International Journal of Innovative
Computing, Information and Control
Volume **9**, Number **5**, May **2013**

 ISSN 1349-4198
pp. **2007–2018**

CHEMICAL OPTIMIZATION PARADIGM APPLIED TO A FUZZY TRACKING CONTROLLER FOR AN AUTONOMOUS MOBILE ROBOT

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Received February 2012; revised June 2012

Abstract. *This paper addresses the tracking problem for the dynamic model of a unicycle mobile robot. A novel optimization method inspired on the chemical reactions is applied to solve this motion problem by integrating a kinematic and a torque controller based on fuzzy logic theory. Computer simulations are presented confirming that this optimization paradigm is able to outperform other optimization techniques applied to this particular robot application.*
Keywords: Fuzzy control, Wheeled mobile robot, Optimization, Chemical reactions, Natural computing

1. **Introduction.** Optimization is an activity carried out in almost every aspect of our life, from planning the best route in our way back home from work to more sophisticated approximations at the stock market, or the parameter optimization for a wave solder process used in a printed circuit board assembly manufacturer optimization theory has gained importance over the last decades. From science to applied engineering (to name a few), there is always something to optimize and of course, more than one way to do it.

In a generic definition, we may say that optimization aims to find the "best" available solution among a set of potential solutions in a defined search space. For almost every problem, there is a solution, not necessarily the best, but we can always find an approximation to the "ideal solution", and while in some cases or processes is still common to use our own experience to qualify a process, a part of the research community have dedicated a considerably amount of time and efforts to developing robust optimization methods for finding optimal solutions in a vast range of applications. On the other hand, it has been stated that is difficult to efficiently solve different problems by applying the same optimization methodology, and even the most robust optimization approaches may be outperformed by other optimization techniques depending on the problem to solve. This has motivated a lot of research work in developing new optimization methods based on diverse natural paradigms, which in this work has led us to propose a new method based on the chemical reactions paradigm.

When the complexity and the dimension of the search space make a problem unsolvable by a deterministic algorithm, probabilistic algorithms deal with this problem by going through a diverse set of possible solutions or candidate solutions. Many metaheuristic algorithms can be considered probabilistic, while they apply probability tools to solve a problem, and metaheuristic algorithms seek good solutions by mimicking natural processes or paradigms. Most of these novel optimization paradigms inspired by nature

were conceived by merely observation of an existing process and their main characteristics were embodied as computational algorithms.

The importance of the optimization theory and its application has grown in the past few decades, from the well known Genetic Algorithm paradigm to PSO, ACO, Harmonic Search, DNA Computing, among others, and they all were introduced with the expectation of improving the results obtained with the existing strategies.

There is no doubt that there could be some optimization strategies presented at some point that were left behind due to their complexity and poor performance. Novel optimization paradigms should be able to perform well in comparison with another optimization techniques and must be "easily adaptable" to different kinds of problems.

Optimization based on chemical processes is a growing field that has been satisfactorily applied to several problems. In [1], A DNA based algorithm was used to solve the small hitting set problem. A catalytic search algorithm was explored in [2], where some physical laws such as mass and energy conservation were taken into account. In [3], the potential roles of energy in algorithmic chemistries were illustrated. An energy framework was introduced, which keeps the molecules within a reasonable length bounds, allowing the algorithm to behave thermodynamically and kinetically similar to real chemistry. A chemical reaction optimization process was applied to the grid scheduling problem in [4], where molecules interact with each other aiming to reach the minimum state of free potential and kinetic energies. The main difference between these metaheuristics is the parameter representation, which can be explicit or implicit.

In this paper, we apply an optimization method inspired on the chemical reactions and its application for the optimization of the tracking controller for the dynamic model of the unicycle mobile robot.

The importance of applying this chemical optimization algorithm is that different methods have been used to solve motion control problems. Kanayama et al. [5] propose a stable tracking control method for a non-holonomic vehicle using a Lyapunov function. Lee et al. [6] solved tracking control using backstepping and in [7] with saturation constraints. Furthermore, most reported designs rely on intelligent control approaches such as fuzzy logic control [8-13] and neural networks [14,15].

However, the majority of the publications mentioned above, have concentrated on kinematic models of mobile robots, which are controlled by the velocity input, while less attention has been paid to the control problems of nonholonomic dynamic systems, where forces and torques are the true inputs: Bloch and Drakunov [16] and Chwa [17], used a sliding mode control to the tracking control problem. Fierro and Lewis [18] propose a dynamical extension that makes possible the integration of kinematics and torque controller for a nonholonomic mobile robot. Fukao et al. [19], introduced an adaptive tracking controller for the dynamic model of mobile robot with unknown parameters using backstepping methodology, which has been recognized as a tool for solving several control problems. Motivated by this, a Mamdani fuzzy logic controller is introduced in order to drive the kinematic model to a desired trajectory in a finite-time, considering the torque as the real input, a chemical reaction optimization paradigm is applied and simulations are shown.

Further publications [20-22] have applied bio-inspired optimization techniques to find the parameters of the membership functions for the fuzzy tracking controller that solves the problem for the dynamic model of a unicycle mobile robot, using a fuzzy logic controller that provides the required torques to reach the desired velocity and trajectory inputs.

In this paper, the main contribution is the representation of the fuzzy controller in the chemical paradigm to search for the optimal parameters. Simulation results show that

the proposed approach outperforms other nature inspired computing paradigms, such as genetic algorithms, particle swarm and ant colony optimization.

The rest of this paper is organized as follows. Section 2 illustrates the proposed methodology. Section 3 describes the problem formulation and control objective. Section 4 describes the proposed fuzzy logic controller of the robot. Section 5 shows some experimental results of the tracking controller and in Section 6 some conclusions and future work are presented.

2. **The Chemical Optimization Paradigm.** The proposed chemical reaction algorithm is a metaheuristic strategy that performs a stochastic search for optimal solutions within a defined search space. In this optimization strategy, every solution is represented as an element (or compound), and the fitness or performance of the element is evaluated in accordance with the objective function. The general flowchart of the algorithm is shown in Figure 1.

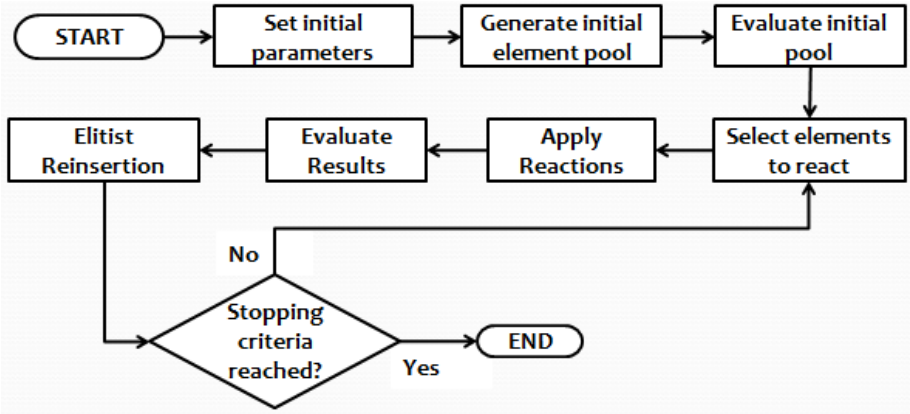


FIGURE 1. General flowchart of the chemical reaction algorithm

The main difference with other optimization techniques [1-4] is that no external parameters are taken into account to evaluate the results, while other algorithms introduce additional parameters (kinetic/potential energies, mass conservation, thermodynamic characteristics, etc.), this is a very straight forward methodology that takes the characteristics of the chemical reactions (synthesis, decomposition, substitution and double-substitution) to find for optimal solution.

This approach is a static population-based metaheuristic that applies an abstraction of the chemical reactions as intensifiers (substitution, double substitution reactions) and diversifying (synthesis, decomposition reactions) mechanisms. The elitist reinsertion strategy allows the permanence of the best elements and thus the average fitness of the entire element pool increases with every iteration. The algorithm may trigger only one reaction or all of them, depending on the nature of the problem to solve, in example; we may use only the decomposition reaction sub-routine to find the minimum value of a mathematical function.

In order to have a better picture of the general schema for this proposed chemical reaction algorithm, a comparison with other nature inspired paradigms is shown in Table 1.

3. **The Mobile Robot.** Mobile robots are non-holonomic systems due to the constraints imposed on their kinematics. The equations describing the constraints cannot be integrated symbolically to obtain explicit relationships between robot positions in local and

TABLE 1. Main elements of several nature inspired paradigms

Paradigm	*Parameter Representation*	*Basic Operations*
GA	*Genes*	*Crossover, Mutation*
ACO	*Ants*	*Pheromone*
PSO	*Particles*	*Cognitive, Social Coefficients*
GP	*Trees*	*Crossover, Mutation (In some cases)*
CRM	*Elements, Compounds*	*Reactions (Combination, Decomposition, Substitution, Double-substitution)*

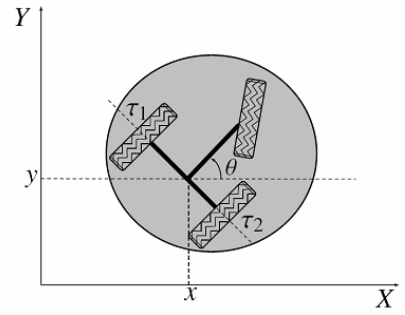


FIGURE 2. Diagram of a wheeled mobile robot

global coordinate's frames. Hence, control problems that involve them have attracted attention in the control community in recent years [23].

The model considered is that of a unicycle mobile robot (see Figure 2) that has two driving wheels fixed to the axis and one passive orientable wheel that are placed in front of the axis and normal to it [24].

The two fixed wheels are controlled independently by the motors, and the passive wheel prevents the robot from overturning when moving on a plane.

It is assumed that the motion of the passive wheel can be ignored from the dynamics of the mobile robot, which is represented by the following set of equations [18]:

$$\dot{q} = \begin{vmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} v \\ w \end{vmatrix}$$

$$M(q)\dot{v} + V(q,\dot{q})v + G(q) = \tau \tag{1}$$

where $q = [x, y, \theta]^T$ is the vector of generalized coordinates which describes the robot position, (x, y) are the Cartesian coordinates, which denote the mobile center of mass and θ is the angle between the heading direction and the x-axis (which is taken counterclockwise form); $v = [v, w]^T$ is the vector of velocities, v and w are the linear and angular velocities respectively; $\tau \in R^r$ is the input vector, $M(q) \in R^{n\times n}$ is a symmetric and positive-definite inertia matrix, $V(q,\dot{q}) \in R^{n\times n}$ is the centripetal and Coriolis matrix, $G(q) \in R^n$ is the gravitational vector. Equation (1) represents the kinematics or steering system of a mobile robot.

Notice that the no-slip condition imposed a non holonomic constraint described by (2), that it means that the mobile robot can only move in the direction normal to the axis of the driving wheels.

$$\dot{y}\cos\theta - \dot{x}\sin\theta = 0 \tag{2}$$

The control objective will be established as follows: Given a desired trajectory $q_d(t)$ and the orientation of the mobile robot we must design a controller that applies an

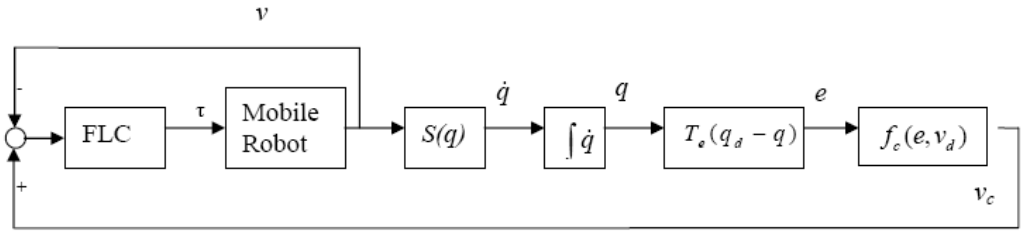


FIGURE 3. Tracking control structure

adequate torque τ such that the measured positions $q(t)$ achieve the desired reference $q_d(t)$ represented as (3):

$$\lim_{t\to\infty} \|q_d(t) - q(t)\| = 0 \tag{3}$$

To reach the control objective, the method is based on the procedure of [18], we are deriving a $\tau(t)$ of a specific $v_c(t)$ that controls the steering system (1) using a Fuzzy Logic Controller (FLC). A general structure of tracking control system is presented in Figure 3.

The control is based on the procedure proposed by Kanayama et al. [5] and Nelson and Cox [25] to solve the tracking problem for the kinematic model $v_c(t)$. Suppose that the desired trajectory q_d satisfies (4):

$$\dot{q}_d = \begin{vmatrix} \cos\theta_d & 0 \\ \sin\theta_d & 0 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} v_d \\ w_d \end{vmatrix} \tag{4}$$

Using the robot local frame (the moving coordinate system x-y in Figure 1), the error coordinates can be defined as (5):

$$e = T_e(q_d - q), \quad \begin{vmatrix} e_x \\ e_y \\ e_\theta \end{vmatrix} = \begin{vmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} x_d - x \\ y_d - y \\ \theta_d - \theta \end{vmatrix} \tag{5}$$

And the auxiliary velocity control input that achieves tracking for (1) is given by (6):

$$v_c = f_c(e, v_d), \quad \begin{vmatrix} v_c \\ w_c \end{vmatrix} = \begin{vmatrix} v_d + \cos e_\theta + k_1 e_x \\ w_d + v_d k_2 e_y + v_d k_3 \sin e_\theta \end{vmatrix} \tag{6}$$

where k_1, k_2 and k_3 are positive gain constants.

The first part for this work is to apply the proposed method to obtain the values of k_i $(i = 1, 2, 3)$ for achieving the optimal behavior of the controller, and the second part is to optimize the fuzzy controller.

4. **Fuzzy Logic Controller.** The purpose of the fuzzy logic controller (FLC) is to find a control input τ such that the current velocity vector v is able to reach the velocity vector v_c and this is denoted as:

$$\lim_{t\to\infty} \|v_c - v\| = 0 \tag{7}$$

The input variables of the FLC correspond to the velocity errors obtained of (6) (denoted as e_v and e_w: linear and angular velocity errors respectively), and 2 outputs variables, the driving and rotational input torques τ (denoted by F and N respectively). The initial membership functions (MF) are defined by 1 triangular and 2 trapezoidal functions for each variable involved. Figure 4(a) and Figure 4(b) depict the MFs in which N, Z, P represent the fuzzy sets (Negative, Zero and Positive respectively) associated to each input and output variable.

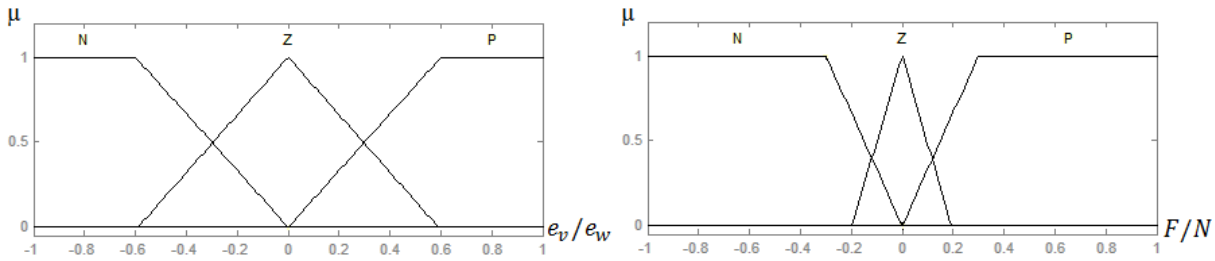


FIGURE 4. Membership functions. (a) Input e_v and e_w, and (b) output variables F and N.

TABLE 2. Fuzzy rule set

e_v/e_w	N	Z	P
N	N/N	N/Z	N/P
Z	Z/N	Z/Z	Z/P
P	P/N	P/Z	P/P

The rule set of the FLC contain 9 rules, which govern the input-output relationship of the FLC and this adopts the Mamdani-style inference engine. We use the center of gravity method to realize defuzzification procedure. In Table 2, we present the rule set whose format is established as follows:

Rule i: If e_v is $G1$ and e_w, is $G2$ then F is $G3$ and N is $G4$

where G1 ... G4 are the fuzzy sets associated to each variable and $i = 1 \ldots 9$. In this case, P denotes "positive", N denotes "negative", and Z denotes "zero".

5. **Experimental Results.** Several tests of the chemical optimization paradigm were made to test the performance of the tracking controller. First, we need to find the values of k_i $(i = 1, 2, 3)$ shown in Equation (6), which shall guarantee convergence of the error e to zero.

For this step of the application, only the decomposition reaction mechanism was triggered and the decomposition factor was varied; this factor is the quantity of resulting elements after applying a decomposition reaction to a determined "compound"; the only restriction here is that let x be the selected compound and x'_i $(i = 1, 2, \ldots, n)$, the resulting elements; the sum of all values found in the decomposition must be equal to the value of the original compound. This is shown in (8).

$$\sum_{i=1}^{n} x'_i = x \tag{8}$$

The test parameters can be observed in Table 3. For statistical purposes, every experiment was executed 35 times.

The decomposition rate represents the percentage of the pool to be considered as a candidate for the decomposition and the selection was made using a stochastic universal sampling technique, which uses a single random value to sample all of the solutions by choosing them at evenly spaced intervals. As mentioned in Section 2, an elitist reinsertion strategy was applied, keeping the compounds/elements with better performance through all the iterations, unless new elements/compounds with better performance are generated.

For example, for a pool containing 5 initial compounds, the vector length of decomposed elements when the decomposition factor is 3 and the decomposition rate is 0.4 will be of 6 elements. After applying these criteria, the need of increasing the initial element pool

TABLE 3. Parameters of the chemical reaction optimization

No.	*Elements*	*Iterations*	*Dec. Factor*	*Dec. Rate*
1	*2*	*10*	*2*	*0.3*
2	*5*	*10*	*3*	*0.3*
3	*2*	*10*	*2*	*0.4*
4	*2*	*10*	*3*	*0.4*
5	*5*	*10*	*2*	*0.4*
6	*5*	*10*	*3*	*0.4*
7	*5*	*10*	*2*	*0.5*
8	*10*	*10*	*2*	*0.5*

TABLE 4. Parameters of the chemical reaction optimization

No.	*Best Error*	*Mean*	k_1	k_2	k_3
1	*0.0086*	*1.1568*	*519*	*46*	*8*
2	*4.79e-004*	*0.1291*	*205*	*31*	*31*
3	*0.0025*	*0.5809*	*36*	*328*	*88*
4	*0.0012*	*0.5589*	*2*	*206*	*0*
5	*0.0035*	*0.0480*	*185*	*29*	*5*
6	*8.13e-005*	*0.0299*	*270*	*53*	*15*
7	*0.0066*	*0.1440*	*29*	*15*	*0*
8	*0.0019*	*0.1625*	*51*	*3*	*0*

TABLE 5. Parameters of the chemical reaction optimization

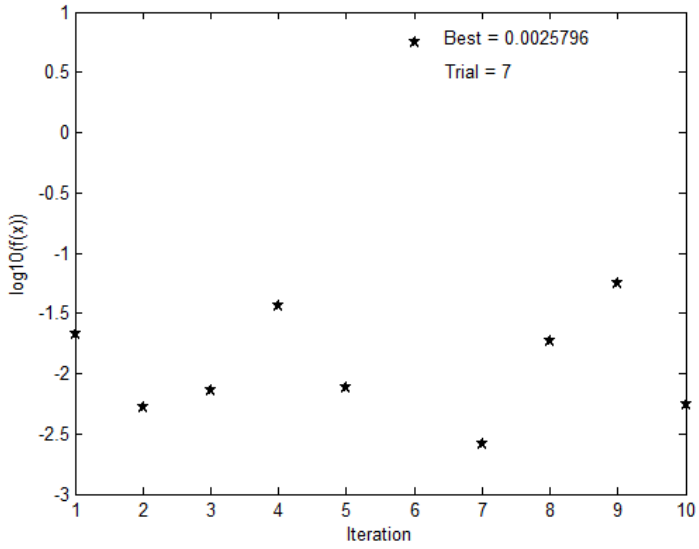
Parameters	*Genetic Algorithm*	*Chemical Optimization Algorithm*
Individuals	*5*	*2*
Iterations	*15*	*10*
Crossover rate	*0.8*	*N/A*
Mutation rate	*0.1*	*N/A*
Decomposition rate	*N/A*	*0.4*
Decomposition factor	*N/A*	*3*
k_1, k_2, k_3	*43, 493, 19*	*36, 328, 88*
Final Error	*0.006734*	*0.0025796*

size was not necessary. Table 4 shows the results after applying the chemical optimization paradigm.

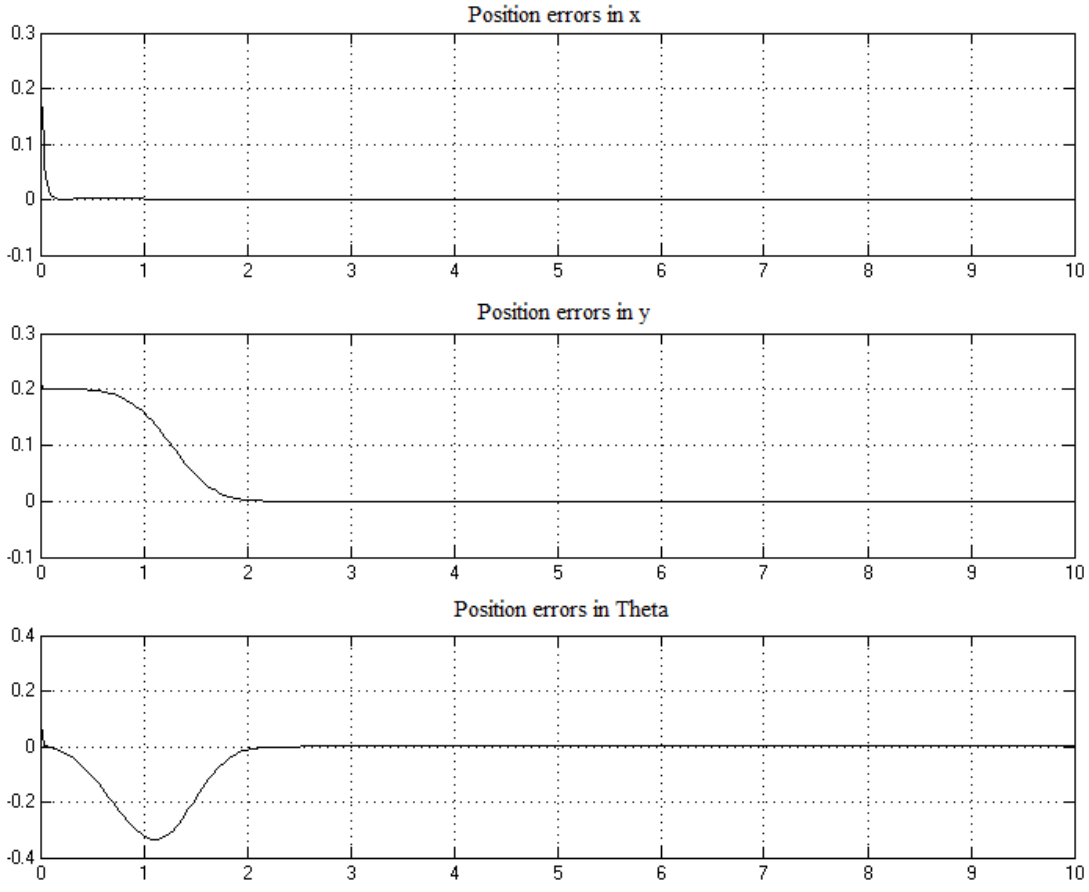
Figure 5 shows the resulting simulation obtained using Simulink for experiment No. 3, which is the best overall result so far, considering the average error and the position errors in x, y and *theta*.

In previous work [20], the gain constant values were found by means of genetic algorithms. Table 5 shows a comparison of the best results obtained with both algorithms; Figure 6 shows the results in Simulink for the experiment with the best overall result, applying GAs as optimization method. We can observe that the result with the chemical optimization outperforms the GA in finding the best gain values.

Once we have found optimal values for the gain constants, the next step is to find the optimal values for the input/output membership functions of the fuzzy controller. Our



(a)



(b)

FIGURE 5. (a) Convergence and (b) x, y and *theta* position errors of simulation No. 3

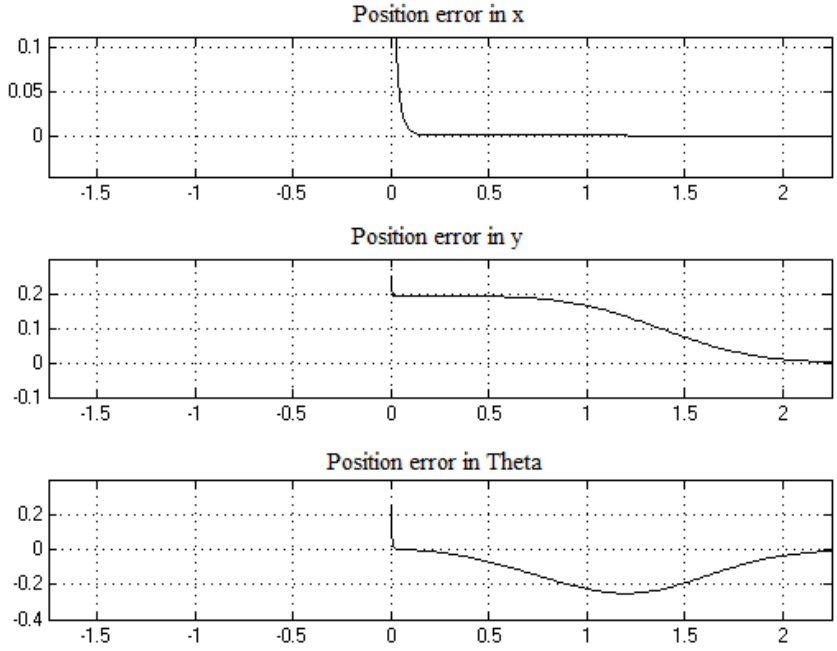


FIGURE 6. Best experiment using GAs

TABLE 6. Simulation parameters of the chemical reaction algorithm

Parameters	*Value*
Elements	*10*
Trials	*15*
Selection Method	*Stochastic Universal Sampling*
k_1, k_2, k_3	*36, 328, 88*
Error	*0.077178*

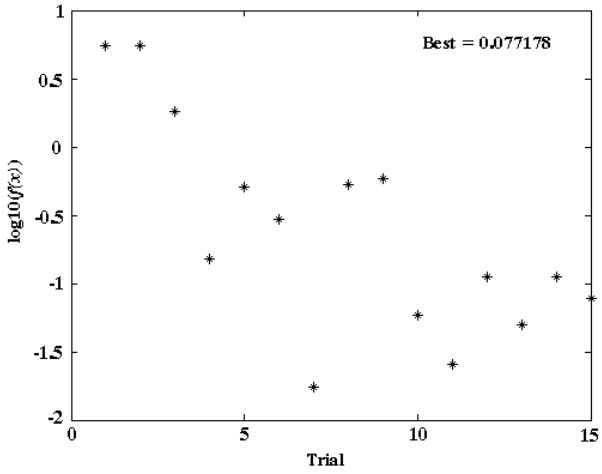


FIGURE 7. Best simulation of experiments with the chemical optimization algorithm

goal is that in the simulations, the linear and angular velocities reach zero. Table 6 shows the parameters used in the first set of simulations and Figure 7 shows the behavior of the chemical optimization algorithm throughout the experiment.

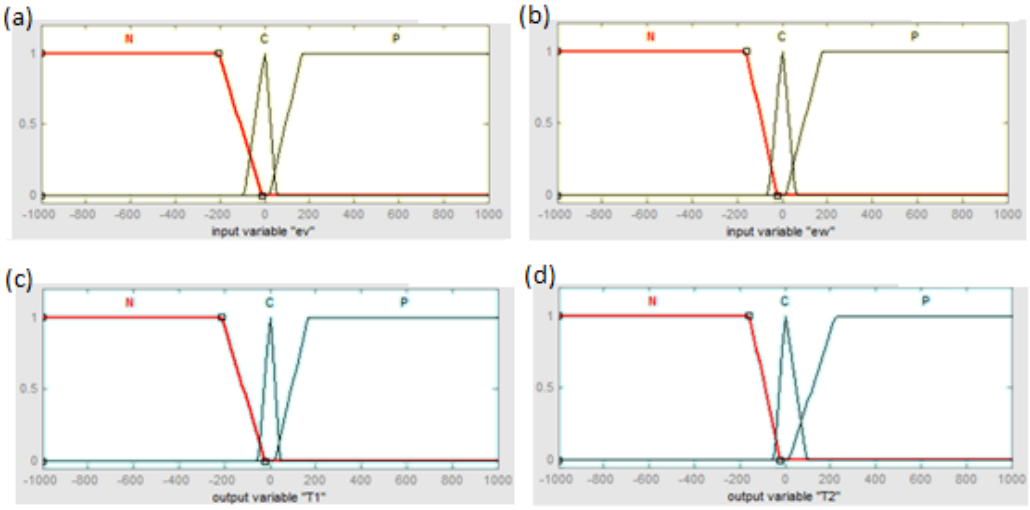
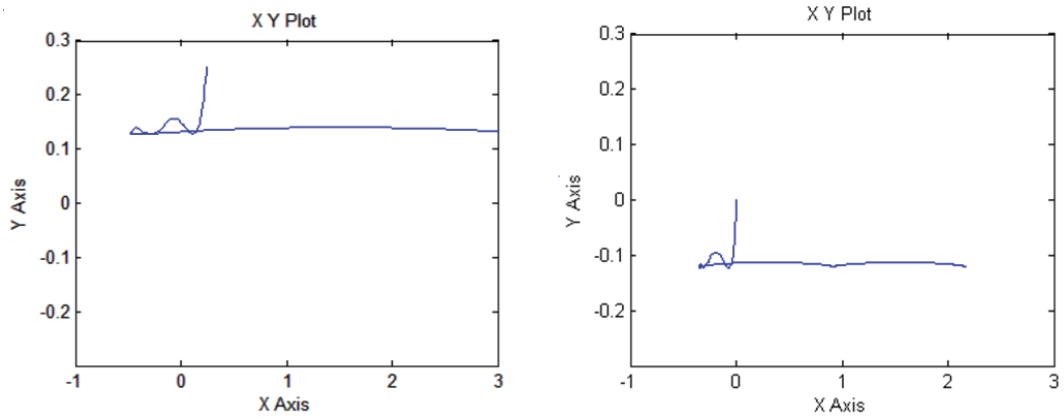


FIGURE 8. Resulting input membership functions: (a) linear and (b) angular velocities and output (c) right and (d) left torque



(a) (b)

FIGURE 9. (a) Obtained trajectory when applying the chemical reaction algorithm, (b) obtained trajectory using genetic algorithms

Figure 8 shows the resulted input and output membership functions found by the proposed optimization algorithm.

Figure 9(a) shows the obtained trajectory when simulating the mobile control system including the obtained input and output membership functions; Figure 9(b) shows the best trajectory reached by the mobile when optimizing the input and output membership functions using genetic algorithms.

In Table 7, a comparison of the proposed optimization method against other nature inspired techniques that have been applied to solve the same problem is shown [20-22]. The tracking error of the proposed method clearly outperforms the other techniques.

6. **Conclusions.** In this paper, we presented simulation results from an optimization method that mimics chemical reactions applied to the problem of tracking control. The goal was to find the gain constants involved in the tracking controller for the dynamic model of a unicycle mobile robot. In the figures of the experiments we are able to note the behavior of the algorithm and the solutions found through all the iterations. Simulation results show that the proposed optimization method is able to outperform the results

TABLE 7. Comparison of results with different optimization methods for the fuzzy optimal control design

Optimization Method	*Parameters*	*Average Tracking Error*
Chemical Optimization	*Number of Elements = 10* *Decomposition rate = 0.4*	*0.0771*
Genetic Algorithm	*Crossover = 0.8, Mutation = 0.1* *Population = 100*	*0.5544*
Particle Swarm Optimization	*C1, C2 dynamic*	*0.1608*
Ant Colony Optimization	*Parameters dynamic*	*0.0866*

previously obtained applying a genetic algorithm optimization technique. The optimal fuzzy logic controller obtained with the proposed chemical paradigm has been able to reach smaller error values in less time than genetic algorithms. Also, the proposed method was able to outperform the particle swarm and ant colony optimization methods. As future work the proposed chemical optimization method will be applied to design optimal type-2 fuzzy controllers for this problem of robot control. The design of optimal type-2 fuzzy controllers has been an important recent problem that has been studied in several works [26-29].

Acknowledgment. This work is partially supported by CONACYT and Tijuana Institute of Technology. The authors also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

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