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NEW ENERGY-TO-PEAK FIR FILTER DESIGN FOR SYSTEMS WITH DISTURBANCE: A CONVEX OPTIMIZATION APPROACH

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Abstract. *This paper proposes a new energy-to-peak filter (EPF) with a finite impulse response (FIR) structure for linear systems with disturbance. The filter is referred to as an energy-to-peak FIR filter (EPFF). The energy-to-peak performance criterion is minimized in filters that show linearity, FIR structure, and quasi-deadbeat property. The EPFF can be obtained by solving a convex optimization problem represented by linear matrix inequalities (LMIs). A numerical example is given to illustrate the validity of the proposed EPFF.*

Keywords: Energy-to-peak filter, Finite impulse response (FIR), Quasi-deadbeat condition, State estimation, Linear systems

1. **Introduction.** The celebrated Kalman filter and its applications have undergone extensive investigation over the past four decades. However, the Kalman filtering scheme is no longer applicable when information regarding external noise is not precisely known. In these cases, alternative approaches can include energy-to-energy ($\mathcal{H}_\infty$) filtering [1, 2, 3, 4] and energy-to-peak ($\mathcal{L}_2 - \mathcal{L}_\infty$ or $l_2 - l_\infty$) filtering [3, 4, 5, 6, 7, 8, 9, 10, 11], but only infinite impulse response (IIR) filters [12, 13, 14, 15, 16, 26] have been introduced for these types of filtering problems. Nevertheless, undesirable signals from past information can accumulate inside IIR filters, so that they may diverge for systems with modeling uncertainties and numerical errors [17, 18, 19].

Filters with finite impulse response (FIR) structure are preferable for nonmodel signal models used in signal processing, because this structure eliminates the accumulation of undesirable effects. The guaranteed stability, the robustness to numerical error and temporary uncertainties, and perfect signal reconstruction (such as a linear phase property) are well-known desirable properties of the FIR structure. Signal models have been represented by general state-space models and several trials have applied the FIR structure to the design of filters including the recursive limited memory filter [20], the optimal FIR filter [21], and unbiased FIR filters [22, 23]. Recently, some robust FIR filters with disturbances and their application to output feedback controls were proposed in [24, 25, 26] and [27, 28], respectively. However, as far as we are aware, no results have yet been published on FIR filters that take into consideration the energy-to-peak performance criterion, even though energy-to-peak filtering is widely used in many practical applications such as electrical circuits, navigation systems, and communication systems, and in estimation of civil structures. Therefore, the energy-to-peak FIR filtering problem still remains unresolved and challenging.

In this paper, a new FIR filter is proposed, based on the energy-to-peak performance criterion. This filter is referred to as an energy-to-peak FIR filter (EPFF) and can be obtained by solving a convex optimization problem via linear matrix inequalities (LMIs). The proposed FPFF is both quasi-deadbeat and optimal *by design* for the energy-to-peak performance criterion. The '*by design*' refers to the fact that both the quasi-deadbeat property and optimality are simultaneously built into the proposed EPFF during its design.

This paper is organized as follows. In Section 2, the EPFF in an LMI form is proposed for discrete-time state-space models. In Section 3, a numerical example is given. Finally, conclusion is stated in Section 4.

2. **New Energy-to-Peak FIR Filter Design.** Consider a linear discrete-time state-space model:

$$x_{k+1} = Ax_k + Gw_k, \tag{1}$$

$$y_k = Cx_k + Dw_k, \tag{2}$$

where $x_k \in \Re^n$, $y_k \in \Re^q$, and $w_k \in \Re^p$ are the state, the output, and the disturbance, respectively. On the horizon $[k-N, k]$ where N is the horizon size, outputs are expressed in terms of the state x_k at the time k as follows:

$$Y_{k-1} = \bar{C}_N x_k + (\bar{G}_N + \bar{D}_N)W_{k-1}, \tag{3}$$

where

$$Y_{k-1} \triangleq [y_{k-N}^T \ y_{k-N+1}^T \ \cdots \ y_{k-1}^T]^T, \tag{4}$$

$$W_{k-1} \triangleq [w_{k-N}^T \ w_{k-N+1}^T \ \cdots \ w_{k-1}^T]^T, \tag{5}$$

and $\bar{C}_N$, $\bar{G}_N$ and $\bar{D}_N$ are obtained from

$$\begin{aligned}
\bar{C}_N &\triangleq \begin{bmatrix} CA^{-N} \\ \vdots \\ CA^{-2} \\ CA^{-1} \end{bmatrix}, \\
\bar{G}_N &\triangleq \begin{bmatrix} CA^{-1}G & CA^{-2}G & \cdots & CA^{-N}G \\ 0 & CA^{-1}G & \cdots & CA^{-N+1}G \\ 0 & 0 & \cdots & CA^{-N+2}G \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & CA^{-1}G \end{bmatrix}, \\
\bar{D}_N &\triangleq \left[\mathrm{diag}(\overbrace{D,\ D,\ \cdots,\ D}^{N})\right].
\end{aligned} \tag{6}$$

The EPFF can be expressed as a linear function of the finite outputs Y_{k-1} on the horizon $[k-N, k]$ as follows:

$$\begin{aligned}
\hat{x}_k &\triangleq \sum_{k-N}^{k-1} H_{k-i} y_i \\
&= HY_{k-1},
\end{aligned} \tag{7}$$

where the gain matrix H is defined as

$$H \triangleq [H_N \; H_{N-1} \; \cdots \; H_1]. \tag{8}$$

From (3), the EPFF (7) can be rewritten as

$$\hat{x}_k = H\bar{C}_N x_k + H(\bar{G}_N + \bar{D}_N)W_{k-1}. \tag{9}$$

We require that the EPFF (7) is independent of any *a priori* information about the horizon initial state x_{k-N}. Furthermore, the EPFF (7) must satisfy

$$\hat{x}_k = x_k \;\text{ for } w_i = 0 \quad (k-N \le i \le k-1). \tag{10}$$

This constraint will be called the quasi-deadbeat constraint [25], which is obtained by setting

$$H\bar{C}_N = I. \tag{11}$$

For a given level $\gamma > 0$, the gain matrix H of the EPFF is determined using the optimization problem based on the following energy-to-peak ($l_2 - l_\infty$) performance criterion:

$$\inf_{H} \sup_{w_k} \frac{\|e_k\|_{l_\infty}^2}{\|w_k\|_{l_2}^2} < \gamma^2, \tag{12}$$

subject to the constraint (11), where $\|e_k\|_{l_\infty}^2 = \sup_{k\ge 0}\{e_k^T e_k\}$ and $\|w_k\|_{l_2}^2 = \sum_{k=0}^{\infty} w_k^T w_k$. Before deriving the EPFF, we introduce the following lemma:

Lemma 2.1. [3, 5] *Let $\gamma > 0$. Then, for the system*

$$x_{k+1} = Ax_k + Bu_k, \tag{13}$$

$$z_k = Cx_k, \tag{14}$$

the following two conditions are equivalent:

(1) *The first is as follows:*

$$\sup_{u_k} \frac{\|z_k\|_{l_\infty}^2}{\|u_k\|_{l_2}^2} < \gamma^2. \tag{15}$$

(2) *There exists a $P = P^T > 0$ such that*

$$\begin{bmatrix} APA^T - P & B \\ B^T & -I \end{bmatrix} < 0,$$

$$\begin{bmatrix} \gamma^2 I & C \\ C^T & P \end{bmatrix} > 0.$$

Using Lemma 2.1, we can present a linear matrix inequality (LMI) problem for the EPFF.

Theorem 2.1. *Assume that $\{A, C\}$ of the model (1)-(2) is observable and $N \ge n$. If the following LMI problem is feasible:*

$$\min_{P>0,\; F} \; \gamma^2$$

subject to

$$\begin{bmatrix} A_u P A_u^T - P & B_u \\ B_u^T & -I \end{bmatrix} < 0, \tag{16}$$

$$\begin{bmatrix} \gamma^2 I & (FM + H_0)(\bar{G}_N + \bar{D}_N) \\ (\bar{G}_N + \bar{D}_N)^T (FM + H_0)^T & P \end{bmatrix} > 0, \tag{17}$$

where

$$A_u \triangleq \begin{bmatrix} 0 & I & 0 & \cdots & 0 \\ 0 & 0 & I & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & I \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \in \Re^{pN \times pN}, \quad B_u \triangleq \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ I \end{bmatrix} \in \Re^{pN \times p}, \tag{18}$$

$H_0 = (\bar{C}_N^T \bar{C}_N)^{-1} \bar{C}_N^T$, *and* M^T *is the bases of the null space of* $\bar{C}_N^T$, *then the optimal gain matrix of the EPFF is given by*

$$H = FM + H_0.$$

Proof: Based on definitions in (18), the disturbance w_k satisfies the following state model on W_{k-1} [24]:

$$W_k = A_u W_{k-1} + B_u w_k. \tag{19}$$

Substituting (11) into (9), we have the following estimation error:

$$\begin{aligned} e_k &\triangleq \hat{x}_k - x_k \\ &= H(\bar{G}_N + \bar{D}_N) W_{k-1}. \end{aligned} \tag{20}$$

Equations (19) and (20) are new state-space equations for obtaining the EPFF. Based on the result of Lemma 2.1, according to the following correspondences:

$$\begin{aligned} A &\longleftarrow A_u, & B &\longleftarrow B_u, \\ C &\longleftarrow H(\bar{G}_N + \bar{D}_N), & x_k &\longleftarrow W_{k-1}, \\ u_k &\longleftarrow w_k, & z_k &\longleftarrow e_k, \end{aligned}$$

the condition (12) is equivalent to the inequality (16) and the following inequality:

$$\begin{bmatrix} \gamma^2 I & H(\bar{G}_N + \bar{D}_N) \\ (\bar{G}_N + \bar{D}_N)^T H^T & P \end{bmatrix} > 0. \tag{21}$$

By minimizing γ^2 subject to (11), (16) and (21), we can obtain the optimal gain matrix H for the EPFF. The equality constraint (11) can be eliminated by computing the null space of $\bar{C}_N^T$. All solutions to the equality constraint $H\bar{C}_N = I$ are parameterized by $H = FM + H_0$, where F is an arbitrary matrix. Note that $\bar{C}_N^T \bar{C}_N$ is guaranteed to be nonsingular if $\{A, C\}$ is observable for $N \geq n$. By replacing H by $FM + H_0$, we can obtain the LMI condition (17) from (21). This completes the proof. ∎

Remark 2.1. *The proposed EPFF can be used in several output feedback control applications. For example, some modeling methods are applied to model physical dynamic systems and the states estimated by the proposed EPFF can be then utilized to achieve certain design objectives by the state feedback control law. Therefore, from the point of view of control, the proposed EPFF for dynamic systems is important for many applications.*

Remark 2.2. *Most existing results on energy-to-peak ($\mathcal{L}_2 - \mathcal{L}_\infty$ or $l_2 - l_\infty$) filtering in the literature were restricted to IIR filters* [3, 4, 5, 6, 7, 8, 9, 10, 11]. *Unfortunately, with the existing results, it is not possible to design an energy-to-peak FIR filter for systems with disturbances. For the first time, this paper presents an energy-to-peak FIR filter for linear systems with disturbances. The proposed result in this paper opens a new path for application of the energy-to-peak stability approach to the derivation of FIR filter for systems with disturbances.*

3. **Numerical Example.** The validity of the EPFF is illustrated by a numerical example that provides a comparison between the proposed EPFF and the existing energy-to-peak filter (EPF) [3, 4] for the following discrete-time state-space model:

$$x_{k+1} = \begin{bmatrix} 0.33 + 3\delta_k & 0.01 + \delta_k \\ 0.01 & 0.9 + 2\delta_k \end{bmatrix} x_k + \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} w_k,$$
$$y_k = \begin{bmatrix} 1 & 0 \end{bmatrix} x_k + \begin{bmatrix} 1 & 0 \end{bmatrix} w_k,$$

where δ_k is a model uncertain parameter, which is assumed to satisfy

$$\delta_k = \begin{cases} 0.1, & 100 \le k \le 150, \\ 0, & \text{otherwise.} \end{cases} \tag{22}$$

Figure 1 compares the estimation errors of the second state for the case where the exogenous input w_k is given by $w_k = \begin{bmatrix} w_{1k} & w_{2k} \end{bmatrix}^T$, where $w_{1k} \sim (0,1)$ and $w_{2k} \sim (0,1)$. This simulation result clearly shows that, due to FIR structure, the estimation error of the EPFF is remarkably smaller than that of the EPF on the interval where modeling uncertainty exists. In addition, the convergence of estimation error is much faster than that of the EPF after temporary modeling uncertainty disappears.

Next, we increase δ_k slightly when $100 \le k \le 150$. Figure 2 shows the estimation errors of the second state when δ_k is given by

$$\delta_k = \begin{cases} 0.12, & 100 \le k \le 150, \\ 0, & \text{otherwise.} \end{cases} \tag{23}$$

From Figure 2, it can be seen that the estimation error of the EPF diverges very fast for higher model uncertain parameter. However, the EPFF is relatively insensitive to the model uncertain parameter.

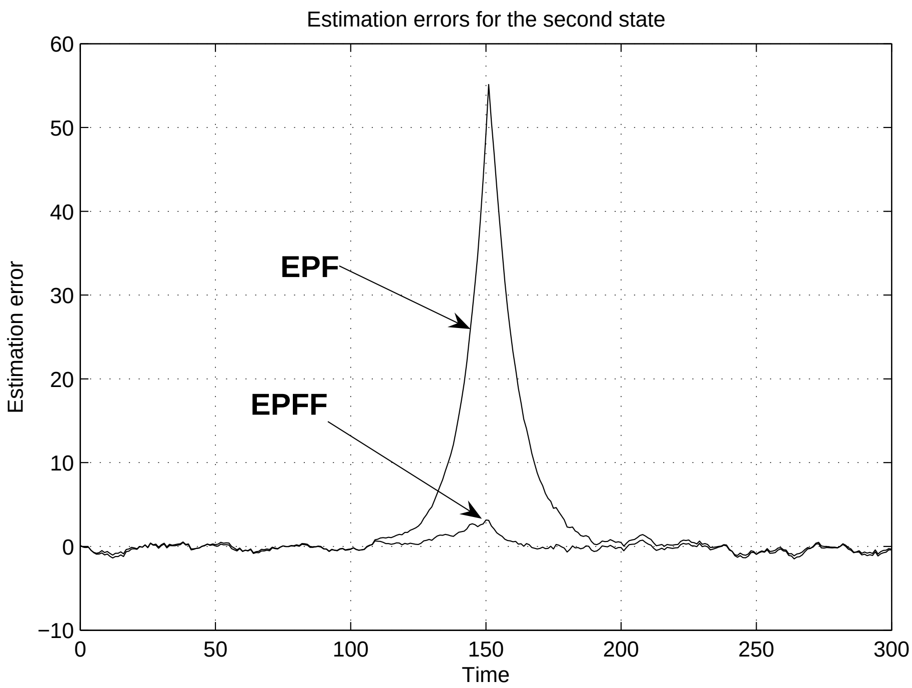

Figure 1. Estimation errors of the EPFF and the EPF when δ_k is given by (22)

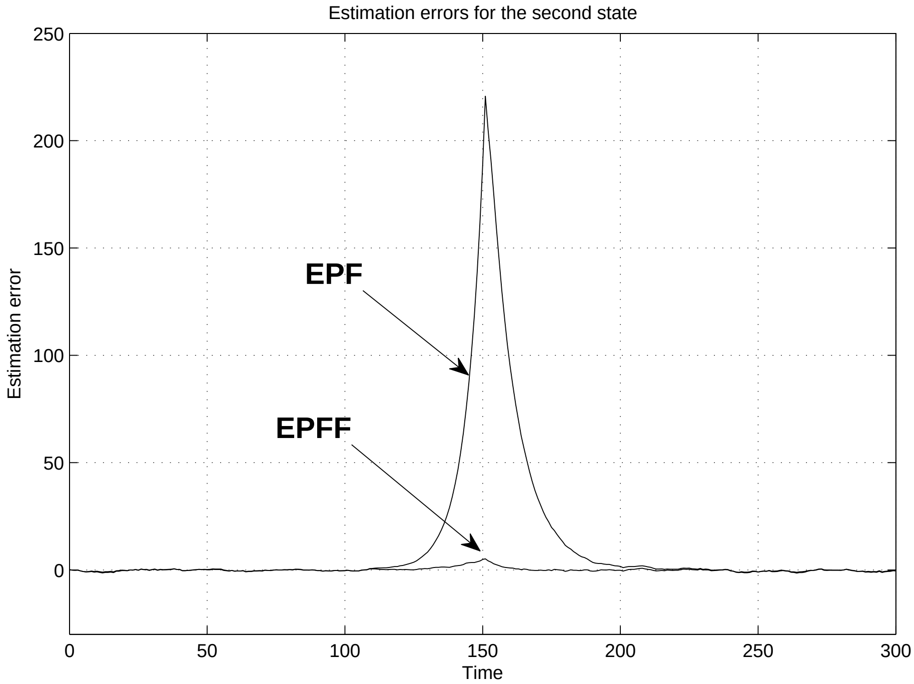

FIGURE 2. Estimation errors of the EPFF and the EPF when δ_k is given by (23)

4. **Conclusion.** This paper proposed a new energy-to-peak filter called the EPFF that allows estimation of unknown signals that can be represented by discrete-time state-space models with disturbance. The EPFF is obtained by solving the LMI problem with the parametrization of the linear equality constraint. The proposed filter does not require *a priori* information of the horizon initial state. The EPFF is linear, with finite outputs on the most recent horizon and exhibits the quasi-deadbeat property. Furthermore, the FIR structure of the EPFF is believed to impart a robustness to the proposed filter against round-off errors or temporary modeling uncertainties, whereas the IIR structure of the existing EPF may show poor robustness and even divergence phenomena. Therefore, we can expect that the proposed EPFF will be useful in many signal processing problems.

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REFERENCES

[1] M. Green and D. J. Limebeer, *Linear Robust Control*, Prentice Hall, Englewood Cliffs, New Jersey, 1995.
[2] K. Zhou, J. C. Doyle and K. Glover, *Robust and Optimal Control*, Prentice Hall, 1996.
[3] K. M. Grigoriadis and J. T. Watson, Reduced-order $\mathcal{H}_\infty$ and $\mathcal{L}_2 - \mathcal{L}_\infty$ filtering via linear matrix inequalities, *IEEE Trans. Aerosp. Electron. Syst.*, vol.33, pp.1326-1338, 1997.
[4] J. T. Watson and K. M. Grigoriadis, Optimal unbiased filtering via linear matrix inequalities, *Systems and Control Letters*, vol.35, pp.111-118, 1998.
[5] R. M. Palhares and P. L. D. Peres, Robust filtering with guaranteed energy-to-peak performance – An LMI approach, *Automatica*, vol.36, pp.851-858, 2000.
[6] H. Gao and C. Wang, Robust $\mathcal{L}_2 - \mathcal{L}_\infty$ fitering for uncertain systems with multiple time-varying state delays, *IEEE Trans. Trans. Circ. Syst. I*, vol.50, pp.594-599, 2003.
[7] H. Gao and C. Wang, Delay-dependent robust $\mathcal{H}_\infty$ and $\mathcal{L}_2 - \mathcal{L}_\infty$ fitering for a class of uncertain nonlinear time-delay systems, *IEEE Trans. Automat. Contr.*, vol.48, pp.1661-1666, 2003.

[8] M. S. Mahmoud, Resilient $\mathcal{L}_2 - \mathcal{L}_\infty$ filtering of polytopic systems with state delays, *IET Control Theory & Applications*, vol.1, no.1, pp.141-154, 2007.
[9] J. Qiu, G. Feng and J. Yang, New results on robust energy-to-peak filtering for discrete-time switched polytopic linear systems with time-varying delay, *IET Control Theory & Applications*, vol.2, no.9, pp.795-806, 2008.
[10] Y. Zhou and J. Li, Energy-to-peak filtering for singular systems: The discrete-time case, *IET Control Theory & Applications*, vol.2, no.9, pp.773-781, 2008.
[11] C. Gong and B. Su, Robust $\mathcal{L}_2 - \mathcal{L}_\infty$ filtering of convex polyhedral uncertain time-delay fuzzy systems, *International Journal of Innovative Computing, Information and Control*, vol.4, no.4, pp.793-802, 2008.
[12] P. Shi, M. Mahmoud, S. Nguang and A. Ismail, Robust filtering for jumping systems with mode-dependent delays, *Signal Processing*, vol.86, pp.140-152, 2006.
[13] D. Du, B. Jiang, P. Shi and S. Zhou, $\mathcal{H}_\infty$ filter of discrete-time switched systems with state delays via switched Lyapunov function approach, *IEEE Trans. on Automatic Control*, vol.52, no.8, pp.1520-1525, 2007.
[14] M. Basin, P. Shi and D. Calderon-Alvarez, Central suboptimal $\mathcal{H}_\infty$ filter design for linear time-varying systems with state and measurement delays, *Int. J. of Systems Science*, vol.41, no.4, pp.411-421, 2010.
[15] J. Zhang, Y. Xia and P. Shi, Parameter-dependent robust $\mathcal{H}_\infty$ filtering for uncertain discrete-time systems, *Automatica*, vol.45, no.2, pp.560-565, 2009.
[16] L. Wu, P. Shi, H. Gao and C. Wang, Robust $\mathcal{H}_\infty$ filtering for 2-D Markovian jump systems, *Automatica*, vol.44, no.7, pp.1849-1858, 2008.
[17] R. J. Fitzgerald, Divergence of the Kalman filter, *IEEE Trans. Automat. Contr.*, vol.16, pp.736-747, 1971.
[18] S. Sangsuk-Iam and T. E. Bullock, Analysis of discrete-time Kalman filtering under incorrect noise covariances, *IEEE Trans. Automat. Contr.*, vol.35, no.12, p.1304, 1990.
[19] M. S. Grewal and A. P. Anderews, *Kalman Filtering – Theory and Practice*, Englewood Cliffs, Prentice-Hall, NJ, 1993.
[20] A. M. Bruckstein and T. Kailath, Recursive limited memory filtering and scattering theory, *IEEE Trans. Inform. Theory*, vol.31, pp.440-443, 1985.
[21] W. H. Kwon and O. K. Kwon, FIR filters and recursive forms for continuous time-invariant state-space models, *IEEE Trans. Automat. Contr.*, vol.32, pp.352-356, 1987.
[22] W. H. Kwon, P. S. Kim and P. Park, A receding horizon Kalman FIR filter for linear continuous-time systems, *IEEE Trans. Automat. Contr.*, vol.44, no.11, pp.2115-2120, 1999.
[23] W. H. Kwon, P. S. Kim and S. H. Han, A receding horizon unbiased FIR filter for discrete-time state space models, *Automatica*, vol.38, no.3, pp.545-551, 2002.
[24] Y. S. Lee, S. H. Han and W. H. Kwon, $\mathcal{H}_2/\mathcal{H}_\infty$ FIR filters for discrete-time state space models, *International Journal of Control, Automation, and Systems*, vol.4, no.5, pp.645-652, 2006.
[25] C. Ahn, S. Han and W. Kwon, $\mathcal{H}_\infty$ FIR filters for linear continuous-time state-space systems, *IEEE Signal Processing Letters*, vol.13, no.9, pp.557-560, 2006.
[26] C. Ahn, Strictly passive FIR filtering for state-space models with external disturbance, *International Journal of Electronics and Communications*, vol.66, no.11, pp.944-948, 2012.
[27] C. Ahn, S. Han and W. Kwon, $\mathcal{H}_\infty$ finite memory controls for linear discrete-time state-space models, *IEEE Trans. Circuits and Systems II*, vol.54, no.2, pp.97-101, 2007.
[28] C. Ahn, Robustness bound for receding horizon finite memory control: Lyapunov-Krasovskii approach, *International Journal of Control*, vol.85, no.7, pp.942-949, 2012.