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METHOD OF STRUCTURAL ANALYSIS FOR STATICALLY INDETERMINATE RIGID FRAMES

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Abstract. *This paper proposes a method for analysis of statically indeterminate structures, considering the shear deformations, which is an extension to the slope-deflection method, which is used to analyze all kinds of structures in the plane. This methodology considers the shear deformation and flexure. The traditional method takes into account only the flexure deformation and without taking into account the shear deformation, this is how it usually develops structural analysis of statically indeterminate rigid frames. It also makes a comparison between the proposed method and the traditional method as can be seen in the results tables of the problems considered, and in the traditional method not all values are on the side of safety. Therefore, the usual practice, without considering the shear deformations will not be a recommended solution. Then is proposed the use of considering shear deformations and also is more attached to real conditions.*
Keywords: Shear deformations, Poisson's ratio, Moment of inertia, Elasticity modulus, Shear modulus, Shear area

1. **Introduction.** In the structural systems analysis has been studied by diverse researchers in the past, making a brief historical review of progress in this subject.

In 1857, Benoit Paul Emile Clapeyron presented to the French Academy his "theorem of three moments" for analysis of continuous beams, and in the same way Bertot had published two years ago in the Memories of the Society of Civil Engineers of France, but without giving some credit. It can be said that from this moment begins the development of a true "Theory of Structures" [1-4].

In 1854 the French Engineer Jacques Antoine Charles Bresse published his book *"Recherches Analytiques sur la Flexion et la Résistance de Pieces Courbés"* in which he presented practical methods for the analysis of curved beams and arcs [1-4].

In 1867 was introduced by the German Emil Winkler (1835-1888), the "Influence Line". He also made important contributions to the Resistance of materials, especially in the flexure theory of curved beams, flexure of beams, resting on elastic medium [1-4].

James Clerk Maxwell (1830-1879), from the University of Cambridge, published what might be called the first systematic method of analysis for statically indeterminate structures, based on the equality of the internal energy of deformation of a loaded structure and the external work done by applied loads, equality had been established by Clapeyron. In his analysis presented in the Theorem of the Reciprocal Deformations, which by in its brevity and lack of illustration, was not appreciated at the time. In another publication later presented his diagram of internal forces to trusses, which combine in one figure all the polygons of forces. The diagram was extended by Cremona, by what is known as the Maxwell-Cremona diagram [1-4].

The Italian Betti in 1872 published a generalized form of Maxwell's theorem, known as the reciprocal theorem of Maxwell-Betti [1-4].

The German Otto Mohr (1835-1918) made great contributions to the Structures Theory. He developed the method for determining the deflections in beams, known as the method of elastic loads or the conjugate beam. He also presented a derivation simpler and more extensive of the general method of Maxwell for analysis in indeterminate structures, using the principles of virtual work. He made contributions in the graphical analysis of deflections of trusses, complemented by Williot diagram, known as the Mohr-Williot diagram of great practical utility. He also earned his famous Mohr Circle for the graphical representation of the stresses in a stress biaxial state [1-4].

Alberto Castigliano (1847-1884) in 1873 introduced the principle of minimum work, which had been previously suggested by Menabrea, and is known as the First Theorem of Castigliano. Later, it presented the Theorem second Castigliano, to find deflections, as a corollary of the first. In 1879 his famous book published in Paris "Thèoreme de l'Equilibre de Systèmes Elastiques et ses Applications", remarkable by its originality and very important in the development of analysis of statically indeterminate structures [1-3].

Heinrich Müller-Breslau (1851-1925), published in 1886 a basic method for analysis of indeterminate structures, but was essentially a variation of those presented by Maxwell and Mohr. He gave great importance the Maxwell's Theorem of Reciprocal Deflections in the assessment of displacement. He discovered that the "influence line" for the reaction or an inner strength of a structure was, on some scale, the elastic produced by an action similar to that reaction, or inner strength. Known as the Müller-Breslau theorem is the basis for other indirect methods of structural analysis using models [1-3].

Hardy Cross (1885-1959) professor at the University of Illinois, published in 1930 his famous moments distribution method, which can be said that it revolutionized the analysis of structures of reinforced concrete by continuous frames and can be considered one of the greatest contributions to the analysis from indeterminate structures. This method of successive approximations evades solving systems of equations, as presented in the methods of Mohr and Maxwell. This method declined popularity with the availability of computers, with which the resolution of equations systems is no longer a problem. The general concepts of the method were later extended in the study on flow of pipes. Later became more popular the methods of Kani and Takabeya also of type iterative and today in disuse [1-6].

In the early 50's, Turner, Clough, Martin and Topp present what may be termed as the beginning of the application to structures of the matrix methods of stiffness, which have gained so much popularity today. Subsequently, it is developed the finite element methods, which have allowed the systematic analysis of large numbers of structures and obtain the forces and deformations in complex systems such as concrete dams used in hydroelectric plants. Its promoters include: Clough, Wilson, Zienkiewics and Gallagher [1,2,7].

The author Luévanos-Rojas [8] developed a method of structural analysis for statically indeterminate beams, and this method takes into account the flexure deformations and shear. That previously was considered only the flexure deformation.

Structural analysis is the study of structures such as discrete systems. The theory of the structures is essentially based on the fundamentals of mechanics with which are formulated the different structural elements. The laws or rules that define the balance and continuity of a structure can be expressed in different ways, including partial differential equations of continuous medium three-dimensional, ordinary differential equations that define a member or the theories various of beams, or simply, algebraic equations for a discrete structure. The more delves into the physics of problem, are developing theories that are most appropriate for solving certain types of structures and that prove more useful for practical calculations. However, in each new theory are made hypotheses about how the system behaves or element. Therefore, we must always be aware of these hypotheses when evaluating results, fruit of the theories that apply or develop [9-11].

Structural analysis can be addressed using three main approaches [12]: a) tensor formulations (Newtonian mechanics and vectorial), b) formulations based on the principles of virtual work, c) formulations based on classical mechanics.

In the design of steel structures, reinforced concrete and prestressed, the study of structural analysis is a crucial stage into its design, since the axial forces, shear forces and moments are those that govern the design of rigid frames and for the case of beams only shear forces and moments, and the damage caused by such effects may become predominant among the various requests to consider for your design.

As regards the conventional techniques of structural analysis of rigid frames, the common practice is to neglect the shear deformations.

This paper proposes to consider the shear deformations and a comparison between the proposed method and the traditional method is realized.

2. Development.

2.1. Theoretical principles.

The scheme of deformation of a structure member is illustrated in Figure 1, which shows the difference between the Timoshenko theory and Euler-Bernoulli theory: the first θ_z and dy/dx does not necessarily coincide, while the second is equal [13].

The fundamental difference between the Euler-Bernoulli theory and Timoshenko's theory is that in the first the relative rotation of the section is approximated by the derivative of vertical displacement, this is an approximation valid only for long members in relation to the dimensions of cross section, and then it happens that due to shear deformations are negligible compared to the deformations caused by moment. On the Timoshenko theory, which considers the deformation due to the shear, i.e., and is valid therefore for short members and long, the equation of the elastic curve is given by the complex system of

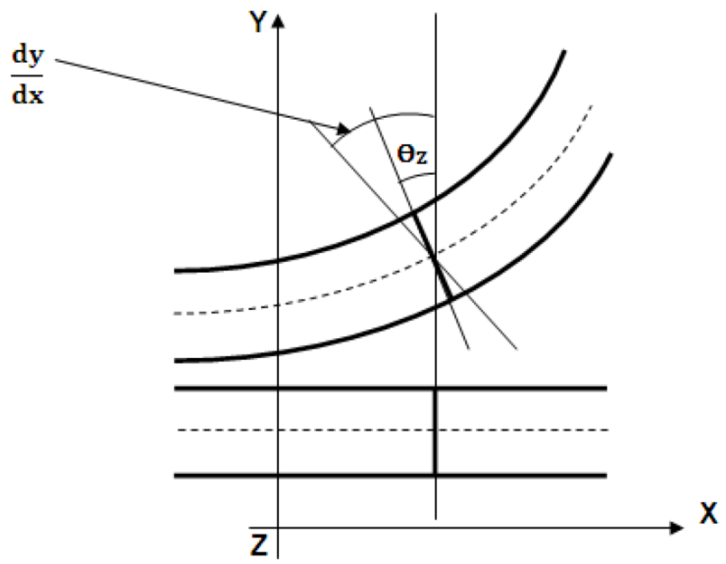

FIGURE 1. Deformation of a structure member

equations:

$$G\left(\frac{dy}{dx}-\theta_z\right)=\frac{V_y}{A_s} \tag{1}$$

$$E\left(\frac{d\theta_z}{dx}\right)=\frac{M_z}{I_z} \tag{2}$$

where G = shear modulus, dy/dx = total rotation around axis "z", θ_z = rotation around axis "z", due to the flexure, V_y = shear force in direction "y", A_s = shear area, $d\theta_z/dx = d^2y/dx^2$, E = elasticity modulus, M_z = moment around axis "z", I_z = moment of inertia around axis "z".

Deriving Equation (1) and substituting into Equation (2), it is arrived at the equation of the elastic curve including the effect of shear stress:

$$\frac{d^2y}{dx^2}=\frac{1}{GA_c}\frac{dV_y}{dx}+\frac{M_z}{EI_z} \tag{3}$$

From Equation (1) is obtained dy/dx:

$$\frac{dy}{dx}=\frac{V_y}{GA_s}+\theta_z \tag{4}$$

And Equation (2) is given θ_z:

$$\theta_z=\int\frac{M_z}{EI_z}dx \tag{5}$$

Now substituting Equation (5) into Equation (4) is:

$$\frac{dy}{dx}=\frac{V_y}{GA_s}+\int\frac{M_z}{EI_z}dx \tag{6}$$

2.2. **General conditions.** The slope-deflection method can be used to analyze all type of beams and rigid frames statically indeterminate. In this method all joints are considered rigid; i.e., the angles between members at the joints are considered not to change in value, when the loads are applied. Thus, the joints at the supports interior of statically indeterminate beams can be considered rigid joints of 180°; and usually the joints in rigid frames are rigid joints of 90°. When beams are deformed, the rigid joints are considered to rotate only as a whole; in other terms, the angles between the tangents to the various branches of the elastic curve in the joint remain the same as in the original undeformed structure.

In the slope-deflection method the rotations and displacements (the end joints are subjected to unequal movements in a direction perpendicular to the axis of the member) of the joints are treated as unknowns. Then the end moments can be expressed in terms of the rotations and displacements. However, to satisfy the condition of equilibrium, the sum of the end moments which any joints exerted on the ends of union of the members must be zero, because the rigid joints in question are subject to the sum of these moments at the ends (Reversed only in the direction). Further, to satisfy the equilibrium condition of cutting, equation of sum in shear forces must be zero. This condition must apply, making a virtual cut in the columns base per level, which provides the additional condition that corresponds to the unknown displacements.

These procedures solve the equations system of rotations and displacements for beams and rigid frames statically indeterminate. Therefore, it is important to remember the hypothesis under which the equations are deduced: a) The material is homogeneous, isotropic and behaves as linear elastic, i.e., the material is of the same nature, have identical physical properties in all directions and stresses that can withstand are directly proportional to the deformations that suffer and the factor of proportionality is called modulus elasticity, E, i.e., $\sigma = E\varepsilon$ (Hooke's Law); b) The principle of the small deformations, which once loaded structure, deformation or linear displacements and angular of the joints and each of the points of its members are rather small in such a way that in form do not change, nor are altered appreciably; c) The principle of effects superposition, that supposes the totals displacements and internal forces total of the structure under a system of loads, can be found separately by the sum of the effects of each one of the considered loads; d) You can only take into account the first order effects such as: internal deformations by flexure always, while the shear deformations can be taken into account or not.

2.3. **Slope-deflection equations.** The slope-deflection equations, the moments that act in the ends of the members are expressed in terms of the rotations, displacements perpendicular to the axis of the bar and the loads on the members. Then, the member AB shown in Figure 2(a) can be expressed in terms of θ_A, θ_B and Δ, also of the applied loads, P_1 and P_2. Counterclockwise end moments acting on the members are considered to be positive and clockwise end moments acting on the members are considered to be negative. Now, with the applied loading on the member, the fixed end moments, M_{FAB} and M_{FBA}, they are moments required to hold the horizontal tangents at the ends fixed in Figure 2(b). Additionally at the fixed moments in the ends, M'_{FAB}, and M'_{FBA}, they are acting on the member in the fixed ends, when the perpendicular displacement to axis the member appears, according to be seen in Figure 2(c). Then the moments, M'_{AB} and M'_{BA}, should be such as to cause rotations of θ_A and θ_B. If θ_{A1} and θ_{B1} are the end rotations caused by M'_{AB}, according to Figure 2(d), thus θ_{A2} and θ_{B2}, due to M'_{BA}, are observed in Figure 2(e).

The conditions required of geometry are [14-18]:

$$\theta_A = -\theta_{A1} + \theta_{A2} \tag{7a}$$

$$\theta_B = \theta_{B1} - \theta_{B2} \tag{7b}$$

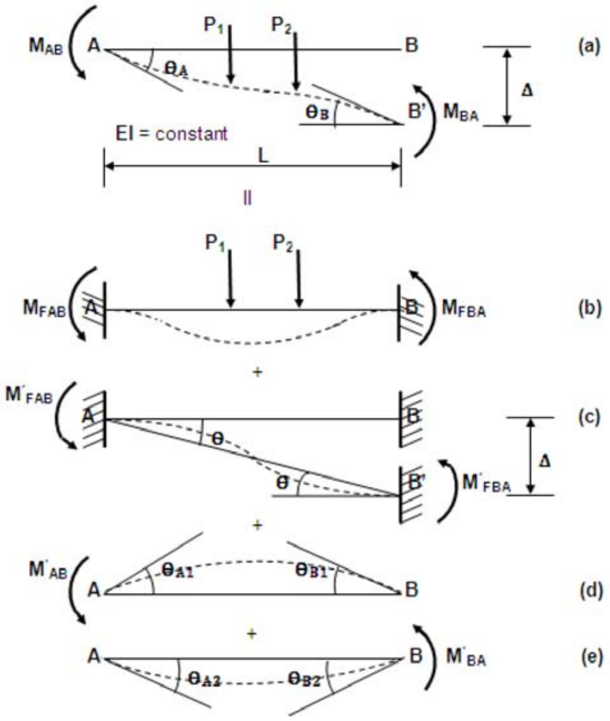

FIGURE 2. Derivation of slope-deflection equations

By superposition:

$$M_{AB} = M_{FAB} + M'_{AB} + M'_{FAB} \tag{8a}$$

$$M_{BA} = M_{FBA} + M'_{BA} + M'_{FBA} \tag{8b}$$

Taking into account the element of Figure 2(c) and supposing that $M'_{FAB} = M'_{FBA}$ and, $V_A = V_B$, doing sum of moments in the point B and is obtained M'_{FAB} in function of V_A:

$$M'_{FAB} = \frac{V_A L}{2} \tag{9}$$

Therefore, the shear forces and moments at a distance "x" are:

$$V_x = V_A \tag{10}$$

$$M_x = V_A \left(\frac{L}{2} - x\right) \tag{11}$$

Substituting M_x and V_x in function of V_A in Equation (6), and separating the shear deformation and flexure to obtain the stiffness due to the displacement, is presented as follows:

Shear deformation:

$$\frac{dy}{dx} = \frac{V_A}{GA_s} \tag{12}$$

Integrating Equation (12) is obtained as follows:

$$y = \frac{V_A}{GA_s} x + C_1 \tag{13}$$

Considering the conditions of border, when $x = 0$; $y = 0$; then $C_1 = 0$.

$$y = \frac{V_A}{GA_s} x \tag{14}$$

Flexure deformation:

$$\frac{dy}{dx} = \frac{V_A}{EI_z} \int \left(\frac{L}{2} - x\right) dx \tag{15}$$

Developing the integral of Equation (15), it is the following expression:

$$\frac{dy}{dx} = \frac{V_A}{EI_z} \left(\frac{L}{2} x - \frac{x^2}{2} + C_2\right) \tag{16}$$

Taking into account the conditions of border, when $x = 0$; $dy/dx = 0$; it is obtained that $C_2 = 0$.

$$\frac{dy}{dx} = \frac{V_A}{EI_z} \left(\frac{L}{2} x - \frac{x^2}{2}\right) \tag{17}$$

Integrating Equation (17) is presented the following:

$$y = \frac{V_A}{EI_z} \left(\frac{L}{4} x^2 - \frac{x^3}{6} + C_3\right) \tag{18}$$

Taking into account the conditions of border, when $x = 0$; $y = 0$; it is obtained that $C_3 = 0$.

$$y = \frac{V_A}{EI_z} \left(\frac{L}{4} x^2 - \frac{x^3}{6}\right) \tag{19}$$

It is developed the sum Equation (14) due to shear deformation and Equation (19) due to flexure presents as follows:

$$y = \frac{V_A}{GA_s} + \frac{V_A}{EI_z} \left(\frac{L}{4} x^2 - \frac{x^3}{6}\right) \tag{20}$$

Replace $x = L$; $y = \Delta$, for to find the displacement in the B support, is as follows:

$$\Delta = \frac{V_A L^3}{12EI_z}\left(\frac{12EI_z}{GA_s L^2} + 1\right) \tag{21}$$

Substituting [19]:

$$\varnothing = \frac{12EI_z}{GA_s L^2} \tag{22}$$

It is obtained G as follows:

$$G = \frac{E}{2(1+\upsilon)} \tag{23}$$

where $\varnothing$ = form factor, υ = Poisson's ratio.

Substituting Equation (22) into Equation (21) and obtaining the value, V_A is of the form:

$$V_A = \frac{12EI_z}{L^3(\varnothing + 1)}\Delta \tag{24}$$

When substituting V_A into Equation (9) is the following way:

$$M'_{FAB} = \frac{6EI_z}{L^2(\varnothing + 1)}\Delta \tag{25}$$

Then like $M'_{FAB} = M'_{FBA}$, appear the following equations:

$$M'_{FAB} = \frac{6EI_z}{L^2(\varnothing + 1)}\Delta \tag{26a}$$

$$M'_{FBA} = \frac{6EI_z}{L^2(\varnothing + 1)}\Delta \tag{26b}$$

Analyzing the element of Figure 2(d) for to find θ_{A1} and θ_{B1} in function of M'_{AB}: It is considered that $V_A = V_B$, doing sum of moments in B and obtaining M'_{AB} in function of V_A is presents:

$$M'_{AB} = V_A L \tag{27}$$

Therefore, the shear forces and moments at a distance "x" are:

$$V_x = \frac{M'_{AB}}{L} \tag{28}$$

$$M_x = \frac{M'_{AB}}{L}(L - x) \tag{29}$$

Substituting Equations (28) and (29) into Equation (6), then separating the shear deformation and flexure for obtain stiffness is presented as follows:

Shear deformation:

$$\frac{dy}{dx} = \frac{M'_{AB}}{GA_s L} \tag{30}$$

Integrating Equation (30) is presented as follows:

$$y = \frac{M'_{AB}}{GA_s L}x + C_1 \tag{31}$$

Taking into account the conditions of border, when $x = 0$; $y = 0$; it is $C_1 = 0$.

$$y = \frac{M'_{AB}}{GA_s L}x \tag{32}$$

Flexure deformation:

$$\frac{dy}{dx} = \frac{M'_{AB}}{EI_z L}\int (L - x)dx \tag{33}$$

It is developed that the integral of Equation (33) is obtained:

$$\frac{dy}{dx} = \frac{M'_{AB}}{EI_zL}\left(Lx - \frac{x^2}{2} + C_2\right) \tag{34}$$

Integrating Equation (34) is presented:

$$y = \frac{M'_{AB}}{EI_zL}\left(\frac{L}{2}x^2 - \frac{x^3}{6} + C_2x + C_3\right) \tag{35}$$

Considering the conditions of border, when $x = 0$; $y = 0$; it is $C_3 = 0$.

$$y = \frac{M'_{AB}}{EI_zL}\left(\frac{L}{2}x^2 - \frac{x^3}{6} + C_2x\right) \tag{36}$$

Now taking into account the conditions of border, when $x = L$; $y = 0$; it is obtained:

$$C_2 = -\frac{L^2}{3} \tag{37}$$

Then, substituting Equation (37) into Equations (34) and (36) is shown as follows:

$$\frac{dy}{dx} = \frac{M'_{AB}}{EI_zL}\left(Lx - \frac{x^2}{2} - \frac{L^2}{3}\right) \tag{38}$$

$$y = \frac{M'_{AB}}{EI_zL}\left(\frac{L}{2}x^2 - \frac{x^3}{6} - \frac{L^2}{3}x\right) \tag{39}$$

Substituting $x = 0$ in Equation (38) to find the rotation in support A due to the flexure deformation θ_{A1F}, it is as follows:

$$\theta_{A1F} = -\frac{M'_{AB}L}{3EI_z} \tag{40}$$

Now substituting $x = L$, in Equation (38) to find the rotation in support B due to the flexure deformation θ_{B1F}, it is obtained as follows:

$$\theta_{B1F} = \frac{M'_{AB}L}{6EI_z} \tag{41}$$

If it is considered that they have his curvature radius in the inferior part. Then, the rotations are positive:

$$\theta_{A1F} = +\frac{M'_{AB}L}{3EI_z} \tag{42a}$$

$$\theta_{B1F} = +\frac{M'_{AB}L}{6EI_z} \tag{42b}$$

The rotation due to the shear deformation, θ_{A1S} and θ_{B1S}, taking into account the curvature radius is:

$$\theta_{A1S} = \frac{dy}{dx} = \frac{M'_{AB}}{GA_sL} \tag{43a}$$

$$\theta_{B1S} = \frac{dy}{dx} = -\frac{M'_{AB}}{GA_sL} \tag{43b}$$

Adding the shear and flexure rotation in the joint A, it is obtained:

$$\theta_{A1} = \theta_{A1F} + \theta_{A1S} \tag{44}$$

Substituting Equations (42a) and (43a) in Equation (44), it is as follows:

$$\theta_{A1} = \frac{M'_{AB}L}{3EI_z} + \frac{M'_{AB}}{GA_sL} \tag{45}$$

The common factor which is obtained in Equation (45) for M'_{AB}, is as follows:

$$\theta_{A1} = \frac{M'_{AB}L}{12EI_z}\left(4 + \frac{12EI_z}{GA_sL^2}\right) \tag{46}$$

Substituting Equation (22) into Equation (46):

$$\theta_{A1} = \frac{M'_{AB}L}{12EI_z}(4 + \varnothing) \tag{47}$$

Adding the shear and flexure rotation in the joint B, and make the simplifications corresponding, it is presented:

$$\theta_{B1} = \frac{M'_{AB}L}{12EI_z}(2 - \varnothing) \tag{48}$$

Analyzing the beam in Figure 2(e) to find θ_{A2} and θ_{B2} in function of M'_{BA} of the same way as was done in Figure 2(d), it is obtains following:

$$\theta_{A2} = \frac{M'_{BA}L}{12EI_z}(2 - \varnothing) \tag{49}$$

$$\theta_{B2} = \frac{M'_{BA}L}{12EI_z}(4 + \varnothing) \tag{50}$$

Now, substituting Equations (47) and (48) into Equation (7a) and Equations (49) and (50) into Equation (7b) presents as follows:

$$\theta_A = -\frac{M'_{AB}L}{12EI_z}(4 + \varnothing) + \frac{M'_{BA}L}{12EI_z}(2 - \varnothing) \tag{51}$$

$$\theta_B = \frac{M'_{AB}L}{12EI_z}(2 - \varnothing) - \frac{M'_{BA}L}{12EI_z}(4 + \varnothing) \tag{52}$$

We develop Equations (51) and (52), to find M'_{AB} and M'_{BA} in function of θ_A and θ_B, it is as it follows:

$$M'_{AB} = \frac{EI_z}{L}\left[-\left(\frac{4+\varnothing}{1+\varnothing}\right)\theta_A - \left(\frac{2-\varnothing}{1+\varnothing}\right)\theta_B\right] \tag{53a}$$

$$M'_{BA} = \frac{EI_z}{L}\left[-\left(\frac{4+\varnothing}{1+\varnothing}\right)\theta_B - \left(\frac{2-\varnothing}{1+\varnothing}\right)\theta_A\right] \tag{53b}$$

Finally substituting, Equations (26a, 26b) and (53a, 53b) into Equation (8a, 8b), respectively, we obtain the slope-deflection equations for statically indeterminate rigid frames:

$$M_{AB} = M_{FAB} + \frac{EI_z}{L}\left[-\left(\frac{4+\varnothing}{1+\varnothing}\right)\theta_A - \left(\frac{2-\varnothing}{1+\varnothing}\right)\theta_B\right] + \frac{6EI_z}{L^2(\varnothing+1)}\Delta \tag{54a}$$

$$M_{BA} = M_{FBA} + \frac{EI_z}{L}\left[-\left(\frac{4+\varnothing}{1+\varnothing}\right)\theta_B - \left(\frac{2-\varnothing}{1+\varnothing}\right)\theta_A\right] + \frac{6EI_z}{L^2(\varnothing+1)}\Delta \tag{54b}$$

3. **Application.** It developed the structural analysis of the steel rigid frame, in three different problems, as shown in Figure 3, by the classic method and the proposed method, i.e., without taking into account and to consider the shear deformations, on the basis of the following data that appear next:

$w = 34.335\text{kN/m}$
$L = 10.00\text{m}; 5.00\text{m}; 3.00\text{m}$
$P = 49.05\text{kN}$
$h = 5.00\text{m}$
$E = 20019.6\text{kN/cm}^2$
Properties of the beam W24X94

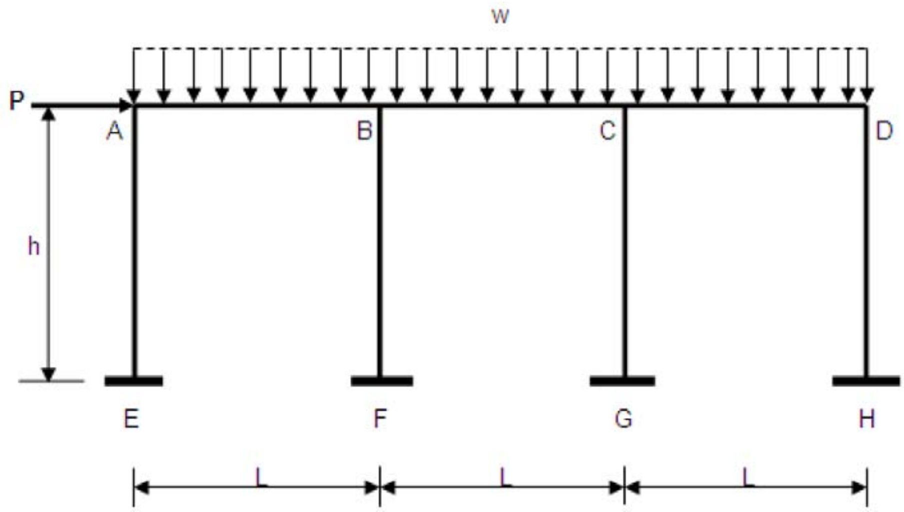

FIGURE 3. Rigid frame of steel of three lengths equal for beams, with uniformly distributed load and a discrete load in joint A

$A = 178.71\text{cm}^2$
$A_c = 80.83\text{cm}^2$
$I_z = 111966\text{cm}^4$
Properties of the column W24X61
$A = 116.13\text{cm}^2$
$A_s = 64.06\text{cm}^2$
$I = 64100\text{cm}^4$
$\upsilon = 0.32$
Known conditions: $\theta_E = \theta_F = \theta_G = \theta_H = 0$
Unknown conditions: θ_A, θ_B, θ_C, θ_D and $\Delta = \Delta_{AE} = \Delta_{BF} = \Delta_{CG} = \Delta_{DH}$
Using Equation (23), it is obtained the shear modulus, as follows:

$$G = \frac{20019.6}{2(1 + 0.32)} = 7583.182\text{kN/cm}^2$$

Once that is obtained the shear modulus is found the form factor through Equation (22) as follows:

For beam of 10.00m is:

$$\varnothing_{AB} = \varnothing_{BC} = \varnothing_{CD} = \frac{12(20019.6)(111966)}{(7583.182)(80.83)(1000)^2} = 0.04388324626$$

For beam of 5.00m is:

$$\varnothing_{AB} = \varnothing_{BC} = \varnothing_{CD} = \frac{12(20019.6)(111966)}{(7583.182)(80.83)(500)^2} = 0.175532985$$

For beam of 3.00m is:

$$\varnothing_{AB} = \varnothing_{BC} = \varnothing_{CD} = \frac{12(20019.6)(111966)}{(7583.182)(80.83)(300)^2} = 0.4875916251$$

For column of 5.00m is:

$$\varnothing_{AE} = \varnothing_{BF} = \varnothing_{CG} = \varnothing_{DH} = \frac{12(20019.6)(64100)}{(7583.182)(64.06)(500)^2} = 0.1267991228$$

The fixed moments for beams with uniformly distributed load are:

For beam of 10.00m is:

$$M_{FAB} = M_{FBC} = M_{FCD} = \frac{wL^2}{12} = +\frac{(34.335)(10.00)^2}{12} = +286.125\text{kN-m}$$

$$M_{FBA} = M_{FCB} = M_{FDC} = -\frac{wL^2}{12} = -\frac{(34.335)(10.00)^2}{12} = -286.125\text{kN-m}$$

For beam of 5.00m is:

$$M_{FAB} = M_{FBC} = M_{FCD} = \frac{wL^2}{12} = +\frac{(34.335)(5.00)^2}{12} = +71.531\text{kN-m}$$

$$M_{FBA} = M_{FCB} = M_{FDC} = -\frac{wL^2}{12} = -\frac{(34.335)(5.00)^2}{12} = -71.531\text{kN-m}$$

For beam of 3.00m is:

$$M_{FAB} = M_{FBC} = M_{FCD} = \frac{wL^2}{12} = +\frac{(34.335)(3.00)^2}{12} = +25.751\text{kN-m}$$

$$M_{FBA} = M_{FCB} = M_{FDC} = -\frac{wL^2}{12} = -\frac{(34.335)(3.00)^2}{12} = -25.751\text{kN-m}$$

Calculation of "EI" is:
For all beams is:

$$EI = (20019.6)(111966) = 2.241514534 \times 10^9\text{kN-cm}^2 = 224151.4534\text{kN-m}^2$$

For all columns is:

$$EI = (20019.6)(64100) = 1.283256360 \times 10^9\text{kN-cm}^2 = 128325.6360\text{kN-m}^2$$

Then, substituting, all these values into the corresponding equations for each member in the traditional method and the proposed method.

The slope-deflection equations, neglecting shear deformations (traditional method) are:

$$M_{AB} = M_{FAB} + \frac{EI_z}{L}(-4\theta_A - 2\theta_B) + \frac{6EI_z}{L^2}\Delta$$

$$M_{BA} = M_{FBA} + \frac{EI_z}{L}(-4\theta_B - 2\theta_A) + \frac{6EI_z}{L^2}\Delta$$

The slope-deflection equations, considering shear deformations (proposed method) are:

$$M_{AB} = M_{FAB} + \frac{EI_z}{L}\left[-\left(\frac{4+\varnothing}{1+\varnothing}\right)\theta_A - \left(\frac{2-\varnothing}{1+\varnothing}\right)\theta_B\right] + \frac{6EI_z}{L^2(\varnothing+1)}\Delta$$

$$M_{BA} = M_{FBA} + \frac{EI_z}{L}\left[-\left(\frac{4+\varnothing}{1+\varnothing}\right)\theta_B - \left(\frac{2-\varnothing}{1+\varnothing}\right)\theta_A\right] + \frac{6EI_z}{L^2(\varnothing+1)}\Delta$$

Once that is obtained the moments in each member as a function of rotations and displacements, it is applied the condition equilibrium of moments at the joints, which are:

Joint A:

$$M_{AB} + M_{AE} = 0$$

Joint B:

$$M_{BA} + M_{BC} + M_{BF} = 0$$

Joint C:

$$M_{CB} + M_{CD} + M_{CG} = 0$$

Joint D:

$$M_{DC} + M_{DH} = 0$$

Below is generated equilibrium condition of shear forces at the base of the frame, as shown in Figure 4, which is:

$$P - H_{EA} - H_{FB} - H_{GC} - H_{HD} = 0$$

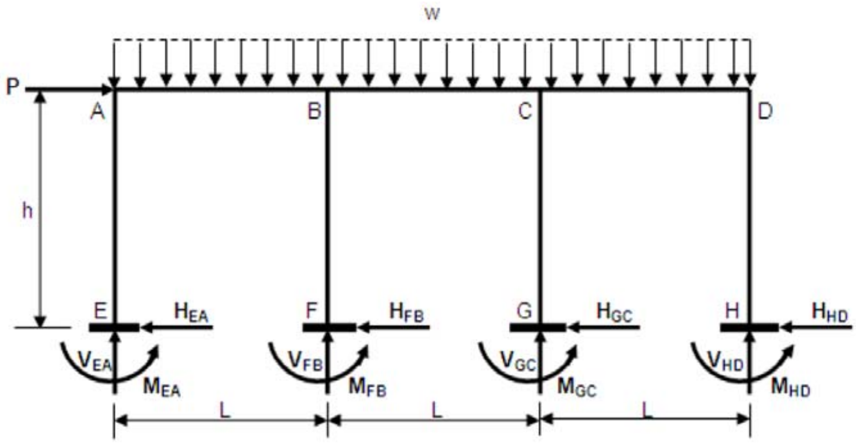

FIGURE 4. Free body diagram of whole frame

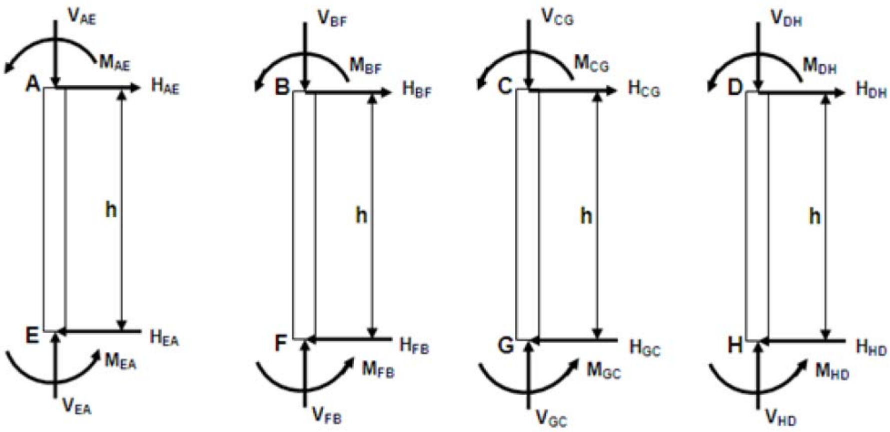

FIGURE 5. Free body diagram of each column

Shear forces on the base of frame are expressed in terms of the final moments, as shown in Figure 5, which are:

$$H_{EA} = \frac{M_{AE} + M_{EA}}{h}; \quad H_{FB} = \frac{M_{BF} + M_{FB}}{h};$$
$$H_{GC} = \frac{M_{CG} + M_{GC}}{h}; \quad H_{HD} = \frac{M_{DH} + M_{HD}}{h}$$

These equations are presented in terms of rotations and displacement, in this case there are 5 equations with 5 unknowns (θ_A, θ_B, θ_C, θ_D and Δ), and these are developed to find their values. Once that are found rotations and the displacement, were subsequently substituted into the slope-deflection equations to localize the final moments at the ends of the members. Now by static equilibrium, shear forces are obtained for each member. Then, it is obtains the diagram of shear forces and moments diagram.

Below are the results in the tables and figures, for the three different cases.

4. **Results.** According to Table 1 and Figure 6 which present the rotations and displacement in each of the joints, it is observed that the values are minor in all cases for the traditional method in absolute value, which does not consider shear deformations, and is quite considerable for short members. For example, for $L = 3.00$m, the major difference exists, is in the joint B, as soon as to the rotation of a 74% and the minor difference, for the horizontal displacement which occurs in the joints A, B, C and D is of 14%.

TABLE 1. The rotations and displacement in each one of the joints

Deformations	Case 1 $L = 10.00$m			Case 2 $L = 5.00$m			Case 3 $L = 3.00$m		
	NSD	CSD	$\frac{\text{NSD}}{\text{CSD}}$	NSD	CSD	$\frac{\text{NSD}}{\text{CSD}}$	NSD	CSD	$\frac{\text{NSD}}{\text{CSD}}$
$\theta_A \times 10^4$	+17.14	+18.57	0.92	+3.91	+4.30	0.91	+1.47	+1.85	0.79
$\theta_B \times 10^4$	−1.92	−2.02	0.95	−0.18	−0.30	0.60	+0.08	+0.31	0.26
$\theta_C \times 10^4$	+3.96	+4.13	0.96	+1.15	+1.19	0.97	+0.47	+0.59	0.80
$\theta_D \times 10^4$	−13.61	−14.50	0.94	−1.59	−1.69	0.94	+0.05	+0.17	0.29
$\Delta \times 10^4$	+13.67	+15.08	0.91	+12.00	+13.57	0.88	+11.25	+13.04	0.86

θ_i = the angle that forms the tangent due to the deformation in the joint i, in radians
Δ = the horizontal displacement that undergoes the joint A, B, C and D, in meters
NSD = Neglecting the shear deformations
CSD = considering the shear deformations
Nomenclature:
+ The rotations are shown as clockwise
− The rotations are shown as counterclockwise

With regard to Table 2 and Figure 7 which show the axial forces in the members between both methods. According to the results, there are also differences, for $L = 3.00$m is lower a 5% of the AE member in the traditional method with respect to the proposed method, and for $L = 10.00$m is higher a 5% of the CD member in the traditional method with respect to the proposed method.

TABLE 2. The normal forces of each member in kN

Normal Force	Case 1 $L = 10.00$m			Case 2 $L = 5.00$m			Case 3 $L = 3.00$m		
	NSD	CSD	$\frac{\text{NSD}}{\text{CSD}}$	NSD	CSD	$\frac{\text{NSD}}{\text{CSD}}$	NSD	CSD	$\frac{\text{NSD}}{\text{CSD}}$
N_{AB}	+86.2	+83.3	1.03	+46.3	+45.9	1.01	+39.7	+39.8	1.00
N_{BC}	+63.4	+61.3	1.03	+30.9	+31.0	1.00	+26.1	+26.4	0.99
N_{CD}	+58.8	+56.1	1.05	+19.7	+19.5	1.01	+13.7	+13.8	0.99
N_{AE}	+150.7	+150.4	1.00	+65.8	+67.6	1.00	+28.3	+29.8	0.95
N_{BF}	+362.5	+361.9	1.00	+186.5	+185.9	1.00	+118.1	+115.7	1.02
N_{CG}	+358.2	+359.4	1.00	+179.3	+179.3	1.00	+103.5	+104.6	0.99
N_{DH}	+158.7	+158.3	1.00	+83.4	+83.6	1.00	+59.2	+59.1	1.00

N_{ij} = normal force the member ij
Nomenclature for the members:
+ Force compression
− Force tension

In Table 3 and Figure 8 which show the shear forces in the ends of the members for the two methods. For example, for $L = 10.00$m is lower a 11% of the CG member in absolute value in the traditional method with respect to the proposed method, and for $L = 10.00$m is greater an 8% in the AE member in the traditional method with respect to the proposed method.

TABLE 3. The shear forces of each member in kN

Shear Force	Case 1 $L = 10.00$m			Case 2 $L = 5.00$m			Case 3 $L = 3.00$m		
	NSD	CSD	$\frac{\text{NSD}}{\text{CSD}}$	NSD	CSD	$\frac{\text{NSD}}{\text{CSD}}$	NSD	CSD	$\frac{\text{NSD}}{\text{CSD}}$
V_{AB}	+150.7	+150.4	1.00	+65.8	+66.3	0.99	+28.3	+29.8	0.95
V_{BA}	−192.6	−193.0	1.00	−105.9	−105.4	1.00	−74.7	−73.2	1.02
V_{BC}	+169.8	+169.0	1.00	+80.6	+80.5	1.00	+43.3	+42.4	1.02
V_{CB}	−173.5	−174.4	1.00	−91.0	−91.2	1.00	−59.7	−60.6	0.99
V_{CD}	+184.7	+185.0	1.00	+88.2	+88.1	1.00	+43.8	+43.9	1.00
V_{DC}	−158.7	−158.3	1.00	−83.4	−83.6	1.00	−59.2	−59.1	1.00
V_{AE}	+37.1	+34.3	1.08	−2.8	−3.1	0.90	−9.3	−9.2	1.01
V_{BF}	−22.8	−22.0	1.03	−15.3	−14.9	1.03	−13.6	−13.4	1.01
V_{CG}	−4.6	−5.2	0.89	−11.3	−11.6	0.97	−12.4	−12.7	0.98
V_{DH}	−58.8	−56.1	1.05	−19.7	−19.5	1.01	−13.7	−13.8	0.99

V_{ij} = shear force the member ij, in the joint i
Nomenclature for beams:
+ Force shear is above of the axis of reference
− Force shear is under of the axis of reference
Nomenclature for columns:
+ Force shear to the right of the axis of reference
− Force shear to the left of the axis of reference
Being: $V_{AE} = -V_{EA}$; $V_{BF} = -V_{FB}$; $V_{CG} = -V_{GC}$ and $V_{DH} = -V_{HD}$.

With respect to Table 4 and Figure 9 which illustrates the negative moments and positive for both methods. As soon as to the results, for $L = 10.00$m is greater a 26% in the EA member at the end E, in absolute value in the traditional method with respect to the proposed method, and for $L = 10.00$m is lower a 40% in the CG member at the C, in absolute value in the traditional method with respect to the proposed method.

5. **Conclusions.** The results of the problem considered, through the application of two different techniques: traditional method (considering flexure deformations) and the proposed method (considering the flexure deformations and shear), allowed to conclude that:

For the traditional method, the rotations and displacement are lower in all cases, with respect to the proposed method. This is a logical situation, since the rigidities are lower when considering shear deformations, because the elements are leaner and have higher rotation and displacement when the load is applied. This condition implies that it must take into account the deformations permitted by building regulations, because in some situation could be presented which does not meet the standards established by these codes.

According to the axial forces, shear forces and moments acting on the members. These mechanical elements are those which govern the design of a structure. The results show that differences exist between the two methods, both on the conservative side as of insecure side with respect to the traditional method. This means that, it is designing wrongly, on one side some members are exceeded in their cross section dimensions and in another situation does not comply with the minimum conditions for that a is satisfactory structure.

TABLE 4. The moments of each member in kN-m

Moment	Case 1 $L = 10.00$m			Case 2 $L = 5.00$m			Case 3 $L = 3.00$m		
	NSD	CSD	$\frac{\text{NSD}}{\text{CSD}}$	NSD	CSD	$\frac{\text{NSD}}{\text{CSD}}$	NSD	CSD	$\frac{\text{NSD}}{\text{CSD}}$
M_{AB}	−137.7	−133.4	1.03	−3.1	−3.3	0.94	+19.5	+18.3	1.07
$M℄_{AB}$	+193.0	+195.8	0.99	+59.9	+60.7	0.99	+31.1	+31.2	1.00
M_{BA}	−347.4	−346.6	1.00	−103.3	−101.0	1.02	−50.2	−46.9	1.07
M_{BC}	−285.6	−286.3	1.00	−64.5	−63.7	1.01	−16.4	−14.2	1.15
$M℄_{BC}$	+134.3	+129.4	1.04	+30.2	+30.7	0.98	+11.0	+12.0	0.91
M_{CB}	−313.0	−313.5	1.00	−90.4	−90.3	1.00	−40.9	−41.4	0.99
M_{CD}	−311.6	−311.2	1.00	−65.2	−64.3	1.01	−11.1	−11.2	0.99
$M℄_{CD}$	+184.9	+187.4	0.99	+48.1	+48.7	0.99	+16.9	+16.8	1.00
M_{DC}	−181.8	−177.5	1.02	−53.3	−53.0	1.01	−34.1	−34.0	1.00
M_{AE}	+137.7	+133.4	1.03	+3.1	+3.3	0.94	−19.5	−18.3	1.07
M_{EA}	−47.8	−38.0	1.26	+16.9	+18.8	0.90	+27.1	+27.8	0.97
M_{BF}	−61.8	−60.2	1.03	−38.8	−37.3	1.04	−33.8	−32.7	1.04
M_{FB}	+52.0	+49.8	1.04	+37.9	+37.2	1.02	+34.2	+34.3	1.00
M_{CG}	−1.4	−2.4	0.60	−25.2	−25.9	0.97	−29.8	−30.1	0.99
M_{GC}	+21.8	+23.6	0.92	+31.1	+32.0	0.97	+32.2	+33.1	0.97
M_{DH}	−181.8	−177.5	1.02	−53.3	−53.0	1.01	−34.1	−34.0	1.00
M_{HD}	+112.0	+103.0	1.09	+45.1	+44.3	1.02	+34.4	+34.9	0.98

M_{ij} = negative moment the member ij, in the joint i
$M℄_{ij}$ = positive moment the member ij
Nomenclature for moment:

- For horizontal members:

+ Moment that is above of the axis of reference (compression in superior fiber and tension in inferior fiber)
− Moment that is under the axis of reference (tension in superior fiber and compression in inferior fiber)

- For vertical members:

+ Moment that is to the right of the axis of reference (compression in right fiber and tension in left fiber).
− Moment that is to the left of the axis of reference (tension in right fibers and compression in left fibers).

Since that the principle in civil engineering, with regard to the structural conditions is that have to be safe and economical.

Therefore, the usual practice of using the traditional method (slope-deflection method considering flexure deformations) is not a recommended solution.

So taking into account the numerical approximation, proposed method (considering the flexure deformations and shear), happens to be the more appropriate method for structural analysis of rigid frames and also more attached to the real conditions.

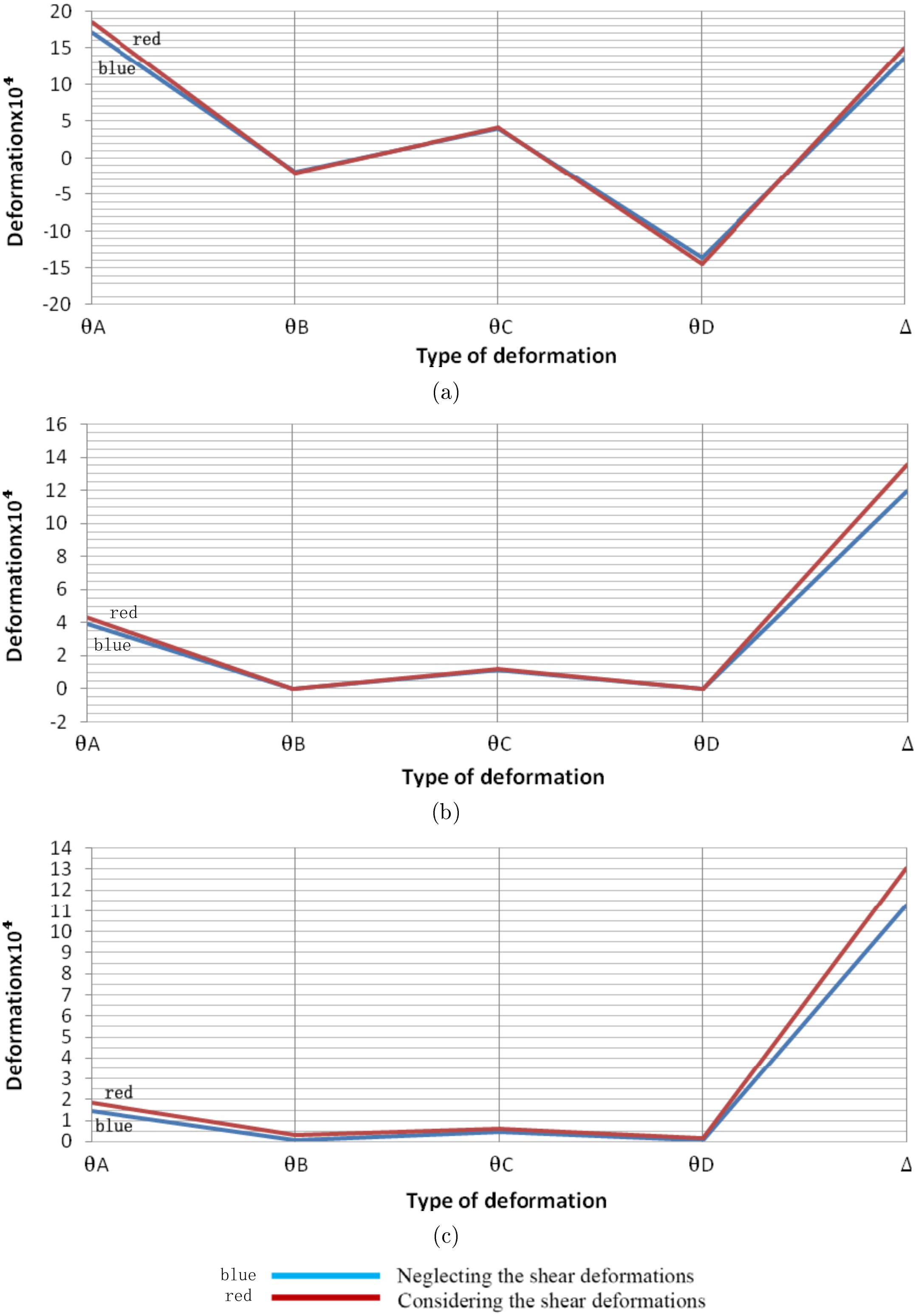

FIGURE 6. Deformations of the rigid frame. (a) For $L = 10.00$m, (b) for $L = 5.00$m, (c) for $L = 3.00$m.

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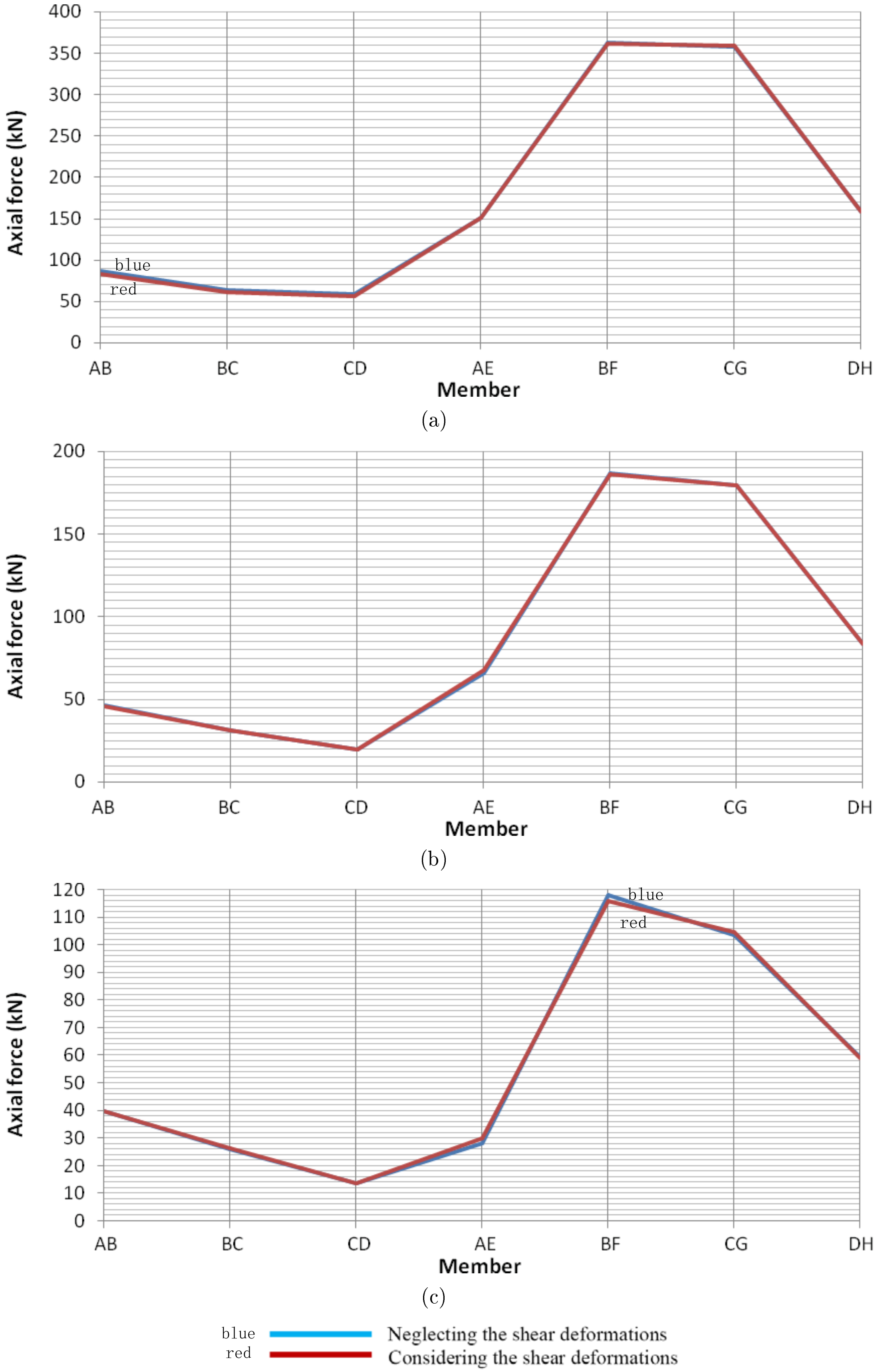

FIGURE 7. Axial forces of the rigid frame. (a) For $L = 10.00$m, (b) for $L = 5.00$m, (c) for $L = 3.00$m.

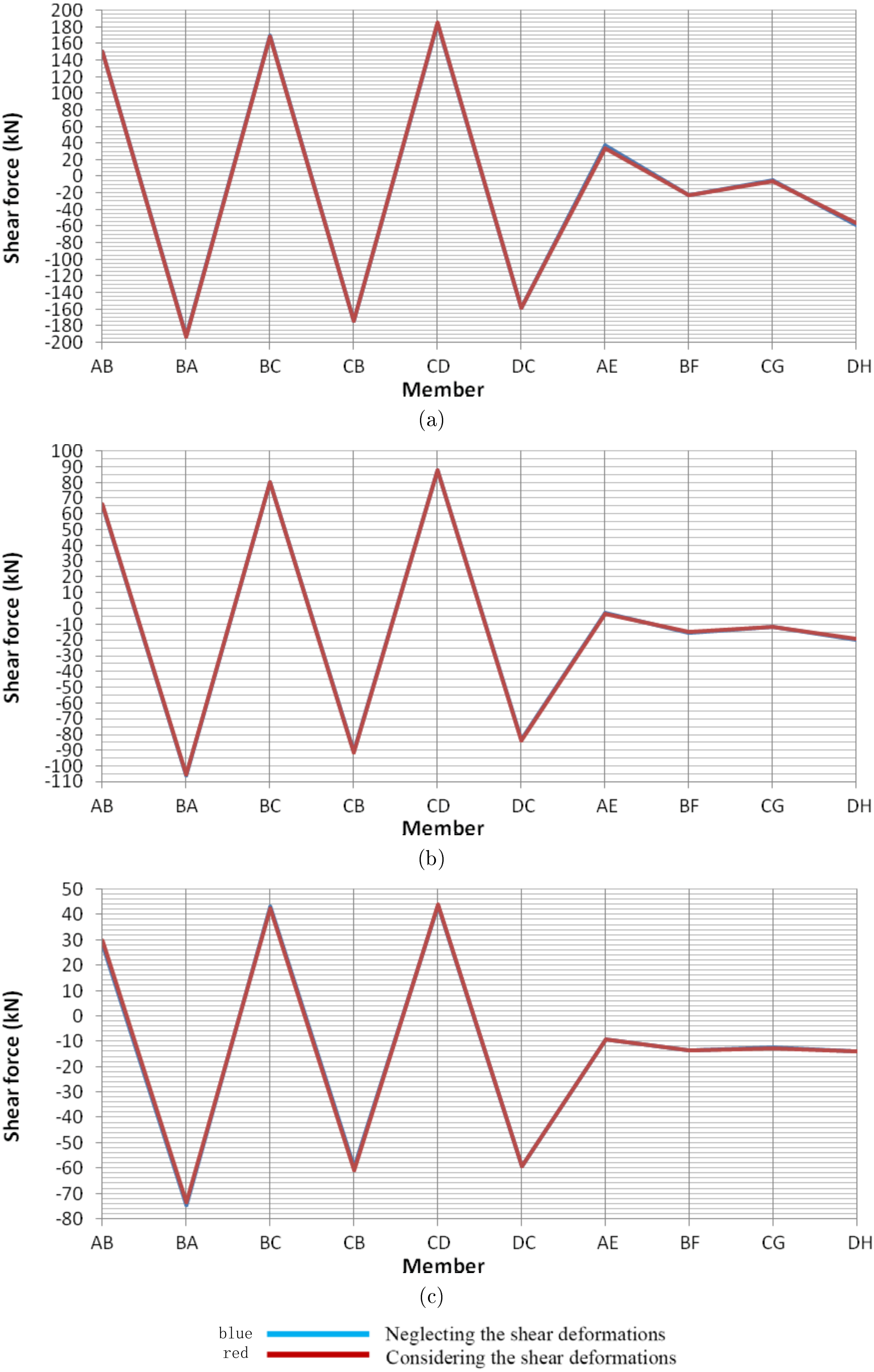

FIGURE 8. Shear forces of the rigid frame. (a) For L = 10.00m, (b) for L = 5.00m, (c) for L = 3.00m.

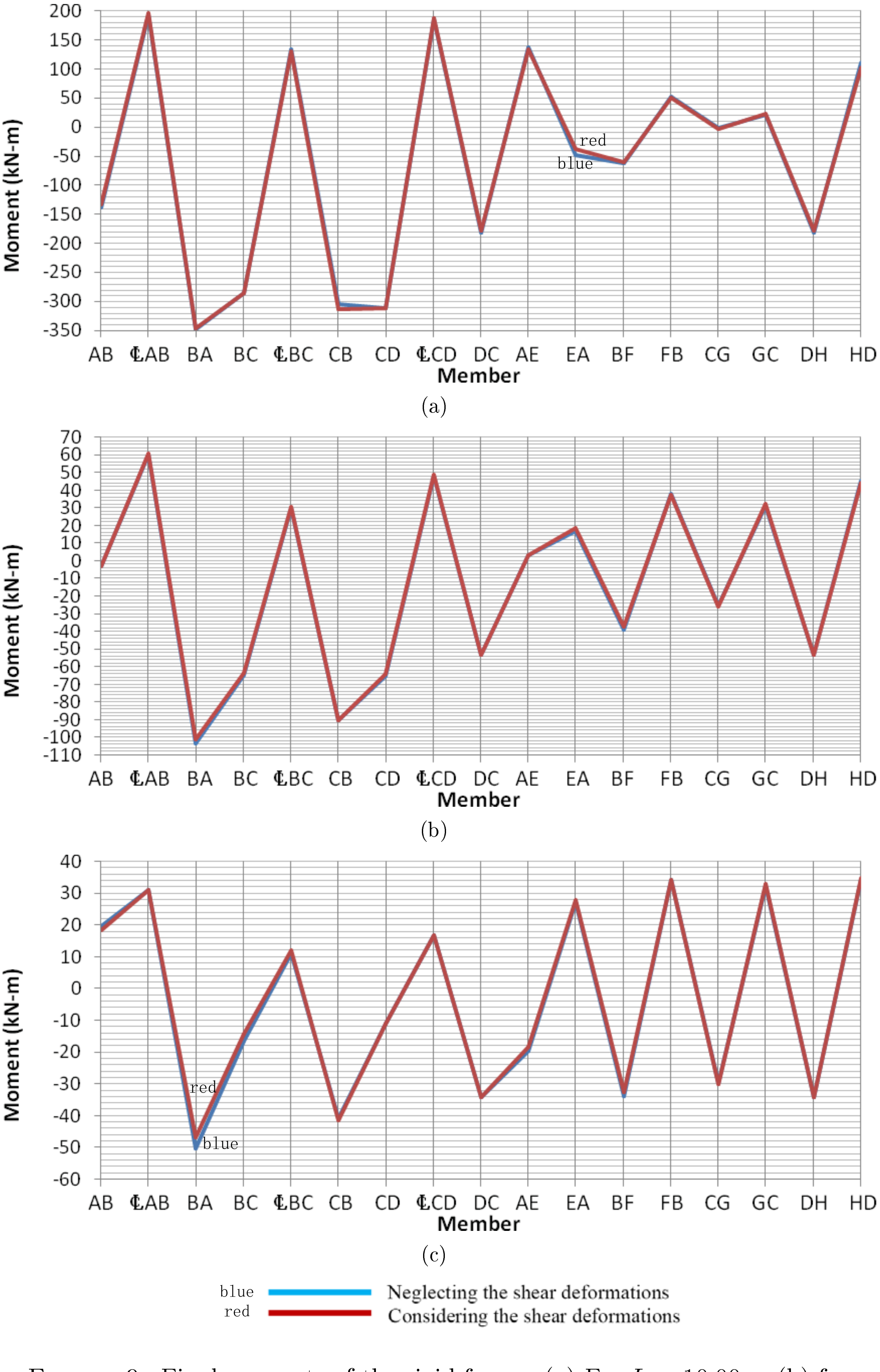

FIGURE 9. Final moments of the rigid frame. (a) For $L = 10.00\text{m}$, (b) for $L = 5.00\text{m}$, (c) for $L = 3.00\text{m}$.

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