International Journal of Innovative
Computing, Information and Control
Volume **9**, Number **7**, July **2013**

pp. 2755–2769

A NOVEL FRAMEWORK OF THEORY ON DISSIPATIVE SYSTEMS

Zhaojing Wu[1] and Hamid Reza Karimi[2]

[1]School of Mathematics and Information Science
Yantai University
No. 32, Qingquan Rd., Laishan District, Yantai 264005, P. R. China
wuzhaojing00@188.com

[2]Department of Engineering
Faculty of Technology and Science
University of Agder
4898 Grimstad, Norway
hamid.r.karimi@uia.no

Received April 2012; revised August 2012

Abstract. *Traditional definitions of dissipativity require an assumption that the admissible control should guarantee that the system has a unique solution and the supply rate is locally integrable. In this paper, by introducing the first exit time from a changeable domain of state and input, a novel dissipativity in the local form and the global sense is defined based on some easily-checked conditions. By the aid of the new definition, a criterion on existence of solution (which will be used before stability analysis) is proposed. All our efforts are to construct a new framework of dissipative-system theory which includes some standard results in traditional dissipative theory and Lyapunov methods as special cases. Therefore, the theoretic analysis of the dissipative systems is more rigorous and the range of applications is significantly widened.*
Keywords: Nonlinear system, Dissipative system, Passivity

1. **Introduction.** In engineering and physics, many researchers pay their attention to dissipative systems that absorb more energy from the external world than they supply, which can be seen as the extension of passivity that was first used to feedback control by [1]. Dissipativity of dynamical systems as it is known in modern system and control community was introduced by [2, 3]. Hill and Moylan carried out an extension of Kalman-Yakubovich-Popov (KYP) Lemma to the case of nonlinear systems with state space representations that are affine in the input in [4, 5]. The authors of [6] further developed a concept of dissipativity for time-variant nonlinear systems and studied the stabilization of such systems. Recently, stochastic dissipativity was researched by [7, 8]. Dissipative techniques have now been widely used as design and analysis tools in many control areas. To name a few, we refer readers to the papers on fully actuated robots manipulators [9], robots with flexible joints [10, 11], fully actuated and underactuated satellites [12], power converters [13, 14, 15], neural networks [16], haptic environments and interfaces [17, 18], process and chemical systems [19, 20, 21], missile guidance [22], magnetically levitated shafts [23], biological and physiological systems [24] and the comprehensive books [25, 26].

In [2], Willems introduced the first definition of dissipativity for a general dynamical system which maps inputs (causes, excitations) into outputs (effects, responses) via a set of intermediate variables (states). This definition depends on the *prior* information: the admissible control should satisfy that the state transition function is well defined and that the supply rate is locally integrable. To obtain more explicit results, by sacrificing some generality, many researchers turned to give definitions of dissipativity for some concrete systems in state space form (e.g., [4, 5, 26]). For the widely used forms such as Definition 4.20 in [26], one should examine the following traditional assumptions (**TA**): *1) the given input belonging to* L_{2e}*, 2) the global existence and uniqueness of solution, and 3) the supply rate function being locally integrable for the given input and the initial state.*

The first condition can be easily verified when the input is an external signal. However, for the interconnection of nonlinear dynamical systems, it becomes more difficult or even impossible to verify this condition since the input of one subsystem usually depends on the state of another subsystem. As for the second one, it should not be examined by Lyapunov function method, which can be regarded as a special dissipative technique (more explanations will be given in Section 3). Therefore, one often counts on the global Lipschitz condition of vector field, which is restrictive since models of many physical systems fail to satisfy it [27, P.94]. The third assumption is difficult to be verified since it usually depends on the first two conditions. In the rest of this paper, just for the convenience, we often say "traditional dissipativity", which means all the definitions (e.g., [2, 3, 4, 5, 6, 26]) with the similar assumptions as the above, to distinguish from the novel definition to be given in this paper.

With the extensions of these traditional methods to general nonlinear systems, more than twenty notions of dissipativity were introduced in literature [25, 26, 28, 29, 30, 31]. Thus, some queries occur: Why the same title of [2, 3] was still used by Willems in paper [32]? Why so many different versions were introduced in literature? Can most of them be unified into one central notion? Why many standard books about nonlinear control systems such as [27, 33, 34] gave few comments about dissipativity?

As an attempt, a notion of dissipativity was presented in [35] in the context of behavioral dynamical systems (see, [32, 36]), where states, inputs and outputs are viewed as behaviors uniformly. The space of admissible inputs is shift-invariant and closed under concatenation, which needs to be verified before dissipativity being applied. How to find a general dissipativity as a central concept to cover most of the existing notions in literature should be further researched. As a key condition to be met in most cases, the imposed assumptions should be reasonable and easy to be checked.

The purpose of this paper is to construct a novel framework of dissipativity theory, which can be used to prove the forward-completion and global stability of systems under some reasonable conditions.

1) The traditional assumptions in **TA** will be replaced by the following preliminary assumptions: *the vector fields and the supply rate satisfy local Lipschitz conditions, and the input is piecewise continuous about time* t, which is very reasonable for most of physical models (for more details, please see [27, P.94]). 2) By defining the first exit time η_l, we introduce a new dissipativity. The interval $[t_0, \eta_l)$ is both existence domain and bounded domain of state and input, and the most importance is that it can change to be the maximal existence domain along with l tending to infinity. 3) The main difference of the new dissipativity from all the other definitions is that the useful information can be extracted out to analyze the existence of solution in addition to the stability. 4) In the novel framework, we pave a way to prove the existence of solution before stability being analyzed.

Rest of this paper is organized as follows. Some preliminaries are given in Section 2. In Section 3, the concept of a new dissipativity is first introduced, and the existence of solution and the stability of systems are analyzed. In Section 4, some notions of passivity, as particular cases of dissipativity, are presented, and two theorems are given, as tools to analyze the stability of interconnected systems. The paper is concluded in Section 5.

Notations: For a vector x, $|x|$ denotes its usual Euclidean norm, x^T denotes its transpose and $\bar{x}_i = (x_1, \cdots, x_i)^T$; $\mathbb{R}^n$ denotes the real n-dimensional space; $\mathbb{R}_+$ denotes the set of all nonnegative real numbers; C^i denotes the set of all functions with continuous ith partial derivative; $\mathcal{K}$ denotes the set of all functions: $\mathbb{R}_+ \to \mathbb{R}_+$, which are continuous, strictly increasing and vanish at zero; $\mathcal{K}_\infty$ denotes the set of all functions which are of class $\mathcal{K}$ and unbounded; $\mathcal{KL}$ denotes the set of all functions $\beta(s,t) : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ which is of class $\mathcal{K}$ for each fixed t, and decreases to zero as $t \to \infty$ for each fixed s; for a real-value function f defined on $\mathbb{R}_+$, it denotes the norms $\|f\| = \left(\int_0^\infty f^2(t)dt\right)^{\frac{1}{2}}$ and $\|f\|_T = \left(\int_0^T f^2(t)dt\right)^{\frac{1}{2}}$; define two sets of functions as $L_2 = \{f : \|f\| < \infty\}$ and $L_{2e} = \{f : \|f\|_T < \infty, \forall T \in \mathbb{R}_+\}$.

2. **Preliminaries.**

Begin with the well-posedness of solution and some existing results about stability, which will be used throughout this paper.

Consider the following time-variant nonlinear system

$$\dot{x} = f(x,t), \quad x(t_0) = x_0, \tag{1}$$

where $x \in \mathbb{R}^n$ is the state and the function $f : \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^n$.

In this section, we will present sufficient conditions for the existence and uniqueness of solution of (1). The following statement comes from Theorem 3.2 in [27].

Lemma 2.1. *If $f(x,t)$ is piecewise-continuous in t and globally Lipschitz in x for all $t \geq t_0$, then there is a unique solution for all $t \geq t_0$.*

For further argument, let us introduce some preliminary notions about time. For any $l \geq 0$, denoting $B_l := \{x : |x| < l\}$, define the first exit time η_l as

$$\eta_l := \eta_l(x_0) = \inf\{t : t \geq t_0,\ x(t) \notin B_l\}, \tag{2}$$

where we set $\inf \emptyset = \infty$ as usual, and the escape time

$$\eta_\infty = \lim_{l\to\infty} \eta_l. \tag{3}$$

Based on Lemma 2.1, there is no difficulty to obtain the following criterion.

Lemma 2.2. *If $f(x,t)$ is piecewise continuous in t and locally Lipschitz in x for all $t \geq t_0$, then there is a unique solution for all $t \in [t_0, \eta_\infty)$.*

Consider a nonlinear system

$$\dot{x} = f(x,u,t), \tag{4}$$

where $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^p$ are the state and the input, respectively. The function $f(x,u,t)$ is locally Lipschitz about x and u, and piecewise continuous about t.

Input-to-state stability (ISS) introduced by Sontag plays an important role in nonlinear controller design, which together with its criterion is recited here with an obvious difference from [37]: the time interval $[t_0, \infty)$ is replaced with $[t_0, \eta_\infty)$.

Definition 2.1. *System (4) is ISS if there exist a class $\mathcal{KL}$ function β and a class $\mathcal{K}$ function γ, such that, for any input $u(\cdot)$ piecewise continuous and bounded on $[t_0, \infty)$, the solution exists and satisfies*

$$|x(t)| \leq \beta(|x(t_0)|, t - t_0) + \gamma\left(\sup\nolimits_{t_0 \leq s \leq t} |u(s)|\right) \tag{5}$$

for all $t \in [t_0, \eta_\infty)$.

Lemma 2.3. *For system (4), if there exists a function $V \in C^1$ (positive definite and radially unbounded) such that, for all $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^p$,*

$$\underline{\alpha}(|x|) \le V(x) \le \bar{\alpha}(|x|), \tag{6}$$

$$\frac{\partial V}{\partial x} f(x,u) \le -\alpha(|x(t)|) + \gamma(u) \tag{7}$$

with a class $\mathcal{K}$ function α and class $\mathcal{K}_\infty$ functions $\underline{\alpha}$, $\bar{\alpha}$, γ, then system (4) is ISS.

It is easy to obtain the following fact.

Proposition 2.1. *If system (4) is ISS with piecewise continuous (ultimate uniformly) bounded input u, then it has a unique solution on $[t_0, \infty)$.*

3. **The Dissipativity.** Great efforts were taken by many researchers to replace the global Lipschitz condition by the local one. It is a standard argument that one can find a Lyapunov function to conclude boundedness of state and use Theorem 3.3 of [27] to show the global existence of solution. Meanwhile, as was pointed out in [26], it is a fundamental property of dissipative systems that one can calculate Lyapunov functions by adding some other conditions, which has even been the main motivation for studying dissipativity, at least in the field of control systems. Therefore, it is unacceptable *logically* to use Lyapunov function methods (see [27]) to examine the global existence and uniqueness of solution to define any dissipativity. To use traditional definitions, one has to count on the global Lipschitz condition to check the global existence and uniqueness of solution in general. In this section, a novel definition of dissipativity will be given, which can be used to prove the global existence of solution and to calculate Lyapunov functions.

3.1. **Definition of dissipativity and its criterion.** Consider a nonlinear system

$$\dot{x} = f(x,u,t), \quad y = h(x,u,t), \tag{8}$$

where $x \in \mathbb{R}^n$ is state, $y \in \mathbb{R}^m$ is output, and $u \in \mathbb{R}^p$ is the dynamic input given by

$$u = u(x, \xi, v), \tag{9}$$

where $\xi \in \mathbb{R}^{n_0}$ is defined by

$$\dot{\xi} = f_0(\xi, x, v_0, t), \tag{10}$$

where $v \in \mathbb{R}^l$ and $v_0 \in \mathbb{R}^{l_0}$ are external disturbances.

The accumulated energy from environment into the system is described by

$$\dot{r}(t) = \varphi(u(t), y(t)), \quad r(t_0) = 0, \tag{11}$$

where $\varphi(u(t), y(t))$ is a supply rate.

In this paper, we only consider the admissible conditions that functions $\bar{f}(x,\xi,v,t) := f(x,u(x,\xi,v(t)),t)$, $\bar{h}(x,\xi,v,t) := h(x,u(x,\xi,v(t)),t)$, $f_0(x,\xi,v_0,t)$ and $\bar{\psi}(x,\xi,v,t) := \psi(u(x,\xi,v(t), h(x,u(x,\xi,v(t)),t))$ are locally Lipschitz in x and ξ, and piecewise continuous in t.

Since u depends on another dynamic ξ, the actors of x and u should be redefined, compared with the traditional understanding to input and output. According to [35, 36], all the states and inputs can be seen uniformly as behaviors of systems. For any $l \ge 0$, define the first exit time η_l as

$$\eta_l = \inf\{t : t \ge t_0,\ |x(t)| \ge l \text{ or } |\xi(t)| \ge l \text{ or } |v(t)| \ge l \text{ or } |v_0(t)| \ge l\}, \tag{12}$$

where $\inf \emptyset = \infty$. By the aid of the first exit time, we can introduce a novel definition of the dissipativity, which is original but different from traditional versions.

By comparison, the maximal existence domain of behaviors $[t_0, \eta_\infty)$ is covered by the maximal existence interval of solution. According to Lemma 2.2, the augmented system consisting of (8)-(11) has a unique solution on $[t_0, \eta_\infty)$. The system is forward-complete if $\eta_\infty = \infty$, for the given v and v_0. The existence of solution to x-system (or ξ-system) should be analyzed in the setting of the augmented (x, ξ)-system, while the input-to-state stability can be analyzed for every subsystems. The stability analysis is only performed for x-system, and that for ξ-system is similar and omitted. As usual, we denote the input (9) as u or $u(t)$ unless otherwise specified.

Definition 3.1 (Dissipativity). *For system (8) with supply rate φ, if there exists a function $V(\cdot) \geq 0$ such that, for every $x(t_0) \in \mathbb{R}^n$, $u \in U$ and all $l \geq 0$,*

$$V(x(\eta_l \wedge t)) - V(x(t_0)) \leq \int_{t_0}^{\eta_l \wedge t} \varphi(u(s), y(s))ds, \tag{13}$$

then system (8) is said to be dissipative with supply rate φ.

Remark 3.1. *From (12), there must exist an escape time (finite or infinite)*

$$\eta_\infty = \lim_{l\to\infty} \eta_l(x_0, u). \tag{14}$$

*From the locally Lipschitz condition of f, h, φ, and according to Lemma 2.2, for any $x_0 \in \mathbb{R}^n$ and $u \in U$, the augmented system consisting of (8) and (11) has a unique solution in the maximal interval of behaviors $[t_0, \eta_\infty)$. Then there is no difficulty to see that the assumptions in **TA** listed in Introduction are satisfied in the interval $[t_0, \eta_l \wedge t)$ for any $l \geq 0$, $t \geq t_0$.*

A crucial role will be played by the following definition, which denotes the energy that may at any time has been extracted from a dynamical system.

Definition 3.2. *The available storage V_a of system (8) is given by*

$$V_a(x) := \sup_{x_0=x, u(\cdot)\in U, t_0\leq t<\eta_\infty} -\int_{t_0}^{t} \varphi(u(s), y(s))ds. \tag{15}$$

The available storage function is an essential function in determining whether or not a system is dissipative. It is the minimum one among the storage functions.

Theorem 3.1. *The available storage function V_a defined in (15) is finite for all $x \in \mathbb{R}^n$ if and only if system (8) is dissipative. Moreover, for dissipative systems, $0 \leq V_a(x) \leq V(x)$ holds for all $x \in \mathbb{R}^n$, and V_a itself is a possible storage function.*

Proof: We can verify that

$$V_a(x) = \sup_{x=x_0, u(\cdot)\in U, t_0\leq t, 0\leq l} -\int_{t_0}^{\eta_l \wedge t} \varphi(u(s), y(s))ds. \tag{16}$$

To show the necessity, assume that $V_a(x) < \infty$ for all $x \in \mathbb{R}^n$. For any $t \geq t' \geq t_0$ and $l \geq l' \geq 0$, since

$$\int_{t_0}^{\eta_l \wedge t} \varphi(u(s), y(s))ds = \int_{t_0}^{\eta_{l'} \wedge t'} \varphi(u(s), y(s))ds + \int_{\eta_{l'} \wedge t'}^{\eta_l \wedge t} \varphi(u(s), y(s))ds, \tag{17}$$

then

$$\begin{aligned} &\sup\nolimits_{x=x_0, u(\cdot)\in U, t_0\leq t, 0\leq l} \left(-\int_{t_0}^{\eta_l \wedge t} \varphi(u(s), y(s))ds\right) \\ &\geq \sup\nolimits_{x=x_0, u(\cdot)\in U, t_0\leq t'\leq t, 0\leq l'} \left(-\int_{t_0}^{\eta_{l'} \wedge t'} \varphi(u(s), y(s))ds\right) \\ &+\sup\nolimits_{x=x(\eta_{l'}\wedge t'), u(\cdot)\in U, (\eta_{l'}\wedge t')\leq t, 0\leq l} \left(-\int_{\eta_{l'}\wedge t'}^{\eta_l \wedge t} \varphi(u(s), y(s))ds\right), \end{aligned} \tag{18}$$

which implies that

$$V_a(x(\eta_{l'} \wedge t')) \leq V_a(x(t_0)) + \int_{t_0}^{\eta_{l'} \wedge t'} \varphi(u(s), y(s))ds, \quad \forall l' \geq 0, \ t' \geq t_0. \tag{19}$$

Thus, one can conclude that (8) is dissipative with storage function V_a.

We next show the sufficiency. From the definition of dissipativity, there exists a function $V(x) \geq 0$ such that

$$0 \leq V(x(\eta_l \wedge t)) \leq V(x(t_0)) + \int_{t_0}^{\eta_l \wedge t} \varphi(u(s), y(s))ds, \tag{20}$$

from which it follows that

$$V_a(x(t_0)) = \sup_{x=x_0, u(\cdot)\in U, t_0 \leq t, 0 \leq l} \left(-\int_{t_0}^{\eta_l \wedge t} \varphi(u(s), y(s))ds \right) \leq V(x(t_0)) < \infty. \tag{21}$$

This completes the proof.

Remark 3.2. *At first sight, one may say that the given definition is difficult to use because it depends on the first exit time which is difficult to calculate. We can say that the importance of the first exit time is only on its applications in the theoretic proof. There is no need to calculate it. In fact, we will use the dissipativity to check the escape time belonging to either of the two cases: $\eta_\infty = \infty$ or $\eta_\infty < \infty$. For the former, we can use Definition 3.1 as the same as traditional ones, and for the latter, we can say that the input makes the state escape in finite time. This comment will be verified by all the forthcoming contents.*

The following example does not satisfy the globally Lipschitz condition. By introducing the first exit time, we can perform the analysis based on Definition 3.1. At the same time, it is easy to understand why there is no need to calculate the first exit time.

Example 3.1. *Consider a 1-dimension system as follows:*

$$\Sigma : \begin{array}{l} \dot{x} = x^3 + xu, \quad x(0) = x_0, \\ y = -\dfrac{x^2}{1+x^4}. \end{array} \tag{22}$$

For any x_0, u and l, define η_l as in (12). It comes from (22) that

$$\begin{aligned} &\int_0^{t\wedge\eta_l} u(s)y(s)ds = -\int_0^{t\wedge\eta_l} (\dot{x}(s) - x^3(s)) \frac{x(s)}{1+x^4(s)} ds \\ \geq &-\frac{1}{2}(\arctan(x^2(t\wedge\eta_l)) - \arctan(x^2(0))) = V(x(t\wedge\eta_l)) - V(x(0)), \end{aligned} \tag{23}$$

where $V(x) = \frac{1}{2}(\frac{\pi}{2} - \arctan(x^2)) > 0$. Thus, system Σ is dissipative with respect to storage function $V(x)$. When $u \geq 0$, it is clear that system Σ blows up in finite time. Therefore, our dissipativity does not naturally imply the global existence of solution.

The weakly-finite-gain stability will be used in the subsequent sections.

Definition 3.3 ([5])**.** *If there exist a function β and a constant k such that, for all u and all x_0,*

$$\|y\|_t^2 \leq k\|u\|_t^2 + \beta(x(t_0)), \tag{24}$$

then system (8) is weakly-finite-gain stable (WFGS).

3.2. **The equivalent definitions of dissipativity.** To examine the consistence to the existing results in dissipative system theory, we shall present the corresponding contents in the new framework and prove them by Definition 3.1. As in [2], the criterion on dissipativity can be simplified for a smooth storage function.

Lemma 3.1. *Consider system (8) with supply rate (11). If there exists a function $V \geq 0$ satisfying*

$$\dot{V}(x(t)) \leq \varphi(u(t), y(t)), \tag{25}$$

then system (8) is dissipative.

Proof: (8) has a unique solution in $t \in [t_0, \eta_l)$ for all $l \geq 0$. Taking integrations on both sides of (25) in interval $[t_0, t \wedge \eta_l)$, one has (13) where the integral is well-defined.

Adding some other conditions to Lemma 3.1, C^1-dissipativity can be extended to the corresponding case in the new dissipativity form, which is presented as follows.

Theorem 3.2. *For system (8) with supply rate (11), there exists a function $V \geq 0$ satisfying*

$$\dot{V}(x(t)) \leq \varphi(u(t), y(t)), \ \ \forall t \in [t_0, \eta_\infty) \tag{26}$$

if and only if system (8) is dissipative with a C^1 storage function.

Proof: The only if part can be proved according to the above lemma. Let us prove (26) under the assumption that (13) holds with a C^1 storage function V. System (8) has a unique solution in $[t_0, \eta_\infty)$. For any $t \in [t_0, \eta_\infty)$, there are t_1 and l such that $\eta_l > t_1 > t > t_0$. From the continuities of $\dot{V}$ and φ, one has

$$\dot{V}(x(t)) = \lim_{t_1 \to t_+} \tfrac{V(x(t_1)) - V(x(t))}{t_1 - t} \leq \lim_{t_1 \to t_+} \varphi(u(t_1), y(t_1)) = \varphi(u(t), y(t)), \tag{27}$$

which results in the conclusion.

The definition of weak dissipativity first appeared in [5] is adapted to our case. We still call it dissipativity since it is equivalent to Definition 3.1.

Definition 3.4. *System (8) is dissipative if there exist a function $\beta : \mathbb{R}^n \to \mathbb{R}$ and a supply rate φ such that*

$$\int_{t_0}^{\eta_l \wedge t} \varphi(u(s), y(s))ds \geq \beta(x_0) \tag{28}$$

for any $t \geq t_0$, $u \in R^p$ and $x_0 \in R^n$.

Theorem 3.3. *Definition 3.4 is equivalent to Definition 3.1.*

Proof: 1. Definition 3.1 $\Longrightarrow$ Definition 3.4. From (13) and $V(\cdot) \geq 0$, one sees that

$$\int_{t_0}^{\eta_l \wedge t} \varphi(u(s), y(s))ds \geq -V(x(t_0)), \tag{29}$$

which means (28) with $\beta = -V$.

2. Definition 3.4 $\Longrightarrow$ Definition 3.1. Let us denote

$$V_a(x) = \sup_{x = x_0, u(\cdot) \in U, t_0 \leq t, 0 \leq l} - \int_{t_0}^{\eta_l \wedge t} \varphi(u(s), y(s))ds,$$

which means that $V_a(\cdot) \geq 0$. For any $t_1 \geq t_0$ it follows from (17)-(19) that

$$V_a(x(\eta_l \wedge t_1)) \leq V_a(x(t_0)) + \int_{t_0}^{\eta_l \wedge t_1} \varphi(u(s), y(s))ds, \ \ \forall l \geq 0,$$

which results in (13). Inequality (28) implies that

$$V_a(x) \le -\beta(x) < \infty, \quad x = x(t_0),$$

so system (8) is dissipative according to Theorem 3.1.

Remark 3.3. *Another frequently used dissipativity was given in* [4]*, which corresponds to Willems' dissipativity with storage function satisfying* $V(0) = 0$*, and the extension to our case is omitted.*

3.3. **The existence and uniqueness of solution.** There are no meaning and means to discuss the existence and uniqueness of solution for an abstract input, even if the system is dissipative. Let $u = v(t)$ be an external reference input or injected disturbance, which is bounded. Given x_0, for any $l \ge 0$, since u is bounded, the first exit time η_l is defined as in (2) and the escape time η_∞ as in (3).

Theorem 3.4. *Suppose system (8) is dissipative and storage function* V *is positive definite and radially unbounded, that is, there exists a class* $\mathcal{K}_\infty$ *function* $\underline{\alpha}(|x|)$ *satisfying* $\underline{\alpha}(|x|) \le V(x)$*. If there exist constants* c *and* d *such that*

$$\varphi(u(t), y(t)) \le cV(x(t)) + d, \tag{30}$$

where d *may depend on* x_0 *and* $\sup_{t\ge t_0}\{|u(t)|\}$*, then system (8) with* $u = v(t)$ *has a unique solution* $x(t) = x(t_0, x_0; t)$ *in* $[t_0, \infty)$*.*

Proof: System (8) has a unique solution in $[t_0, \eta_\infty)$ according to Lemma 2.2. To get the result, let us prove $\eta_\infty = \infty$ by contradiction. In the case of $c \le 0$, from the dissipativity of (8), one has

$$V(x(\eta_\infty \wedge t)) \le V(x(t_0)) + (\eta_\infty \wedge t)d - t_0 d < \infty. \tag{31}$$

If $\eta_\infty < \infty$, then $\infty = V(x(\eta_\infty)) < \infty$, a contradiction. In the case of $c > 0$, according to Bellmam-Gronwall Lemma [38, P.101], it comes from the dissipativity that

$$\left(V(x(\eta_l \wedge t)) + \frac{d}{c}\right) \le \left(V(x(t_0)) + \frac{d}{c}\right) e^{c(\eta_l \wedge t - t_0)}, \tag{32}$$

and by letting $l \to \infty$, we have

$$\left(V(x(\eta_\infty \wedge t)) + \frac{d}{c}\right) \le \left(V(x(t_0)) + \frac{d}{c}\right) e^{c(\eta_\infty \wedge t - t_0)}. \tag{33}$$

If $\eta_\infty < \infty$, choosing $t \ge \eta_\infty$ leads to $\infty = V(x(\eta_\infty)) < \infty$, a contradiction too. Therefore, $\eta_\infty = \infty$.

3.4. **Stability analysis.** As was pointed out by Willems, the interest in dissipative systems stems mainly from their applications on the stability analysis of control systems. It can be performed by adding some reasonable conditions to dissipativity.

1. When $u = v(t)$ is an external reference input or injected disturbance, one usually concerns the ISS property.

Theorem 3.5. *Suppose system (8) is dissipative. If* $V \in C^1$ *is positive definite, radially unbounded and there exist a class* $\mathcal{K}$ *function* α *and a class* $\mathcal{K}_\infty$ *function* γ *such that*

$$\varphi(u(t), y(t)) \le -\alpha(|x(t)|) + \gamma(|u(t)|), \tag{34}$$

then system (8) is ISS.

Proof: From Theorem 3.2 and (34), we have

$$\dot{V}(x(t)) \leq -\alpha(|x(s)|) + \gamma(|u(t)|), \quad \forall t \in [t_0, \eta_\infty), \tag{35}$$

then, further following the line of [39], we can obtain the result.

2. When $u = 0$, $f(0,0) = 0$ which means that the trivial solution $x = 0$ is the equilibrium of system (8). The following results are presented.

Theorem 3.6. *Suppose system (8) is dissipative. For storage function V, there exist class $\mathcal{K}_\infty$ functions $\underline{\alpha}(|x|)$ and $\bar{\alpha}(|x|)$ such that $\underline{\alpha}(|x|) \leq V(x) \leq \bar{\alpha}(|x|)$.*

1. *If (34) holds with $u = 0$ and α is nonnegative, then $x = 0$ is globally stable. If α is positive then $x = 0$ is globally asymptotically stable.*
2. *If (30) holds with $c < 0$ and $d = 0$, then $x = 0$ is globally exponentially stable.*

Proof: In the above two cases, the existence and the uniqueness are guaranteed by Theorem 3.4, then $\eta_\infty = \infty$. The rest part of proof can be found in any standard nonlinear systems such as [27], which is omitted.

Remark 3.4. *Suppose when $u = v$ (constant), $x = x_e$ is one equilibrium but not the unique one of system (8). Only local stability can be considered. If the conditions of Theorem 3.6 hold in a compacted region $\Omega \in \mathbb{R}^n$, then the corresponding results are in the local sense.*

3.5. **Nonexistence of solution and instability of equilibrium.** In the above subsections, the dissipativity serves as an important tool to prove the existence of solution and stability of equilibrium. Naturally, this prompts us to construct tools to prove the nonexistence of solution and instability of equilibrium by contrasting to the dissipativity.

Theorem 3.7. *For system (8), if for some $x(t_0) \in \mathbb{R}^n/\{0\}$ and $u = v(t)$,*

$$V(x(\eta_l \wedge t)) - V(x(t_0)) \geq \int_{t_0}^{\eta_l \wedge t} \varphi(u(s), y(s))ds \tag{36}$$

holds with radially unbounded function $V(\cdot) > 0$ and

$$\varphi(u(t), y(t)) \geq cV^\alpha(x(t)), \tag{37}$$

where $c > 0$, $\alpha > 1$, then system (8) escapes in finite time, i.e., $\eta_\infty < \infty$.

Proof: For any $0 \leq l < \infty$, when $t < \eta_l$, one has $|x| < l$. Since $f(x,u)$ is locally Lipschitz in (x,u) and $u = v(t)$ is piecewise continuous in t, then (8) has a unique solution in $t \in [t_0, \eta_\infty)$, according to Lemma 2.2. Next, we will prove $\eta_\infty < \infty$ by contradiction. From (36), one has, for any $0 \leq l < \infty$,

$$V(x(\eta_l \wedge t)) - V(x(t_0)) \geq \int_{t_0}^{\eta_l \wedge t} cV^\alpha(x(s))ds. \tag{38}$$

Construct an integral equation as follows:

$$q(x(\eta_l \wedge t)) - q(x(t_0)) = \int_{t_0}^{\eta_l \wedge t} cq^\alpha(x(s))ds, \quad q(x(t_0)) = V(x(t_0)), \tag{39}$$

which implies that

$$V(x(\eta_l \wedge t)) \geq q(x(\eta_l \wedge t)), \quad \forall t \geq t_0. \tag{40}$$

From (39), we can see that

$$\dot{q}(x(\eta_l \wedge t)) = cq^\alpha(x(\eta_l \wedge t)), \quad \forall l \geq 0. \tag{41}$$

If $\eta_\infty = \infty$, by letting $l \to \infty$, (41) becomes

$$\dot{q}(x(t)) = cq^\alpha(x(t)), \quad q(x(t_0)) = V(x(t_0)) \tag{42}$$

whose solution is

$$q(x(t)) = \left(\frac{1}{(1-\alpha)c(t-t_0) + V^{-\alpha+1}(x(t_0))}\right)^{\frac{1}{\alpha-1}} \tag{43}$$

which escapes to finite at time $t_b = t_0 + \frac{V^{-\alpha+1}(x(t_0))}{(1-\alpha)c}$. Since $\eta_\infty = \infty$, there exists a larger number l_b such that $\eta_{l_b} \geq t_b$. Then, by taking $l > l_b$, $t = \eta_{l_b}$ in (40), we have

$$\infty > V(x(t_b)) \geq q(x(t_b)) = \infty, \tag{44}$$

which leads to a contradiction, then $\eta_\infty < \infty$ holds. Therefore, system (8) blows up in finite time.

As far as the instability of equilibrium is concerned, we only need to show it in the case of $\eta_\infty = \infty$ (otherwise, $x(t)$ escapes in finite time, obvious instability). Thus, there is no new contents deserving to be added to the existing results on instability [40, P.206].

4. **Passivity and Stability Analysis of Interconnected System.** In this section, the new dissipativity will be used to define various types of strict passivity that are used to present weakly-finite-gain stability result and passive theorem for interconnection of dynamical systems. These are very desirable materials to display the superiority of Definition 3.1 to traditional definitions.

4.1. **Passivity.** Consider a nonlinear system affine in input as follows:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u, \\ y &= h(x) + j(x)u, \end{aligned} \tag{45}$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^m$ and $u \in U \subset \mathbb{R}^m$ are the state, the output and the input, respectively. Functions f, g, h and j are locally Lipschitz in their variables.

The following general supply rate was introduced in [4],

$$\varphi = y^T Q y + u^T R u + 2y^T S u, \tag{46}$$

where $Q = Q^T$, $R = R^T$ and $S = S^T$, which is useful to distinguish types of strictly passive systems and will be used in the passive theorem to be presented in the subsequent subsection, see also [26, P.209].

Definition 4.1. *Assume that system (45) is dissipative with supply rate (46). If $Q = 0$, $R = 0$, $S = \frac{1}{2}I_m$, system (45) is said to be passive. If $Q = 0$, $R = -\varepsilon I_m$, $\varepsilon > 0$, $S = \frac{1}{2}I_m$, the system is said to be input strictly passive (ISP). If $R = 0$, $Q = -\delta I_m$, $\delta > 0$, $S = \frac{1}{2}I_m$, the system is said to be output strictly passive (OSP). If $Q = -\delta I_m$, $\delta > 0$, $R = -\varepsilon I_m$, $\varepsilon > 0$, $S = \frac{1}{2}I_m$, the system is said to be very-strictly passive (VSP).*

As application of the general supply rate, the relation between WFGS and OSP is explained as follows.

Theorem 4.1. *If system (45) is OSP with radially unbounded storage function V and input $u \in L_{2e}$, then it has a unique solution in $[t_0, \infty)$ and is WFGS.*

Proof: Using Young inequality, we have

$$\int_{t_0}^{\eta_l \wedge t} u^T(s) y(s) ds \leq \frac{1}{2\delta} \|u\|^2_{\eta_l \wedge t} + \frac{\delta}{2} \|y\|^2_{\eta_l \wedge t}, \tag{47}$$

where $\|f\|_t = \left[\int_{t_0}^t |f(s)|^2 ds\right]^{\frac{1}{2}}$. Substituting it to the definition of OSP leads to

$$V(x(\eta_l \wedge t)) + \frac{\delta}{2}\|y\|^2_{\eta_l \wedge t} \le \frac{1}{2\delta}\|u\|^2_{\eta_l \wedge t} + V(x(t_0)). \tag{48}$$

By letting $l \to \infty$ in the above inequality, it follows that

$$V(x(\eta_\infty \wedge t)) \le \frac{1}{2\delta}\|u\|^2_{\eta_\infty \wedge t} + V(x(t_0)). \tag{49}$$

Suppose $\eta_\infty < \infty$, then, by taking $t \ge \eta_\infty$, it comes from the radial unboundedness of V and $u \in L_{2e}$ that

$$\infty = V(x(\eta_\infty)) \le \frac{1}{2\delta}\|u\|^2_{\eta_\infty} + V(x(t_0)) < \infty, \tag{50}$$

which is a contradiction. Therefore, $\eta_\infty = \infty$, which implies that system (45) has a unique solution in $[t_0, \infty)$. Letting $l \to \infty$ in (48), we have

$$\frac{\delta}{2}\|y\|^2_t \le \frac{1}{2\delta}\|u\|^2_t + V(x(t_0)), \tag{51}$$

which means that system (45) is WFGS.

To obtain more accurate stability results for the interconnected system, we propose the following strict passivity with dissipation rate depending on the states. Its original form can be found in [41].

Definition 4.2. *System (45) is said to be state strictly passive (SSP) if it is dissipative with the storage function $V(\cdot)$ satisfying $V(0) = 0$, the supply rate $\varphi = y^T u - \psi$ where dissipation rate function $\psi > 0$ is locally Lipschitz, that is, for every $x(t_0) \in \mathbb{R}^n$, $u \in U$ and all $t_0 \le t < \infty$ and $0 \le l < \infty$, there holds*

$$V(x(\eta_l \wedge t)) - V(x(t_0)) \le \int_{t_0}^{\eta_l \wedge t} (u(s)^T y(s) - \psi(x(s)))ds. \tag{52}$$

4.2. **Stability analysis of interconnected systems.** In this subsection, we consider the feedback interconnection of dissipative systems

$$\Sigma_1 \quad : \quad \dot{x}_1 = f_1(x) + g_1(x)u_1, \quad y_1 = m_1(x) \tag{53}$$

$$\Sigma_2 \quad : \quad \dot{x}_2 = f_2(x) + g_2(x)u_2, \quad y_2 = m_2(x) \tag{54}$$

connected by the relations

$$u_1 = -y_2 + v_1, \quad u_2 = y_1, \tag{55}$$

where the functions f_i, g_i, m_i $(i = 1, 2)$ satisfy locally Lipschitz conditions and v_1 is a new input. For the interconnected system, define the overall variables as

$$x = (x_1, x_2)^T, \quad y = y_1, \quad u = v_1.$$

WFGS of interconnected system: New input $v_1 \in L_{2e}$ is piecewise continuous function in t. The first exit time η_l is given by

$$\eta_l = \inf\{t \ge t_0 : |x_1(t)| \ge l \ge 0 \text{ or } |x_2(t)| \ge l \ge 0 \text{ or } |v_1(t)| \ge l \ge 0\}. \tag{56}$$

Theorem 4.2. *Suppose that both system Σ_1 and Σ_2 are VSP, i.e.,*

$$\begin{aligned} &V_i(x_i(\eta_l \wedge t)) - V_i(x_i(t_0)) \\ &\le \int_{t_0}^{\eta_l \wedge t} (u_i(s)^T y_i(s) - \varepsilon_i u_i^T(s) u_i(s) - \delta_i y_i^T(s) y_i(s))ds \quad (i = 1, 2) \end{aligned} \tag{57}$$

with

$$\varepsilon_i + \delta_i > 0.$$

If V_1 and V_2 are radially unbounded and

$$\delta_2 \geq 0, \quad \varepsilon_1 \geq 0, \quad \varepsilon_2 + \delta_1 > 0,$$

then the closed-loop system has a unique solution in $[t_0, \infty)$ and is WFGS.

Proof: Adding up inequalities in (57) gives

$$V(x(\eta_l \wedge t)) - V(x(t_0)) \leq \int_{t_0}^{\eta_l \wedge t} (u^T(s)y(s) - (\delta_1 + \varepsilon_2)y^T(s)y(s))ds, \tag{58}$$

where $V(x) = V_1(x_1) + V_2(x_2)$. According to Theorem 4.1, we can obtain the results.

Asymptotic stability of equilibrium of the interconnected systems: New input $v_1 \equiv 0$. Originated from [41], the following result is presented for the case without global Lipschitz condition.

Theorem 4.3. *Suppose that system Σ_1 is SSP with storage function V_1 (and dissipation rate ψ_1) independent of x_2. Likewise, suppose system Σ_2 is passive with storage function V_2 independent of x_1. Storage function V_i is positive definite and radially unbounded. Then the interconnected system with input v_1 and output y_1 is passive. Moreover, when $v_1 \equiv 0$, the equilibrium $x = 0$ is globally stable and $\lim_{t\to\infty} x_1(t) = 0$.*

Proof: In view of (55), from the passivity, we have

$$\begin{aligned} V_1(x_1(\eta_l \wedge t)) - V_1(x_1(t_0)) &\leq \int_{t_0}^{\eta_l \wedge t} y_1^T(s)(v_1(s) - y_2(s))ds - \int_{t_0}^{\eta_l \wedge t} \psi_1(x_1(s))ds, \\ V_2(x_2(\eta_l \wedge t)) - V_2(x_2(t_0)) &\leq \int_{t_0}^{\eta_l \wedge t} y_2^T(s)y_1(s)ds. \end{aligned} \tag{59}$$

Adding up these two inequalities gives

$$V(x(\eta_l \wedge t)) - V(x(t_0)) \leq \int_{t_0}^{\eta_l \wedge t} y^T(s)u(s)ds - \int_{t_0}^{\eta_l \wedge t} \psi(x(s))ds, \tag{60}$$

where $\psi(x) = \psi_1(x_1)$, $V(x) = V_1(x_1) + V_2(x_2)$. Since $\psi(x) = \psi_1(x_1)$ is only positive semi-definite about x, then the overall system is passive. When $v_1 \equiv 0$, from (60), we have

$$\varphi(x(s)) = -\psi_1(x_1(s)) \leq 0. \tag{61}$$

Since storage function V is positive definite and radially unbounded, then the overall system has a unique solution (thus $\eta_\infty = \infty$) and the equilibrium is globally stable according to Theorem 3.6. Again from (60), we have

$$\int_{t_0}^{\eta_l \wedge t} \psi_1(x_1(s))ds \leq V(x(t_0)), \tag{62}$$

in which making $l \to \infty$ and $t \to \infty$ gives

$$\int_{t_0}^{\infty} \psi_1(x_1(s))ds \leq V(x(t_0)). \tag{63}$$

Next, following the same line as Appendix A in [41], we have $\lim_{t\to\infty} x_1(t) = 0$.

5. **Conclusions.** In the traditional dissipative system theory, conditions in **TA** were imposed on the original system to obtain the dissipativity – a tool to analyze the stability, then most nonlinear physical models that do not satisfy these conditions are excluded out before the dissipativity is used. Great efforts have been taken by many researchers to overcome these difficulties [29, 35]. In this paper, a novel dissipativity is defined in a bounded existence domain of input and state under locally Lipschitz conditions that can be satisfied by most physical models. It is of most importance that our definition can be extended to the stochastic case. In deed, it is the difficulty of extension of traditional definitions to the stochastic case that stimulates the authors to try a deterministic version to relax the strict assumptions in **TA** [8, 42, 43, 44]. It has not been a systematic theory until the other important issues such as KYP lemma, passivity-based feedback control, inverse optimal control, H_2/H_∞ control, adaptive control using passivity and its applications to Hamiltonian control systems are considered (see [25, 26, 41, 45, 46, 47, 48]). All these directions are under current research.

Acknowledgment. This work is supported by the National Natural Science Foundation of China (61273128) and Norwegian Center for Offshore Wind Energy under grant 193821/S60 from Research Council of Norway.

REFERENCES

[1] V. M. Popov, *Hyperstability of Control Systems*, Springer-Verlag New York, Inc., Secaucus, NJ, USA, 1973.
[2] J. C. Willems, Dissipative dynamical systems, part I: General theory, *Archive for Rational Mechanics and Analysis*, vol.45, no.5, pp.321-351, 1972.
[3] Dissipative dynamical systems, part II: Linear systems with quadratic supply rates, *Archive for Rational Mechanics and Analysis*, vol.45, no.5, pp.352-393, 1972.
[4] D. Hill and P. Moylan, The stability of nonlinear dissipative systems, *IEEE Transactions on Automatic Control*, vol.21, no.5, pp.708-711, 1976.
[5] Connections between finite-gain and asymptotic stability, *IEEE Transactions on Automatic Control*, vol.25, no.5, pp.931-936, 1980.
[6] C. I. Byrnes, A. Isidori, and J. C. Willems, Passivity, feedback equivalence, and the global stabilization of minimum phase nonlinear systems, *IEEE Transactions on Automatic Control*, vol.36, no.11, pp.1228-1240, 1991.
[7] W. H. Zhang and B. S. Chen, State feedback H_∞ control for a class of nonlinear stochastic systems, *SIAM Journal on Control and Optimization*, vol.44, no.6, pp.1973-1991, 2006.
[8] Z. J. Wu, M. Y. Cui, X. J. Xie and P. Shi, Theory of stochastic dissipative systems, *IEEE Transactions on Automatic Control*, vol.56, no.7, pp.1650-1655, 2011.
[9] B. Brogliato, I.-D. Landau and R. Lozano-Leal, Adaptive motion control of robot manipulators: A unified approach based on passivity, *International Journal of Robust and Nonlinear Control*, vol.1, no.3, pp.187-202, 1991.
[10] B. Brogliato, R. Ortega and R. Lozano, Global tracking controllers for flexible-joint manipulators: A comparative study, *Automatica*, vol.31, pp.941-956, 1995.
[11] R. Lozano and B. Brogliato, Adaptive control of robot manipulators with flexible joints, *IEEE Transactions on Automatic Control*, vol.37, no.2, pp.174-181, 1992.
[12] O. Egeland and J.-M. Godhavn, Passivity-based adaptive attitude control of a rigid spacecraft, *IEEE Transactions on Automatic Control*, vol.39, no.4, pp.842-846, 1994.
[13] D. J. Jeltsema, R. Ortega and J. M. A. Sherpen, On passivity and power balance inequalities of nonlinear RLC circuits, *IEEE Transactions on Circuits and Systems I-Fundamental Theory and Applications*, vol.50, no.9, pp.1174-1179, 2003.
[14] D. J. Jeltsema and J. M. A. Sherpen, Tuning of passivity-preserving controllers for switched-mode power converters, *IEEE Transactions on Automatic Control*, vol.49, no.8, pp.1333-1344, 2004.
[15] H. Sira-Ramirez, R. P. Moreno, R. Ortega and M. G. Esteban, Passivity-based controllers for the stabilization of dc to dc power converters, *Automatica*, vol.33, no.4, pp.499-513, 1997.

[16] T. Hayakawa, W. M. Haddad, J. M. Bailey and N. Hovakimyan, Passivity-based neural network adaptive output feedback control for nonlinear nonnegative dynamical systems, *IEEE Transactions on Neural Networks*, vol.16, no.2, pp.387-398, 2005.

[17] J. Ryu, B. Hannaford, D. Kwon and J. Kim, A simulation/experimental study of the noisy behavior of the time-domain passivity controller, *IEEE Transactions on Robotics*, vol.21, no.4, pp.733-741, 2005.

[18] X. Shen and M. Goldfarb, On the enhanced passivity of pneumatically actuated impedance-type haptic interfaces, *IEEE Transactions on Robotics*, vol.22, no.3, pp.470-480, 2006.

[19] H. Sira-Ramirez, Passivity versus flatness in the regulation of an exothermic chemical reactor, *European Journal of Control*, vol.6, pp.1-17, 2000.

[20] H. Sira-Ramirez and M. Angulo-Nunez, Passivity based control of nonlinear chemical processes, *International Journal of Control*, vol.68, no.5, pp.971-976, 1997.

[21] B. Ydstie and A. Alonso, Process systems and passivity via the clausius-planck inequality, *Systems & Control Letters*, vol.30, no.5, pp.253-264, 1997.

[22] N. Léchevin, C. A. Rabbath and P. Sicard, A passivity perspective for the synthesis of robust terminal guidance, *IEEE Transactions on Control Systems Technology*, vol.13, no.5, pp.760-765, 2005.

[23] J. D. Miras and A. Charara, A vector oriented control for a magnetically levitated shaft, *IEEE Transactions on Magnetics*, vol.34, no.4, pp.2039-2041, 1998.

[24] W. Haddad, V. Chellaboina and T. Rajpurobit, Dissipativity theory for nonnegative and compartmental dynamical systems with time delay, *IEEE Transactions on Automatic Control*, vol.49, no.5, pp.747-751, 2004.

[25] A. van der Schaft, *L_2-Gain and Passivity in Nonlinear Control*, Springer-Verlag New York, Inc., Secaucus, NJ, USA, 1999.

[26] B. Brogliato, R. Lozano, B. Maschke and O. Egeland, *Dissipative Systems Analysis and Control: Theory and Applications*, Springer-Verlag New York, Inc., Secaucus, NJ, USA, 2000.

[27] H. K. Khalil, *Nonlinear Systems*, 3nd Edition, Englewood Cliffs, Prentice Hall, NJ, 2002.

[28] R. Sepulchre, R. Jankovic and P. V. Kokotović, *Constructive Nonlinear Control*, Springer, New York, 1997.

[29] H. Pota and P. Moylan, Stability of locally dissipative interconnected systems, *IEEE Transactions on Automatic Control*, vol.38, no.2, pp.308-312, 1993.

[30] P. V. Kokotović and M. Arcak, Constructive nonlinear control: A historical perspective, *Automatica*, vol.37, no.5, pp.637-662, 2001.

[31] W. M. Haddad and V. Chellaboina, *Nonlinear Dynamical Systems and Control: A Lyapunov-Based Approach*, Princeton University Press, New Jersey, 2008.

[32] J. C. Willems, Dissipative dynamical systems, *European Jonurnaol of Control*, vol.13, no.2-3, pp.134-151, 2007.

[33] E. Sontag, *Mathematical Control Theory. Deterministic Finite-Dimensional Systems*, Springer-Verlag, New York, 1990.

[34] S. Sastry, *Nonlinear Systems: Analysis, Stability, and Control*, Springer, New York, 1999.

[35] A. L. Chen, G.-Q. Chen and R. A. Freeman, Stability of nonlinear feedback systems: A new small-gain theorem, *SIAM Journal on Control and Optimization*, vol.46, no.6, pp.1995-2012, 2007.

[36] J. C. Willems, Paradigms and puzzles in the theory of dynamical systems, *IEEE Transactions on Automatic Control*, vol.36, no.3, pp.259-294, 1991.

[37] E. D. Sontag, Smooth stabilization implies coprime factorization, *IEEE Transactions on Automatic Control*, vol.34, no.4, pp.435-443, 1989.

[38] P. Ioannou and J. Sun, *Robust Adaptive Control*, Prentice-Hall, New Jersey, 1996.

[39] E. D. Sontag and Y. Wang, On characterizations of the input-to-state stability property, *Systems & Control Letters*, vol.24, no.5, pp.351-359, 1995.

[40] S. Sastry and M. Bodson, *Adaptive Control: Stability, Convergence and Robustness*, Englewood Cliffs, Prentice Hall, NJ, 1989.

[41] M. Krstić, I. Kanellakopoulos and P. V. Kokotović, *Nonlinear and Adaptive Control Design*, John Wiley, New York, 1995.

[42] Z. J. Wu, X. J. Xie and S. Y. Zhang, Adaptive backstepping controller design using stochastic small-gain theorem, *Automatica*, vol.43, no.4, pp.608-620, 2007.

[43] Z. J. Wu, X. J. Xie, P. Shi and Y. Q. Xia, Backstepping controller design for a class of stochastic nonlinear systems with markovian switching, *Automatica*, vol.45, no.4, pp.997-1004, 2009.

[44] Z. J. Wu, J. Yang and P. Shi, Adaptive tracking for stochastic nonlinear systems with markovian switching, *IEEE Transactions on Automatic Control*, vol.55, no.9, pp.2135-2141, 2010.

[45] M. Krstić, I. Kanellakopoulos and P. V. Kokotović, Passivity and parametric robustness of a new class of adaptive systems, *Automatica*, vol.30, no.11, pp.1703-1716, 1994.
[46] M. Krstić and P. Tsiotras, Inverse optimal stabilization of a rigid spacecraft, *IEEE Transactions on Automatic Control*, vol.44, no.5, pp.1042-1049, 1999.
[47] J. Zhao and D. J. Hill, On stability, L2-gain and H_∞ control for switched systems, *Automatica*, vol.44, no.5, pp.1220-1232, 2008.
[48] X. M. Sun, J. Zhao and D. J. Hill, Stability and L2-gain analysis for switched delay systems: A delay-dependent method, *Automatica*, vol.42, no.10, pp.1769-1774, 2006.