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SLIDING MODE VARIABLE STRUCTURE I/O FEEDBACK LINEARIZATION DESIGN FOR THE SPEED CONTROL OF PMSM WITH LOAD TORQUE OBSERVER

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Abstract. *Based on the mathematical model of the permanent magnet synchronous motor (PMSM), a novel sliding mode variable structure input-output feedback linearization controller is proposed. A precision linearization method is employed to achieve input-output linearization and decoupling control of the motor, which is decoupled into a second-order linear speed subsystem and a first-order linear d-axis current subsystem. Sliding mode variable structure is simple, and easy to combine with other intelligent methods, especially it has robustness to external disturbances. So the combination of sliding mode variable structure and feedback linearization could make the design method robust and quick dynamic. At the same time, an observer based on extended state observer is designed to evaluate the load torque of the system online. Theoretical analysis and matlab simulation results prove the effectiveness of the proposed method.*

Keywords: PMSM, Sliding mode variable structure, Input-output feedback linearization, Speed controller, Load torque observer

1. **Introduction.** Permanent magnet synchronous motors (PMSMs) have been widely used in applications such as machine tools, steel mills, electric vehicles owing to their good performance provided by their rugged construction, easy maintenance, high efficiency, and high torque to current ratio, low moment of inertia.

Some control techniques have been developed to regulate these motor drives in high-performance applications. One of the most popular techniques is the Field-Oriented control method, which could make the control of PMSM linearly as a DC motor [1]. And, many nonlinear and intelligent methods have been made on the motor drives, such as optimal control, fuzzy control, neural control, adaptive backstepping control [2-4]. Among of them, fuzzy controller [2] has great robustness as its design is independent of the controller system; however, there is difficulty for the determination of fuzzy rules. Neural control [3] is intelligent, and the control performance is good at dynamic and steady state, but this method is complex with calculation. Adaptive backstepping strategy is extensively used in the control of motor drives [4], but the method needs the additional observers for load torque or motor parameters.

Recently, feedback linearization has emerged as a very useful control law for electric drives [5-10]. Several authors use the exact I/O feedback linearization technique to uncouple the control of the motor drivers. It consists in exactly linearization the motor drivers by feedback and transformation, so that well-known linear control strategies can be used on the whole state space. Consequently, in ideal case, the controllers of each output variable could be designed separately. Different control methods can be associated to the exact I/O feedback linearization technique. For example, the authors in [6] use a

PI controller to obtain the control law. In [7], a state feedback control law is obtained by pole placement. However, the conventional feedback linearization controller may fail to meet the high performance requirements of industrial servo since it is highly vulnerable to parameter perturbations and unknown external disturbances of the plant.

As is well known, sliding mode variable structure is simple, and easy to combine with other intelligent methods; especially it has robustness to external disturbances [11-17]. In order to improve the robustness of feedback linearization controller, a sliding mode I/O feedback linearization controller is employed as the speed and current tracking for PMSM, which could ensure a high precision control of velocity with steady state error being zero despite the motor parameters and load torque variations. The overall stability of the controller is proved by the Lyapunov theory.

Besides, an information on the acceleration (dw/dt) is needed for the state feedback. It is, however, required to estimate an unknown load torque. Extended state observer (ESO) is proposed by Han [18], which is part of ADRC (Active Disturbance Rejection Control) controller [19], and is used to estimate the disturbances of the nonlinear system. In this paper, the load torque is estimated by extended state observer (ESO).

2. **Modeling of PMSM.**

For a nonsalient PMSM motor, $L_d = L_q = L$, choosing (i_d, i_q, w_r) as state variables, the PMSM system can be written in the following explicit form:

$$\frac{dw_r}{dt} = \frac{3p_n\phi_f}{2J}i_q - \frac{B}{J}w_r - \frac{1}{J}T_L \tag{1}$$

$$\frac{di_d}{dt} = -\frac{R_s}{L}i_d + p_n w_r i_q + \frac{1}{L}u_d \tag{2}$$

$$\frac{di_q}{dt} = -\frac{R_s}{L}i_q - p_n w_r i_d - \frac{p_n w_r}{L}\phi_f + \frac{1}{L}u_q \tag{3}$$

where u_d, u_q, i_d, i_q, L_d, L_q are the stator voltages, stator currents, stator inductances in the frame of d-q respectively; w_r is the rotor speed; p_n is the number of the pole pair; R_s is stator resistance; ϕ_f is magnet flux linkage; T_L is the load torque; J is the motor inertia; B is friction coefficient.

Based on Equations (1)-(3), we could design the controller based on conventional I/O feedback linearization and the proposed sliding mode I/O feedback linearization controller.

3. **Input-Output Feedback Linearization Control.**

3.1. **Input-output feedback linearization theory.**

Consider a multiple-input multiple-output (MIMO) system as follows:

$$\dot{x} = f(x) + \sum_{i=1}^{m} g_i(x) \cdot u_i \tag{4}$$

$$y_1 = h_1(x)$$

$$\cdots$$

$$y_m = h_m(x)$$

where x is the $n \times 1$ state vector, u is the $m \times 1$ control input vector, y is the $m \times 1$ vector of the system outputs, $f(x), g_1(x), \cdots, g_m(x)$ are smooth vector fields and $h_1(x), \cdots, h_m(x)$ are continuous functions.

In nonlinear control the relative degree is an important theoretical concept, which is related to the number of times that the system outputs y_i have to be differentiated until the inputs u_i explicitly appear in the expression. Thus, assuming that r_i is the smallest integer such that at least one of the inputs explicitly appears in $y_i^{(r_i)}$, then

$$y_i^{(r_i)} = L_f^{r_i} h_i + \sum_{j=1}^{m} L_{g_i} L_f^{r_i-1} h_i u_j \tag{5}$$

where the Lie Derivative $L_{g_i} L_f^{r_i-1} h_i(x) \neq 0$ for at least one j. Performing the above procedure for each output y_i leads to

$$\begin{bmatrix} y_1^{(r_1)} \\ \cdots \\ y_m^{(r_m)} \end{bmatrix} = \begin{bmatrix} L_f^{r_1} h_1 \\ \cdots \\ L_f^{r_m} h_m \end{bmatrix} + \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_1 & \cdots & L_{g_m} L_f^{r_1-1} h_1 \\ \cdots & \cdots & \cdots \\ L_{g_1} L_f^{r_m-1} h_m & \cdots & L_{g_m} L_f^{r_m-1} h_m \end{bmatrix} \begin{bmatrix} u_1 \\ \cdots \\ u_m \end{bmatrix} \tag{6}$$

The I/O transformation can be obtained by setting the control vector u as

$$\begin{bmatrix} u_1 \\ \cdots \\ u_m \end{bmatrix} = \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_1 & \cdots & L_{g_m} L_f^{r_1-1} h_1 \\ \cdots & \cdots & \cdots \\ L_{g_1} L_f^{r_m-1} h_m & \cdots & L_{g_m} L_f^{r_m-1} h_m \end{bmatrix}^{-1} \begin{bmatrix} v_1 - L_f^{r_1} h_1(x) \\ \cdots \\ v_m - L_f^{r_m} h_m(x) \end{bmatrix} \tag{7}$$

Then we could get a linear differential relation between the output y and the new input v as

$$\begin{bmatrix} y_1^{(r_1)} \\ \cdots \\ y_m^{(r_m)} \end{bmatrix} = \begin{bmatrix} v_1 \\ \cdots \\ v_m \end{bmatrix} \tag{8}$$

After the application of an exact I/O feedback linearization, each output (y_i) depends only on its associated control input (v_i).

3.2. **Conventional input-output feedback linearization control of PMSM.** In order to avoid any zero dynamics and to get a total input-output linearization, the d-axis current and rotor speed are chosen as outputs. For convenience, we define the outputs of the system state variables i_d and w_r as follows:

$$y_1 = h_1(x) = i_d \tag{9}$$

$$y_2 = h_2(x) = w_r \tag{10}$$

Differentiating the output y_1, we could get

$$\dot{y}_1 = \frac{di_d}{dt} = -\frac{R_s}{L} i_d + p_n w_r i_q + \frac{1}{L} u_d \tag{11}$$

From above equation, we could find the relative degree value for the output y_1 is $r_1 = 1$. For the output y_2, the second-order time derivative of y_2 is

$$\begin{aligned} \ddot{y}_2 &= L_f(L_f h_2) + L_{g1}(L_f h_2) u_1 + L_{g2}(L_f h_2) u_2 \\ &= \frac{3p_n\phi_f}{2J}\left[-\frac{R}{L} i_q - \frac{p_n\phi_f}{L} w_r - p_n w_r i_d + \frac{1}{L} u_q\right] - \frac{B}{J}\dot{w}_r \\ &= \frac{3p_n\phi_f}{2JL}\left[-R i_q - p_n\phi_f w_r - L p_n w_r i_d\right] - \frac{B}{J}\dot{w}_r + \frac{3p_n\phi_f}{2JL} u_q \end{aligned} \tag{12}$$

Based on Equation (12), we could find the relative degree of the output of y_2 is $r_2 = 2$. So the vector relative degree of the system is $r = (r_1, r_2) = (1, 2)$. Then, the relationship between the outputs and inputs of the model can be obtained as follows:

$$\begin{bmatrix} \dot{y}_1 \\ \ddot{y}_2 \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} + D \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{13}$$

where $A_1 = -\frac{R_s}{L} i_d + p_n w_r i_q$, $A_2 = \frac{3p_n\phi_f}{2JL}\left[-Ri_q - p_n\phi_f w_r - Lp_n w_r i_d\right] - \frac{B}{J}\dot{w}_r$, $D = \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & \frac{3p_n\phi_f}{2JL} \end{bmatrix}$, and the inverse of matrix D is $D^{-1} = \begin{bmatrix} L & 0 \\ 0 & \frac{2JL}{3p_n\phi_f} \end{bmatrix}$, which is nonsingular since $det(D) = \frac{3p_n\phi_f}{2JL^2} \neq 0$. Then based on feedback linearization theory, the dq axis voltage control are given as follows:

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = D^{-1}\left[-\begin{bmatrix} A_1 \\ A_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}\right] \tag{14}$$

which lead to the following input-output relation between the output y_i and the new inputs v_i

$$\begin{bmatrix} \dot{y}_1 \\ \ddot{y}_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{15}$$

The new control inputs v_1, v_2 are given as follows:

$$v_1 = \dot{y}_1^* + K_{10}(y_1^* - y_1) \tag{16}$$

$$v_2 = K_{20}(y_2^* - y_2) + K_{21}(\dot{y}_2^* - \dot{y}_2) + \ddot{y}_2^* \tag{17}$$

where K_{10}, K_{20}, K_{21} are the gains. Based on Equations (14), (16) and (17), the control inputs for the PMSM are

$$u_1 = R_s i_d - Lp_n w_r i_q + Lv_1 \tag{18}$$

$$u_2 = Ri_q + p_n\phi_f w_r + Lp_n w_r i_d + \frac{2BL}{3p_n\phi_f}\dot{w}_r + \frac{2JL}{3p_n\phi_f}v_2 \tag{19}$$

4. **Sliding Mode I/O Feedback Linearization Control of PMSM.**

4.1. **Sliding mode I/O feedback linearization design.**

Due to the fact that the values of the system parameters are not exactly known and may change; therefore, the conventional I/O feedback linearization control of PMSM may fail to have good current and speed tracking performance. In order to improve the robustness of feedback linearization controller, a sliding mode I/O feedback linearization controller is designed as the current and speed tracking for PMSM drive system.

From Section 3, we could find the vector relative degree is $r = (1, 2)$ in the current and speed control system of PMSM. And in this part the objective is to design an equilibrium surface so that the state trajectories of the system have the desired behavior when restricted to the surface. Therefore, we adopt the following surfaces with the errors of the d-axis current and motor speed:

$$s_1 = K_{10}\int e_1 dt + e_1 \tag{20}$$

$$s_2 = K_{20}\int e_2 dt + K_{21}e_2 + \dot{e}_2 \tag{21}$$

where $e_1 = y_1^* - y_1$, $e_2 = y_2^* - y_2$, K_{10}, K_{20} and K_{21} are positive constants. If the system states operate on the surface, then $s_1 = s_2 = 0$ and $\dot{s}_1 = \dot{s}_2 = 0$, yields

$$\begin{aligned}\dot{s}_1 &= \dot{e}_1 + K_{10}e_1 = v_1 - \dot{y}_1 \\ &= v_1 + \frac{R_s}{L}i_d - p_n w_r i_q - \frac{1}{L}u_d\end{aligned} \tag{22}$$

$$\begin{aligned}\dot{s}_2 &= \ddot{e}_2 + K_{21}\dot{e}_2 + K_{20}e_2 = v_2 - \ddot{y}_2 \\ &= v_2 - \frac{3p_n\phi_f}{2JL}\left[-Ri_q - p_n\phi_f w_r - Lp_n w_r i_d\right] + \frac{B}{J}\dot{w}_r - \frac{3p_n\phi_f}{2JL}u_q\end{aligned} \tag{23}$$

The equivalent control concept of a sliding surface is the continuous control that allows for the maintenance of the state trajectory on the sliding surface $s = \dot{s} = 0$. Then we can obtain the equivalent control from Equations (22) and (23),

$$u_{1eq} = R_s i_d - Lp_n w_r i_q + L\dot{y}_1^* + LK_{10}(y_1^* - y_1) \tag{24}$$

$$\begin{aligned}u_{2eq} =& R_s i_q + p_n\phi_f w_r + Lp_n w_r i_d + \frac{2BL}{3p_n\phi_f}\dot{w}_r \\ &+ \frac{2JL}{3p_n\phi_f}\left[K_{20}(y_2^* - y_2) + K_{21}(\dot{y}_2^* - \dot{y}_2) + \ddot{y}_2^*\right]\end{aligned} \tag{25}$$

Now, in order to drive the state variable to the sliding surface $s_1 = s_2 = 0$, the following control laws are defined:

$$u_d = u_1 = u_{1eq} + u_{1s} = u_{1eq} + L(\rho_1 sign(s_1) + \lambda_1 s_1) \tag{26}$$

$$u_q = u_2 = u_{2eq} + u_{2s} = u_{2eq} + \frac{2JL}{3p_n\phi_f}(\rho_2 sign(s_2) + \lambda_2 s_2) \tag{27}$$

where u_{1s}, u_{2s} are switching functions that ensure the state trajectories are kept on the sliding surfaces. Then the reaching law can be derived and given as

$$\dot{s}_1 = -\rho_1 sign(s_1) - \lambda_1 s_1 \tag{28}$$

$$\dot{s}_2 = -\rho_2 sign(s_2) - \lambda_2 s_2 \tag{29}$$

Theorem 4.1. *Considering the permanent magnet synchronous motor given in Equations (1)-(3), if the following assumptions (A1), (A2) and (A3) are verified, the control law (26) and (27) leads the speed error* $e = w_r^* - w_r$ *tending to zero.*

A1. The state variables i_d, i_q, w_r *are measurable and available for feedback.*

A2. The signal w_r *is differentiable in a numerical way and bounded with the derivatives.*

A3. The variations of armature resistance R_s*, motor inertia* J*, and load torque disturbance* T_L *are within bounds.*

Proof: In order to prove the stability of the designed controller, the Lyapunov's function is chosen as

$$V = \frac{1}{2}s^T s \tag{30}$$

And the derivative of Equation (30) is

$$\begin{aligned}\dot{V} &= s^T\dot{s} \\ &= s_1\dot{s}_1 + s_2\dot{s}_2 \\ &= s_1\left[v_1 + \frac{R_s}{L}i_d - p_n w_r i_q - \frac{1}{L}u_{1eq} - \rho_1 sign(s_1) - \lambda_1 s_1\right]\end{aligned}$$

$$
\begin{aligned}
&+ s_2\left[v_2 - \frac{3p_n\phi_f}{2JL}(-Ri_q - p_n\phi_f w_r - Lp_n w_r i_d)\right] \\
&+ s_2\left[\frac{B}{J}\dot{w}_r - \frac{3p_n\phi_f}{2JL}u_{2eq} - \rho_2 sign(s_2) - \lambda_2 s_2\right] \\
=& -\rho_1 s_1 sign(s_1) - \lambda_1 s_1^2 - \rho_2 s_2 sign(s_2) - \lambda_2 s_2^2 \\
\le& 0
\end{aligned} \tag{31}
$$

Due to the Lyapunov's function is negative definite, the control system is stable and the systems states will converge towards the sliding mode surface in the limited time.

4.2. **Continuous approximation of switching control law.** Sliding mode variable structure causes the chattering phenomenon in the control law (26) and (27), and in order to smooth the control signal, and the sign function is replaced by saturation function $sat(\cdot)$, defined as

$$
sat(s,\varsigma) = \begin{cases} s/\varsigma & (|s| \le \varsigma) \\ ssign(s) & (|s| > \varsigma) \end{cases} \tag{32}
$$

where ς represents the thickness of the boundary layer neighboring the switching surface, then we could get $ssat(s,\varsigma) \ge 0$. And, the control law given in Equations (26) and (27) can be furtherly modified using the saturation function as

$$
u_d = R_s i_d - Lp_n w_r i_q + L\dot{y}_1^* + LK_{10}(y_1^* - y_1) + L\rho_1 sat(s_1,\varsigma_1) + L\lambda_1 s_1 \tag{33}
$$

$$
\begin{aligned}
u_q =& R_s i_q + p_n\phi_f w_r + Lp_n w_r i_d + \frac{2BL}{3p_n\phi_f}\dot{w}_r \\
&+ \frac{2JL}{3p_n\phi_f}\left[K_{20}(y_2^* - y_2) + K_{21}(\dot{y}_2^* - \dot{y}_2) + \ddot{y}_2^* + \rho_2 sat(s_2,\varsigma_2) + \lambda_2 s_2\right]
\end{aligned} \tag{34}
$$

4.3. **Load torque observer of PMSM based on ESO.** For the control schemes proposed in this paper, an information on the acceleration (dw/dt) is needed for the state feedback and can be calculated from Equation (1). It is, however, required to make the load torque known. In this paper the load torque is estimated by extended state observer (ESO).

The construction of ESO is derived from a system with the following expression:

$$
\begin{aligned}
x'^{(n)} &= f'(x', x'^{(1)}, \cdots, x'^{(n-1)}, t) + w(t) + bu' \\
&= d + bu'
\end{aligned} \tag{35}
$$

where f' is an unknown function, w is an unknown disturbance, u' is the input to the system, and b is a system parameter.

Then the $n+1$th-order ESO for this nth-order system is designed as follows:

$$
\begin{cases}
\dot{z}_1 = z_2 - g_1(z_1 - x'(t)) \\
\qquad \vdots \\
\dot{z}_n = z_{n+1} - g_n(z_1 - x'(t)) + bu' \\
\dot{z}_{n+1} = -g_{n+1}(z_1 - x'(t))
\end{cases} \tag{36}
$$

where, $z_1 \to x'(t)$, $\cdots$, $z_n \to x'^{n-1}(t)$, $z_{n+1} \to d(t)$. z_{n+1} is the estimate signal of the unknown function f' and the disturbance w.

In Equation(36), the $g_i(\cdot)$ $(i = 1, 2, \cdots, n+1)$ is a nonlinear function, is chosen as $\beta_i \cdot fal_i(\cdot)$, where β_i is a constant, and the fal_i function is expressed as

$$fal_i(\varepsilon, \alpha_i, \delta_i) = \begin{cases} |\varepsilon|^{\alpha_i} sign(\varepsilon) & (|\varepsilon| > \delta_i) \\ \varepsilon / \delta_i^{1-\alpha_i} & (|\varepsilon| \le \delta_i) \end{cases} \tag{37}$$

From Equation (1), we can find the mechanical dynamic equation of PMSM is a first-order controlled object. By defining

$$d = \frac{1}{J}(-T_L - Bw_r) \tag{38}$$

then Equation (1) becomes

$$\dot{w}_r = \frac{3p_n\phi_f}{2J} i_q + d \tag{39}$$

Here, d is regarded as the disturbance of speed loop. Then the two-order ESO for this one-order object is constructed as

$$\begin{cases} \varepsilon = w_r - z_{21} \\ \dot{z}_{21} = z_{22} - \beta_1 fal_1(\varepsilon, \alpha_1, \delta_1) + bu' \\ \dot{z}_{22} = -\beta_2 fal_2(\varepsilon, \alpha_2, \delta_2) \end{cases} \tag{40}$$

where, w_r is the motor speed, ε is speed error, z_{21} is the tracking signal for w_r, z_{22} is the observed value of the disturbance d, u' represents ${i_q}^*$, is the system control variable, $b = 3p_n\phi_f/2J$. β_1, β_2 are gains of output error, δ_1, δ_2 are the filtering factor, α_1, α_2 are the nonlinear factor. fal_1 and fal_2 are as shown in Equation (37), where the sign function $sign(\varepsilon)$ in fal_1 and fal_2 are also replaced by the saturation function $sat(\varepsilon, \mu_1)$ and $sat(\varepsilon, \mu_2)$.

Therefore, based on Equation(40), due to the motor speed w_r is known, then the load torque can be estimated by z_{22} in the extended state observer.

5. **Simulation Results and Discussions.** In this section, we will study the speed regulation performance of the proposed sliding mode I/O feedback linearization controller under motor parameters and load torque variations by means of simulation examples. The control block diagram of the whole system is shown in Figure 1. Traditional Field-Oriented Control theory and SVPWM method are used in the control of the two-level three-phase inverter.

The normal parameters of a PMSM used are as follows: the base speed (w_{b0}) is 2000 *rpm*, stator resistance (R_{s0}) is 0.9585 Ω, d-axis and q-axis inductance (L_{d0}) are 0.00525 H, magnet flux linkage (ϕ_{f0}) is 0.1827 Wb, number of pole pair (p_n) is 4, motor inertia (J_0) is 0.0006329 Kg· m^2, friction coefficient (B_0) is 0.0003035 N· m·s.

During the simulation, the reference speed (w_r^*) changes in ramp profile from 0 rad/s to 100 rad/s, at $t = 0.05$ s, reaches 100 rad/s, stay at 100 rad/s after $t = 0.05$ s. The motor starts with initial load torque 0 Nm. The reference d-axis current i_d^* is given as $i_d^* = 0$.

The parameters of the proposed controller are chosen as: $K_{10} = 10$, $\rho_1 = 500$, $r_1 = 500$, $\varsigma_1 = 0.1$, $K_{20} = 10^6$, $K_{21} = 8000$, $\rho_2 = 1050$, $r_2 = 15000$, $\varsigma_2 = 0.1$. The parameters of ESO are $\beta_1 = 3000$, $\beta_2 = 815000$, $\alpha_1 = 0.75$, $\alpha_2 = 0.5$, $\delta_1 = 0.01$, $\delta_2 = 0.01$, $\mu_1 = \mu_2 = 0.01$. The parameters of conventional feedback linearization controller are chosen as: $K'_{10} = 10$, $K'_{20} = 10^6$, $K'_{21} = 8000$.

Case 1: The stator resistance steps from normal R_{s0} to $10 * R_{s0}$ at $t = 0.09$ s, back to R_{s0} at $t = 0.13$ s. Figure 2a and Figure 2b show the comparation between sliding mode I/O feedback linearization controller and conventional I/O feedback linearization controller. The maximum speed tracking error is about 0.08 rad/s for the conventional I/O feedback linearization controller, in contrast, speed tracking error is much smaller for the proposed method when the stator resistance varied.

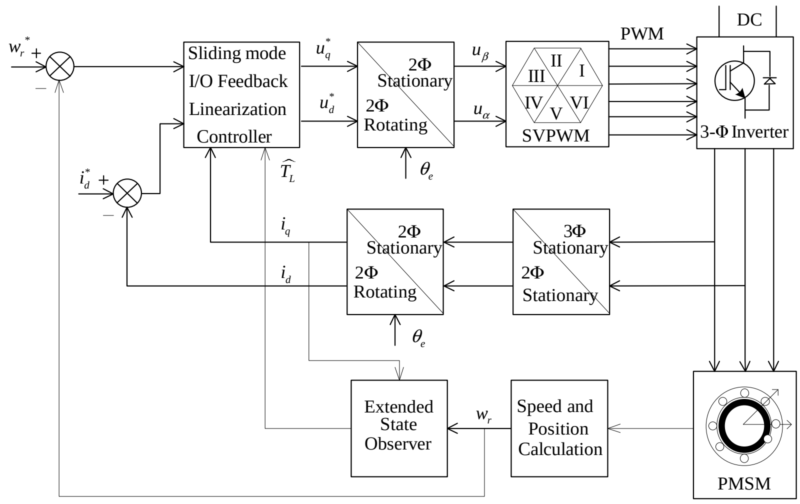

FIGURE 1. Block diagram of PMSM controlled by sliding mode I/O feedback linearization with extended state observer

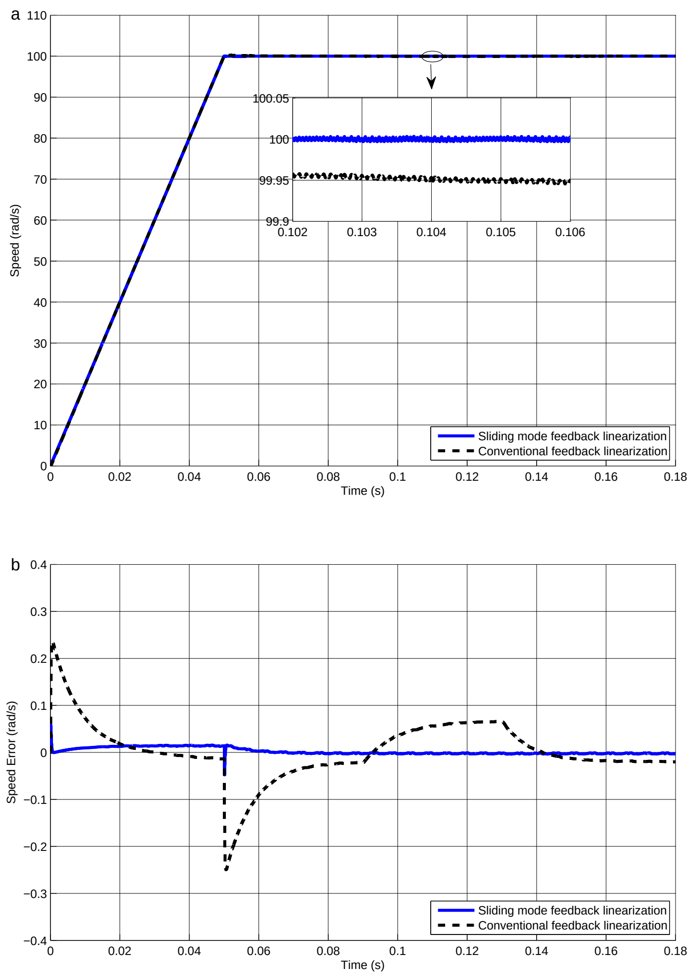

FIGURE 2. Speed tracking response between the proposed and conventional controller in the situation of stator resistance variation

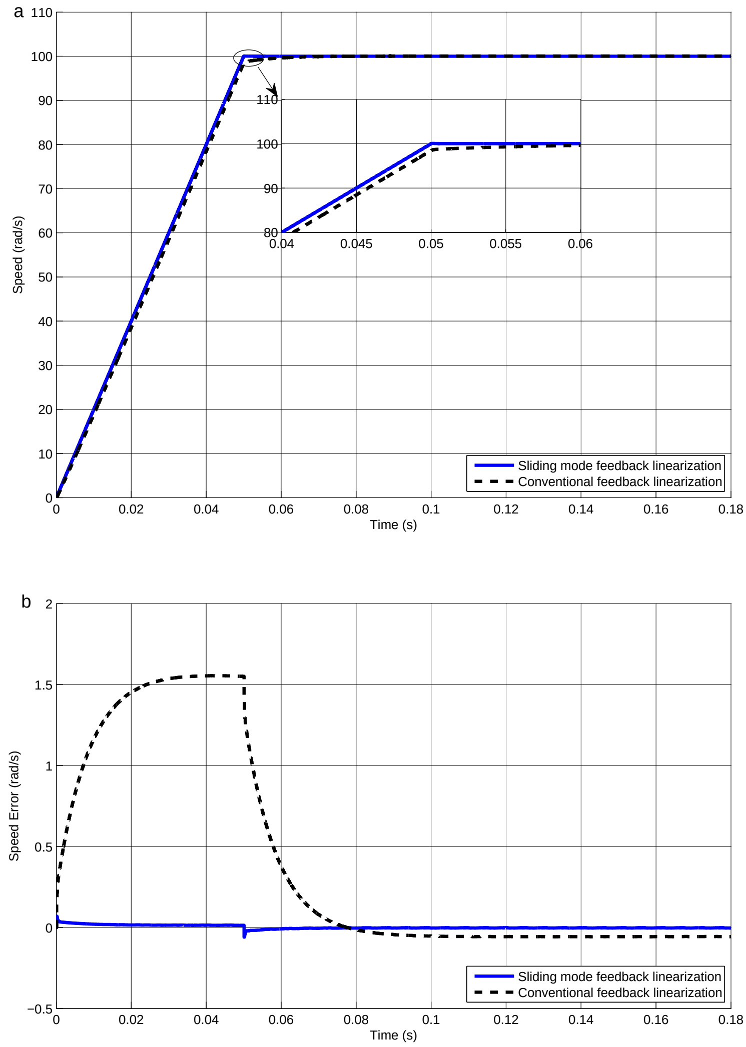

FIGURE 3. Speed tracking response between the proposed and conventional controller in the situation of motor Inertia variation

Case 2: The motor inertia steps from normal J_0 to $1.1{*}J_0$ at $t = 0$ s. Figure 3a and Figure 3b show the comparation between sliding mode I/O feedback linearization controller and conventional I/O feedback linearization controller. The maximum speed tracking error is about 1.6 rad/s for the conventional method at the start of the motor, while the proposed method has much better tracking performance, even the motor inertia varied.

Case 3: The load torque steps from 0 Nm to 1 Nm at $t = 0.09$ s, back to 0 Nm at $t = 0.13$ s. Figure 4a and Figure 4b show the comparation between sliding mode I/O feedback linearization controller and conventional I/O feedback linearization controller, Figure 5 shows the real load torque and observed load torque based on the extended state observer. The maximum speed tracking error is about 2.6 rad/s for the conventional method when the load torque varied, while speed tracking error is about 0.2 rad/s for the proposed method, and the adjust time for the proposed method is also much shorter than the conventional method.

6. **Conclusion.** In this paper, a sliding mode I/O feedback linearization controller is designed as controller for speed and current tracking of the PMSM, which could enhance

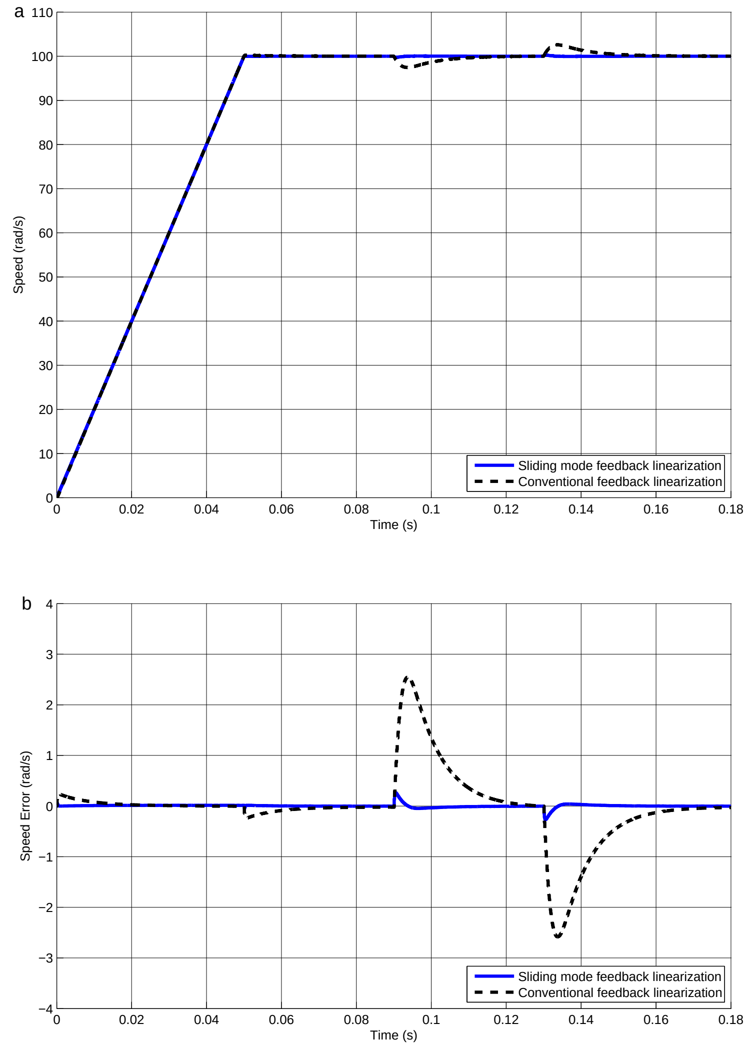

FIGURE 4. Speed tracking response between the proposed and conventional controller in the situation of load torque variation

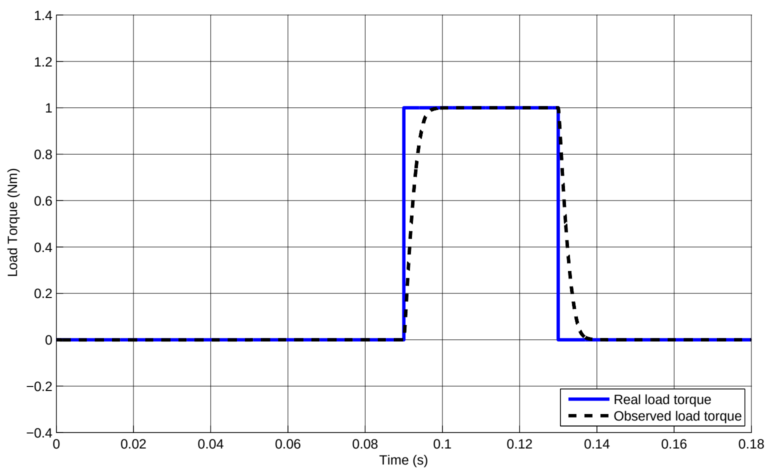

FIGURE 5. Load torque observer based on ESO

the quick response and anti-disturbance ability of the control system. The proof of the stability of the controller is proved based on Lyapunov function. Besides, in this paper, extended state observer (ESO) is designed for the estimation of load torque.

Then by simulation examples, it is evident that the proposed controller shows better speed tracking performance at both dynamic and steady state than conventional I/O feedback linearization controller in the situation of motor parameters and load torque variations. The designed extended state observer is an effective observer for the load torque. Thus simulation results have verified the proposed whole system has great robust to external disturbances.

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