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A MULTI-ROBOT, COOPERATIVE, AND ACTIVE SLAM ALGORITHM FOR EXPLORATION

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Abstract. *The paper proposes a cooperative and active SLAM (simultaneous localization and mapping) algorithm for exploration of unknown environment by using multiple robots. In the exploration process, it is desired that the task can be accomplished efficiently and with high quality. The paper formulates the exploration problem as an efficiency optimization problem that is subject to localization quality constraints. The proposed approach allows the dispersion of multiple robots in the cooperative and active sense with a guaranteed level of localization and mapping accuracy. The paper further investigates the use of adaptive strategy for balancing the need of exploration and localization. A rendezvous technique is provided to deal with the problem of limited communication. Simulation results demonstrate that the proposed approach outperforms existing methods.*
Keywords: Active SLAM, Multiple robots, Exploration, Relocalization, Global optimization

1. **Introduction.** Autonomous mobile robots have been employed in a wide range of tasks such as search and rescue, planetary and underwater exploration, as well as surveillance and reconnaissance. An autonomous robot must have the capability of efficiently and effectively exploring its surrounding environment, performing simultaneous localization and mapping (SLAM), and navigating in the environment. It is well known that multi-robot systems have several advantages over single-robot systems, such as faster task completion, more efficient localization, and higher fault tolerance [1,2]. The paper develops an algorithm for multi-robot exploration that combines the concepts of SLAM, active path planning, and cooperation among multiple robots.

Robot dispersion is a key requirement in many applications such as search and rescue. However, in existing active SLAM approaches for multi-robot exploration [3,4], the robots are spread over the environment only at a local level, i.e., the robots attempt to avoid overlap in the exploration area rather than being assigned to different regions of the environment to minimize the overall exploration time. In addition, there is a lack of mechanism to balance the exploration performance and localization quality. Moreover, existing approaches typically assume that the communication range is unlimited. The paper addressed the aforementioned limitations through the development of a multi-robot, cooperative, and active SLAM algorithm for exploration. Firstly, in the exploration phase, a global optimization scheme is proposed to disperse the robots in the environment without assuming a priori knowledge of the environment. Secondly, an adaptive strategy is developed for automatically adjusting the threshold of the robot pose and map uncertainty constraints in order to better balance the needs of exploration and relocalization. Finally, a rendezvous technique is designed to deal with the limited communication range by allowing robots to temporarily move out of communication range and rejoin the group later.

The rest of this paper is organized as follows. Section 2 reviews related work in the field of mobile robot exploration. Section 3 formulates the active SLAM problem with multiple robots. Section 4 describes the proposed approach, followed by the simulation results and discussions in Section 5. Section 6 gives the conclusions.

2. **Related Work.** In existing exploration methods, robot poses are usually assumed to be known, which is not true in many real situations. To account for unknown robot poses, several SLAM methods [5-9] have been developed. However, these algorithms are passive in the sense that the trajectory of the robot is not actively controlled. It should be noted that the path control strategy can have a substantial impact on the quality of the resulting map [10]. Therefore, active SLAM methods have been recently proposed to efficiently explore the environment while collecting data to maximize the accuracy of the resulting map [10-15]. Nevertheless, most active SLAM methods only consider the single-robot case. It is desirable to extend active SLAM to multi-robot scenarios. Up to now, only a few authors considered the active SLAM algorithm for multiple robots [3]. The method in [3] does not have the relocalization phase thus does not guarantee that the robots will not get lost.

The core problem in multi-robot exploration is to choose appropriate target points for robots so that they simultaneously explore different regions of the environment. In existing multi-robot exploration methods, the coordination level varies from no coordination [16] to intensive cooperation [7,8,17]. Several methods including integer programming [8] and auction scheme [7,17] have been proposed for the assignment of robots to targets. Other studies [18] obtain the robot cooperation by employing potential field methods with several basic behaviours such as moving to frontier, avoiding obstacles, and avoiding robots. The idea of spreading multiple robots for better coverage was explored in [5-7]. However, the robot dispersion is only at a local level. In [19], the robots are globally scattered in the environment. Nevertheless, a priori knowledge of the environment is assumed.

Existing multi-robot exploration methods typically assume that the robots under consideration are within the communication range [20-23]. More recently, a role-based exploration strategy has been proposed that allows robots to explore beyond the communication range [24]. Robots either act as explorers or as relays to explore the environment and return information to a central command center. Rendezvous points between explorers and their corresponding relays are dynamically set during the exploration process. Extensions of such a method can be found in [25,26], in which a new rendezvous point selection procedure and dynamic team hierarchies are adopted, respectively. However, existing cooperative methods that allow robots to go beyond the communication range typically assume that perfect sensor data and localization are available. Existing exploration approaches are briefly summarized in Table 1.

Table 1. Comparison of existing exploration methods

Number of robots	**Cooperation**	**SLAM**	**References**
Single	No	Active	[10],[11], [12], [13], [14], [15]
Multiple	No	No	[16]
Multiple	Yes	No	[17], [18], [19], [20], [21], [22], [23], [24], [25], [26]
Multiple	Yes	Passive	[5], [6], [7], [8]
Multiple	Yes	Active	[3], [4], this paper

3. **Active SLAM Problem with Multiple Robots.** In multi-robot SLAM problems, autonomous vehicles with known models incrementally build a map of the unknown environment while simultaneously using the map to compute absolute vehicle locations. For simplicity, a two-dimensional SLAM problem is considered. Each robot is equipped with a sensor that is capable of measuring the relative range and angle between any individual landmark/robot and the robot itself.

In multi-robot exploration problems, robots are required to coordinate with one another to efficiently and completely explore an unknown environment in a way that minimizes the exploration time while maintaining the accuracy of the robot pose and map estimates. To this end, the following constrained optimization problem is formulated:

$$\begin{aligned}&\text{min exploration time}\\&\text{Subject to } trace(\mathbf{P}_r) + \frac{1}{L}\sum_{i=1}^{L} trace(\mathbf{P}_i) < \alpha,\ r = 1, 2, \ldots, R\end{aligned} \tag{1}$$

where the exploration time is the time needed to completely explore the environment; $\mathbf{P}_r$ and $\mathbf{P}_i$ are the pose error covariance of the rth robot and the position error covariance of the ith landmark, respectively; R and L are respectively the number of robots and the number of landmarks observed; and α is a pre-defined threshold.

4. **Proposed Approach for Active SLAM with Multiple Robots.** The problem (1) is a complex optimization problem. In this section, an algorithm is proposed to solve the multi-robot exploration problem. Previously, an approach that relies on the alternation of exploration phase and relocalization phase was proposed to address the problem [4]. In the section, several revisions are made to the method for the determination of the optimal solution to the constrained optimization problem (1). In the exploration phase, robots with low pose and map uncertainty cooperatively explore the environment with a balance of information gain, localization quality, and navigation cost. A global optimization strategy is proposed to allocate the exploration task equally among robots. Whenever the robot pose and map uncertainty estimated by a robot exceeds a pre-defined threshold, the robot switches to the relocalization phase to revisit previously seen landmarks or to meet other robots in order to relocalize itself. During the exploration process, robots may switch between the two phases to minimize the exploration time while maintaining the localization and mapping accuracy. To prevent the robots from oscillating between the two phases, an adaptive strategy is proposed to automatically adjust the threshold of the uncertainty constraints. A rendezvous technique is designed to deal with the problem of limited communication.

A flowchart of the proposed approach is shown in Figure 1. At any point in time, the task of a robot is determined to be either exploration or relocalization. If the exploration task is executed, an exploration target is selected for the robot. If the robot receives a meeting request from another robot, instead of proceeding to the exploration target, the robot responds to the request by moving to the meeting point or standing still. The robot then waits for the designated robot for a certain amount of time before returning to its exploration/relocalization task. Whenever the uncertainty of the robot pose and map exceeds a pre-defined threshold, the relocalization task is executed by determining a relocalization target for the robot. If the relocalization task involves another robot, a request is sent to that robot. If a robot moves out of communication range, it later returns to the rendezvous point, which is determined before leaving the group. In Figure 1, t_0 is the time at which a robot moves out of the communication range of other robots, τ is the duration which robots are allowed to be out of communication range, and T

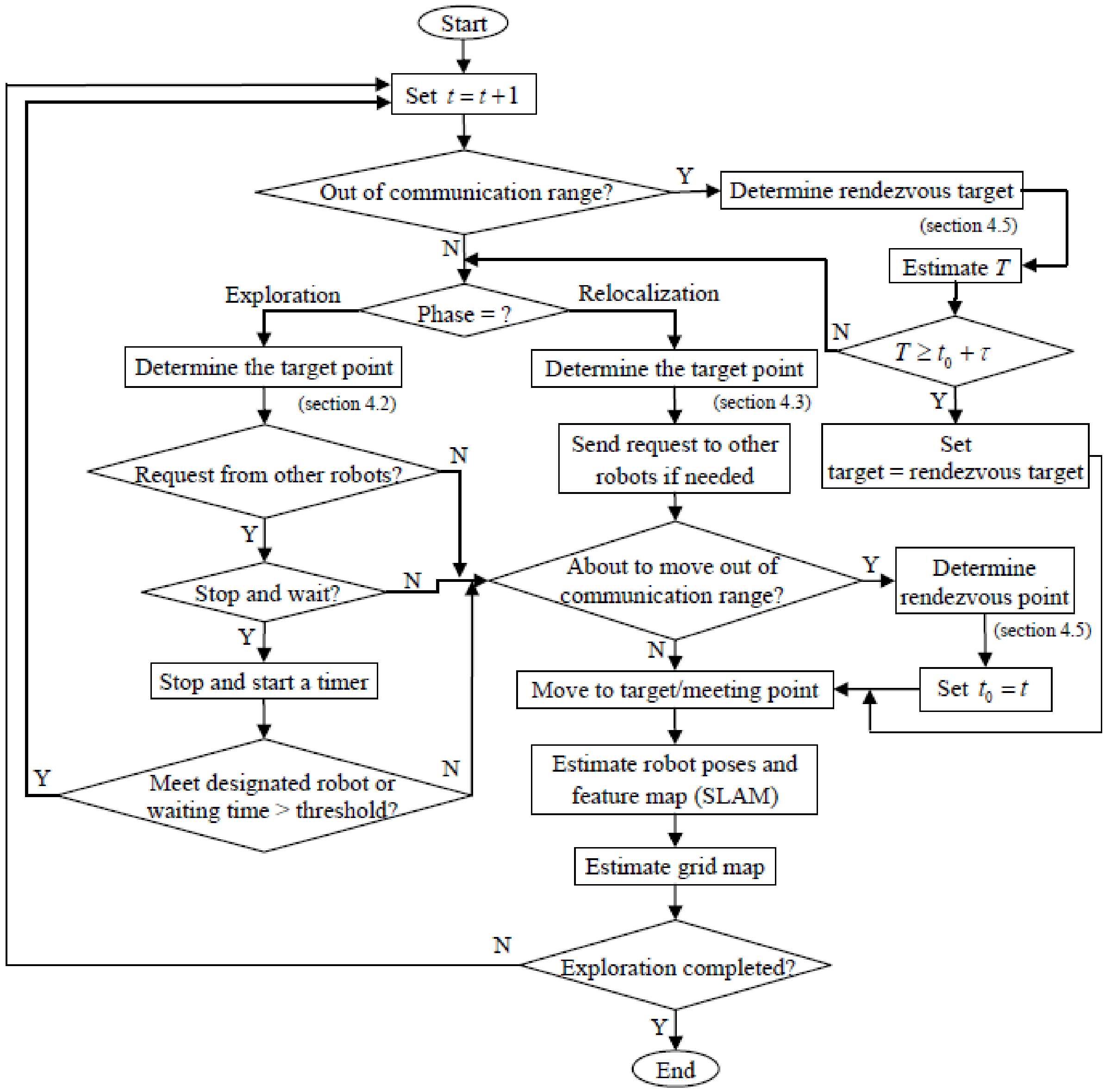


FIGURE 1. Flowchart of the proposed approach for each robot

is the estimated time at which the robot can reach the rendezvous target. During the exploration process, SLAM operations are performed to obtain estimates of the robot poses and the map. A coarse occupancy grid map is also estimated for exploration and path planning. The procedure is repeated until the environment has been completely explored.

During the exploration process, robots simultaneously perform SLAM and decide the next action. To this end, the extended Kalman filter (EKF)-based SLAM algorithm [27] with a feature map is adopted. In existing exploration methods, SLAM algorithms with occupancy grid maps are often employed. However, in large-scale outdoor environments with few landmarks/structures, SLAM with a high-resolution grid map may be difficult to apply for real-time exploration due to its high computational load. In such environments, feature-based SLAM algorithms are a better choice. It should be noted that other SLAM algorithms ([28,29], for example) and map representation ([30], for example) can also be used.

4.1. **Exploration phase.** The key problem in multi-robot exploration is the selection of appropriate target points for robots. More specifically, robots must balance information gain, localization quality, and navigation cost. A global optimization strategy with respect to the dispersion of the robots is proposed to allocate the exploration task equally among robots. A priori knowledge of the environment is not required. This global strategy is a key improvement over those in [4].

A frontier-based exploration strategy [16] is adopted. An occupancy gird map [30,31] is utilized for the exploration purpose, i.e., to distinguish between explored and unexplored areas, and the navigation purpose. To reduce computational load, the grid map is built at a low resolution and estimated using a simple counting method [32]. This coarse grid map was found to be sufficient for exploration and navigation.

The global optimization scheme for robots dispersion is as follows. Firstly, frontier cells extracted from the estimated grid map are grouped into exploration points (targets), which are then clustered into separate borders, as illustrated in Figure 2. If the number of separate borders is less than the number of robots, the targets are divided into as many borders as available robots using the K-means clustering algorithm [33]. Secondly, an integer programming problem is solved to determine the optimal assignment from robots to borders that minimizes the total travelling distance. This global assignment plays an important role in distributing robots over the environment, which is a key requirement in many robot applications, such as search and rescue. Solving this integer programming problem does not take much time because the number of separate borders is usually small and the travelling distances are easily computed. If the number of separate borders is greater than the number of robots, some borders are left unassigned (referred to as *free borders*).

The global assignment from robots to borders attempts to disperse robots equally in the environment. However, robot separation may degrade the efficiency of robot localization because the robots cannot help one another to relocalize. Therefore, besides being allowed to freely choose a target from its border or free borders, a robot is also allowed to choose, with a penalty, a target from other robots' borders. The penalty coefficient is defined as:

$$\beta(r,t) = \begin{cases} 1, & \text{if target } t \text{ belongs to } r\text{th robot's border or free borders} \\ \delta, & \text{otherwise} \end{cases} \tag{2}$$

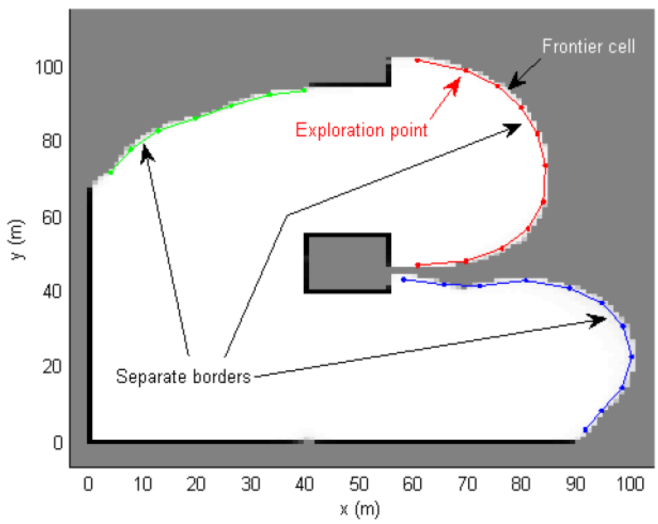


FIGURE 2. Frontier cells, exploration points, and separate borders

where $\delta \geq 1$ is a mission-dependent constant. For example, if the robots should stay close enough to help one another to relocalize, δ is kept close to 1. Otherwise, if the process and measurement errors are small and the landmark density is sufficiently high, δ is set to be much greater than 1 to force the robots to disperse. The penalty coefficient defined above is then utilized in the assignment of robots to targets as an integration of global information into the local level. Besides this global information, the expected payoffs and costs associated with moving to the proposed locations must be calculated. Assume that there are R_{exp} robots and T targets under consideration. R_{exp} can be smaller than R because some robots may not be in the exploration phase at that time. In the exploration phase, an $R_{\text{exp}} \times T$ assignment matrix $\boldsymbol{A}$, which contains only 0's and 1's, is determined. When $\boldsymbol{A}(r,t) = 1$, the rth robot is assigned to move to the tth target. Each robot is assigned to exactly one target, and each target has at most one robot assigned to it. The assignment matrix is determined to maximize the following objective function:

$$J_{exploration} = \sum_{r=1}^{R_{\text{exp}}} \sum_{t=1}^{T} \boldsymbol{A}(r,t) \left[\frac{w_I U_I(r,t)}{\beta(r,t)} + \frac{w_{L1} U_{L1}(r,t)}{\beta(r,t)} - \beta(r,t) w_{N1} C_{N1}(r,t) \right] \quad (3)$$

where $U_I(r,t)$ is the utility of information, $U_{L1}(r,t)$ is the utility of localizability, $C_{N1}(r,t)$ is the cost of navigation, and w_I, w_{L1}, and w_{N1}, respectively, are the associated weighting coefficients. The utility of information $U_I(r,t)$ is the expected area that the rth robot will explore at the frontier when it reaches the tth target. The area is assessed by using an estimate of the size of the unknown area visible from the tth target. The utility of localizability $U_{L1}(r,t)$ [13] is used to distinguish between target points with different localization qualities. The localization quality is defined as the minimum covariance achievable by relocalizing a lost vehicle at a given location by observing only the landmarks visible from that location. The cost of navigation $C_{N1}(r,t)$ is the minimal path length for the rth robot to move to the tth target. The minimal path can be efficiently computed using the method in [5,6]. The objective function is evaluated with respect to each robot-target pair. The determination of the assignment matrix $\boldsymbol{A}$ becomes an integer programming problem. Standard tools such as branch-and-bound algorithms can then be used to find the assignment matrix. The robots are then directed to the assigned target points. It should be noted that when $\delta = 1$, the objective Function (3) reduces to the objective function in [4].

4.2. **Relocalization phase.** The relocalization phase helps maintain the localization and mapping accuracy by forcing a robot to revisit previously seen landmarks or to meet another robot whenever its pose and map uncertainty exceeds a pre-defined threshold. Because the exploration phase does not guarantee that the robots will not get lost, the relocalization phase is especially necessary when measurement errors are large or landmarks are sparse.

The relocalization phase is briefly described below. The relocalization phase is applied for each individual robot. For relocalization, a landmark with low position uncertainty can serve as a target point. The robot can also reduce its pose uncertainty by meeting other robots which have good pose knowledge and are in its communication range. The target point for relocalization is determined by optimizing an objective function that is related to the cost of navigation C_{N2}, loss of efficiency L, utility of localizability U_{L2}, and distance to the nearest exploration point D. The objective function of the rth robot is given by:

$$J_{relocalization} = w_{L2} U_{L2} - w_{N2} C_{N2} - w_{Loss} L - w_D D \quad (4)$$

where w_{L2}, w_{N2}, w_{Loss}, and w_D are weighting coefficients. The utility of localizability U_{L2} is used as in the exploration phase to distinguish between target points with different localization qualities. The cost of navigation C_{N2} is given by the minimal path length from the position of the rth robot to the selected landmark or to the meeting point with another robot. The loss of efficiency L, if any, is due to the interruption of the exploration task of another robot involved in the relocalization phase. The distance D from the relocalization target to the nearest exploration point (the nearest frontier) is used to assess the effort needed to go back to perform exploration. More specifically, robots prefer relocalization targets which are close to the frontier. Once the robot is forced to enter the relocalization phase, the objective function $J_{relocalization}$ in (4) as a function of all candidate target points is optimized. After determining the target point, the robot is controlled to move to the target point. If control actions involve another robot, a request is sent to the designated robot to perform the relocalization operation.

4.3. **Adaptive threshold selection.** With a fixed uncertainty threshold α in (1), robots may get stuck in regions with few or no landmarks by repeatedly switching between the exploration and relocalization phases, as illustrated in Figure 3. In this figure, a robot is in its exploration phase, moving from the left side towards exploration point A. At point B, its pose and map uncertainty exceeds α; it thus switches to the relocalization phase and heads to a good landmark (point C). At point D, its pose and map uncertainty becomes small enough to switch back to the exploration phase. Assume that the new exploration point is still A; the robot thus moves towards this point. At point E, the robot pose and map uncertainty again exceeds α, so the robot switches to the relocalization phase and heads to point C. The robot then oscillates between the exploration and relocalization phases.

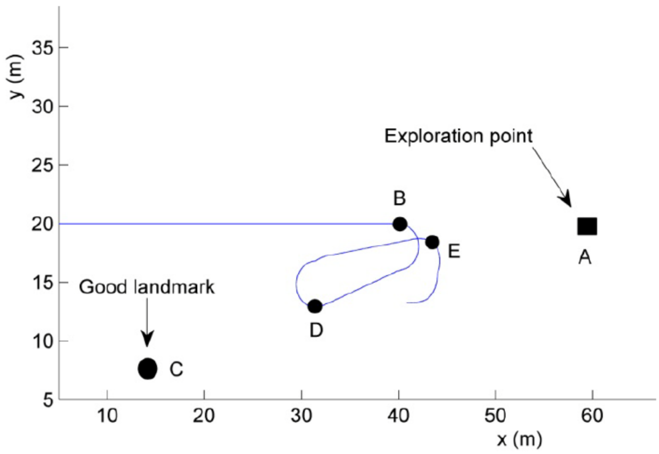


FIGURE 3. Robot getting stuck in a region with few landmarks

To overcome this problem, the uncertainty threshold α in the proposed approach is temporarily increased (within a permissible range) in such a situation to allow the robot to pass this region. In particular, at point B, the robot stores the current exploration point A as an unreached exploration point and at point D it stores the current relocalization point C as an old relocalization point. Having stored the unreached exploration point and the old relocalization point, now at point E, the robot knows that the situation is the same as that at point B. Therefore, it increases the threshold α to pass this region. In the proposed approach, each robot has its own uncertainty threshold and only the threshold of the concerned robot is increased.

Temporarily increasing the threshold α allows robots to pass regions with few or no landmarks. However, the threshold α should be reduced as soon as possible to avoid large error in the subsequent exploration process. In the method, α is reduced whenever the robot pose uncertainty decreases and the landmark density in the neighboring region of the robot is sufficiently high.

4.4. **Limited communication range.** In many exploration algorithms, it is assumed that a wireless network is always established among robots. However, communication range is limited in real situations. One way to deal with this problem is to keep robots close to one another. Nevertheless, this strategy can significantly degrade the efficiency of exploration. Another way is to allow robots to move out of the communication range, as in [6]. However, when a robot does not know where other robots are, it can repeat the route of other robots, leading to an overlap in exploration.

To overcome the above problems, the proposed approach allows robots to temporarily move out of communication range and then rendezvous with one another after a specific time. If a robot moves out of communication range, the centralized approach presented above cannot work for the whole system, but it can be applied to subsystems consisting of robots remaining in communication range. The idea of periodic connectivity was utilized in [34] to deal with the multi-robot path planning problem in a known environment.

The amount of time for which robots are allowed to be out of communication range should depend on the communication range, sensor range, robot velocity, and even the environment. Figure 4 illustrates the sensor and communication ranges of two robots at the beginning (solid line) and sometime later (dashed line). The minimum time required for the robot R1 to overlap the area covered by the robot R2 is given by:

$$\tau_0 = \frac{d}{v} = \frac{R_C - 2R_S}{v} \tag{5}$$

where R_C and R_S are the communication and sensor ranges, respectively. Therefore, the maximum time for robots to be allowed to move out of communication range is τ_0. In practice, however, this value is very conservative and thus it only serves as a lower bound in the proposed approach.

After moving out of the communication range for a preset time τ, robots have to return to a pre-arranged meeting point to exchange information obtained during that time in order to avoid overlap in exploration. The meeting point should be chosen such that: (i) the localization quality at this point is high, (ii) total travelling distance from all robot locations to this point is short, and (iii) distances to exploration targets excluding current targets of all robots are short. More specifically, landmarks with low position uncertainties and the central location of the robot team can serve as target points. Similar to the exploration and relocalization phases, the target point for rendezvous is determined by optimizing an objective function that is related to the utility of localizability U_{L3}, cost

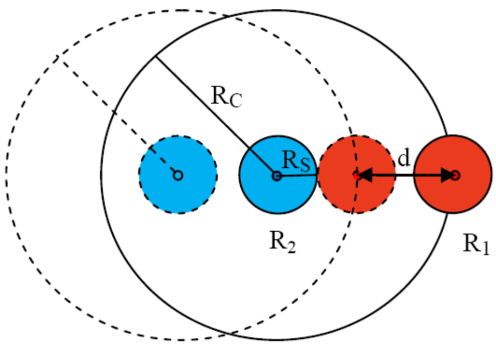


FIGURE 4. Minimum distance required for a robot to overlap another robot's coverage

of navigation C_{N3}, and distance to the nearest exploration point D (excluding current destinations of all robots). The objective function of the ith target is given by:

$$J_{rendezvous} = w_{L3}U_{L3} - w_{N3}C_{N3} - w_{D3}D \quad (6)$$

where w_{L3}, w_{N3}, and w_{D3} are weighting coefficients. The cost of navigation C_{N3} is given by the total travelling distance from the locations of all robots to the target point. U_{L3} and D are similar to those in (4).

When a robot is about to move out of communication range, the objective function $J_{rendezvous}$ in (6) as a function of all candidate target points is optimized. All robots then store this location as well as the current time and begin to explore beyond the communication range within a preset time τ.

When gathering, robots may not need to reach the predetermined rendezvous point to reestablish communication. The procedure for returning to the rendezvous point for a two-robot team is described as follows. Let O be the rendezvous point obtained by maximizing (6) and t_0 be the time at which a robot moves out of communication range. To ensure that the two robots can communicate again at the latest at time $t_0 + \tau$ each robot (with its own estimated map) manages to return to its rendezvous target, i.e., a point on the circumference centered at O with diameter $R_c/2$. In Figure 5, A and B are rendezvous targets for Robot 1 and Robot 2, respectively.

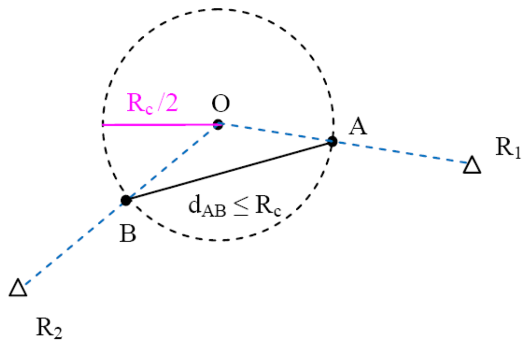


FIGURE 5. Gathering procedure for a 2-robot team

The procedure for reestablishing the communication network for a robot team with more than two robots is more complicated. Figure 6 plots a three-robot team with rendezvous point O. To ensure that the wireless mobile network can be reestablished at the latest at time $t_0 + \tau$, each robot (with its own estimated map) manages to return to its rendezvous target (say, points A, B, and C for robots R_1, R_2, and R_3, respectively) at time $t_0 + \tau$. Now assume that at time t ($t_0 < t < t_0 + \tau$), R_1 decides to move to A while R_2 is still exploring (because $R_2B < R_1A$). Also assume that at that time, R_1 and R_2 are within communication range, and thus R_2 knows that R_1 is moving to A. Let B' be a point on the circumference centered at A with diameter R_c. It is better, in terms of navigation cost, for R_2 to choose B' instead of B as a target for the rendezvous phase. Therefore, R_2 can choose to move either to B or B' to maximize the following objective function:

$$J_{gathering} = w_{L4}U_{L4} - w_{N4}C_{N4} - w_{D4}D \quad (7)$$

where w_{L4}, w_{N4}, and w_{D4} are weighting coefficients; U_{L4}, C_{N4} and D are similar to those in (4). It should be noted that R_2 considers moving to B' if and only if $R_2A > R_c$. If $R_2A \leq R_c$, B' is replaced by R_2's current position. After optimizing (7), if R_2 chooses its current position (i.e., do nothing for rendezvous), it will continue doing its current task

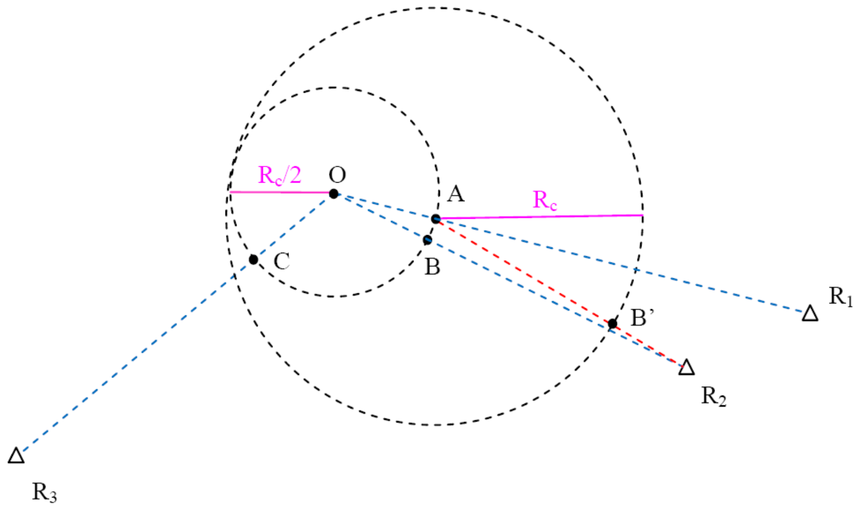


FIGURE 6. Gathering procedure for a 3-robot team

(exploration or relocalization). This behavior can be generally applied to any robot which knows other robots' targets for the rendezvous phase.

The rendezvous phase ends whenever all robots reestablish the wireless mobile network. After exchanging information collected during the period of exploration beyond communication range, robots return to the exploration or relocalization phases. To ensure robustness with respect to robot failure, when returning to the rendezvous point, a robot waits for other robots for a specified time before continuing its exploration/relocalization mission.

5. **Simulation Results and Discussion.** The section presents simulation results to assess the proposed method. In the simulations, it is assumed that a three-robot team cooperatively explores the environment. The mean velocity of the robots is 3 m/s and the sensor range is 20 m. The destination is recomputed whenever a robot reaches its current destination or a robot has moved 5 m. The following control noise and measurement noise covariance matrices are used:

$$\boldsymbol{Q}\begin{bmatrix} \sigma^2_{velocity} & 0 \\ 0 & \sigma^2_{steering} \end{bmatrix} = \begin{bmatrix} (0.3\text{ m/s})^2 & 0 \\ 0 & (3^0)^2 \end{bmatrix}$$

$$\boldsymbol{R} = \begin{bmatrix} \sigma^2_{range} & 0 \\ 0 & \sigma^2_{bearing} \end{bmatrix} = \begin{bmatrix} (0.1\text{ m})^2 & 0 \\ 0 & (1^0)^2 \end{bmatrix}$$

To quantify the performance of the proposed approach, the root mean square (RMS) errors are evaluated. Let $\boldsymbol{p}_{r,j}(k)$ be the true position of the rth robot at time k of the jth run. Also, let $\hat{\boldsymbol{p}}_{r,j}(k)$ be the estimate of $\boldsymbol{p}_{r,j}(k)$. The position estimation error is given by $\tilde{\boldsymbol{p}}_{r,j}(k) = \boldsymbol{p}_{r,j}(k) - \hat{\boldsymbol{p}}_{r,j}(k)$. The RMS error in robot localization for the whole robot team is computed as:

$$e = \frac{1}{J}\left[\sum_{j=1}^{J}\sqrt{\frac{1}{KR}\sum_{k=1}^{K}\sum_{r=1}^{R}\|\tilde{\boldsymbol{p}}_{r,j}(k)\|^2}\right]$$

where J is the total number of simulation runs and K is the number of time steps. The error in the estimation of landmark position is evaluated in a similar manner. Let $\boldsymbol{q}_{l,j}$ be the true position of the lth landmark of the jth run and let $\hat{\boldsymbol{q}}_{l,j}(K)$ be the final estimate of $\boldsymbol{q}_{l,j}$. The landmark estimation error is given by $\tilde{\boldsymbol{q}}_{l,j} = \boldsymbol{q}_{l,j} - \hat{\boldsymbol{q}}_{l,j}(K)$. The RMS error in landmark estimation is given by:

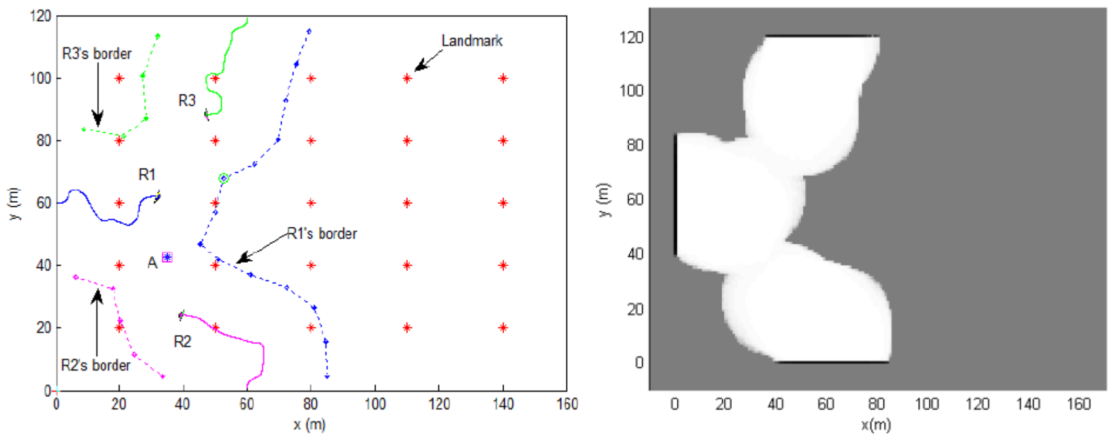
$$\eta = \frac{1}{J}\left[\sum_{j=1}^{J}\sqrt{\frac{1}{L}\sum_{l=1}^{L}\|\tilde{\boldsymbol{q}}_{l,j}\|^2}\right]$$

5.1. **Global exploration strategy.** In this simulation, the environment is assumed to contain 25 landmarks located in an area of 160 m × 120 m. The four methods listed in Table 2 were tested. Figures 7 and 8 show snapshots of the results and the obtained trajectories, respectively. At the beginning, all robots are in the exploration phase. At epoch 639, the pose and map uncertainty of Robot 2 exceeds the predefined threshold, and hence it switches to the relocalization phase by rendezvousing with Robot 1 at point A (Figure 7(a)). As shown in Figure 8, after meeting Robot 1 at A at epoch 694, the position errors of Robot 2 are reduced. The two robots then return to the exploration phase. The proposed global exploration strategy attempts to distribute robots over the environment, as shown in Figure 7(a), where each separate border is assigned to a robot.

For simulation analysis, 20 independent runs for each method were performed. Table 3 lists position errors of the robots and errors in landmark estimation. The number of epochs, which is related to exploration time, is also given in Table 3. Method D yields the largest errors because it does not use the relocalization phase and the utility of localizabilty in the exploration phase. Its exploration time is not the smallest because its robots-target assignments are suboptimal. For method C, with the utility of localizability, robots prefer exploration targets with high localization quality and thus errors are smaller.

Table 2. Summary of methods used for testing global exploration strategy

Method	Use of utility of localizability in the exploration phase	Use of relocalization phase	Use of global exploration strategy
A (proposed)	Yes	Yes	Yes
B [4]	Yes	Yes	No
C [13]	Yes	No	No
D [5,6]	No	No	No



(a) (b)

Figure 7. Snapshots of typical results for proposed method in rectangular environment: (a) robot trajectories, borders, and landmarks and (b) occupancy grid map

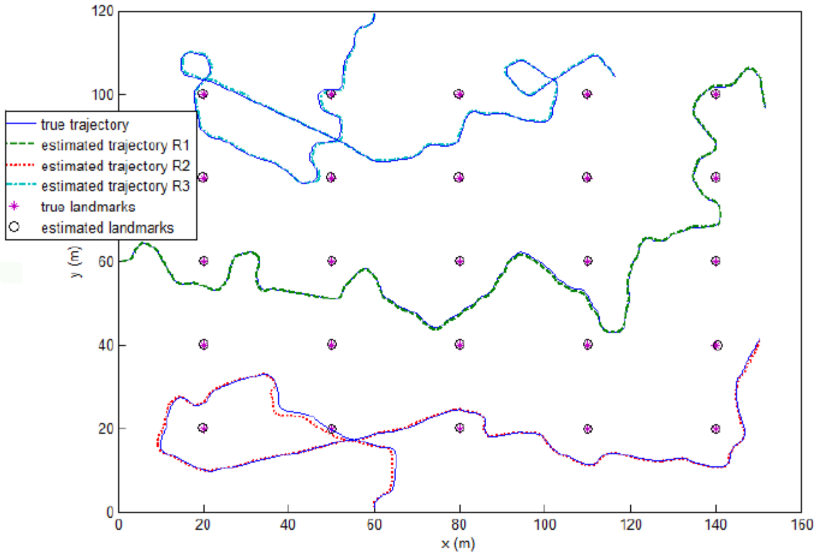


FIGURE 8. Obtained trajectories for proposed method in rectangular environment

TABLE 3. Position errors and exploration time for rectangular environment

Method	Robot RMS error (m) e		Landmark RMS error (m) η		Exploration time (epochs)	
	Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation
A	0.548	0.244	0.242	0.152	3898	293
B	0.558	0.168	0.209	0.134	4333	611
C	0.880	0.410	0.317	0.127	3876	585
D	1.669	1.058	0.383	0.658	4009	548

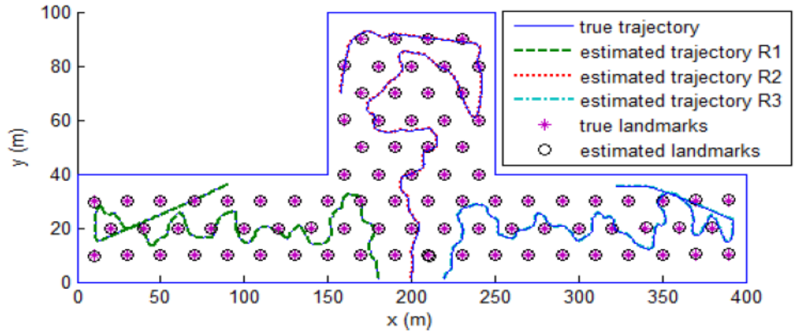


FIGURE 9. Typical results for proposed method in T-shaped environment

Methods A and B yield the smallest errors due to their use of the utility of localizability and relocalization phase. The proposed global exploration strategy has the second-lowest exploration time.

The four methods were compared again in an environment containing 86 landmarks with a T-shaped area of 22,000 m^2, as shown in Figure 9. Estimation errors and exploration time are shown in Table 4, with 20 independent runs used for each method. In this environment, it is assumed that robot dispersion over the environment is the main concern; thus, δ is set to be much greater than 1. Again, robot dispersion is very important in applications such as search and rescue. In this environment, method A yields the best

TABLE 4. Position errors and exploration time for T-shaped environment

Method	Robot RMS error (m) e		Landmark RMS error (m) η		Exploration time (epochs)	
	Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation
A	0.382	0.182	0.352	0.170	4587	401
B	0.266	0.093	0.228	0.095	5185	899
C	0.288	0.132	0.246	0.123	6476	684
D	0.276	0.132	0.216	0.081	6279	488

robot dispersion and the smallest exploration time because robots always separate at the T-junction. Here, robot dispersion and exploration time are traded off against estimation error. The estimation errors for method A are slightly larger than those for methods B, C, and D.

5.2. **Adaptive uncertainty threshold.** In this simulation, an environment with 59 landmarks and an area of 200 m $\times$ 200 m was used. The proposed method is compared with the method in which the uncertainty threshold in (1) is fixed. Typical results obtained for the two methods are shown in Figures 10 and 11, respectively. For simulation analysis, 20 independent runs for each method were performed. Table 5 compares the robot position errors, landmark errors, and exploration time for these two methods. For the method with a fixed uncertainty threshold, Robot 1 and Robot 3 get stuck in regions with sparse landmarks, leading to a long exploration time. In this case, the oscillation between exploration and relocalization phases significantly degrades the estimation quality. For the proposed method, the uncertainty thresholds of Robot 1 and Robot 3 are temporarily increased to allow the robots to pass these regions, and thus the exploration time is much smaller. Moreover, these two robots have a chance to rendezvous with each other or with Robot 2 in the relocalization phase, and thus they obtain better estimates. Therefore, the proposed method outperforms the method with a fixed uncertainty threshold in terms of both exploration time and estimation error.

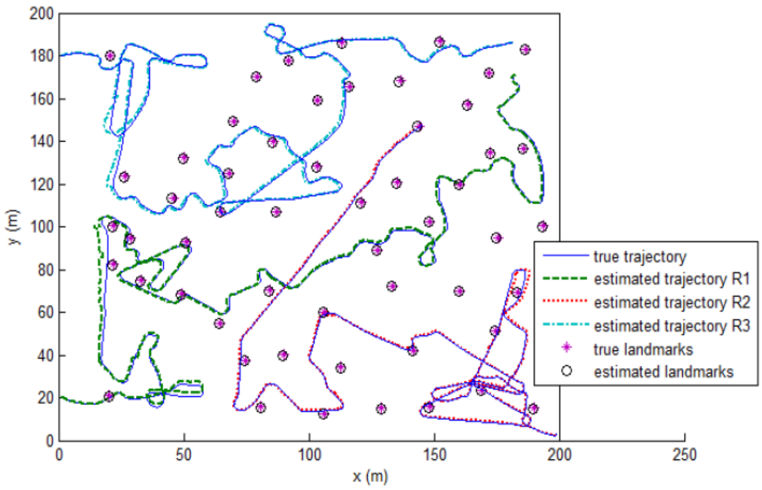


FIGURE 10. Typical results for proposed method

5.3. **Limited communication range.** In this simulation, an environment with 141 landmarks and an area of 160 m $\times$ 120 m was used. The communication range is set to 80 m, i.e., 40% of the maximum distance in the environment. This range is chosen based on [6], in which it was claimed that if the communication range equals to 30% of

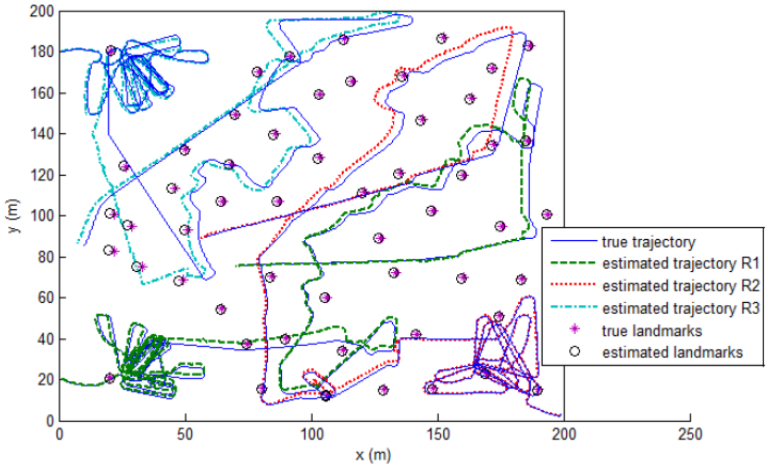


FIGURE 11. Typical results for method with a fixed uncertainty threshold

TABLE 5. Position errors and exploration time for proposed method and method with a fixed uncertainty threshold

Method	Robot RMS error (m) e		Landmark RMS error (m) η		Exploration time (epochs)	
	Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation
Fixed uncertainty threshold	2.055	1.669	0.762	0.827	13271	410
Proposed	1.330	0.425	0.433	0.173	10191	789

the maximum distance in the environment then the results obtained are the same as those of the case with unlimited communication. Table 6 summarizes the three tested methods. Typical results for the method E and the method proposed here are shown in Figures 12 and 13, respectively. For simulation analysis, 20 independent runs for each method were performed. Table 7 compares the robot position errors, landmark errors, and exploration time for these methods.

The results show that, without the relocalization phase and the utility of localizability in the exploration phase, the method F yields the largest errors. For the methods E and F, robots are allowed to move out of communication range without rendezvous, and thus there can be overlap in exploration. In particular, for the method E, Robot 2 and Robot 3 spend a lot of time repeating Robot 1's route, leading to the longest exploration time (Figure 12). In contrast, the rendezvous scheme in the proposed method allows robots to exchange information obtained during the time outside communication range, avoiding

TABLE 6. Summary of methods used for testing limited communication range

Method	Use of utility of localizabilty and relocalization phase	Rendezvous
A (proposed)	Yes	Yes
E (without rendezvous)	Yes	No
F [6]	No	No

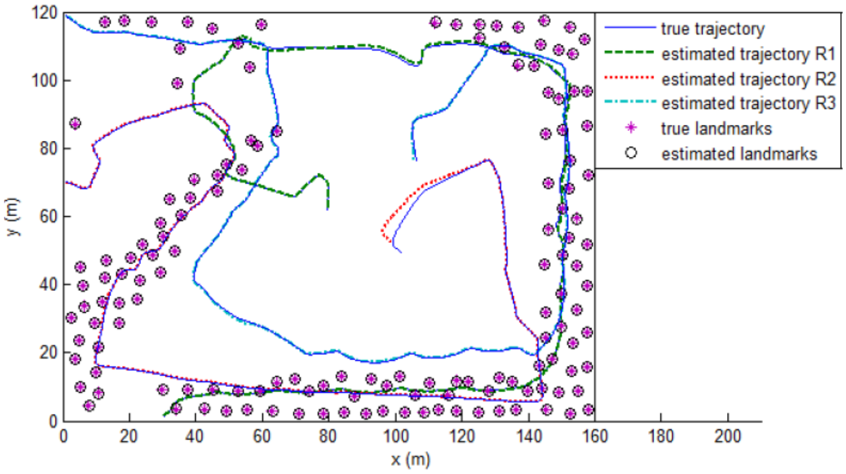


Figure 12. Typical results for method E

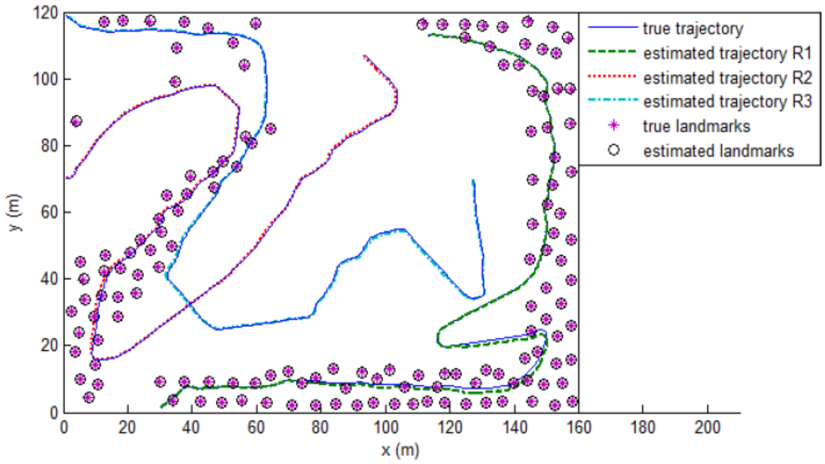


Figure 13. Typical results for the proposed method

Table 7. Position errors and exploration time for the three methods

Method	Robot RMS error (m) e		Landmark RMS error (m) η		Exploration time (epochs)	
	Mean	Standard deviation	Mean	Standard deviation	Mean	Standard deviation
A	0.849	0.428	0.341	0.257	4279	128
E	0.954	0.451	0.189	0.123	5090	404
F	1.907	1.550	0.622	1.070	4303	583

overlap in exploration. As expected, the proposed method yields the shortest exploration time and small estimation errors.

6. **Conclusions.** A multi-robot, cooperative, and active SLAM algorithm for exploration is presented. In the exploration phase, a global optimization scheme allocates the exploration task equally among robots, without assuming a priori knowledge of the environment. An adaptive strategy is employed to automatically adjust the threshold of the robot pose and map uncertainty constraints in order to prevent the robots from oscillating between the exploration and relocalization phases. Finally, a rendezvous technique is

utilized to deal with the limited communication range by allowing robots to temporarily move out of communication range. Simulation results show that the proposed approach outperforms existing methods.

The proposed approach relies on the use of a centralized optimization method to determine the target point for each robot. In the future, distributed control schemes will be investigated to facilitate the design of robot path control to balance map quality, position accuracy, and overall exploration time.

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