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FORECASTING INTERMITTENT DEMAND BY MARKOV CHAIN MODEL

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Abstract. *The inventory control of the products which have intermittent demand is essential for many organizations since these items have a low lead time demand but a high price. Since the intermittent demand pattern is irregular, the estimation of the lead time demand is challenging. A modified Markov chain model (MMCM) has been proposed for modeling and estimating intermittent demand data, motivated by a case study. The performance of MMCM and the traditional methods have been compared by accuracy measures. The results reveal that the proposed method is a good competitor or even better than other methods.*
Keywords: Forecasting, Intermittent, Lead time, Markov chain model, Transition probability

1. **Introduction.** One of the most critical issues of inventory management is demand forecasting. Variation in demand increases the difficulty of determining the precise amount of inventory both to avoid stockouts and to satisfy the customer fill rate. The inventory control problem is getting complicated by the fact that demand is uncertain or the variation of demand is highly volatile.

The most important factor that makes the demand volatile is the sequence of zero values in a demand series. The products, which are not demanded very frequently, mostly have high percentage of zero demand. Such items are also referred to as slow-moving items. Slow-moving items are defined as inventories that have a slower turnover rate than the average turnover for the entire inventory [1,2]. These items are classified as lumpy which means that there is great variability among the non-zero values [2]. The slow-moving demand with a large proportion of zero values is described as intermittent [1]. Traditionally, whether to determine a data set is intermittent or not, two measures are computed, namely the average inter demand interval (ADI) and the coefficient of variation squared (CV^2). ADI measures the average number of time periods between two successive demands and CV represents the standard deviation of demand values divided by the average demand over a number of time periods [3].

The inventory control of the products which have intermittent demand is essential to many organizations, since excess inventory leads to high holding costs and stockouts can have a great impact on operations performance. The difficulty in assessing good strategies for the management of these items lies in their specific nature. Since intermittent demands are highly stochastic and have a large percentage of zero values, the estimation of the lead time demand distribution is particularly intricate [4].

Markov chains are useful tools to model many practical systems such as manufacturing and inventory systems [5,6]. Applications of Markov chains and higher order Markov chain models are mostly used to model categorical data sequences. However, we realize that the structure of intermittent demand data allows us to use higher order Markov chain models to obtain accurate forecasts. In this study, we propose a modified Markov chain model which provides accurate forecast results for intermittent demand data.

In the following section, the literature is briefly reviewed. In the third section, the structure of the higher order Markov chain model is described. The proposed modification in forecasting procedure, namely modified Markov chain model is explained in detail with an example. Lastly, the accuracy measures used in this study are reviewed in this section. The application of the modified Markov chain model and comparisons with other forecasting methods are provided in section four. Finally, the fifth section covers overall evaluations and future research directions.

2. **Related Literature.** It is possible to classify the literature about intermittent demand into two main groups. The first group includes the literature about the efficient inventory management of products that have intermittent demand. With the assumption of the demand follows Poisson distribution, the studies which optimize the system by continuous review inventory policy [7-11], the papers which assume the demand is compound Bernoulli process and compute the reorder point [12,13], the papers which examine the system using periodic review inventory policy [14-16] can be classified in this group. In addition, a combined forecast-inventory management approach is presented in [17]. The authors examined the performance of two different (s,Q) inventory models. One of them assumes that the lead time demand follows Normal distribution, whereas the other one assumes Gamma distribution for the lead time demand. The authors conclude that the second model yields a service level close to the desired level. An empirical comparison of different reorder point methods are given by [4]. The authors constructed the lead time demand distribution with respect to different approaches and gave an optimization by the decomposition approach.

The papers that address the problem of forecasting intermittent demand and examine how to obtain accurate demand forecasts can be classified as a second group of the literature. When demand is intermittent, the classical approach which assumes that the demands in each time period are independent and lead time demand follows Normal distribution gives unsatisfactory results. Since intermittent demand pattern has a long stream of zero values, it is difficult to estimate the lead time demand distribution. Different approaches that estimate the lead time demand have been developed.

Single Exponential Smoothing (SES) and Simple Moving Averages (SMA) are used to model intermittent demand data and give satisfactory results in practice [18]. In 1972 Croston's method which is superior to the exponential smoothing is developed by [19], assuming that the demand is Bernoulli process and the demand size is assumed to have a Normal distribution. According to Croston's method, separate exponential smoothing estimates of the average size of the demand and the average demand interval are made after demand occurs. If no demand occurs, the estimates do not change. Certain limitations of Croston's method are identified in [20]. The authors quantify the bias associated with Croston's method and they present a modification that gives approximately unbiased demand estimates. [18] proposes a new method of forecasting and also gives a discussion about the comparison of some forecasting methods and accuracy of resulting estimates.

Another comparison of different forecasting methods for the management of spare parts is provided in [21]. The results of [21] show that the use of the SES method to be questionable since it consistently creates poor forecasting performance which remains poor as the demand variability increases. And also the authors conclude that the weighted moving average is much superior to exponential smoothing and its superiority increases with the increase of seasonal period length.

One of the most important forecasting methods is Willemain's bootstrap method [2] which captures the autocorrelations between successive demands using first order Markov chain. According to this method, there are two states of Markov chain which correspond to zero demand and non-zero demand. The transition probabilities are calculated according to demand data and the sequence of zero and non-zero values is generated consistent with the transition probabilities. Once the sequence is obtained, each non-zero value is replaced with a numerical value by bootstrapping. That is the demand value is obtained by sampling at random with replacement from the set of observed non-zero demands.

The two-state Markov chain is used to estimate intermittent demand by [2] and this idea gives the motivation of our study. Since Markov chains keep and transfer the information of history to the following steps, they are useful tools for modeling and forecasting industrial systems. This fact is realized by [2]; however, they used first order Markov chain. The order of the chain determines that how many steps back the transferred information comes from. Our research question is whether we can use Markov chain not only to forecast the sequence of zero and non-zero demands, but also to obtain the forecasts of the demand values. Namely, the usage of higher order Markov chain model is examined to model the dependence structure of intermittent demand pattern. The special structure of the intermittent demand data allows us to employ each demand value as a state of a Markov chain. Then it is possible to forecast the next state by the Markov chain model. The forecast of the next state is the forecast of the next demand value, at the same time. The order of Markov chain is determined depending on the analyzed data set. When applying higher order Markov chain model to intermittent data, a modification is needed for the procedure of obtaining forecast values. The proposed modification can be explained in the following manner. The basic idea is that the states are the demand sizes rather than the categories; however, we need to categorize demand sizes in some cases. Recall that, intermittent demand structure involves demand values such as 0, 1, or 2 with high frequencies, whereas higher demand values are faced with lower frequencies. Two cases are possible here. The first case is for low demand sizes with high frequencies. In this case, since each state corresponds to a demand value, the estimation of the next state simply gives the estimation of the next demand size. Higher demand sizes with low frequencies correspond to the second case. Low or zero frequencies affect the transition probability matrix and sometimes result in finding zero transition probabilities. This is undesirable because it affects the ergodicity and the calculation of steady state probabilities of the related Markov chain. Hence these demands are grouped into one state. And when estimating the next state, the occurrences of the demands grouped in the same state is treated with equal probability.

To summarize, this paper proposes a modified higher order Markov chain model to obtain accurate lead time demand (LTD) forecasts for the intermittent data, motivated by a case study. The modified Markov chain model was obtained for 695 data sets and forecast values were calculated. In addition, LTD forecasts were computed for the same group of data by using other forecasting methods (Croston's method, Syntetos and Boylan approximation and Markov bootstrap method) as well. A comparison among the forecast values obtained by modified Markov chain method and those obtained by other forecasting methods is presented. The validity of the forecasts was compared using various accuracy measures. The statistical results of both forecast values and validity values show that the proposed method is a good competitor or even better than other methods.

In the following section, after a brief introduction about Markov chain model, the proposed modification will be presented when using the Markov chain model for the intermittent data and obtaining forecast values. Besides, the accuracy measures used will be explained.

3. Estimation of Intermittent Demand with Markov Chain Model.

In a first order Markov chain model, the estimation of the next state is obtained depending only on the one-step transition probability matrix, whereas k-step transition matrix is needed for kth-order Markov chain model. The higher order (kth-order) Markov chain models were first proposed by Raftery [22]. Raftery's model is extended by [23,24] to a more general higher order Markov chain model given in (1).

$$\hat{X}^{(n)} = \sum_{i=1}^{k} \lambda_i \hat{Q}_i X^{(n-i)} \tag{1}$$

$\hat{X}^{(n)}$ is the state vector which is the prediction of the next state at time n. $\hat{Q}_i$ is the i-step transition probability matrix and λ_i are the weights given by [24] as nonnegative real numbers such that

$$\sum_{i=1}^{k} \lambda_i = 1 \tag{2}$$

λ_i $(i = 1, 2, \ldots, k)$ can be estimated by the maximum likelihood estimation or obtained by solving a linear programming (LP) model which is proposed by [23].

3.1. Modified Markov chain model.

Accurate estimates of intermittent demands cannot be obtained by applying Markov chain model because of the high percentage of zero demands. A modification is needed when obtaining forecast values. The steps of the algorithm proposed are described in this section. All computations were performed by a computer code written in Matlab.

Step 1. The frequency of each demand value is obtained for data set j $(j = 1, 2, \ldots, 695)$.

While some demand values (such as 0, 1, 2 and 3) are frequently observed, the frequencies of some demand values (such as $8, 9, \ldots, 12$) are low. Each demand value with high frequency corresponds to a state of a Markov chain. The demands with low frequencies are collected under a single group in order to be reconsidered in the procedure of forecasting.

Step 2. The states of the Markov chain are determined.

Since Markov chain model used to model mostly in categorical data, the states of the Markov chain are the categorical variables. However, in modified Markov model, the states include the demand information.

Step 3. The one-step and two-step transition probability matrices are computed.

Step 4. Steady state probabilities are calculated.

Step 5. The linear programming (LP) model is constructed.

The steady state probabilities and the transition probabilities are the parameters of the LP model and the λ_is are the decision variables.

Step 6. Values are calculated as a solution of the LP model.

The number of values determines the order of the Markov chain model. First order Markov chain model is required for some of the data sets whereas second order Markov chain model is constructed for some of them.

Step 7. By using values and the transition probability matrices, the Markov chain model is constructed for data set j.

Step 8. According to the resulting Markov chain model, the demand forecasts are obtained.

The modification of the Markov chain model is mostly included in this step. [23] states that, according to k-th order state probability distribution, the prediction of the next state ($\hat{X}^{(n)}$) must be taken as the state with the maximum probability. The highest probability obviously states that the related state is the most probably to occur but if there are some states which have non-zero probabilities, these states are also likely to occur relative to their probabilities. We claim that when estimating the next state, considering all the states relative to their probabilities will yield better forecast values. Therefore, in the procedure we proposed we have considered not only the state with the highest probability but also all the states in proportion to their probabilities when estimating the next state of the Markov chain. If more than one demand with a low frequency corresponds to a state, it is thought that the demands within this state occur with an equal probability. Consequently, modified Markov chain model determines the next state taking into account all states relative to their probabilities whereas in Markov chain model the state with maximum probability determines the next state.

Step 9. The accuracy measure r is calculated for the forecast values.

The accuracy measure used for the Markov chain model, which is represented by r, is computed for the forecast values. The computation of accuracy measures are defined in Section 3.3, in detail.

Step 10. Repeat Steps 8 and 9 until obtain the highest r value.

Since the determination of forecast values depends on the probabilities of states, different sequences will be obtained whenever Step 8 is repeated. Step 8 is repeated for several times and the data set that gives the highest r value is recorded as the final forecast values.

Step 11. The lead time demands (LTD) are computed consistent with the lead time information for data set j.

Each data set may have different lead time. Summation of monthly demand forecasts along with the lead time results in forecasts of LTD.

Step 12. Steps 8-11 are repeated many times for the same data set and the LTD distribution is obtained.

The flowchart of the computer code is given in Appendix I. Even though the program contains numerous steps, the results are obtained in a few minutes and no difficulty is experienced in practice.

3.2. **An example.** Let us consider the frequency distribution of a sample data set presented in Table 1, as an example. This data set is organized as presented in Table 2 and considered as a 5-state Markov chain.

1,1,3,1,8,2,1,0,0,0,0,0,5,0,0,0,2,9,0,0,1,1,1,1,1,4,0,0,1,1,0,2,0,0,1,1,3,1,0,0,2,2,1,1,0,1, 0,0,2,0,0,0,3,1,1,0,4,0,1,0,0,0,0,3,2,0

Table 1. Frequency distribution of the sample data set

Demand	0	1	2	3	4	5	6	7	8	9
Frequency	30	20	7	4	2	1	0	0	1	1
Probability	0.455	0.303	0.106	0.060	0.030	0.015	0.000	0.000	0.015	0.015

Table 2. Markov chain states according to Table 1

State	1	2	3	4	5
Frequency	30	20	7	4	5

TABLE 3. Forecast results obtained by modified Markov chain model

Demand	0	1	2	3	4	5	6	7	8	9
Frequency	668	409	121	68	39	4	3	2	4	2
Probability	0.506	0.310	0.092	0.051	0.029	0.003	0.002	0.002	0.003	0.002

States 1 to 4 corresponds to the demand values 0 to 3. The demands 4 or more are encountered less frequently. Hence, these data are grouped in to one state and state 5 covers all demands greater than or equal to 4. The probability of encountering state 5 is considered the probability of encountering demands greater than or equal to 4.

Transition probability matrices and steady state probabilities are calculated. The computer code takes the probabilities as the parameters of the LP model; first constructs the LP model and then solves. The solution of the LP model determines the order of the Markov chain model. Since λ_1 and λ_2 are found as 0.9106 and 0.0894 respectively for the sample data set, second order Markov chain model is constructed as in (3) and forecast values are obtained successively starting from the third data value according to the resulting model.

$$\hat{X}^{(n)} = 0.9106\hat{Q}_1 X^{(n-1)} + 0.0894\hat{Q}_2 X^{(n-2)} \tag{3}$$

The first and the second data are used to obtain the forecast value of the third data. $\hat{X}^{(3)}$ is a column vector of dimension five, whose components are the transition probabilities to each state. Markov chain model selects the maximum probability among these probabilities and corresponding state is the forecast of the next state. However, according to modified Markov chain model that we proposed, the next state is forecasted considering the probabilities of each state. To this end, a demand sequence in which each demand value appears depending on its probability of occurrence is generated. A demand is randomly chosen from this sequence and this demand value determines the next state. The less frequent demand values are not ignored in this manner. Since the next state is determined randomly, when we repeat this procedure, we obtain a different forecast values. Hence, this procedure is repeated many times (500 times in this study) for the same demand series. The accuracy measure r is computed for each demand series and the forecast data that have the best consistence is the final forecast values. Summing the monthly demand data along with the lead time, the lead time demand forecasts are obtained.

This procedure must be repeated many times to attain lead time demand distribution in the long run. It is acceptable that the procedure is repeated 20 times. Even though the lead time is 8 months, if 20 repetitions are performed, 1180 data will be obtained and it is sufficient to derive the lead time demand distribution. The lead time is taken as 1 month for the example data set and the forecast results are provided in Table 3. The consistence between the real and the forecasted data can clearly be seen from Tables 1 and 3.

3.3. **Accuracy of estimates.** To evaluate the performance and effectiveness of the higher order Markov chain model, a prediction accuracy r is defined by [23] as

$$r = \frac{1}{T}\sum_{t=n+1}^{T} \delta_t \tag{4}$$

where T is the length of the data sequence and

$$\delta_t = \begin{cases} 1, & \text{if } \hat{X}_{(t)} = X_{(t)} \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

Since delta (δ) measures the consistency between the forecast values and the actual ones, higher delta values are desired.

There are many forecasting accuracy measures to assess the performance of forecasting methods. See [25] for more detailed information. The mean absolute scaled error (MASE) defined as a scale-independent measure for intermittent data series by [25] was used in this study to compare the accuracy of different forecasting methods. A scaled error (q_t) is computed as given in (6).

$$q_t = \frac{e_t}{\frac{1}{n-1} \times \sum_{i=2}^{n} |Y_i - Y_{i-1}|} \tag{6}$$

where Y_i is the ith demand observation, n is the length of the data sequence, and e_t is the difference between the real value Y_t and the forecast value $\hat{Y}_t$. The scaled error is less than 1 if it arises from a better forecast than the average one-step naive forecast computed depending on the sample [25]. MASE is a mean value which is computed as given in (7).

$$MASE = mean\left(|q_t|\right) \tag{7}$$

To assess the performance of LTD forecast, the mean absolute percentage error of LTD (MAPELTD) is used. The absolute percentage error of lead time demand (APELTD) is calculated as given in (8) for the non-zero LTD values [26]. MAPELTD is the average of APELTD.

$$APELTD = \sum_{i=1}^{m} \frac{\left|LTD\hat{Y}_i - LTDY_i\right|}{LTDY_i} \times 100 \tag{8}$$

where $LTDY_i$ is the ith lead time demand observation, $LTD\hat{Y}_i$ is the forecast of LTD, and m is the length of LTD data sequence. When LTD is zero, APELTD is not defined. MAPELTD is

$$MAPELTD = mean\left(APELTD\right) \tag{9}$$

4. **A Case Study.** The monthly demand data of 695 different products are obtained from a medium size enterprise which has been active in Turkey since 2000. Each data sequence is for 66 months and has different lead times. The forecast values were calculated by using modified Markov chain model (MMCM) for demand data of each product. The accuracy of forecasts was calculated. On the other hand, forecasts were computed for the same group of data by using other forecasting methods also, such as the Markov bootstrap method (MB), Syntetos-Boylan Approximation (SBA), and Crostons method (CM). We proposed a comparison between the forecasts computed by MMCM and those computed with the other forecasting methods.

4.1. **Demand structure.** First, in order to classify the demand as intermittent, the values of ADI and CV^2 for each data set are calculated. Some statistics related with the values of ADI and CV^2 are presented in Table 4. Furthermore, the zero demand rates and the mean of non-zero demands for each data set are calculated and the statistics related with these variables are also provided in the last two columns of Table 4. Each product has a different lead time and the lead time values change from 1 month to 8 months.

4.2. **Calculation of demand forecasts by modified Markov chain method.** The modified Markov chain model was applied to all data sets selected for the study as explained in Section 3.1. Data sets are grouped according to their lead times. The number of data set in each group and the statistics for the computed LTD forecast values for each group are provided in the following tables. A comparison in terms of zero demand percentage is provided in Table 5. The zero demand rates observed in the sequence of

TABLE 4. Summary statistics for the monthly intermittent demand data

	ADI	CV^2	Zero demand rate	Mean of non-zero demands
Mean	1.553	0.745	0.371	1.848
Stdev of mean	0.218	0.350	0.056	0.224
min. of means	1.095	0.163	0.235	1.379
max. of means	2.358	1.482	0.519	2.361

TABLE 5. Statistics for the mean zero demand rate

		Original Data		Estimated Data		
Lead Time	Num. of samples	mean zero demand rate	st. dev. of mean	mean zero demand rate	st. error of mean	0.95 Confidence Interval (CI)
1	94	0.3707	0.0658	0.3869	0.1004	(0.3666; 0.4072)
2	97	0.1462	0.0451	0.1620	0.0567	(0.1507; 0.1733)
3	94	0.0582	0.0416	0.0675	0.0408	(0.0593; 0.0758)
4	95	0.0197	0.0256	0.0296	0.0276	(0.0241; 0.0352)
5	94	0.0072	0.0170	0.0148	0.0171	(0.0114; 0.0183)
6	91	0.0023	0.0096	0.0055	0.0087	(0.0037; 0.0072)
7	80	0.0004	0.0026	0.0036	0.0047	(0.0026; 0.0046)
8	50	0.0010	0.0072	0.0014	0.0032	(0.0005; 0.0023)

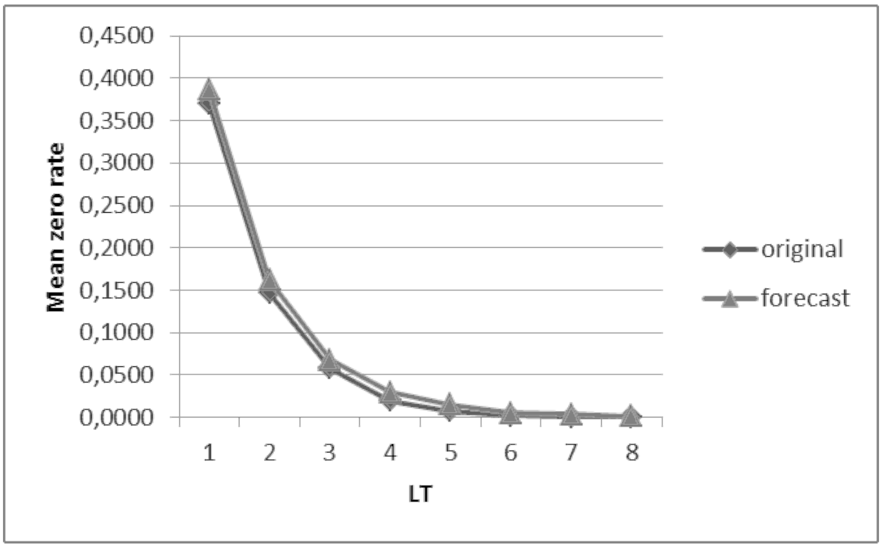

FIGURE 1. Mean zero demand rate vs lead time

demands for each lead time group was computed and the mean and standard deviation for this value were provided. As it is realized from Table 5, the mean zero demand rate for the real data is fairly close to those for the demand forecast data.

The consistence of mean zero demand rates between the real and the forecast data is seen on Figure 1, as well. Since the lead time demand values are compared, the frequency of the zero demands approaches zero as the lead time increases.

In Table 6, the comparison among the same data groups is this time given in terms of the means of non-zero demands. When the means of non-zero demands of the real data and the forecast data is compared, it is clearly perceived that these values are fairly close for each lead time group. This fact is also shown in Figure 2.

Table 6. Statistics for the mean non-zero demand

		Original Data		Estimated Data with MMCM		
Lead Time	Num. of samples	mean non -zero demand	st. dev. of mean	mean non -zero demand	st. error of mean	0.95 Confidence Interval (CI)
1	94	1.8845	0.2402	1.6926	0.2459	(1.6429; 1.7423)
2	97	2.7303	0.3912	2.4684	0.3330	(2.4021; 2.5347)
3	94	3.7076	0.4829	3.2902	0.4942	(3.1903; 3.3901)
4	95	4.7976	0.6096	4.2684	0.6211	(4.1435; 4.3933)
5	94	5.7501	0.8055	5.1371	0.9382	(4.9474; 5.3268)
6	91	6.8077	1.0658	6.1392	1.0756	(5.9182; 6.3602)
7	80	8.1962	1.1698	6.9911	1.1846	(6.7315; 7.2507)
8	50	9.3498	1.0580	8.2584	1.1918	(7.9280; 8.5887)

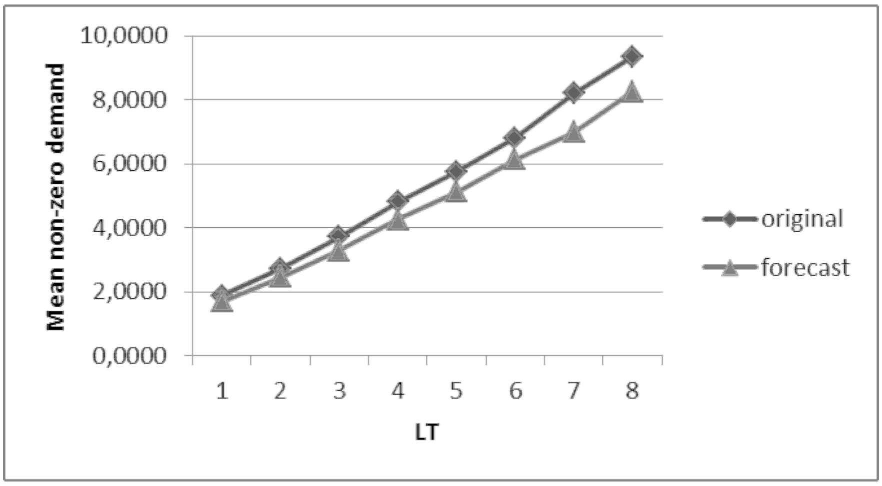

Figure 2. Mean non-zero demand vs lead time

Table 7. Statistics for accuracy measures (L = 1-8 months)

	r	MASE	MAPE
Mean	0.503	0.738	0.582
Standard error	0.038	0.109	0.153
0.95 CI for mean	$(0.501; 0.506)$	$(0.730; 0.746)$	$(0.571; 0.593)$

4.3. **Accuracy of demand forecasts.** To assess the accuracy of estimates, r and MASE values are calculated. Besides, for non-zero LTD estimates, MAPELTD is computed and presented in Table 7.

The mean r value, which measures the agreement of the forecasts obtained by means of the modified Markov chain model with the original data set, was found as 0.503. It was observed that, the highest and the lowest rate of the r value were found as 0.64 and 0.40, respectively. This indicates that the agreement of forecast values with the real data is high; namely 50.3 percent on the average. In parallel with this, the MASE value to be smaller than 1 demonstrates that the forecasts are reliable.

4.4. **Results and discussion: a comparison with other methods.** To make a comparison, forecast values were also obtained with the Markov bootstrap, SBA and Crostons method for the same 695 data sets. Tables 8 and 9 present the mean MASE and MAPELTD values, respectively. In each table, we can make a comparison among the

TABLE 8. A comparison of MASE values (L = 1-8 months)

	MMCM	SBA	Croston
Mean MASE	0.738	0.945	0.738
Standard error	0.109	0.468	0.073
0.95 CI for mean	(0.730; 0.746)	(0.911; 0.980)	(0.732; 0.743)

TABLE 9. A comparison of MAPELTD values (L = 1-8 months)

	MMCM	MB	SBA	Croston
Mean	0.582	0.613	0.788	0.839
Standard error	0.153	0.162	0.402	0.754
0.95 CI for mean	(0.571; 0.593)	(0.601; 0.625)	(0.758; 0.818)	(0.783; 0.895)

accuracy of the MMCM and other forecasting methods. MASE values are not computed for MB method since this method concerns with estimation of the lead time demand distribution rather than the single demand values.

5. **Conclusions.** We examine and propose the utilization of the higher order Markov chain model to forecast the intermittent demand data. A modification is also proposed for the procedure of obtaining the demand forecasts by Markov chain model. The modified Markov chain model suggests considering not only the state with the highest probability but also every state in proportion to their probabilities when forecasting the next state. The comparisons with other methods and the statistical results showed that the modified Markov chain model yielded valid and accurate forecasts regarding intermittent demand forecasting.

This work is motivated by a case study and the intermittent demand data for different products with different lead times were used. The lead time demand forecasts were obtained by the modified Markov chain model. The order and the parameters of the modified Markov chain model are determined as a result of the LP model. The first order Markov chain model ($\lambda_1 = 1, \lambda_2 = 0$), was used for some data sets, whereas the second order model ($0 < \lambda_1 < 1, 0 < \lambda_2 < 1$), was used for some of them. The forecast results and the validity of forecasts were presented comparatively with the other forecasting methods. For both one period forecasts and lead time demand forecasts, the modified Markov chain model gives accurate estimates.

All the procedures were performed by a computer code written in Matlab. The demand data were read from an excel file and after the computations final forecast results were also written to an excel file. Although there are many computations in the background, performing all the calculations by computer code provides with the computations completed in a short time.

As a result, although higher order Markov chain models are mostly used to model categorical data sequences, the characteristics of the intermittent data enable us to use this method to obtain accurate forecasts. For the intermittent demand structure the sequence of demand is critical as well as the percentage of zero values and the mean of non-zero demands. Modified Markov chain model makes possible to forecast both the sequence and the values of demand. The accuracy measure r, which measures the consistence, was computed as 0.50 on the average for the forecast values. In this case we can conclude that the forecast values reflect the real data pattern well and can be used for effective inventory management.

Another issue which makes using Markov chain model to forecast intermittent data critical is that, multivariate Markov chain model enables us to model interaction of different demand sequences. There are numerous examples in which the demands of different products are influenced by each other. It is the subject of a future research whether the multivariate Markov chain model is an alternative method that yields valid forecasts when the demands for a product is influenced by the demand for another product.

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REFERENCES

[1] E. A. Silver, Operations research in inventory management: A review and critique, *Operational Research*, vol.29, pp.628-645, 1981.
[2] T. R. Willemain, C. N. Smart and H. F. Schwarz, A new approach to forecasting intermittent demand for service parts inventories, *International Journal of Forecasting*, vol.20, pp.375-387, 2004.
[3] E. Ferrari, A. Pareshi, A. Regattieri and A. Persona, Statistical management and modeling for demand of spare parts, *Springer Handbook of Engineering Statistics*, vol.3, pp.905-929, 2006.
[4] E. Porras and R. Dekker, An inventory control system for spare parts at a refinery: An empirical comparison of different re-order point methods, *European Journal of Operational Research*, vol.184, pp.101-132, 2008.
[5] W. Ching, E. Fung and M. Ng, A higher order Markov chain models for categorical data sequences, *Journal of Operational Research Society*, vol.54, pp.291-298, 2008.
[6] W. Ching, W. Yuen and A. Loh, An inventory model with returns and lateral transshipments, *Journal of Operational Research Society*, vol.54, pp.636-641, 2004.
[7] A. C. Heyvaert and A. Hurt, Inventory management of slow-moving parts, *Operations Research*, vol.4, pp.572-580, 1956.
[8] G. Feeney and C. Sherbrooke, The (s-1,s) inventory policy under compound Poisson demand, *Management Science*, vol.12, pp.391-411, 1966.
[9] R. M. Hill, Continuous review, lost sales inventory models with Poisson demand, *European Journal of Operations Research Society*, vol.176, pp.956-963, 2007.
[10] T. Williams, Stock control with sporadic and slow-moving demand, *Journal of the Operational Research Society*, vol.35, pp.939-948, 1984.
[11] R. Adelson, Compound poisson distributions, *Operational Research Quarterly*, vol.17, pp.73-75, 1966.
[12] W. Dunsmuir and R. Snyder, Control of inventories with intermittent demand, *European Journal of Operational Research*, vol.40, pp.16-21, 1989.
[13] F. Janssen, R. Heuts and T. de Kok, On the (R,s,Q) inventory model when demand is modeled as a compound Bernoulli process, *European Journal of Operational Research*, vol.104, pp.423-436, 1998.
[14] C. Schultz, Forecasting and inventory control for sporadic demand under periodic review, *Journal of the Operational Research Society*, vol.38, pp.453-458, 1987.
[15] B. Sani and B. Kingsman, Selecting the best periodic inventory control and demand forecasting methods for low demand items, *Journal of the Operational Research Society*, vol.48, pp.700-713, 1997.
[16] R. H. Teunter, A. A. Syntetos and M. Z. Babai, Determining order-up-to levels under periodic review for compound binomial (intermittent) demand, *European Journal of Operational Research*, vol.203, pp.619-624, 2010.
[17] L. Strijbosch, R. Heuts and E. van der Schoot, A combined forecast-inventory control procedure for spare parts, *Journal of the Operational Research Society*, vol.51, pp.1184-1192, 2000.
[18] A. Syntetos and J. Boylan, The accuracy of intermittent demand estimates, *International Journal of Production Economics*, vol.71, pp.303-314, 2005.
[19] J. D. Croston, Forecasting and stock control for intermittent demands, *Operational Research Quarterly*, vol.23, pp.289-303, 1972.
[20] A. Syntetos and J. Boylan, On the bias of intermittent demand estimates, *International Journal of Production Economics*, vol.71, pp.457-466, 2001.
[21] A. A. Ghobbar and C. H. Friend, Evaluation of forecasting methods for intermittent parts demand in the field of aviation: A predictive model, *Computers and Operations Research*, vol.30, pp.2097-2114, 2003.

[22] A. Raftery, A model for higher order Markov chains, *Journal of Royal Statistical Society, Series B*, vol.47, pp.528-539, 1985.
[23] W. K. Ching and M. K. Ng, *Markov Chains: Models, Algorithms and Applications*, Springer, 2006.
[24] W. K. Ching, M. K. Ng and E. S. Fung, Higher order multivariate Markov chains and their applications, *Linear Algebra and Its Applications*, vol.428, pp.492-507, 2008.
[25] R. J. Hyndman, Another look at forecast accuracy metrics for intermittent demand, *The International Journal of Applied Forecasting*, vol.4, pp.43-46, 2006.
[26] Z. S. Hua, B. Zhang, J. Yang and D. S. Tan, A new approach of forecasting intermittent demand for spare parts inventories in the process industries, *Journal of the Operational Research Society*, vol.58, pp.52-61, 2007.

Appendix I. Estimation of the Demand Procedure Using Markov Chain Model.