_6(s) \end{array} \right] \left[\begin{array}{c} r(s) \\ d_1(s) \\ d_2(s) \end{array} \right] \tag{4}$$

are stable, the two-degrees-of-freedom control system in Figure 1 is stable.

According to [3, 4, 5, 6, 7, 8, 9, 10], when the plant $G(s)$ has a periodic disturbance $d_1(s)$ with period T and uncertainty, if the output $y(s)$ is to follow the periodic reference input $r(s)$ with period T with a small steady-state error, the feedback controller $C_2(s)$ must be written in the following form:

$$C_2(s) = C_{21}(s) + C_{22}(s)C_r(s), \tag{5}$$

where $C_{21}(s) \in R(s)$ and $C_{22}(s) \in R(s)$, satisfying $C_{22}(s) \neq 0$. $C_r(s)$ is an internal model for the periodic reference input $r(s)$ with period T and is written as

$$C_r(s) = \frac{e^{-sT}}{1 - q(s)e^{-sT}}, \tag{6}$$

where $q(s) \in R(s)$ is a proper low-pass filter satisfying $q(0) = 1$. The feedback controller $C_2(s)$ defined by (5) is called a modified repetitive controller [3, 4, 5, 6, 7, 8, 9, 10].

Using $C(s)$ in (1) with the modified repetitive controller $C_2(s)$ in (5), transfer functions from the periodic reference input $r(s)$ to the output $y(s)$ in Figure 1, from the disturbance $d_1(s)$ to the output $y(s)$ in Figure 1 and from the disturbance $d_2(s)$ to the output $y(s)$ in Figure 1 have infinite numbers of poles. As noted above, the transfer functions need finite numbers of poles to make the input-output and disturbance attenuation characteristics easy to specify.

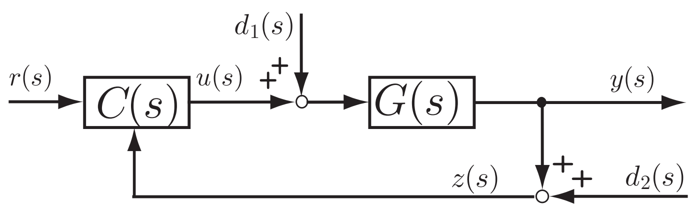

Figure 1. Two-degrees-of-freedom control system

To satisfy this practical requirement, Yamada et al. defined a stabilizing two-degrees-of-freedom simple repetitive controller as follows [19].

Definition 2.1. *(Stabilizing two-degrees-of-freedom simple repetitive controllers)* [19]
We call the controller $C(s)$ in (1) a "stabilizing two-degrees-of-freedom simple repetitive controller" if the following requirements are met.

1. *The feedback controller $C_2(s)$ in (1) works as a modified repetitive controller. That is, the feedback controller $C_2(s)$ is written as (5), where $C_{21}(s) \in R(s)$, $C_{22}(s) \neq 0 \in R(s)$, $C_r(s)$ is described by (6) and $q(s) \neq 0 \in R(s)$ satisfies $q(0) = 1$.*
2. *The controller $C(s)$ ensures that transfer functions from the periodic reference input $r(s)$ to the output $y(s)$ in Figure 1, from the disturbance $d_1(s)$ to the output $y(s)$ in Figure 1 and from the disturbance $d_2(s)$ to the output $y(s)$ in Figure 1 have finite numbers of poles. That is, $V_i(s)$ $(i = 4, 5, 6)$ in (4) have finite numbers of poles.*
3. *The two-degrees-of-freedom control system in Figure 1 is stable. That is, all transfer functions $V_i(s)$ $(i = 1, \ldots, 6)$ in Figure 1 are stable.*
4. *The transfer function $V_{er}(s)$ from the periodic reference input $r(s)$ to the error $e(s) = r(s) - y(s)$ in Figure 1 satisfies*
$$V_{er}(s_i) \simeq 0 \quad (\forall i = 0, 1, \ldots, n), \tag{7}$$
where s_i $(i = 0, 1, \ldots, n)$ are the frequency components of the periodic reference input $r(s)$ given by
$$s_i = j\frac{2\pi}{T}i \quad (i = 0, 1, \ldots, n), \tag{8}$$
and s_n is the maximum-frequency component of the periodic reference input $r(s)$.

In addition, Yamada et al. examined the parameterization of all stabilizing two-degrees-of-freedom simple repetitive controllers such that the input-output characteristic and the disturbance attenuation characteristic can be specified separately [19]. However, they omitted the complete proof because of space limitations. The objectives in this paper are to give the complete proof, to propose a design method for the control system using the parameterization in [19] and for the control characteristics of the control system using the parameterization in [19], and to illustrate the effectiveness of the proposed method using a numerical example and an application of reducing rotational unevenness in motors.

3. **Parameterization of all Stabilizing Two-Degrees-of-Freedom Simple Repetitive Controllers.** According to [19], the parameterization of all stabilizing two-degrees-of-freedom simple repetitive controllers can be summarized by the following theorem.

Theorem 3.1. *The controller $C(s)$ is a stabilizing two-degrees-of-freedom simple repetitive controller if and only if*
$$C(s) = \begin{bmatrix} C_1(s) & -C_2(s) \end{bmatrix}, \tag{9}$$
where
$$C_1(s) = \frac{Q_1(s)}{Y(s) - N(s)\left(Q_2(s) + \bar{Q}_2(s)e^{-sT}\right)} \tag{10}$$
and
$$C_2(s) = \frac{X(s) + D(s)\left(Q_2(s) + \bar{Q}_2(s)e^{-sT}\right)}{Y(s) - N(s)\left(Q_2(s) + \bar{Q}_2(s)e^{-sT}\right)}. \tag{11}$$

Here, $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = \frac{N(s)}{D(s)}. \tag{12}$$

$X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ are functions satisfying

$$X(s)N(s) + Y(s)D(s) = 1, \tag{13}$$

and $Q_1(s) \in H_\infty$ is any function that has finite numbers of poles and satisfies

$$1 - N(s_i)Q_1(s_i) \simeq 0 \quad (\forall i = 0, 1, \ldots, n). \tag{14}$$

$Q_2(s) \in RH_\infty$ and $\bar{Q}_2(s) \neq 0 \in RH_\infty$ are any functions satisfying

$$\frac{N(0)\bar{Q}_2(0)}{Y(0) - N(0)Q_2(0)} = 1. \tag{15}$$

The proof of this theorem requires the following lemma.

Lemma 3.1. *Consider the following unity feedback control system:*

$$\begin{cases} \tilde{y}(s) = \tilde{G}(s)\tilde{u}(s) \\ \tilde{u}(s) = \tilde{C}(s)\left(\tilde{r}(s) - \tilde{y}(s)\right) \end{cases}, \tag{16}$$

where $\tilde{G}(s) \in R(s)$ is the plant, $\tilde{C}(s) \in R(s)$ is the controller, $\tilde{u}(s) \in R(s)$ is the control input, $\tilde{y}(s) \in R(s)$ is the output and $\tilde{r}(s) \in R(s)$ is the reference input. The unity feedback control system in (16) is stable if and only if $\tilde{C}(s)$ is

$$\tilde{C}(s) = \frac{\tilde{X}(s) + \tilde{D}(s)\tilde{Q}(s)}{\tilde{Y}(s) - \tilde{N}(s)\tilde{Q}(s)}, \tag{17}$$

where $\tilde{N}(s) \in RH_\infty$ and $\tilde{D}(s) \in RH_\infty$ are coprime factors of $\tilde{G}(s)$ on RH_∞ satisfying

$$\tilde{G}(s) = \frac{\tilde{N}(s)}{\tilde{D}(s)}, \tag{18}$$

$\tilde{X}(s) \in RH_\infty$ and $\tilde{Y}(s) \in RH_\infty$ are functions satisfying

$$\tilde{X}(s)\tilde{N}(s) + \tilde{Y}(s)\tilde{D}(s) = 1, \tag{19}$$

and $\tilde{Q}(s) \in RH_\infty$ is any function [18].

Using Lemma 3.1, we present the proof of Theorem 3.1.

Proof: First, the necessity is shown. We show that if the controller $C(s)$ in (9) stabilizes the two-degrees-of-freedom control system in Figure 1 and ensures that the transfer functions $V_i(s)$ $(i = 4, 5, 6)$ of the two-degrees-of-freedom control system in Figure 1 have finite numbers of poles, then $C(s)$ takes the form (9), (10) and (11). From the assumption that the controller $C(s)$ in (9) with $C_2(s)$ in (5) ensures that the transfer functions $V_i(s)$ $(i = 4, 5, 6)$ of the two-degrees-of-freedom control system in Figure 1 have finite numbers of poles, we know that

$$\begin{aligned} V_4(s) &= \frac{C_1(s)G(s)}{1 + C_2(s)G(s)} \\ &= \frac{\left(1 - q(s)e^{-sT}\right)C_1(s)G(s)}{1 + C_{21}(s)G(s) - \left\{(1 + C_{21}(s)G(s))q(s) - C_{22}(s)G(s)\right\}e^{-sT}}, \end{aligned} \tag{20}$$

$$\begin{aligned} V_5(s) &= \frac{G(s)}{1+C_2(s)G(s)} \\ &= \frac{\left(1-q(s)e^{-sT}\right)G(s)}{1+C_{21}(s)G(s)-\{(1+C_{21}(s)G(s))\,q(s)-C_{22}(s)G(s)\}\,e^{-sT}} \end{aligned} \tag{21}$$

and

$$\begin{aligned} V_6(s) &= -\frac{C_2(s)G(s)}{1+C_2(s)G(s)} \\ &= -\frac{\left\{C_{21}(s)-(C_{21}(s)q(s)-C_{22}(s))\,e^{-sT}\right\}G(s)}{1+C_{21}(s)G(s)-\{(1+C_{21}(s)G(s))\,q(s)-C_{22}(s)G(s)\}\,e^{-sT}} \end{aligned} \tag{22}$$

have finite numbers of poles. This implies that

$$C_{22}(s) = \frac{(1+C_{21}(s)G(s))q(s)}{G(s)} \tag{23}$$

is satisfied; that is, $C_2(s)$ is necessarily

$$C_2(s) = \frac{C_{21}(s)G(s)+q(s)e^{-sT}}{G(s)\left(1-q(s)e^{-sT}\right)}. \tag{24}$$

From the assumption that $C(s)$ in (5) makes the two-degrees-of-freedom control system in Figure 1 stable, we know that $V_i(s)$ $(i=1,\ldots,6)$ are stable. From simple manipulation and (24), we have

$$V_1(s) = \frac{C_1(s)}{1+C_2(s)G(s)} = \frac{C_1(s)\left(1-q(s)e^{-sT}\right)}{1+C_{21}(s)G(s)}, \tag{25}$$

$$V_2(s) = -\frac{C_2(s)G(s)}{1+C_2(s)G(s)} = -\frac{C_{21}(s)G(s)+q(s)e^{-sT}}{1+C_{21}(s)G(s)}, \tag{26}$$

$$V_3(s) = -\frac{C_2(s)}{1+C_2(s)G(s)} = -\frac{C_{21}(s)G(s)+q(s)e^{-sT}}{(1+C_{21}(s)G(s))\,G(s)}, \tag{27}$$

$$V_4(s) = \frac{C_1(s)G(s)}{1+C_2(s)G(s)} = \frac{C_1(s)G(s)\left(1-q(s)e^{-sT}\right)}{1+C_{21}(s)G(s)}, \tag{28}$$

$$V_5(s) = \frac{G(s)}{1+C_2(s)G(s)} = \frac{G(s)\left(1-q(s)e^{-sT}\right)}{1+C_{21}(s)G(s)} \tag{29}$$

and

$$V_6(s) = -\frac{C_2(s)G(s)}{1+C_2(s)G(s)} = -\frac{C_{21}(s)G(s)+q(s)e^{-sT}}{1+C_{21}(s)G(s)}. \tag{30}$$

From the assumption that all transfer functions in (25) $\sim$ (30) are stable, $C_{21}(s)$ is an internally stabilizing controller for $G(s)$. From Lemma 3.1, $C_{21}(s)$ must take the following form:

$$C_{21}(s) = \frac{X(s)+D(s)Q_2(s)}{Y(s)-N(s)Q_2(s)}, \tag{31}$$

where $Q_2(s) \in RH_\infty$. From the assumption that the transfer functions in (26) and (30) are stable, we know that $q(s)/(1+C_{21}(s)G(s))$ is stable. This implies that any unstable poles of $q(s)$ are included in those of $C_{21}(s)$. That is, $q(s)$ takes the following form:

$$q(s) = \frac{\hat{Q}_2(s)}{Y(s) - N(s)Q_2(s)}, \tag{32}$$

where $\hat{Q}_2(s) \in RH_\infty$ is any function satisfying $\hat{Q}_2(s) \neq 0$ because $q(s) \neq 0$.

Because the transfer function in (27) is stable, $q(s)/\{(1 + C_{21}(s)G(s))G(s)\}$ is stable. From (31) and (32), we have

$$\frac{q(s)}{(1 + C_{21}(s)G(s))\,G(s)} = \frac{D^2(s)\hat{Q}_2(s)}{N(s)}. \tag{33}$$

From the assumption that $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ are coprime and the transfer function in (33) is stable, $\hat{Q}_2(s)$ can be written in the following form:

$$\hat{Q}_2(s) = N(s)\bar{Q}_2(s), \tag{34}$$

where $\bar{Q}_2(s) \in RH_\infty$ is any function satisfying $\bar{Q}_2(s) \neq 0$, because $\hat{Q}_2(s) \neq 0$. Substituting (23), (31), (32) and (34) into (5), we have (11). From the assumption that the transfer function in (25) written as

$$\frac{C_1(s)(1 - q(s)e^{-sT})}{1 + C_{21}(s)G(s)} = \left\{Y(s) - N(s)\left(Q_2(s) + \bar{Q}_2(s)e^{-sT}\right)\right\} D(s)C_1(s) \tag{35}$$

is stable, we know that $C_1(s)$ is written as

$$C_1(s) = \frac{\bar{Q}_1(s)}{\left\{Y(s) - N(s)\left(Q_2(s) + \bar{Q}_2(s)e^{-sT}\right)\right\} D(s)}, \tag{36}$$

where $\bar{Q}_1(s) \in H_\infty$. In addition, because the transfer function in (28) written as

$$\frac{C_1(s)G(s)(1 - q(s)e^{-sT})}{1 + C_{21}(s)G(s)} = \frac{N(s)\bar{Q}_1(s)}{D(s)} \tag{37}$$

is stable, we have

$$\bar{Q}_1(s) = D(s)Q_1(s), \tag{38}$$

where $Q_1(s) \in H_\infty$. Substituting (38) into (36), we have (10). Thus, we have shown that if the controller $C(s)$ is a stabilizing two-degrees-of-freedom simple repetitive controller, then $C_1(s)$ and $C_2(s)$ are written as (10) and (11).

Next, we show that (14) and (15) are satisfied. From (28), (10) and (11), the transfer function $V_{er}(s)$ from the periodic reference input $r(s)$ to the error $e(s) = r(s) - y(s)$ is written as

$$V_{er}(s) = \frac{e(s)}{r(s)} = 1 - \frac{C_1(s)G(s)}{1 + C_2(s)G(s)} = 1 - N(s)Q_1(s). \tag{39}$$

From the definition of stabilizing two-degrees-of-freedom simple repetitive controllers in Definition 2.1 and (39), (14) holds true. In addition, because $q(0) = 1$, from (32) and (34), (15) is satisfied. Thus, the necessity has been shown.

Next, the sufficiency is shown. If $C_1(s)$ and $C_2(s)$ in (9) take the forms (10) and (11), then the controller $C(s)$ stabilizes the two-degrees-of-freedom control system in Figure 1, ensures that the transfer functions $V_i(s)$ $(i = 4, 5, 6)$ in (4) have finite numbers of poles, $C_2(s)$ also takes the form in (5), and $q(s)$ in (6) satisfies $q(0) = 1$. After simple manipulation, we have

$$V_1(s) = \frac{C_1(s)}{1 + C_2(s)G(s)} = D(s)Q_1(s), \tag{40}$$

$$V_2(s) = -\frac{C_2(s)G(s)}{1 + C_2(s)G(s)} = -\left\{X(s) + D(s)\left(Q_2(s) + \bar{Q}_2(s)e^{-sT}\right)\right\} N(s), \tag{41}$$

$$V_3(s) = -\frac{C_2(s)}{1 + C_2(s)G(s)} = -\left\{X(s) + D(s)\left(Q_2(s) + \bar{Q}_2(s)e^{-sT}\right)\right\} D(s), \tag{42}$$

$$V_4(s) = \frac{C_1(s)G(s)}{1 + C_2(s)G(s)} = N(s)Q_1(s), \tag{43}$$

$$V_5(s) = \frac{G(s)}{1 + C_2(s)G(s)} = \left\{Y(s) - N(s)\left(Q_2(s) + \bar{Q}_2(s)e^{-sT}\right)\right\} N(s) \tag{44}$$

and

$$V_6(s) = -\frac{C_2(s)G(s)}{1 + C_2(s)G(s)} = -\left\{X(s) + D(s)\left(Q_2(s) + \bar{Q}_2(s)e^{-sT}\right)\right\} N(s). \tag{45}$$

Because $N(s) \in RH_\infty$, $D(s) \in RH_\infty$, $X(s) \in RH_\infty$, $Y(s) \in RH_\infty$, $Q_1(s) \in H_\infty$, $Q_2(s) \in RH_\infty$ and $\bar{Q}_2(s) \in RH_\infty$, the transfer functions in (40) $\sim$ (45) are stable. In addition, from the assumption that $Q_1(s)$ has a finite numbers of poles and from the above argument, the transfer functions $V_i(s)$ $(i = 4, 5, 6)$ in (43), (44) and (45) of the two-degrees-of-freedom control system in Figure 1 have finite numbers of poles. From (43) and (14), the transfer function $V_{er}(s)$ from the periodic reference input $r(s)$ to the error $e(s) = r(s) - y(s)$, written as

$$V_{er}(s) = \frac{e(s)}{r(s)} = 1 - N(s)Q_1(s), \tag{46}$$

satisfies (7).

Next, we show that the feedback controller $C_2(s)$ in (11) works as a modified repetitive controller. The controller is rewritten in the form in (5), where

$$C_{21}(s) = \frac{X(s) + D(s)Q_2(s)}{Y(s) - N(s)Q_2(s)}, \tag{47}$$

$$C_{22}(s) = \frac{\bar{Q}_2(s)}{\left(Y(s) - N(s)Q_2(s)\right)^2} \tag{48}$$

and

$$q(s) = \frac{N(s)\bar{Q}_2(s)}{Y(s) - N(s)Q_2(s)}. \tag{49}$$

From the assumptions that $\bar{Q}_2(s) \neq 0$ and (48), $C_{22}(s) \neq 0$ holds true. In addition, from (49) and the assumption in (15), $q(0) = 1$ is satisfied. These expressions imply that the feedback controller $C_2(s)$ in (11) works as a modified repetitive controller. Thus, the sufficiency has been shown.

We have thus proved Theorem 3.1.

4. **Control Characteristics.** In this section, we describe the control characteristics of the two-degrees-of-freedom control system in Figure 1 using the stabilizing two-degrees-of-freedom simple repetitive controller $C(s)$ in (9) with the feed-forward controller $C_1(s)$ in (10) and the feedback controller $C_2(s)$ in (11).

First, we consider the input-output characteristic. The transfer function from the periodic reference input $r(s)$ to the error $e(s) = r(s) - y(s)$ is written as

$$\frac{e(s)}{r(s)} = 1 - N(s)Q_1(s). \tag{50}$$

From (50), because $Q_1(s)$ is chosen to satisfy (14), the output $y(s)$ follows the periodic reference input $r(s)$ with a small steady-state error.

Next, we consider the disturbance attenuation characteristic. The transfer function from the disturbance $d_1(s)$ to the output $y(s)$ is written as

$$\begin{aligned}\frac{y(s)}{d_1(s)} &= \left\{Y(s) - N(s)\left(Q_2(s) + \bar{Q}_2(s)e^{-sT}\right)\right\}N(s) \\ &= \left\{1 - \frac{N(s)\bar{Q}_2(s)e^{-sT}}{Y(s) - N(s)Q_2(s)}\right\}\left(Y(s) - N(s)Q_2(s)\right)N(s). \end{aligned} \tag{51}$$

From (51), for s_i $(i = 0, 1, \ldots, n)$ in (8) of the frequency components of the disturbance $d_1(s)$ that are the same as those of the periodic reference input $r(s)$, if

$$1 - \frac{N(s_i)\bar{Q}_2(s_i)}{Y(s_i) - N(s_i)Q_2(s_i)} \simeq 0 \quad (\forall i = 0, \ldots, n), \tag{52}$$

then the disturbance $d_1(s)$ is attenuated effectively. For s_d of the frequency components of the disturbance $d_1(s)$ that are different from those of the periodic reference input $r(s)$, that is, $s_d \neq s_i$, even if

$$1 - \frac{N(s_d)\bar{Q}_2(s_d)}{Y(s_d) - N(s_d)Q_2(s_d)} \simeq 0, \tag{53}$$

the disturbance $d_1(s)$ cannot be attenuated because

$$e^{-s_d T} \neq 1 \tag{54}$$

and

$$1 - \frac{N(s_d)\bar{Q}_2(s_d)e^{-s_d T}}{Y(s_d) - N(s_d)Q_2(s_d)} \neq 0. \tag{55}$$

To attenuate the frequency components s_d of the disturbance $d_1(s)$ that are different from those of the periodic reference input $r(s)$, we must choose $Q_2(s)$ to satisfy

$$Y(s_d) - N(s_d)Q_2(s_d) \simeq 0. \tag{56}$$

Thus, the role of $Q_1(s)$ in (10) is different from the roles of $Q_2(s)$ and $\bar{Q}_2(s)$ in (10) and (11). The role of $Q_1(s)$ is to specify the input-output characteristic for the periodic reference input $r(s)$, whereas the role of $\bar{Q}_2(s)$ is to specify the disturbance attenuation characteristic for the frequency components of the disturbance $d_1(s)$ that are the same as those of the periodic reference input $r(s)$, and the role of $Q_2(s)$ is to specify the disturbance attenuation characteristic for the frequency components of the disturbance $d_1(s)$ that are different from those of the periodic reference input $r(s)$. That is, we find that the two-degrees-of-freedom simple repetitive control system can specify the input-output characteristic and the disturbance attenuation characteristic separately.

5. **Design Procedure.** In this section, we present a design procedure for stabilizing the two-degrees-of-freedom simple repetitive controller satisfying Theorem 3.1, as follows.

Procedure

Step 1) Obtain coprime factors $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ of $G(s)$ that satisfy (12).

Step 2) Choose $X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ to satisfy (13).

Step 3) Choose $Q_2(s) \in RH_\infty$ so that for the frequency component s_d of the disturbance $d_1(s)$, $|Y(s_d) - N(s_d)Q_2(s_d)|$ is effectively small. $Q_2(s)$ is chosen according to

$$Q_2(s) = \frac{Y(s)}{N_o(s)} q_{d1}(s), \tag{57}$$

where $N_o(s) \in RH_\infty$ is an outer function of $N(s)$ satisfying

$$N(s) = N_i(s)N_o(s), \tag{58}$$

$N_i(s) \in RH_\infty$ is an inner function satisfying $N_i(0) = 1$, $q_{d1}(s)$ is a low-pass filter satisfying $q_{d1}(0) = 1$ so that

$$q_{d1}(s) = \frac{1}{(1 + s\tau_{d1})^{\alpha_{d1}}} \tag{59}$$

is valid, α_{d1} is an arbitrary positive integer that ensures that $q_{d1}(s)/N_o(s)$ is proper and $\tau_{d1} \in R$ is any positive real number satisfying

$$1 - N_i(s_d)\frac{1}{(1 + s_d\tau_{d1})^{\alpha_{d1}}} \simeq 0. \tag{60}$$

Step 4) Choose $\bar{Q}_2(s) \in RH_\infty$ so that for the frequency components s_i $(i = 0, 1, \ldots, n)$ of the disturbance $d_1(s)$, $1 - N(s_i)\bar{Q}_2(s_i)/(Y(s_i) - N(s_i)Q_2(s_i)) \simeq 0$ is satisfied. To design $\bar{Q}_2(s)$ to hold $1 - N(s_i)\bar{Q}_2(s_i)/(Y(s_i) - N(s_i)Q_2(s_i)) \simeq 0$, $\bar{Q}_2(s) \in RH_\infty$ is chosen according to

$$\bar{Q}_2(s) = \frac{Y(s) - N(s)Q_2(s)}{N_o(s)} q_{d2}(s), \tag{61}$$

where $q_{d2}(s)$ is a low-pass filter satisfying $q_{d2}(0) = 1$ so that

$$q_{d2}(s) = \frac{1}{(1 + s\tau_{d2})^{\alpha_{d2}}} \tag{62}$$

is valid, α_{d2} is an arbitrary positive integer that ensures that $q_{d2}(s)/N_o(s)$ is proper and $\tau_{d2} \in R$ is any positive real number satisfying

$$1 - N_i(s_i)\frac{1}{(1 + s_i\tau_{d2})^{\alpha_{d2}}} \simeq 0 \quad (\forall i = 0, 1, \ldots, n). \tag{63}$$

Step 5) Choose $Q_1(s) \in H_\infty$ so that for s_i $(i = 0, 1, \ldots, n)$ of the periodic reference input $r(s)$, $1 - N(s_i)Q_1(s_i) \simeq 0$ $(\forall i = 0, 1, \ldots, n)$ is satisfied. To design $Q_1(s)$ to hold $1 - N(s_i)Q_1(s_i) \simeq 0$ $(\forall i = 0, 1, \ldots, n)$, $Q_1(s) \in H_\infty$ is chosen according to

$$Q_1(s) = \frac{1}{N_o(s)} q_r(s), \tag{64}$$

where $q_r(s)$ is a low-pass filter satisfying $q_r(0) = 1$ so that

$$q_r(s) = \frac{1}{(1 + s\tau_r)^{\alpha_r}} \tag{65}$$

is valid, α_r is an arbitrary positive integer that ensures $q_r(s)/N_o(s)$ is proper and $\tau_r \in R$ is any positive real number satisfying

$$1 - N_i(s_i)\frac{1}{(1+s_i\tau_r)^{\alpha_r}} \simeq 0 \quad (\forall i = 0, \ldots, n). \tag{66}$$

6. **Numerical Example.** In this section, a numerical example is presented to demonstrate the effectiveness of the proposed method.

We consider the problem of obtaining the parameterization of all stabilizing two-degrees-of-freedom simple repetitive controllers for the plant $G(s)$ written as

$$G(s) = \frac{s+5}{(s-2)(s+9)}, \tag{67}$$

which follows the periodic reference input $r(t)$ with period $T = 1$[sec].

A pair of coprime factors $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ of $G(s)$ in (67) satisfying (12) is given by

$$N(s) = \frac{s+5}{s^2+13s+42} \tag{68}$$

and

$$D(s) = \frac{s^2+7s-18}{s^2+13s+42}. \tag{69}$$

$X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ satisfying (13) are derived as

$$X(s) = -\frac{70.78s+624.3}{s^2+13s+44.46} \tag{70}$$

and

$$Y(s) = \frac{s^2+19s+69.68}{s^2+13s+44.46}. \tag{71}$$

From Theorem 3.1, the parameterization of all stabilizing two-degrees-of-freedom simple repetitive controllers for $G(s)$ in (67) is given by (9), where $Q_1(s) \in H_\infty$ in (10) is any function that has a finite number of poles and satisfies (14), $Q_2(s) \in RH_\infty$, and $\bar{Q}_2(s) \neq 0 \in RH_\infty$ in (11) are any functions satisfying (15).

In order that the disturbances

$$d_1(t) = \sin(4\pi t) \tag{72}$$

and

$$d_1(t) = \sin(\pi t) \tag{73}$$

can be attenuated effectively and for the output $y(t)$ to follow the periodic reference input

$$r(t) = \sin(2\pi t) \tag{74}$$

with a small steady-state error, $Q_2(s)$, $\bar{Q}_2(s)$ and $Q_1(s)$ are chosen by (57), (61) and (64), respectively, where

$$q_r(s) = \frac{1}{0.001s+1}, \tag{75}$$

$$q_{d1}(s) = \frac{1}{0.02s+1}, \tag{76}$$

$$q_{d2}(s) = \frac{1}{0.01s+1}, \tag{77}$$

$$N_i(s) = 1 \tag{78}$$

and

$$N_o(s) = N(s). \tag{79}$$

Using these parameters, we have a stabilizing two-degrees-of-freedom simple repetitive controller.

The response to the error $e(t) = r(t) - y(t)$ in Figure 1 for the periodic reference input $r(t)$ in (74) derived using this controller is shown in Figure 2. In the figure, the dotted line shows the response to the periodic reference input $r(t)$ in (74) and the solid line shows the response to the error $e(t) = r(t) - y(t)$. Thus, Figure 2 shows that the output $y(t)$ follows the periodic reference input $r(t)$ with a small steady-state error.

Next, the disturbance attenuation characteristic derived using the designed controller $C(s)$ is shown. The response of the output $y(t)$ to the disturbance $d_1(t)$ in (72), in which

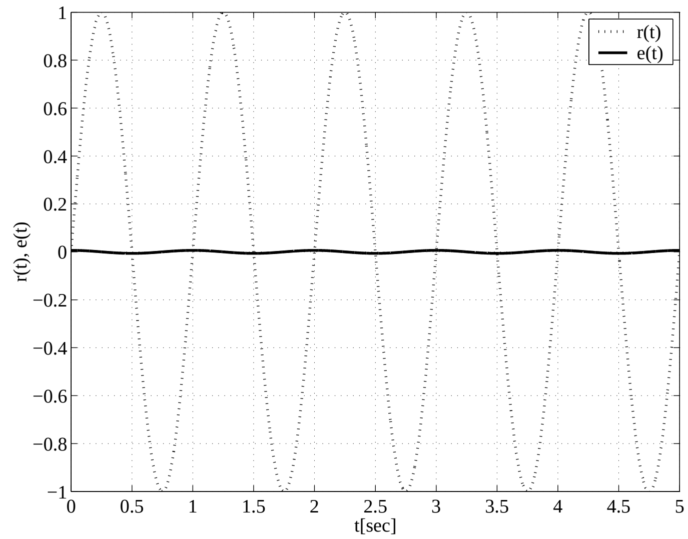

FIGURE 2. Response to the error $e(t) = r(t) - y(t)$ for the periodic reference input $r(t) = \sin(2\pi t)$

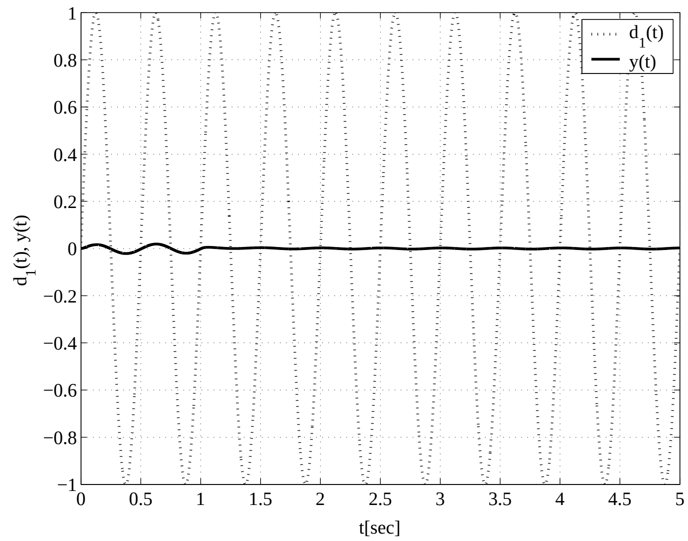

FIGURE 3. Response of the output $y(t)$ to the disturbance $d_1(t) = \sin(4\pi t)$

the frequency component is equivalent to that of the periodic reference input $r(t)$, is shown in Figure 3. In the figure, the dotted line shows the response to the disturbance $d_1(t)$ in (72) and the solid line shows that of the output $y(t)$. Thus, Figure 3 shows that the disturbance $d_1(t)$ in (72) is attenuated effectively.

Finally, the response of the output $y(t)$ to the disturbance $d_1(t)$ in (73) having a frequency component different from that of the periodic reference input $r(t)$ is shown in Figure 4. In the figure, the dotted line shows the response to the disturbance $d_1(t)$ in (73) and the solid line shows the output $y(t)$. Thus, Figure 4 shows that the disturbance $d_1(t)$ in (73) is attenuated effectively.

To demonstrate the effectiveness of the proposed method, its result is compared with the response of the output using the one-degree-of-freedom simple repetitive control system in [14]. According to [14], the parameterization of all stabilizing one-degree-of-freedom simple repetitive controllers $\bar{C}(s)$ that can stabilize the control system written as

$$\begin{cases} y(s) = G(s)\left(u(s) + d_1(s)\right) \\ u(s) = \bar{C}(s)(r(s) - y(s)) \end{cases} \tag{80}$$

is given by (11), that is, $\bar{C}(s)$ is written as

$$\bar{C}(s) = \frac{X(s) + D(s)\left(Q_2(s) + \bar{Q}_2(s)e^{-sT}\right)}{Y(s) - N(s)\left(Q_2(s) + \bar{Q}_2(s)e^{-sT}\right)}, \tag{81}$$

where $Q_2(s) \in RH_\infty$ and $\bar{Q}_2(s) \neq 0 \in RH_\infty$ are any functions satisfying (15). We design a stabilizing one-degree-of-freedom simple repetitive controller $\bar{C}(s)$ using the same parameters as those used to design the feedback controller $C_2(s)$ of the stabilizing two-degrees-of-freedom simple repetitive controller, that is, $\bar{C}(s) = C_2(s)$.

Using this one-degree-of-freedom simple repetitive controller, the response to the error $e(t) = r(t) - y(t)$ for the periodic reference input $r(t)$ in (74) is shown in Figure 5. In the figure, the dotted line shows the response for the periodic reference input $r(t)$ and the solid line shows that to the error $e(t)$. Thus, Figure 5 shows that the output $y(t)$ follows the periodic reference input $r(t)$ in (74) with a small steady-state error.

Next, using the designed one-degree-of-freedom simple repetitive controller $C(s)$, the disturbance attenuation characteristics are shown. The response of the output $y(t)$ to the disturbance $d_1(t)$ in (72) having a frequency component equivalent to that of the periodic

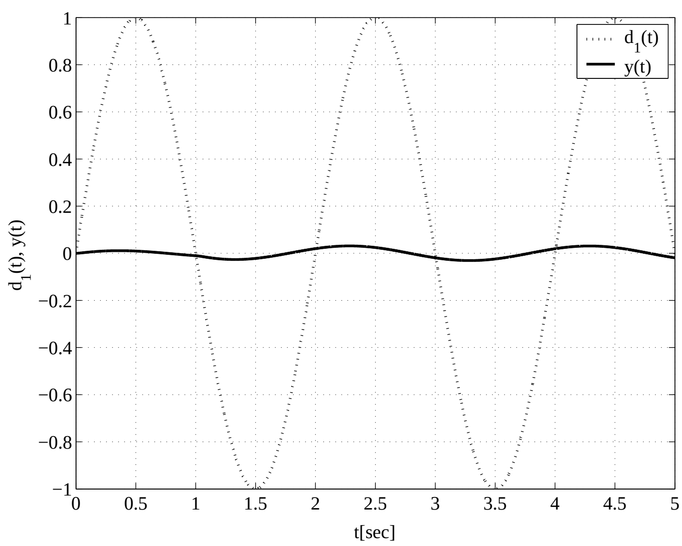

FIGURE 4. Response of the output $y(t)$ to the disturbance $d_1(t) = \sin(\pi t)$

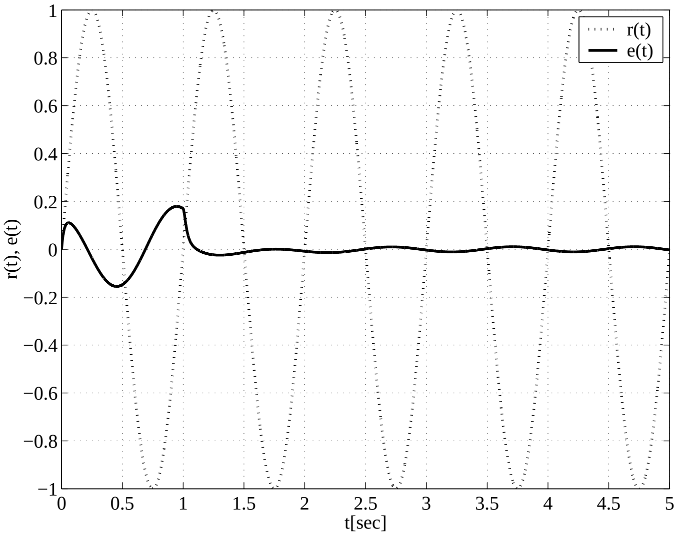

FIGURE 5. Response to the error $e(t) = r(t) - y(t)$ for the periodic reference input $r(t) = \sin(2\pi t)$ using a one-degree-of-freedom control system

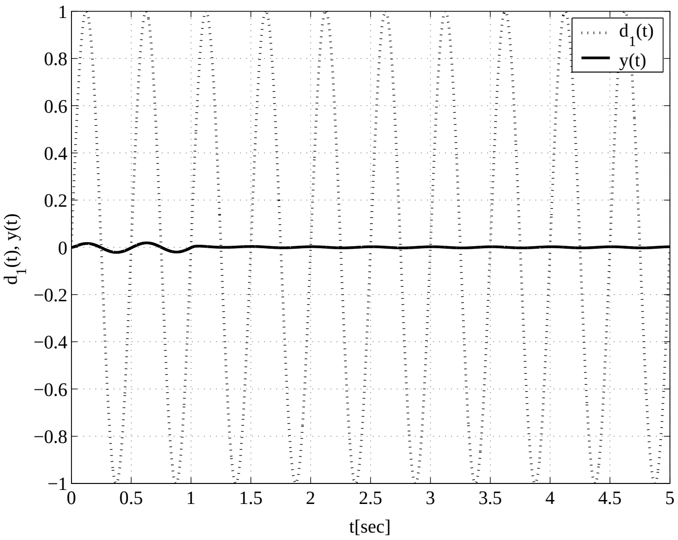

FIGURE 6. Response of the output $y(t)$ to the disturbance $d_1(t) = \sin(4\pi t)$ using a one-degree-of-freedom control system

reference input $r(t)$ is shown in Figure 6. In the figure, the dotted line shows the response to the disturbance $d_1(t)$ in (72) and the solid line shows that of the output $y(t)$. Thus, Figure 6 shows that the disturbance $d_1(t)$ in (72) is attenuated effectively.

Finally, the response of the output $y(t)$ to the disturbance $d_1(t)$ in (73) with a different frequency component from that of the periodic reference input $r(t)$ is shown in Figure 7. In the figure, the dotted line shows the response to the disturbance $d_1(t)$ in (73) and the solid line shows that of the output $y(t)$. Figure 7 shows that the disturbance $d_1(t)$ in (73) is attenuated effectively.

The comparison of Figure 3 and Figure 4 with Figure 6 and Figure 7 shows that the two-degrees-of-freedom control system and the one-degree-of-freedom control system have similar disturbance attenuation characteristics. In addition, the comparison of Figure 2 with Figure 5 shows that the convergence speed of the two-degrees-of-freedom control system is faster than that of the one-degree-of-freedom control system. The reason is

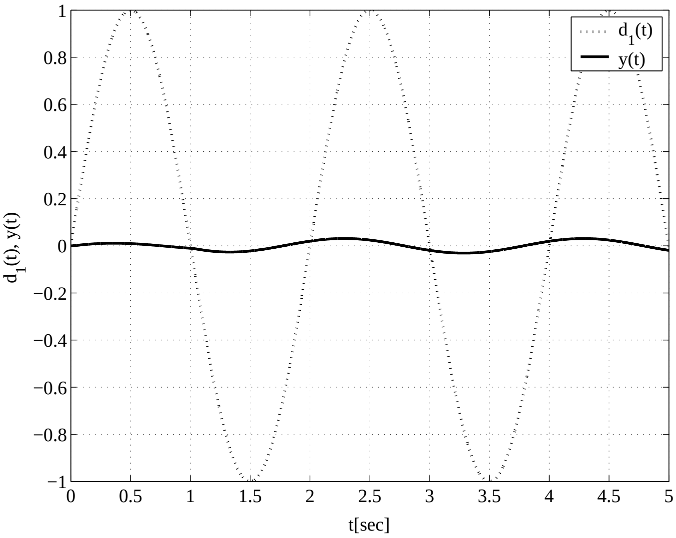

FIGURE 7. Response of the output $y(t)$ to the disturbance $d_1(t) = \sin(\pi t)$ using a one-degree-of-freedom control system

that the input-output characteristic of the two-degrees-of-freedom control system can be specified independently of the disturbance attenuation characteristic. That is, the input-output characteristic can be improved using the feed-forward controller.

Therefore, using the method shown here, a stabilizing two-degrees-of-freedom simple repetitive controller can be easily designed.

7. **Application: Reducing Rotational Unevenness in Motors.** In this section, to demonstrate the effectiveness of the parameterization of all stabilizing two-degrees-of-freedom simple repetitive controllers for real plants, we present an application of reducing variations in rotation in motors.

7.1. **Motor control experiment and problem description.** A motor control experiment is illustrated in Figure 8. The motor control experiment consists of a direct-current motor with an optical encoder of 1000 [counts/revolution] and a wheel that has a diameter of 50.7 [mm], a width of 10.3 [mm] and mass of 72.5 [g] attached to the motor. We denote

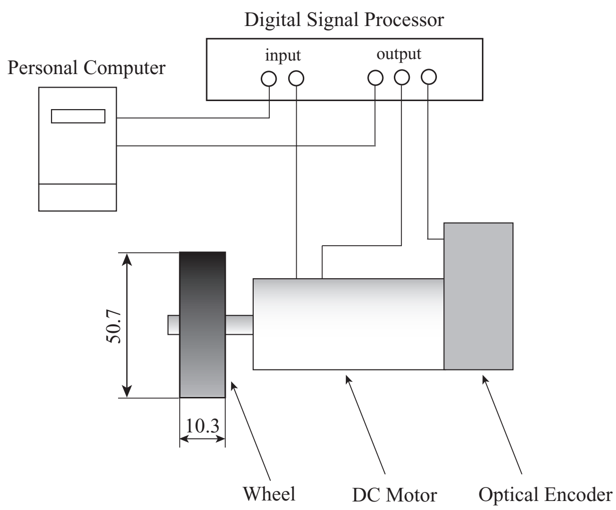

FIGURE 8. Motor control experiment

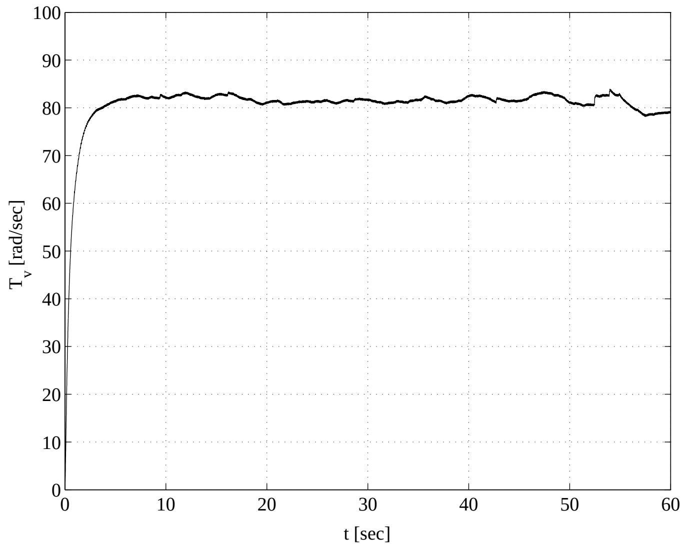

FIGURE 9. Response of T_v when $V_m = 2.1$[V]

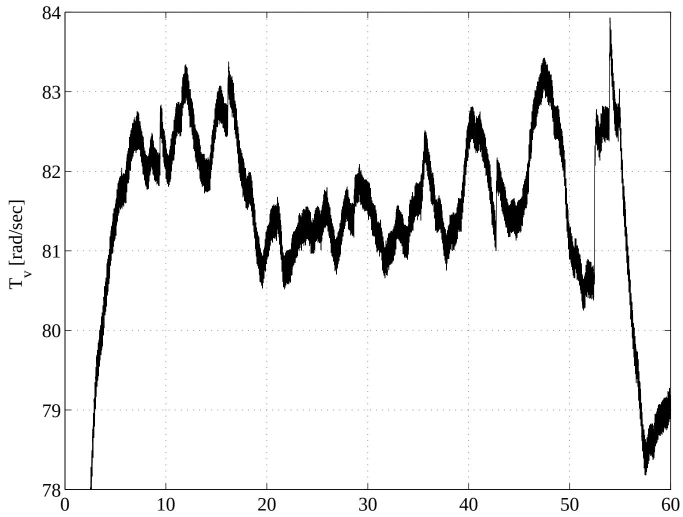

FIGURE 10. Magnified detail drawing showing Figure 9 between 78[rad/sec] and 84[rad/sec]

by T_v[rad/sec] the estimated value of the angular velocity of the wheel calculated from the measurement of the angle of the wheel. V_m denotes a control input for the direct-current motor, and the available voltage of V_m is $-24[\mathrm{V}] \leq V_m \leq 24[\mathrm{V}]$. When we set $V_m = 2.1$[V], the response of T_v, which is the angular velocity of the wheel, is shown in Figure 9 and Figure 10. Figure 9 and Figure 10 show that there are disturbances including rotational variation in the motor. Because the rotational unevenness in the motor depends on the angle of the motor, the disturbance is considered to be a periodic disturbance.

The problem considered in this experiment is to design a control system to attenuate periodic disturbances including the rotational variation in the motor by parameterizing all stabilizing two-degrees-of-freedom simple repetitive controllers described in this paper, to create an effective compensator for attenuating periodic disturbances.

7.2. **Experimental results.** In this subsection, we present the experimental results for controlling the angular velocity in the motor control experiment in Figure 8 using the parameterization of all stabilizing two-degrees-of-freedom simple repetitive controllers.

From Figure 9, we find that the transfer function from V_m to T_v is

$$T_v = \frac{39}{1+0.60s}V_m. \tag{82}$$

T_v and V_m are considered to be the output $y(s)$ and the control input $u(s)$ in the control system. $G(s)$ is then written as

$$G(s) = \frac{39}{1+0.60s} \in RH_\infty. \tag{83}$$

The reference input $r(s)$ is chosen as $r(t) = v_r = 100$[rad/sec]. The period T of the disturbance $d(t)$ caused by the rotational variation in the motor is

$$T = \frac{2\pi}{v_r} = \frac{2\pi}{100}. \tag{84}$$

To attenuate the periodic disturbance $d(t)$ with period T, we design a two-degrees-of-freedom simple repetitive controller $C(s)$ in (9). Coprime factors $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ of the plant $G(s)$ in (83) on RH_∞ are given by

$$N(s) = \frac{65}{s+1} \tag{85}$$

and

$$D(s) = \frac{s+1.67}{s+1}. \tag{86}$$

A pair of $X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ satisfying $N(s)X(s) + D(s)Y(s) = 1$ is written as

$$X(s) = \frac{0.0068}{s+1} \tag{87}$$

and

$$Y(s) = \frac{s-0.33}{s+1}. \tag{88}$$

Using these parameters, the parameterization of all stabilizing two-degrees-of-freedom simple repetitive controllers for $G(s)$ in (83) is given by (9), where $Q_1(s) \in H_\infty$ in (10) is any function that has a finite number of poles and satisfies (14), $Q_2(s) \in RH_\infty$, and $\bar{Q}_2(s) \neq 0 \in RH_\infty$ in (11) are any functions satisfying (15).

$Q_2(s)$, $\bar{Q}_2(s)$ and $Q_1(s)$ are selected using (57), (61) and (64), respectively, where

$$q_r(s) = \frac{1}{0.3s+1}, \tag{89}$$

$$q_{d1}(s) = \frac{1}{0.015s+1}, \tag{90}$$

$$q_{d2}(s) = \frac{1}{0.77s+1} \tag{91}$$

and

$$N_o(s) = N(s). \tag{92}$$

Using these parameters, we have a stabilizing two-degrees-of-freedom simple repetitive controller.

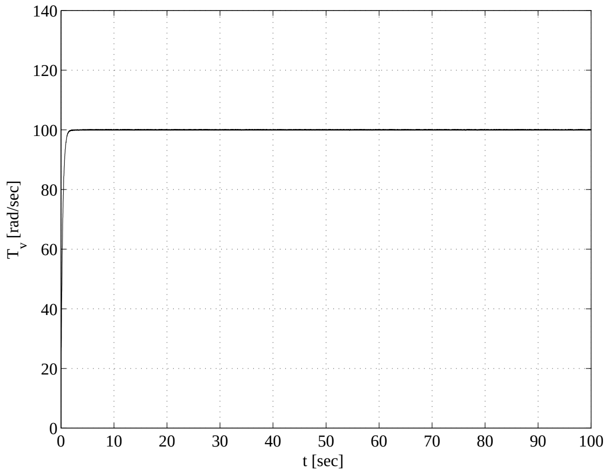

FIGURE 11. Response of the output $y(t)$, which is the angular velocity of the wheel T_v, for the reference input $r(t) = 100$[rad/sec] using the two-degrees-of-freedom simple repetitive controller

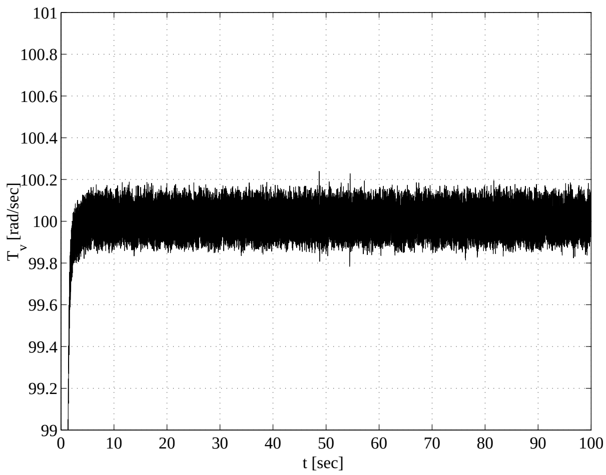

FIGURE 12. Magnified plot of Figure 11 between 99[rad/sec] and 101[rad/sec]

With this controller $C(s)$, the response of the output $y(t)$, which is the angular velocity of the wheel T_v, for the reference input $r(t) = 100$[rad/sec] is shown in Figure 11, Figure 12 and Figure 13. Figure 11, Figure 12 and Figure 13 show that the output $y(t)$, which is the angular velocity of the wheel T_v, follows the reference input $r(t) = 100$[rad/sec] with a small steady-state error. In addition, the disturbance $d(t)$ that includes the rotational variation in the motor is attenuated effectively.

To demonstrate the effectiveness of the two-degrees-of-freedom simple repetitive controller, the response is compared with the response when using the parameterization of all stabilizing one-degree-of-freedom simple repetitive controllers in (80), where one-degree-of-freedom controller $\bar{C}(s)$ is written as (81), where $Q_2(s) \in RH_\infty$ and $\bar{Q}_2(s) \neq 0 \in RH_\infty$ are any functions satisfying (15).

To ensure a fair comparison of the two-degrees-of-freedom simple repetitive controller and the one-degree-of-freedom simple repetitive controller, we design a stabilizing one-degree-of-freedom simple repetitive controller using the same parameters as were used to design the feedback controller $C_2(s)$ of the stabilizing two-degrees-of-freedom simple repetitive controller $C(s)$. The response of the output $y(t)$, which is the angular velocity of the

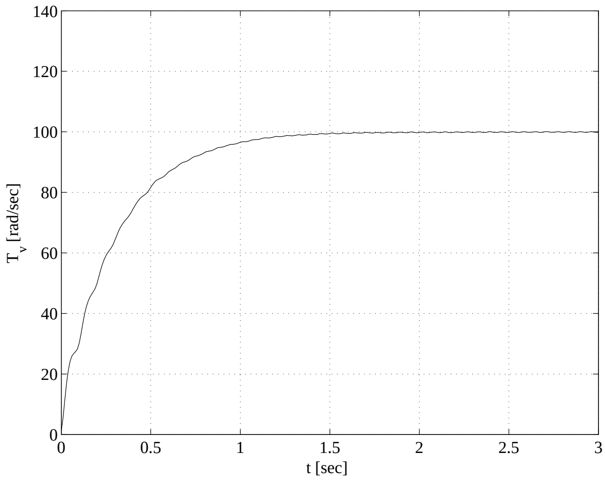

FIGURE 13. Magnified plot of Figure 11 between 0[sec] and 3[sec]

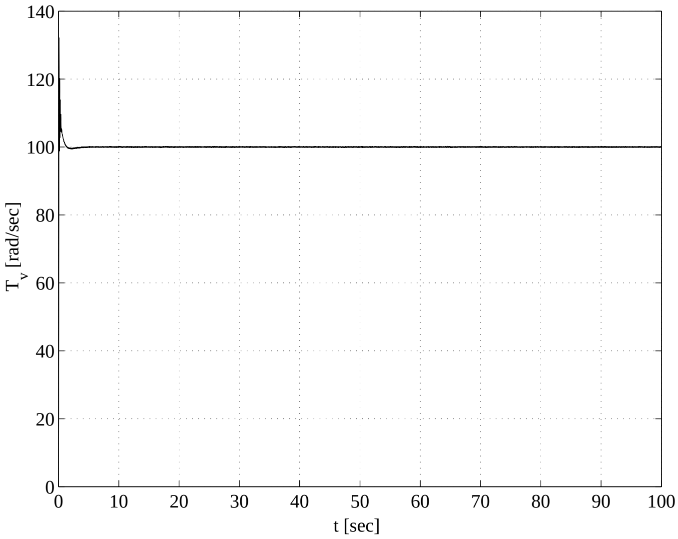

FIGURE 14. Response of the output $y(t)$, which is the angular velocity of the wheel T_v, for the reference input $r(t) = 100$[rad/sec] using the one-degree-of-freedom simple repetitive controller

wheel T_v, for the reference input $r(t) = 100$[rad/sec], derived using the designed stabilizing one-degree-of-freedom simple repetitive controller $\bar{C}(s)$, is shown in Figure 14, Figure 15 and Figure 16. Figure 14, Figure 15 and Figure 16 show that the output $y(t)$, which is the angular velocity of the wheel T_v, follows the reference input $r(t) = 100$[rad/sec] with a small steady-state error. In addition, the disturbance $d(t)$ that includes the rotational variation of the motor is attenuated effectively.

The comparison of Figure 12 and Figure 13 with Figure 15 and Figure 16 shows that the two-degrees-of-freedom control system and the one-degree-of-freedom control system have the same disturbance attenuation characteristics. In addition, the two-degrees-of-freedom control system follows the reference input $r(t) = 100$[rad/sec] without overshoot and reduces the vibration in transition zones. As shown, advantages of the two-degrees-of-freedom simple repetitive control system include that the transient characteristics can be improved using the feed-forward controller and the system is easy to design. This result illustrates that the two-degrees-of-freedom simple repetitive control system is more

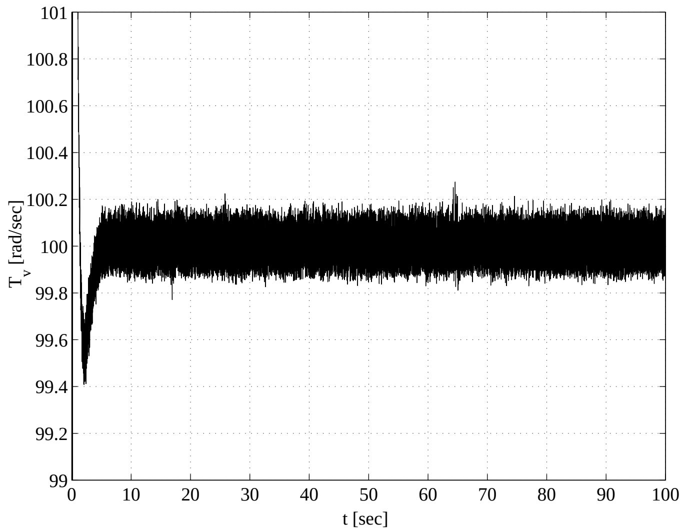

FIGURE 15. Magnified plot of Figure 14 between 99[rad/sec] and 101[rad/sec]

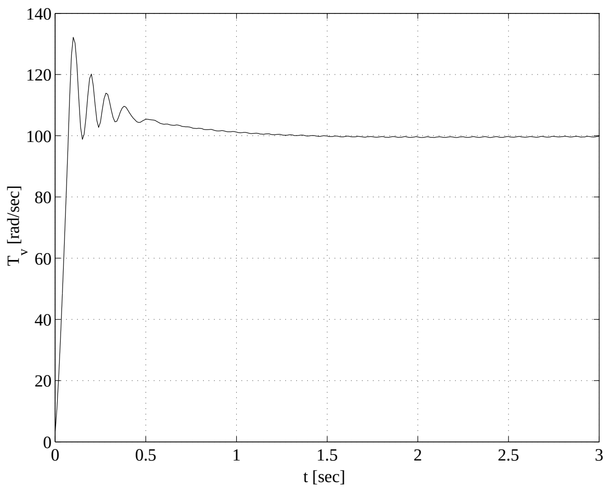

FIGURE 16. Magnified plot of Figure 14 between 0[sec] and 3[sec]

effective for the reduction of rotational unevenness in motors than the one-degree-of-freedom simple repetitive control system.

Thus, we have demonstrated the effectiveness of the control system employing the parameterization of all stabilizing two-degrees-of-freedom simple repetitive controllers in (9) with (10) and (11) for real plants.

8. **Conclusions.** In this paper, we have given a complete proof of the parameterization of all stabilizing two-degrees-of-freedom simple repetitive controllers such that the feedback controller works as a stabilizing modified repetitive controller and transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles. The control characteristics of a two-degrees-of-freedom simple repetitive control system were presented, along with a design procedure for a stabilizing two-degrees-of-freedom simple repetitive controller.

Design methods for conventional simple repetitive control systems [14] cannot specify the input-output characteristic and the disturbance attenuation characteristic separately.

However, our proposed method can easily design a simple repetitive control system that has the desired input-output and disturbance attenuation characteristics.

Finally, a numerical example and an application for the reduction of rotational unevenness in motors were presented to illustrate the effectiveness of the proposed method. Advantages of the two-degrees-of-freedom simple repetitive control system are that its input-output characteristics can be improved using the feed-forward controller and the system is easy to design. This control system is expected to have practical applications in, for example, engines, electrical motors and generators, converters, and other machines that perform cyclic tasks.

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