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ADAPTIVE DECENTRALIZED SLIDING MODE NEURAL NETWORK CONTROL OF A CLASS OF NONLINEAR INTERCONNECTED SYSTEMS

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Abstract. *In this paper, a completely Decentralized control method (DNNS) for a class of large-scale interconnected systems is developed based on the combination of the sliding mode control with the Neural Network (NN). The standard sliding mode control (SMC) can be used; however, for systems with unknown interconnection terms and in the presence of large uncertainties, the result controller is with higher switching gain and introduces higher amplitude of chattering, or may completely diverge. In this study, the NN is used to predict the unknown interconnection terms and the unknown part of model for each subsystem, and hence it enables a lower switching gain to be used. The stability is shown by the Lyapunov theory and the control action used did not exhibit any chattering behaviour. The effectiveness of the designed DNNS is illustrated in simulations by a comparison with standard SMC technique.*
Keywords: Adaptive decentralized control, Neural network, Sliding mode control, Interconnected nonlinear systems

1. **Introduction.** Large-scale systems are often modeled as non linear dynamic equations composed of interconnections of lower-dimensional subsystems. Decentralized adaptive control technique [7-9,13-16] has received great attention, due to its advantages such as ease of design and improving transient performance. Based on conventional adaptive approach, several results on global stability and steady state tracking were reported [13,14]. However, transient performance is not ensured. Recently, many results based on backstepping technique are proposed; see for example [15,16]. They are only applicable to systems with interactions effects bounded by static functions of subsystem outputs; this is restrictive as it is a kind of matching condition. Model reference adaptive control, based on design for decentralized systems, has been studied in [1-4]. These approaches, however, are limited to decentralized systems with linear subsystems. Sliding mode control has been used by many authors [6,17]; however, strict conditions are imposed on the nominal subsystems together with some limitations on the admissible interconnections

as the linearity assumptions or assumed bounded. On the other hand, the value of the switching gain used depends on the bounds of system interconnections. Hence, systems with large interconnections and uncertainties need to use a controller with higher switching gain and thus produces higher amplitude of chattering. The boundary layer approach can eliminate the chattering effects in the control action. However, systems with large interconnections will need a thicker boundary layer. It only guarantees that the state is driven to the boundary layer, but not to the sliding surface. As a result, if we continuously increase the boundary layer thickness due to the presence of large uncertainties, we are actually reducing the feedback system to a system with no sliding mode.

NN-based controls have been closely scoped out in NN control applications [5,10,11], and most of them get the results that the tracking errors can be asymptotically converged to zero. However, the considered uncertainties are small or some gains parameters are sufficiently large in the case of large uncertainties. In this work, we consider the combination of sliding mode with a boundary layer approach and neural network technique to control a class of Nonlinear interconnected systems. We exploit the function approximation capabilities of the neural network functions to approximate the unknown part of dynamics and unknown interconnection terms. This provides a better description of the plant, and hence enables a lower switching gain to be used. The proposed control consists of the so called equivalent control added to robust control term; the neural network that predicted unknown terms are incorporated in the equivalent control component, thus enabling the robust component to be used with a small gain which is responsible to compensate only the network errors prediction. The paper is organized as follow: Section 2 describes the problem under investigation; in Section 3, we introduce the proposed adaptive decentralized neural network sliding mode control. Simulation results are given to demonstrate the effectiveness of the proposed approach in Section 4. Finally, a concluding remark is given in Section 5.

2. **Problem Formulation of a Class of Decentralized Systems.** Here we consider the large-scale nonlinear system composed of N interconnected subsystems. Each of subsystems Σ_i is described as:

$$\begin{cases} \dot{x}_{i1} = x_{i2} \\ \quad \cdots \\ \dot{x}_{id_i} = f_i(\underline{x}_i) + b_i u_i + \Delta_i(\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_N) \\ y_i = x_{i1} \end{cases} \tag{1}$$

where $\underline{x}_i = [x_{i1}, x_{i2}, \ldots, x_{id_i}]^T$ is the state vector, $u_i \in \Re$ is the control signal input and $y_i \in \Re$ is the output of the plant for the subsystem Σ_i. The nonlinear function $f_i(.)$ is unknown and $\Delta_i(\underline{x}) \in \Re$, are interconnection among subsystems $(i = 1, 2, \ldots, N)$.

Assume that the given reference y_{ir} is bounded and has up to $(d_i - 1)$ bounded derivatives. Define the tracking error of the ith subsystem as:

$$e_i = y_{ir} - y_i \tag{2}$$

We assume that we know the nominal model of each subsystem and for all $\underline{x}_i$ and b_i is a non null constant, then we can rewrite the dynamics $f_i(.)$ as follows:

$$f_i(\underline{x}_i) = f_{iN}(\underline{x}_i) + f_{ik}(\underline{x}_i) \tag{3}$$

where $f_{iN}(\underline{x}_i)$ represent the known dynamics and $f_{ik}(\underline{x}_i)$ represent the unknown part of dynamics.

3. Decentralized Adaptive Control Using Neural Network and Sliding Mode Techniques.

3.1. **Neural network design.** Let Ω_i be a compact simply connected set of $\Re^{d_i}$, and $F_i(\underline{x}_i)$ be a continuous function from Ω_i to $\Re$. Then for any given positive constant ε_{iM} there exist ideal parameters W_i^* and V_i^* such that:

$$F_i(\underline{x}_i) = W_i^* \sigma(V_i^* \underline{x}_i) + \varepsilon_i \tag{4}$$

where the approximation error ε_i is bounded as follows:

$$|\varepsilon_i| < \varepsilon_{iM} \tag{5}$$

where W_i^* and V_i^* are the weights matrix of the output and first layer respectively that minimize $|\varepsilon_i|$, and $\sigma(.)$ represents the hidden-layer activation function considered as a sigmoid function given by:

$$\sigma(s) = \frac{1}{1+e^{-s}} \tag{6}$$

In real control, the ideal W_i^* and V_i^* are unknown and need to be estimated in control design. Let $\hat{W}_i$ and $\hat{V}_i$ be the estimates of W_i^* and V_i^* respectively and the weights estimation errors are $\tilde{W}_i = W_i^* - \hat{W}_i$ and $\tilde{V}_i = V_i^* - \hat{V}_i$.

Then the estimation of $F_i(\underline{x}_i)$ is given by

$$\hat{F}_i(\underline{x}_i) = \hat{W}_i^T \sigma(\hat{V}_i^T \underline{x}_i) \tag{7}$$

Assumption 1: the ideal neural network parameters are bounded by some positive values, that

$$\|W_i^*\| \leq W_{iM} \text{ and } \|V_i^*\| \leq V_{iM} \tag{8}$$

The network weights are adjusted during online implementation. The method used is based on the gradient descent method (GD), which is a simple and fast method for online adaptation.

The essence of GD consists of iteratively adjusting the weights in the direction opposite to the gradient of E, so as to reduce the discrepancy according to:

$$\frac{\partial w_{kj}}{\partial t} = -\eta_k \frac{\partial E}{\partial w_{kj}} \tag{9}$$

where $\eta_k > 0$ is the usual learning rate, and the gradient terms $\frac{\partial E}{\partial w_{kj}}$ can be derived using the backpropagation algorithm [12]. The cost function E is defined as the error index and the least square error criterion is often chosen as follows:

$$E = \frac{1}{2} \sum_{k=1}^{2} \varepsilon_k^2 \tag{10}$$

3.2. **Decentralized adaptive controller design.** For the system defined in (1) with relative degree d_i, the sliding variable can be defined as:

$$S_i = \sum_{j=0}^{d_i-1} \alpha_{ij} e_i^{(j)} \text{ with } \alpha_{i0} = 1 \tag{11}$$

The selection of α_{ij} must satisfy the following Hurwitz polynomial:

$$s_i^{d_i} + \sum_{j=0}^{d_i-1} \alpha_{ij} s_i^j = 0 \tag{12}$$

The sliding variable derivative is:

$$\begin{aligned}\dot{S}_i &= e_i^{(d_i)} + \sum_{j=0}^{d_i-1} \alpha_{ij} e_i^{(j+1)} \\ &= f_{iN}(\underline{x}_i) + f_{ik}(\underline{x}_i) + \Delta_i(\underline{x}) + b_i u_i + \sum_{j=0}^{d_i-1} \alpha_{ij} e_i^{(j+1)}\end{aligned} \tag{13}$$

To ensure that a sliding mode exists on a switching surface, and this switching surface can be reached in finite time, one has to satisfy the η-reachability condition given below:

$$S_i \dot{S}_i < -\eta \, |S_i| \tag{14}$$

where η is a small positive constant.

The standard sliding mode control law that satisfies (14) can be given by:

$$u_i = b_i^{-1} \left(-f_{iN}(\underline{x}_i) + \sum_{j=0}^{d_i-1} \alpha_{ij} e_i^{(j+1)} - \gamma_i \, sat(S_i) \right) \tag{15}$$

where *sat* is the saturation function, given by

$$sat(S_i) = \begin{cases} S_i/\delta & \text{if } |S_i| < \delta \\ sgn(S_i) & \text{otherwise} \end{cases} \tag{16}$$

with δ being the boundary layer thickness.

The positive switching gain to compensate the uncertainties and the interconnection terms is: γ_i which is designed as:

$$\eta + B_i + C_i < \gamma_i \tag{17}$$

with B_i and C_i being respectively the upper bounds of uncertainties and interconnections. When the uncertainties and the interconnections terms are large, γ_i become large and thus, produces higher amplitude of chattering. The thickness of the boundary layer required to achieve elimination of chattering is proportional to the amplitude of chattering, which is in turn, proportional to the value of the switching gain γ_i. However, if we continuously increase the boundary layer thickness, we are actually changing the control system to a system without sliding mode. As a conclusion, the standard SMC with boundary layer technique, can resolve the problem only for systems with small uncertainties and interconnections. For systems with large uncertainties and interconnections, we propose in this work using neural networks to predict theme, so that the system uncertainties can be kept small. Let denote:

$$\begin{aligned}&\varepsilon_{if}(\underline{x}_i) = f_{ik}(\underline{x}_i) - \hat{f}_{ik}(\underline{x}_i) \text{ with } |\varepsilon_{if}| < \varepsilon_{if}^* \text{ and} \\ &\varepsilon_{i\Delta}(\underline{x}) = \Delta_i(\underline{x}) - \hat{\Delta}_i(\underline{x}) \text{ with } |\varepsilon_{i\Delta}| < \varepsilon_{i\Delta}^*\end{aligned}$$

where ε_{if}^* and $\varepsilon_{i\Delta}^*$ are the upper bound of the network errors prediction.

Theorem 3.1. *Consider the interconnected system modeled by (1) in the presence of large uncertainties and interconnections. If the system control is designed as:*

$$u_i = (b_i)^{-1} \left(-\left(f_{iN}(\underline{x}_i) + \hat{f}_{ik}(\underline{x}_i) + \hat{\Delta}_i(\underline{x}) \right) + \sum_{j=0}^{d_i-1} \alpha_{ij} e_i^{(j+1)} - k_i \, sat(S_i) \right) \tag{18}$$

with:

$$(\eta + \varepsilon_{if}^* + \varepsilon_{i\Delta}^*) < k_i \tag{19}$$

The trajectory tracking errors will converge to zero in finite time.

Proof: Consider the candidate Lyapunov function: $V_i = \frac{1}{2}S_i^2$

$$\dot{V}_i = S_i\dot{S}_i \tag{20}$$

Replacing the expression of $\dot{S}_i$ given in (13) we have:

$$\dot{V}_i = S_i(f_{iN}(\underline{x}_i) + f_{ik}(\underline{x}_i) + \Delta_i(\underline{x}) + b_i u_i + \sum_{j=0}^{d_i-1} \alpha_{ij} e_i^{(j+1)} \tag{21}$$

By replacing the expression of u_i given in the theorem we have:

$$\begin{aligned} \dot{V}_i &= S_i\left(f_{ik}(\underline{x}_i) - \hat{f}_{ik}(\underline{x}_i) + \Delta_i(\underline{x}) - \hat{\Delta}_i(\underline{x})\right) - k_i sat(S_i)) \\ &= S_i(\varepsilon_{if} + \varepsilon_{i\Delta}) - k_i S_i sat(S_i) \\ &\leq |S_i|\,|\varepsilon_{if}| + |S_i|\,|\varepsilon_{i\Delta}| - k_i S_i sat(S_i) \\ &\leq |S_i|\,(\varepsilon_{if}^* + \varepsilon_{i\Delta}^*) - k_i S_i sat(S_i) \end{aligned} \tag{22}$$

By choosing $\eta + \varepsilon_{if}^* + \varepsilon_{i\Delta}^* < k_i$, we have:

For any $\delta_i > 0$, if $S_i \geq \delta_i$, $sat(S_i) = sign(S_i)$, the function $\dot{V}_i \leq (\varepsilon_{if}^* + \varepsilon_{i\Delta}^* - k_i)\,|S_i| < -\eta\,|S_i|$. However, in a small δ_i-vicinty of the origin, $sat(S_i) = \frac{S_i}{\delta_i}$ is continuous, and the system trajectories are confined to a boundary layer of sliding mode manifold $S_i = 0$.

4. **Simulation Results.** To show the performance of the presented controller we consider a classical tested for nonlinear decentralized control: the double inverted pendulum [5]. The equations which describe the motion of the pendulums are defined by

$$\dot{x}_{11} = x_{12},\ \dot{x}_{12} = \left(\frac{m_1 gr}{j_1} - \frac{kr^2}{4j_1}\right)\sin(x_{11}) + \frac{kr}{2j_1}(l-b) + \frac{u_1}{j_1} + \frac{kr^2}{4j_1}\sin(x_{21}) \tag{23}$$

$$\dot{x}_{21} = x_{22},\ \dot{x}_{22} = \left(\frac{m_2 gr}{j_2} - \frac{kr^2}{4j_2}\right)\sin(x_{21}) + \frac{kr}{2j_2}(l-b) + \frac{u_2}{j_2} + \frac{kr^2}{4j_2}\sin(x_{12}) \tag{24}$$

where $x_{11} = \phi_1$ and $x_{21} = \phi_2$ are the angular displacements of the pendulums from vertical. The parameters $m_1 = 2$kg and $m_2 = 2.5$kg are the pendulum end masses, $j_1 = 0.5$kg and $j_2 = 0.625$kg are the moments of inertia, the constant of connecting spring is $k = 100$N/m, the pendulum height is $r = 0.5$m, the natural length of the spring is $l = 0.5$m and the gravitational acceleration is $g = 9.81$m/s^2. The distance between the pendulum hinges is defined as $b = 0.4$m (with $b < l$ in this example), so that the pendulum links repels each other when both are in the upright position. The control parameters for simulation are chosen as follows: $\alpha_{i1} = 6$ and $\alpha_{i2} = 10$ $(i = 1, 2)$.

In simulation we have chosen the initial values of $(x_{11}, x_{12}, x_{21}, x_{22})^T = (\pi, \pi, -\pi, -\pi)^T$.

The considered uncertainty is a vector random noise with the magnitude equal to 0.1.

The simulation results are given in Figures 1 and 2.

The proposed decentralized adaptive sliding mode neural network control scheme achieves good performance, when it is compared with standard sliding mode control, as can be seen from the simulation results. Only the nominal functions are used in SMC, and the result sliding mode controller totally failed to control the system.

5. **Conclusions.** In this paper, we have presented an adaptive decentralized control for a class of large-scale nonlinear interconnected systems. The designed method is a combination of sliding mode control with a boundary layer approach and neural network. The latter is employed to approximate the unknown interconnection terms and the nonlinear model function unknown part. This provides a better description of the plant, and hence enables a lower switching gain to be used despite the presence of large uncertainties. As

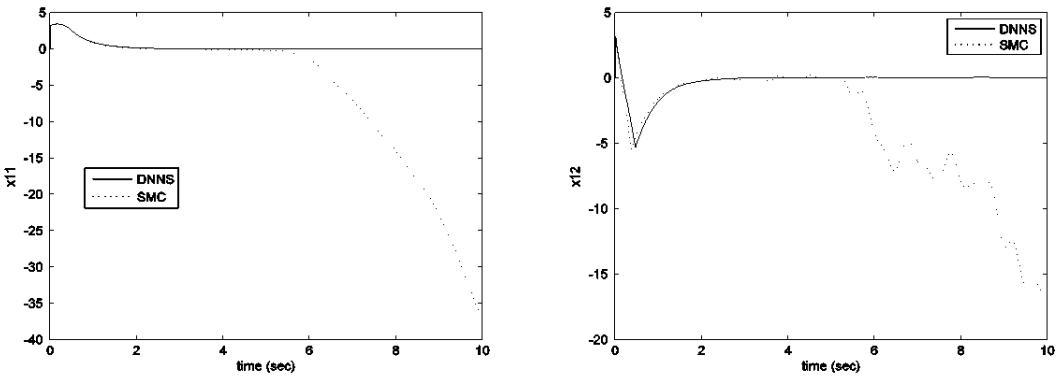

FIGURE 1. State of the subsystem1

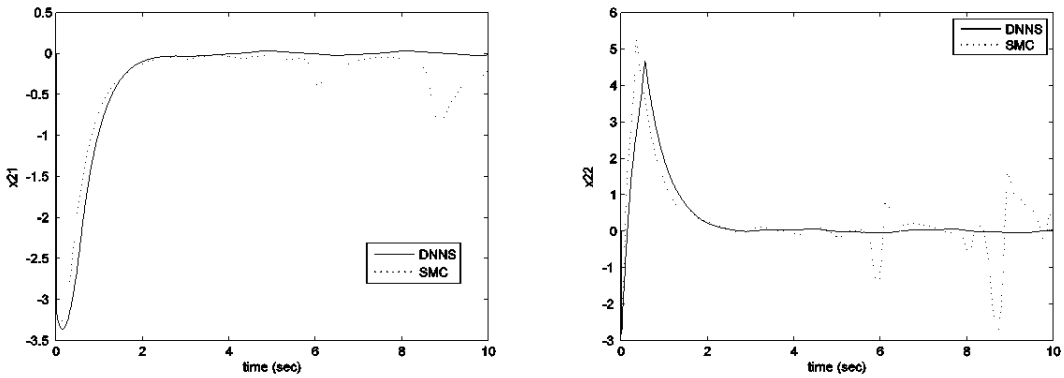

FIGURE 2. State of the subsystem2

a result, we obtained the convergence of tracking errors to zero without any chattering behavior. When comparing the results of the proposed method to that of the standard sliding mode technique where only nominal functions are used, the latter either totally failed to control the system. Simulation results showed the effectiveness of the proposed DNNS controller.

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