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NEW SETS OF CRITERIA FOR EXPONENTIAL $L_2 - L_\infty$ STABILITY OF TAKAGI-SUGENO FUZZY SYSTEMS COMBINED WITH HOPFIELD NEURAL NETWORKS

Choon Ki Ahn[1] and Moon Kyou Song[2,*]

[1]School of Electrical Engineering
Korea University
Anam-dong, Seongbuk-Gu, Seoul 136-701, Korea
hironaka@korea.ac.kr

[2]Division of Electronics and Control Engineering
Wonkwang University
344-2 Shinyong-dong, Iksan 570-749, Korea
[*]Corresponding author: mksong@wonkwang.ac.kr

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Abstract. *In this paper, we propose new sets of criteria for exponential robust stability of Takagi-Sugeno (T-S) fuzzy Hopfield neural networks. The $L_2 - L_\infty$ approach is applied to obtain new sets of stability criteria, under which T-S fuzzy Hopfield neural networks reduce the effect of external input to a prescribed level. These sets of criteria are presented based on the matrix norm and linear matrix inequality (LMI). The proposed sets of criteria also guarantee exponential stability for T-S fuzzy Hopfield neural networks without external input.*

Keywords: Exponential $L_2 - L_\infty$ stability, Takagi-Sugeno (T-S) fuzzy Hopfield neural network, Matrix norm, Linear matrix inequality (LMI)

1. **Introduction.** The Hopfield neural networks have received much attention due to extensive applications in several signal processing problems such as image processing, optimization, fixed point computations, and other areas. In the past decade, stability analysis has been addressed extensively for Hopfield neural networks, and many research results have been presented in the literature [1-3].

Takagi-Sugeno (T-S) fuzzy systems have been widely used in industrial applications and academic research. In general, T-S fuzzy systems use a set of fuzzy rules to represent various nonlinear systems in terms of a set of local linear systems that are smoothly connected by fuzzy membership functions [4]. These T-S fuzzy systems can also be used to describe many complex nonlinear systems by having a set of Hopfield neural networks as their consequent parts [5,6]. Some stability criteria for Hopfield neural networks based on T-S fuzzy systems have been presented previously [7-13]. Recently, some results on learning and identification for T-S fuzzy Hopfield neural networks with external inputs or disturbances were proposed in [14-16].

Real physical systems always have external disturbances and model uncertainties. This fact has led to a recent interest in the $L_2 - L_\infty$ approach [17-26], which is accepted as an important concept for analysis of the stability of various dynamical systems. This paper provides an answer to the question of whether a $L_2 - L_\infty$ stability criterion can be obtained for T-S fuzzy Hopfield neural networks. To the best of our knowledge, the $L_2 - L_\infty$ analysis of T-S fuzzy Hopfield neural networks has not yet been reported in the literature.

In this paper, we propose new sets of $L_2 - L_\infty$ stability criteria for T-S fuzzy Hopfield neural networks. The sets of conditions proposed in this paper are a new contribution to the stability analysis of T-S fuzzy Hopfield neural networks. The proposed sets of criteria are based on the matrix norm and linear matrix inequality (LMI) and, they ensure that the T-S fuzzy Hopfield neural networks attenuate the effect of an external input to a prescribed level. This paper is organized as follows. In Section 2, new sets of $L_2 - L_\infty$ stability criteria are derived. Finally, conclusions are presented in Section 3.

2. **New Sets of Exponential $\boldsymbol{L_2 - L_\infty}$ Stability Criteria.** Consider the following T-S fuzzy Hopfield neural network:

Fuzzy Rule $\boldsymbol{i}$:

$$\text{IF } \omega_1 \text{ is } \mu_{i1} \text{ and ... } \omega_s \text{ is } \mu_{is} \text{ THEN}$$

$$\dot{x}(t) = A_i x(t) + W_i \phi((x(t)) + J(t) \tag{1}$$

$$z(t) = H_i x(t), \tag{2}$$

where $x(t) = [x_1(t) \dots x_n(t)]^T \in R^n$ is the state vector, $z(t) \in R^p$ is a linear combination of the states, $A_i = diag\{-a_{(i,1)}, \dots, -a_{(i,n)}\} \in R^{n\times n}$, $(a_{(i,k)} > 0, k = 1, \dots, n)$ is the self-feedback matrix, $W_i \in R^{n\times n}$ is the connection weight matrix, $\phi(x(t)) = [\phi_1(x(t)) \dots \phi_n(x(t))]^T : R^n \to R^n$ is the nonlinear function vector satisfying the global Lipschitz condition with Lipschitz constant $L_\phi > 0$, $J(t) \in R^n$ is an external input vector, $H_i \in R^{p\times n}$ is a known constant matrix, ω_j $(j = 1, \dots, s)$ is the premise variable, μ_{ij} $(i = 1 \dots, r, j = 1, \dots, s)$ is the fuzzy set that is characterized by membership function, r is the number of IF-THEN rules, and s is the number of premise variables. A singleton fuzzifier, product fuzzy inference, and weighted average defuzzifier are used to infer the T-S fuzzy Hopfield neural network (1) and (2), as follows:

$$x(t) = \sum_{i=1}^{r} h_i(\omega)[A_i x(t) + \omega_i \phi(x(t)) + J(t)], \tag{3}$$

$$z(t) = \sum_{i=1}^{r} h_i(\omega) H_i x(t), \tag{4}$$

where $\omega = [\omega_1, \dots, \omega_s]$, $h_i(\omega) = w_i(\omega)/\sum_{j=1}^{r} w_j(\omega)$, $w_i : R^s \to [0, 1]$ $(i = 1, \dots, r)$ is the membership function of the system with respect to the fuzzy rule i. $h_i(\omega)$ satisfies $h_i(\omega) \ge 0$ and $\sum_{i=1}^{r} h_i(\omega) = 1$.

Let $\gamma > 0$ be a prescribed level of noise attenuation. In this paper, we find sets of criteria such that the T-S fuzzy Hopfield neural network (3) and (4) with $J(t) = 0$ is exponentially stable and

$$\sup_{t\ge 0}\{\exp(\kappa t) z^T(t) z(t)\} < \gamma^2 \int_0^\infty \exp(\kappa t) J^T(t) J(t) dt, \tag{5}$$

under zero-initial conditions for all nonzero $J(t) \in L_2[0, \infty)$, where $L_2[0, \infty)$ is the space of square integrable vector functions over $[0, \infty)$ and κ is a positive constant.

In the following theorem, we obtain a set of $L_2 - L_\infty$ stability criteria for the T-S fuzzy Hopfield neural network (3) and (4).

Theorem 2.1. *For a given level $\gamma > 0$, the T-S fuzzy Hopfield neural network (3) and (4) is exponentially $L_2 - L_\infty$ stable if*

$$\|W_i\| < \frac{1}{L_\phi} \sqrt{\frac{k_i - \|P\|^2 - (1+\kappa)\|P\|}{\|P\|}}, \tag{6}$$

$$\|P\| < \frac{-(1+\kappa) + \sqrt{(1+\kappa)^2 + 4k_i}}{2}, \quad k_i > 0, \quad P = P^T > 0, \tag{7}$$

$$\|H_i\| \, \|H_j\| \le \gamma^2 \lambda_{\min}(P), \tag{8}$$

for $i = 1, \ldots, r$ and $j = 1, \ldots, r$, where $\lambda_{\min}(\cdot)$ is the minimum eigenvalue of the matrix and P satisfies the Lyapunov inequality $A_i^T P + PA_i < -k_i I$.

Proof: First, consider the Lyapunov function $V(t) = \exp(\kappa t) x^T(t) P x(t)$. Its time derivative along the trajectory of (3) satisfies

$$\begin{aligned} \dot{V}(t) < \sum_{i-1}^{r} h_i(\omega) \exp(\kappa t)\{-k_i x^T(t) x(t) \\ + \kappa x^T(t) P x(t) + 2x^T(t) P W_i \phi(x(t)) + 2x^T(t) P J(t)\}. \end{aligned} \tag{9}$$

If we apply Young's inequality [26,27], we have

$$\begin{aligned} 2x^T(t) P W_i \phi(x(t)) &\le x^T(t) P x(t) + (PW_i\phi(x(t)))^T P^{-1} (PW_i \phi(x(t))) \\ &\le \|P\| \, \|x(t)\|^2 + L_\phi^2 \|P\| \, \|W_i\|^2 \, \|x(t)\|^2 \end{aligned} \tag{10}$$

and

$$2x^T(t) P J(t) \le x^T(t) P P^T x(t) + J^T(t) J(t) \le \|P\|^2 \, \|x(t)\|^2 + \|J(t)\|^2 . \tag{11}$$

If we substitute (10) and (11) into (9), we have

$$\begin{aligned} \dot{V}(t) &= \sum_{i=i}^{r} h_i(\omega) \exp(\kappa t)\{-(k_i - \|P\|^2 - \|P\| - \kappa \|P\| - L_\phi^2 \|P\| \, \|W_i\|^2) \, \|x(t)\|^2 \\ &\quad + \|J(t)\|^2\} \\ &= -\sum_{i=1}^{r} h_i(\omega) \exp(\kappa t)(k_i - \|P\|^2 - \|P\| - \kappa \|P\| - L_\phi^2 \|P\| \, \|W_i\|^2) \, \|x(t)\|^2 \\ &\quad + \sum_{i=1}^{r} h_i(\omega) \exp(\kappa t) \, \|J(t)\|^2. \end{aligned} \tag{12}$$

If the following condition is satisfied:

$$k_i - \|P\|^2 - (1+\kappa) \|P\| - L_\phi^2 \|P\| \, \|W_i\|^2 > 0, \tag{13}$$

for $i = 1, \ldots, r$ we have [26]

$$\begin{aligned} \dot{V}(t) &< \sum_{i=1}^{r} h_i(\omega) \exp(\kappa t) \, \|J(t)\|^2 \\ &= \exp(\kappa t) \, \|J(t)\|^2 . \end{aligned} \tag{14}$$

For $i = 1, \ldots, r$, the following two inequalities

$$\begin{aligned} \|W_i\|^2 &< \frac{k_i - \|P\|^2 - (1+\kappa) \|P\|}{L_\phi^2 \|P\|}, \\ \|P\| &< \frac{-(1+\kappa) + \sqrt{(1+\kappa)^2 + 4k_i}}{2}, \end{aligned} \tag{15}$$

imply the condition (13). Thus, we obtain (6) and (7). Under the zero-initial condition, we have $V(t)|_{t=0} = 0$ and $V(t) \ge 0$. If we define [26]

$$\Phi(t) = V(t) - \int_0^t \exp(\kappa\sigma) J^T(\sigma) J(\sigma) d\sigma, \tag{16}$$

then, for any nonzero $J(t)$, we obtain

$$\begin{aligned}\Phi(t) &= V(t) - V(t)|_{t=0} - \int_0^t \exp(\kappa\sigma)J^T(\sigma)J(\sigma)d\sigma \\ &= \int_0^t [\dot{V}(\sigma) - \exp(\kappa\sigma)J^T(\sigma)J(\sigma)]d\sigma.\end{aligned}$$

From (14), we have $\Phi(t) < 0$. This means that [26]

$$V(t) < \int_0^t \exp(\kappa\sigma)J^T(\sigma)J(\sigma)d\sigma.$$

Condition (8) implies that

$$\begin{aligned}\exp(\kappa t)z^T(t)z(t) &= \exp(\kappa t)\sum_{i=1}^r h_i(\omega)\sum_{j=1}^r h_j(\omega)x^T(t)H_i^T H_j x(t) \\ &\le \exp(\kappa t)\sum_{i=1}^r h_i(\omega)\sum_{j=1}^r h_j(\omega)\,\|H_i\|\,\|H_j\|\,\|x(t)\|^2 \\ &\le \gamma^2\exp(\kappa t)\sum_{i=1}^r h_i(\omega)\sum_{j=1}^r h_j(\omega)\lambda_{\min}(P)\,\|x(t)\|^2 \\ &\le \gamma^2\exp(\kappa t)\sum_{i=1}^r h_i(\omega)\sum_{j=1}^r h_j(\omega)x^T(t)Px(t) \\ &= \gamma^2 V(t) \\ &< \gamma^2\int_0^t \exp(\kappa\sigma)J^T(\sigma)J(\sigma)d\sigma \\ &\le \gamma^2\int_0^\infty \exp(\kappa\sigma)J^T(\sigma)J(\sigma)d\sigma.\end{aligned} \tag{17}$$

Taking the supremum over $t > 0$ leads to (5). This completes the proof.

Corollary 2.1. *When $J(t) = 0$, the condition (6)-(8) ensures that the T-S fuzzy Hopfield neural network (3) and (4) is exponentially stable.*

Proof: When $J(t) = 0$, from (14), we have $\dot{V}(t) < 0$. This implies that $V(t) < V(0) = x^T(0)Px(0)$ for any $t \ge 0$. We also have

$$V(t) \ge \lambda_{\min}(P)\exp(\kappa t)\,\|x(t)\|^2\,. \tag{18}$$

It immediately follows from (18) that

$$\|x(t)\| < \sqrt{\frac{x^T(0)Px(0)}{\lambda_{\min}(P)}}\exp(-\kappa t/2). \tag{19}$$

This relation ensures that the T-S fuzzy Hopfield neural network (3) and (4) is exponentially stable. This completes the proof.

Now, we are ready to state a new set of LMI based criteria for the exponential $L_2 - L_\infty$ stability of the T-S fuzzy Hopfield neural network (3) and (4). These LMI criteria can be readily facilitated via standard numerical algorithms [28,29].

Theorem 2.2. *For a given level $\gamma > 0$, the T-S fuzzy Hopfield neural network (3) and (4) is exponentially $L_2 - L_\infty$ stable if a positive symmetric matrix P and a positive scalar $\grave{o}$ exist, such that*

$$\begin{bmatrix} A_i^T P + PA_i + \kappa P + \grave{o} L_\phi^2 I & PW_i & P \\ W_i^T P & -\grave{o} I & 0 \\ P & 0 & -I \end{bmatrix} < 0, \tag{20}$$

$$P - \frac{1}{\gamma^2} H_i^T H_j > 0, \tag{21}$$

for $i = 1, \ldots, r$ and $j = 1, \ldots, r$.

Proof: Consider the Lyapunov function $V(t) = \exp(\kappa t) x^T(t) P x(t)$. If we apply Young's inequality [26,27], we have

$$\grave{o}[L_\phi^2 x^T(t)x(t) - \phi^T(x(t))\phi(x(t))] \geq 0. \tag{22}$$

Using (22), the time derivative of $V(t)$ along the trajectory of (3) [26] is

$$\begin{aligned} \dot{V}(t) = & \sum_{i=1}^{r} h_i(\omega) \exp(\kappa t) \{x^T(t)[A_i^T P + PA_i + \kappa P]x(t) \\ & + 2x^T(t) PW_i \phi(x(t)) + 2x^T(t) PJ(t)\} \\ \leq & \sum_{i=1}^{r} h_i(\omega) \exp(\kappa t) \{x^T(t)[A_i^T P + PA_i + \kappa P]x(t) \\ & + 2x^T(t) PW_i \phi(x(t)) + 2x^T(t) PJ(t) + \grave{o}[L_\phi^2 x^T(t)x(t) - \phi^T(x(t))\phi(x(t))]\} \end{aligned} \tag{23}$$

$$= \sum_{i=1}^{r} h_i(\omega) \exp(\kappa t) \begin{bmatrix} x(t) \\ \phi(x(t)) \\ J(t) \end{bmatrix}^T \begin{bmatrix} A_i^T P + PA_i + \kappa P + \grave{o} L_\phi^2 I & PW_i & P \\ W_i^T P & -\grave{o} I & 0 \\ P & 0 & -I \end{bmatrix}$$
$$\begin{bmatrix} x(t) \\ \phi(x(t)) \\ J(t) \end{bmatrix} + \sum_{i=1}^{r} h_i(\omega) \exp(\kappa t) J^T(t) J(t).$$

If the LMI (20) is satisfied for $i = 1, \ldots, r$ and $j = 1, \ldots, r$, we have

$$\begin{aligned} \dot{V}(t) < & \sum_{i=1}^{r} h_i(\omega) \exp(\kappa t) J^T(t) J(t) \\ = & \exp(\kappa t) J^T(t) J(t). \end{aligned} \tag{24}$$

Under the zero-initial condition, we have $V(t)|_{t=0} = 0$ and $V(t) \geq 0$. If we define [26]

$$\Phi(t) = V(t) - \int_0^t \exp(\kappa\sigma) J^T(\sigma) J(\sigma) d\sigma, \tag{25}$$

then, for any nonzero $J(t)$, we obtain

$$\begin{aligned} \Phi(t) = & V(t) - V(t)|_{t=0} - \int_0^t \exp(\kappa\sigma) J^T(\sigma) J(\sigma) d\sigma \\ = & \int_0^t [\dot{V}(\sigma) - \exp(\kappa\sigma) J^T(\sigma) J(\sigma)] d\sigma. \end{aligned}$$

Form (24), we have $\Phi(t) < 0$. This means that [26]

$$V(t) < \int_0^t \exp(\kappa\sigma) J^T(\sigma) J(\sigma) d\sigma.$$

The LMI (21) implies that

$$\begin{aligned}
\exp(\kappa t) z^T(t) z(t) &= \exp(\kappa t) \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\omega) h_j(\omega) x^T(t) H_i^T H_j x(t) \\
&< \gamma^2 \exp(\kappa t) \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(\omega) h_j(\omega) x^T(t) P x(t) \\
&= \gamma^2 V(t) \\
&< \gamma^2 \int_0^t \exp(\kappa\sigma) J^T(\sigma) J(\sigma) d\sigma \\
&\leq \gamma^2 \int_0^\infty \exp(\kappa\sigma) J^T(\sigma) J(\sigma) d\sigma.
\end{aligned} \tag{26}$$

Taking the supremum over $t > 0$ leads to (5). This completes the proof.

Corollary 2.2. *When $J(t) = 0$, the LMI conditions (20) and (21) ensure that the T-S fuzzy Hopfield neural network (3) and (4) is exponentially stable.*

Proof: When $J(t) = 0$, from (24), we have $\dot{V}(t) < 0$. This implies that $V(t) < V(0) = x^T(0)Px(0)$ for any $t \geq 0$. We also have

$$V(t) \geq \lambda_{\min}(P) \exp(\kappa t) \|x(t)\|^2 . \tag{27}$$

It immediately follows from (27) that

$$\|x(t)\| < \sqrt{\frac{x^T(0)Px(0)}{\lambda_{\min}(P)}} \exp(-\kappa t/2). \tag{28}$$

This relation ensures that the T-S fuzzy Hopfield neural network (3) and (4) is exponentially stable. This completes the proof.

Remark 2.1. *Most existing results on stability analysis for Takagi-Sugeno fuzzy systems combined with Hopfield neural networks in the literature were restricted to systems without external disturbances. Unfortunately, with the existing results, it is not possible to analyze robust stability for Takagi-Sugeno fuzzy systems combined with Hopfield neural networks with external disturbances. For the first time, this paper presents the $L_2 - L_\infty$ approach to robust stability analysis for Takagi-Sugeno fuzzy systems combined with Hopfield neural networks with disturbances. The proposed results in this paper open a new path for application of the exponential $L_2 - L_\infty$ approach to the derivation of new stability criteria for Takagi-Sugeno fuzzy systems combined with Hopfield neural networks.*

Remark 2.2. *The proposed exponential $L_2 - L_\infty$ stability criteria can be used in several control applications. For example, T-S fuzzy Hopfield neural networks are applied to model unknown nonlinear systems with disturbances and then these neural networks can be utilized to design new nonlinear $L_2 - L_\infty$ control laws to achieve certain design objectives. Here, we can check $L_2 - L_\infty$ stability of introduced T-S fuzzy Hopfield neural networks with disturbances. Therefore, from the point of view of control, the exponential $L_2 - L_\infty$ stability criteria for T-S fuzzy Hopfield neural networks is of significance for many applications.*

3. **Conclusions.** In this paper, we have established new sets of exponential $L_2 - L_\infty$ stability criteria for T-S fuzzy Hopfield neural networks. These sets of criteria were based on the matrix norm and LMI and they ensured that the T-S fuzzy Hopfield neural networks reduced the effect of external input on the state vector. These sets of criteria also guaranteed exponential stability for T-S fuzzy Hopfield neural networks without external input.

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