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METHOD OF SUCCESSIVE APPROXIMATIONS FOR STATICALLY INDETERMINATE RIGID FRAMES INCLUDING A NEW VARIABLE

Arnulfo Luévanos Rojas[1], Nataliya I. Kalashnykova[2]
Alberto Diosdado Salazar[1], Ramón Luévanos Rojas[1]
and Facundo Cortés Martinez[1]

[1]Facultad de Ingeniería, Ciencias y Arquitectura
Universidad Juárez del Estado de Durango
Gómez Palacio, Durango, México
arnulfol_2007@hotmail.com

[2]Department of Physics and Mathematics
Universidad Autónoma de Nuevo León
San Nicolás de los Garza, Nuevo León, México

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Abstract. *This paper proposes a method of successive approximations for analysis of statically indeterminate rigid frames including a new variable; that variable is the shear deformations, which is an extension to the method of successive approximations which bears the name Hardy Cross method, which is used to analyze all kinds of structures in the plane. This methodology considers the deformations by flexure and shear, which is the innovative part of this research. The classical method takes into account only the deformations by flexure and without taking into account the deformations by shear; this is how it usually develops structural analysis of statically indeterminate rigid frames. It also makes a comparison between the proposed method and the classical method as can be seen in the results tables of the problems considered; in the classical method not all values are of the side of safety. Therefore, the usual practice, without considering the deformations by shear will not be a recommended solution. Then is proposed the use of method developed in this paper, because deformations by shear is taken into account and also is more attached to real conditions, since shear forces of any type of structure are present.*
Keywords: Deformations by shear, Poisson's ratio, Successive approximation method, Stiffness factor, Factor of carry-over, Distribution factor

1. **Introduction.** Hardy Cross (1885-1959) professor at the University of Illinois, in 1930 presented in his work, the analysis for continuous rigid frames, the method of successive approximations that bears his name, can be said that it revolutionized the structures analysis for continuous frames of reinforced concrete and can be considered one of the greatest contributions to the analysis of indeterminate structures. This method of successive approximations evades solving equations systems, as presented in the methods of Mohr and Maxwell. The general concepts of the method of successive approximations were extended subsequently in the study of pipes flow. Later became more popular the methods of Kani and Takabeya also of type iterative [1-6].

Method of Cross is a procedure designed to solve the problem of reticular structures. The calculation is relatively simple, without the appearance in develop complex integrations or complicated systems of equations. Moreover, once understood the mechanism of the method, operations are reduced to addition, subtraction, multiplication and division. Also, is not required remember anything of memory. If the tables of moments are available, stiffness factors and factors of carry-over can be solved any structure [1-6].

Method of Cross is a procedure of successive approximations, which does not mean that is approximates. It means that the precision degree in the calculation can be as elevated as the calculist want, between greater number of cycles have been made, the greater the accuracy [1-6].

The method permits to follow step by step process of moment distribution in the structure, giving a clear physical meaning to the mathematical operations that are performed [1-6].

Luévanos-Rojas developed a method of structural analysis for beams and rigid frames statically indeterminate, in this method takes into account the deformations by flexure and shear. That previously was considered only the deformations by flexure [7,8].

Structural analysis is the study of structures such as discrete systems [9-14]. The theory of the structures is essentially based on the fundamentals of mechanics with which are formulated the different structural elements. The laws or rules that define the balance and continuity of a structure can be expressed in different ways, including partial differential equations of continuous mediums three-dimensional, ordinary differential equations that define a member or the theories several of beams, or simply, algebraic equations for a discrete structure [15,16]. The more delves into the physics of problem, are being developed theories that are most appropriate for solving certain types of structures and that prove more useful for practical calculations. However, in each new theory are made hypotheses about how the system behaves or element. Therefore, we must always be aware of these hypotheses when evaluating results, fruit of the theories that apply or develop [1-8].

Structural analysis can be addressed using three main approaches: a) tensor formulations (Newtonian mechanics and vectorial), b) formulations based on the principles of virtual work, c) formulations based on classical mechanics [17-19].

As regards the conventional techniques of structural analysis for rigid frames using the method of successive approximations, the common practice is to consider only deformations by flexure (classic method) [1-8].

This paper proposes the successive approximation method considering the deformations by flexure and shear, for structural analysis of rigid frames. In this paper is presented developed mathematic for obtaining the stiffness factor, the carry-over factor and distribution factor. Also this paper shows a comparison between the classic method and the proposed method through a practical example to observe the differences.

2. **Development.**

2.1. **Theoretical principles.**

In the scheme of deformation of a structure member in Figure 1 is illustrated the difference between the Timoshenko theory and Euler-Bernoulli theory: the first "θ_Z" and "dy/dx" not necessarily coincides, while in the second are equal [7,8,20].

The fundamental difference between the Euler-Bernoulli theory and Timoshenko's theory is that in the first the relative rotation of the section is approximated by the derivative of vertical displacement, this is an approximation valid only for long members in relation to the dimensions of cross section, and then it happens that due to shear deformations are negligible compared to the deformations caused by moment. On the Timoshenko theory, which considers the deformation due to the shear, i.e., and is valid therefore for short members and longs, the equation of the elastic curve is given by the equations system

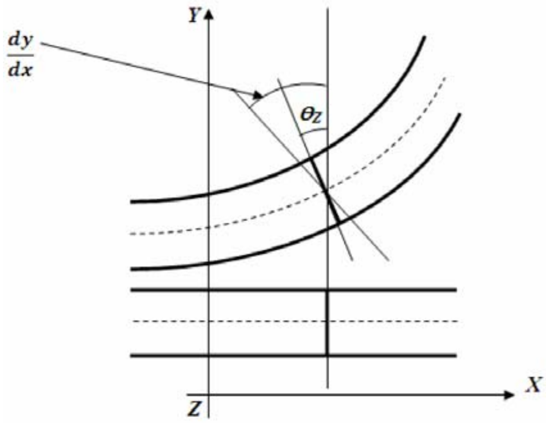

FIGURE 1. Deformation of a structure member

most complex:

$$G\left(\frac{dy}{dx} - \theta_Z\right) = \frac{V_y}{A_s} \quad (1)$$

$$E\left(\frac{d\theta_Z}{dx}\right) = \frac{M_z}{I_z} \quad (2)$$

where: G = shear modulus, dy/dx = total rotation around axis "Z", θ_Z = rotation around axis "Z", due to the flexure, V_y = shear force in direction "Y", A_s = shear area, $d\theta_Z/dx = d^2y/dx^2$, E = elasticity modulus, M_z = moment around axis "Z", I_z = moment of inertia around axis "Z".

Deriving Equation (1) and substituting into Equation (2), it is arrived at the equation of the elastic curve including the effect of shear stress:

$$\frac{d^2y}{dx^2} = \frac{1}{GA_s}\frac{dV_y}{dx} + \frac{M_z}{EI_z} \quad (3)$$

From Equation (1) is obtained dy/dx:

$$\frac{dy}{dx} = \frac{V_y}{GA_s} + \theta_Z \quad (4)$$

And of Equation (2) is given θ_Z:

$$\theta_Z = \int \frac{M_z}{EI_z} dx \quad (5)$$

Now substituting Equation (5) into Equation (4) is:

$$\frac{dy}{dx} = \frac{V_y}{GA_s} + \int \frac{M_z}{EI_z} dx \quad (6)$$

2.2. **General description of the proposed method.** The method of successive approximations can be used to analyze all types of beams or rigid frames statically indeterminate.

2.2.1. *Rigid frames without sidesway in the joints.* The application of the moment-distribution method to the analysis of statically indeterminate frames wherein no joint movements or "sidesway" is involved is very similar to that of beams, except that in the case of frames there are frequently more than two members meeting in one joint. In such cases the unbalance at any joint is distributed to the ends of the several members meeting at the joint in the ratio of their relative stiffnesses. There are several ways in which the work

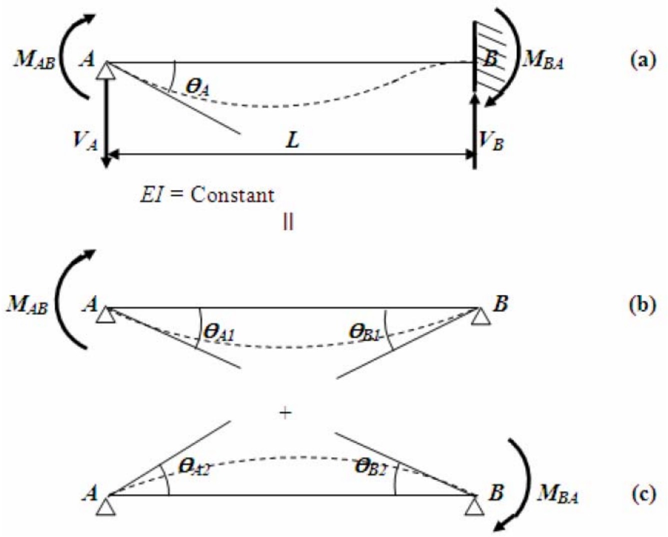

FIGURE 2. Derivation of moment-distribution equations

for the moment distribution may be arranged, but the tabular form in which all members meeting at the same joint are grouped together is used here and is suggested as the most convenient form.

In order to develop the method, it will be helpful to consider the following problem: If a clockwise moment of M_{AB} is applied at the simple support of a straight member of constant cross section simply supported at one end and fixed at the other, find the rotation θ_A at the simple support and the moment M_{BA} at the fixed end, as shown in Figure 2.

The additional end moments, M_{AB} and M_{BA}, should be such as to cause rotations of θ_A and θ_B, respectively. If θ_{A1} and θ_{B1} are the end rotations caused by M_{AB}, according to Figure 2(b), and θ_{A2} and θ_{B2} by M_{BA}, they are observed in Figure 2(c).

The conditions of geometry required are [7,8,21-24]:

$$\theta_A = \theta_{A1} - \theta_{A2} \tag{7}$$

$$0 = \theta_{B1} - \theta_{B2} \tag{8}$$

The beam of Figure 2(b) is analyzed to find θ_{A1} and θ_{B1} in function of M_{AB}:

It is considered that $V_A = V_B$, doing the sum of moments in B and value of M_{AB} in function of V_A is obtained:

$$M_{AB} = V_A L \tag{9}$$

Therefore, the shear forces and moments at a distance "x" are:

$$V_x = \frac{M_{AB}}{L} \tag{10}$$

$$M_x = \frac{M_{AB}}{L}(L - x) \tag{11}$$

where: V_x is shear force at a distance "x" and M_x is the moment at a distance "x".

Substituting M_x and V_x in function of M_{AB} into Equation (6) and is separated the deformation by shear and flexure to obtain the stiffness, it is presented as follows:

- Deformation by shear:

$$\frac{dy}{dx} = \frac{M_{AB}}{GA_s L} \tag{12}$$

Integrating Equation (12) is presented as follows:

$$y = \frac{M_{AB}}{GA_sL}x + C_1 \tag{13}$$

Considering the conditions of border, when $x = 0$; $y = 0$; is presented of the following way $C_1 = 0$, and is substituted into Equation (13):

$$y = \frac{M_{AB}}{GA_sL}x \tag{14}$$

- Deformation by flexure:

$$\frac{dy}{dx} = \frac{M_{AB}}{EI_zL}\int (L - x)dx \tag{15}$$

Developing the integral, it is obtained:

$$\frac{dy}{dx} = \frac{M_{AB}}{EI_zL}\left(Lx - \frac{x^2}{2} + C_2\right) \tag{16}$$

Integrating Equation (16), it is shown:

$$y = \frac{M_{AB}}{EI_zL}\left(\frac{L}{2}x^2 - \frac{x^3}{6} + C_2x + C_3\right) \tag{17}$$

Considering the conditions of border, when $x = 0$; $y = 0$; is presented of the following way $C_3 = 0$, and substituted into Equation (17):

$$y = \frac{M_{AB}}{EI_zL}\left(\frac{L}{2}x^2 - \frac{x^3}{6} + C_2x\right) \tag{18}$$

Now considering the conditions of border, when $x = L$ and $y = 0$, is presented $C_2 = -L^2/3$, and substituting into Equation (16) and (18) shown below:

$$\frac{dy}{dx} = \frac{M_{AB}}{EI_zL}\left(Lx - \frac{x^2}{2} - \frac{L^2}{3}\right) \tag{19}$$

$$y = \frac{M_{AB}}{EI_zL}\left(\frac{L}{2}x^2 - \frac{x^3}{6} - \frac{L^2}{3}x\right) \tag{20}$$

Substituting $x = 0$ into Equation (19) to find the rotation in support A due to the deformation by flexure θ_{A2F}, it is as follows:

$$\theta_{A1F} = -\frac{M_{AB}L}{3EI_z} \tag{21}$$

Substituting $x = L$ into Equation (19) to find the rotation in support B due to the deformation by flexure θ_{B2F}, it is obtained as follows:

$$\theta_{B1F} = \frac{M_{AB}L}{6EI_z} \tag{22}$$

If it is considered that they have his curvature radius in the inferior part. Then, the rotations are positive:

$$\theta_{A1F} = +\frac{M_{AB}L}{3EI_z} \tag{23}$$

$$\theta_{B1F} = +\frac{M_{AB}L}{6EI_z} \tag{24}$$

The rotation due to deformation by shear θ_{A1S} and θ_{B1S}, taking into account the curvature radius is:

$$\theta_{A1S} = \frac{dy}{dx} = \frac{M_{AB}}{GA_sL} \tag{25}$$

$$\theta_{B1S} = \frac{dy}{dx} = -\frac{M_{AB}}{GA_sL} \tag{26}$$

Summing the rotation by flexure and shear in the joint A, it is obtained:

$$\theta_{A1} = \theta_{A1F} + \theta_{A1S} \tag{27}$$

Substituting Equations (23) and (25) into Equation (27), it is as follows:

$$\theta_{A1} = +\frac{M_{AB}L}{3EI_z} + \frac{M_{AB}}{GA_sL} \tag{28}$$

The common factor is obtained of Equation (28) for M_{AB}, it is as follows:

$$\theta_{A1} = \frac{M_{AB}L}{12EI_z}\left(4 + \frac{12EI_z}{GA_sL^2}\right) \tag{29}$$

By replacing [7,8,18,19,25]:

$$\varnothing = \frac{12EI_z}{GA_sL^2} \tag{30}$$

where $\varnothing$ = form factor.

It is obtained "G" as follows:

$$G = \frac{E}{2(1+\nu)} \tag{31}$$

where ν = Poisson's ratio.

Then, substituting Equation (30) into Equation (29), it is obtained:

$$\theta_{A1} = \frac{M_{AB}L}{12EI_z}(4+\varnothing) \tag{32}$$

Summing the rotation by flexure and shear in the joint B, and making the simplifications corresponding, it is presented:

$$\theta_{B1} = \frac{M_{AB}L}{12EI_z}(2-\varnothing) \tag{33}$$

Analyzing the beam in Figure 2(c) to find θ_{A2} and θ_{B2} in function of M_{BA}, it is making of the same way as was done in Figure 2(b), it is obtained the following:

$$\theta_{A2} = \frac{M_{BA}L}{12EI_z}(2-\varnothing) \tag{34}$$

$$\theta_{B2} = \frac{M_{BA}L}{12EI_z}(4+\varnothing) \tag{35}$$

Now, substituting Equations (33) and (35) into Equation (8), it is as follows:

$$0 = \frac{M_{AB}L}{12EI_z}(2-\varnothing) - \frac{M_{BA}L}{12EI_z}(4+\varnothing) \tag{36}$$

Find M_{BA} in function of M_{AB}:

$$M_{BA} = \left(\frac{2-\varnothing}{4+\varnothing}\right)M_{AB} \tag{37}$$

Also, substituting Equations (32) and (34) into Equation (7), it is as follows:

$$\theta_A = \frac{M_{AB}L}{12EI_z}(4+\varnothing) - \frac{M_{BA}L}{12EI_z}(2-\varnothing) \tag{38}$$

Substituting Equation (37) into Equation (38), it is obtained:

$$\theta_A = \frac{M_{AB}L}{12EI_z}(4+\varnothing) - \left\{\left(\frac{2-\varnothing}{4+\varnothing}\right)M_{AB}\right\}\frac{L(2-\varnothing)}{12EI_z} \tag{39}$$

Find M_{AB} in function of θ_A:

$$M_{AB} = \left(\frac{4+\varnothing}{1+\varnothing}\right)\frac{EI_z}{L}\theta_A \tag{40}$$

Thus, for a span AB which is simply supported at A and fixed at B, a clockwise rotation of θ_A can be effected by applying a clockwise moment of $M_{AB} = [(4+\varnothing)/(1+\varnothing)](EI_z/L)\theta_A$ at A, which in turn induces a clockwise moment of $M_{BA} = [(2-\varnothing)/(4+\varnothing)]M_{AB}$ on the member at B. The expression, $[(4+\varnothing)/(1+\varnothing)](EI_z/L)$ is usually called the stiffness factor, which is defined as the moment required to be applied at A to cause a rotation of 1 rad at A of a span AB simply supported at A and fixed at B; the number $[(2-\varnothing)/(4+\varnothing)]$ is the carry-over factor, which is the radio of the moment induced at B to the moment applied at A.

2.2.2. *The fixed-end moments due to sidesway in the joints.* In order to obtain the fixed-end moments due to sidesway in the joints, it will be helpful to consider the following problem, as shown in Figure 3.

Taking into account the member of Figure 3 and supposing that $M''_{FAB} = M''_{FBA}$ and, $V_A = V_B$, doing sum of moments in the point B and is obtained M''_{FAB} in function of V_A:

$$M''_{FAB} = \frac{V_A L}{2} \tag{41}$$

Therefore, the shear forces and moments at a distance "x" are:

$$V_x = V_A \tag{42}$$

$$M_x = V_A\left(\frac{L}{2} - x\right) \tag{43}$$

Substituting M_x and V_x in function of V_A into Equation (6), and separating the shear deformation and flexure to obtain the stiffness due to the displacement, it is presented as follows:

- Deformation by shear:

$$\frac{dy}{dx} = \frac{V_A}{GA_s} \tag{44}$$

Integrating Equation (44) is obtained as follows:

$$y = \frac{V_A}{GA_s}x + C_1 \tag{45}$$

Considering the conditions of border, when $x = 0$; $y = 0$; then $C_1 = 0$.

$$y = \frac{V_A}{GA_s}x \tag{46}$$

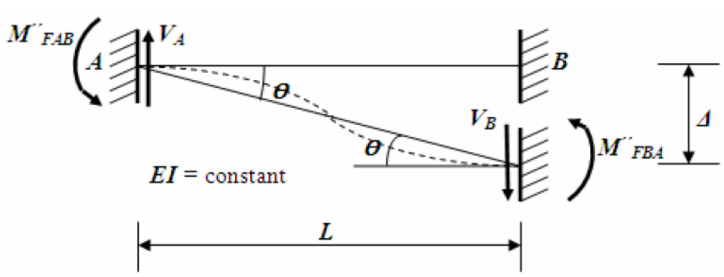

FIGURE 3. The fixed-end moments due to sidesway in the joints

- Deformation by flexure:

$$\frac{dy}{dx} = \frac{V_A}{EI_z} \int \left(\frac{L}{2} - x \right) dx \tag{47}$$

Developing the integral of Equation (47), it is the following expression:

$$\frac{dy}{dx} = \frac{V_A}{EI_z} \left(\frac{L}{2}x - \frac{x^2}{2} + C_2 \right) \tag{48}$$

Taking into account the conditions of border, when $x = 0$; $dy/dx = 0$; it is obtained that $C_2 = 0$.

$$\frac{dy}{dx} = \frac{V_A}{EI_z} \left(\frac{L}{2}x - \frac{x^2}{2} \right) \tag{49}$$

Integrating Equation (49) is presented the following:

$$y = \frac{V_A}{EI_z} \left(\frac{L}{4}x^2 - \frac{x^3}{6} + C_3 \right) \tag{50}$$

Taking into account the conditions of border, when $x = 0$; $y = 0$; it is obtained that $C_3 = 0$.

$$y = \frac{V_A}{EI_z} \left(\frac{L}{4}x^2 - \frac{x^3}{6} \right) \tag{51}$$

It is developed the sum of Equation (46) due to deformation by shear and Equation (51) due to deformation by flexure and presents as follows:

$$y = \frac{V_A}{GA_s}x + \frac{V_A}{EI_z} \left(\frac{L}{4}x^2 - \frac{x^3}{6} \right) \tag{52}$$

Substituting $x = L$; $y = \Delta$, into Equation (52) for to find the displacement in the B support, it is as follows:

$$\Delta = \frac{V_A L^3}{12EI_z} \left(\frac{12EI_z}{GA_s L^2} + 1 \right) \tag{53}$$

It is replaced Equation (30) into Equation (53) and is obtained value of V_A of the form follows:

$$V_A = \frac{12EI_z}{L^3 (\varnothing + 1)} \Delta \tag{54}$$

Substituting Equation (54) into Equation (41) is presented the following way:

$$M''_{FAB} = \frac{6EI_z}{L^2 (\varnothing + 1)} \Delta \tag{55}$$

Then the equations of fixed-end moments due to sidesway in the joints $M''_{FAB} = M''_{FBA}$ is shown as follows:

$$M''_{FAB} = \frac{6EI_z}{L^2 (\varnothing + 1)} \Delta \tag{56}$$

$$M''_{FBA} = \frac{6EI_z}{L^2 (\varnothing + 1)} \Delta \tag{57}$$

2.2.3. *Rigid frames with sidesway in the joints.* The application of the moment-distribution method to the analysis of statically indeterminate frames in which sidesway or joints movements are involved consists in the following:

1. The joints are first held against sidesway. The fixed-end moments as caused by the applied loading are distributed, and a first set of balanced end moments are obtained.

2. The unloaded frame is assumed to have a certain amount of sidesway which will cause a set of fixed-end moments. These fixed-end moments are then distributed, and a second set of balanced end moments are obtained.

3. The resulting set of end moments can be obtained by adding the first set and the product of a ratio and the second set, the ratio being determined by use of shear conditions, as will be explained.

Take, for example, the rigid frame shown in Figure 4. It is required to analyze this statically indeterminate frame by the moment-distribution method. The given frame shown in Figure 4(a) is equivalent to the sum of Figure 4(b) and Figure 4(c). In Figure 4(b) the joints B and C are held against sidesway by the fictitious support at C, the horizontal reaction of which is denoted as H'_C. If the fictitious support at C is removed,

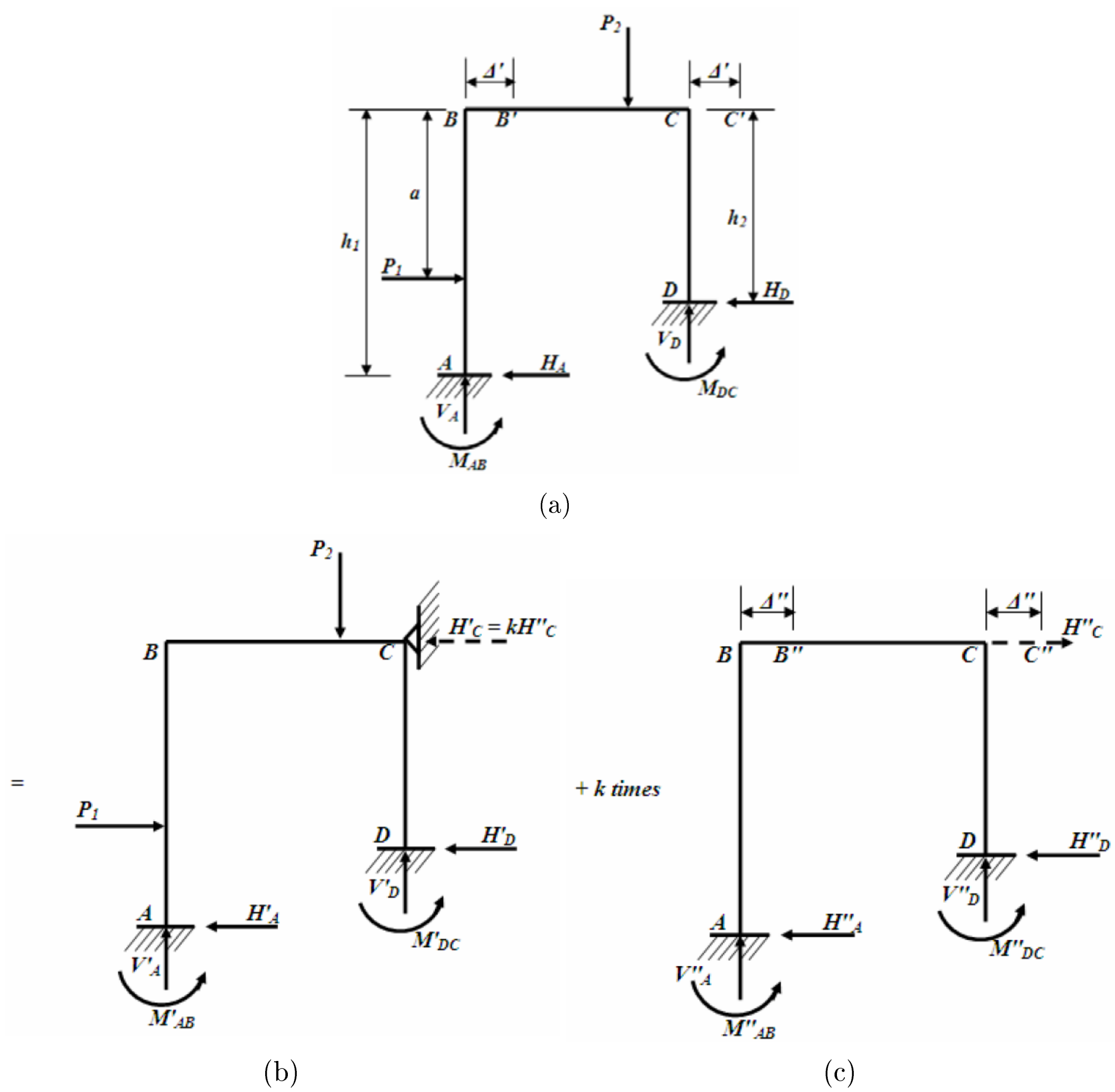

FIGURE 4. The rigid frame. (a) The rigid frame is shown, (b) the joints are held against sidesway, (c) the unloaded frame with sidesway.

the force H'_C would act at joint C. In Figure 4(c), Δ'' is the sidesway caused by any arbitrary force H''_C. If H'_C is equal to kH''_C, where k is the unknown ratio, the actual amount of the sidesway, Δ', must be equal to $k\Delta''$. Let M'_{AB}, M'_{BA}, M'_{BC}, M'_{CB}, M'_{CD}, and M'_{DC} be the balanced moments obtained by distributing the fixed-end moments due to the applied loading while only permitting joints B and C to rotate but holding their locations in Figure 4(b). Let M''_{AB}, M''_{BA}, M''_{BC}, M''_{CB}, M''_{CD}, and M''_{DC} be the balanced moments obtained by distributing the fixed-end moments due to any assumed amount Δ'' of the horizontal movement of joint B or C.

The shear condition required of the frame shown in Figure 4(a) is:

$$H_A + H_D = P_1 \tag{58}$$

since

$$H_A = \frac{M_{AB} + M_{BA}}{h_1} + \frac{P_1 a}{h_1} \tag{59}$$

and

$$H_D = \frac{M_{CD} + M_{DC}}{h_2} \tag{60}$$

The shear condition becomes:

$$\frac{M_{AB} + M_{BA}}{h_1} + \frac{P_1 a}{h_1} + \frac{M_{CD} + M_{DC}}{h_2} = P_1 \tag{61}$$

Also, by superposition,

$$\begin{aligned} M_{AB} &= M'_{AB} + k(M''_{AB}); \quad & M_{BA} &= M'_{BA} + k(M''_{BA}) \\ M_{BC} &= M'_{BC} + k(M''_{BC}); \quad & M_{CB} &= M'_{CB} + k(M''_{CB}) \\ M_{CD} &= M'_{CD} + k(M''_{CD}); \quad & M_{DC} &= M'_{DC} + k(M''_{DC}) \end{aligned} \tag{62}$$

By substituting Equation (62) into Equation (61),

$$\frac{(M'_{AB} + M'_{BA}) + k(M''_{AB} + M''_{BA})}{h_1} + \frac{P_1 a}{h_1} + \frac{(M'_{CD} + M'_{DC}) + k(M''_{CD} + M''_{DC})}{h_2} = P_1 \tag{63}$$

The unknown ratio k can then be found by solving Equation (63). Once k is known, all end moments acting on the frame of Figure 4(a) can be found from Equation (62).

Where two or more unknown movements of sidesway are involved, the resulting set of end moments can be expressed as the sum of (1) the balanced end moments by distributing the fixed-end moments due to the applied loading, and (2) the products of an unknown ratio and the balanced end moments found by distributing the fixed-end moments due to a certain amount of the first movement in sidesway, and (3) the products of a second unknown ratio and the balanced end moments due to a certain amount of the second movement in sidesway, and so on. The unknown ratios are determined from the shear conditions.

3. **Application.** It developed the structural analysis of a steel rigid frame, in three different problems, as shown in Figure 5, by the classic method and the proposed method, i.e., without taking into account and to consider the shear deformations, on the basis of the followings data that appear next:

$L = 10.00\text{m}; 5.00\text{m}; 3.00\text{m}$
$P = 49.05\text{kN}$
$h = 5.00\text{m}$
$E = 20019.6\text{kN/cm}^2$
Properties of the beam $W24X94$
$A = 178.71\text{cm}^2$

$A_s = 80.83\text{cm}^2$
$I_z = 111966\text{cm}^4$
Properties of the column $W24X61$
$A = 116.13\text{cm}^2$
$A_s = 64.06\text{cm}^2$
$I_z = 64100\text{cm}^4$
$\nu = 0.32$

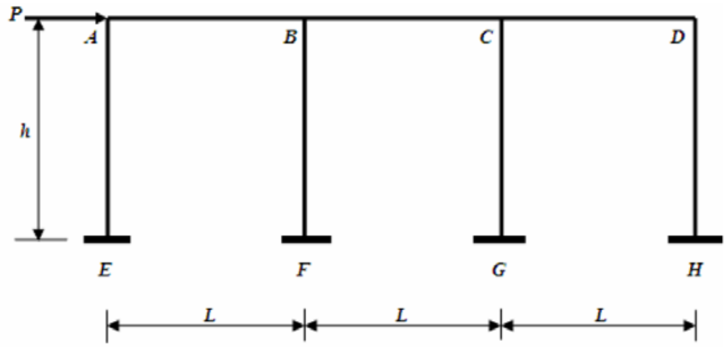

FIGURE 5. Rigid frame of steel of three lengths equal for beams, with a discrete load in joint A

Using Equation (31), it is obtained the shear modulus, as follows:

$$G = 7583.182\text{kN/cm}^2$$

Once that is obtained the shear modulus is found the form factor through Equation (30) as follows:

For beam of 10.00m is:

$$\varnothing_{AB} = \varnothing_{BC} = \varnothing_{CD} = 0.04388324731$$

For beam of 5.00m is:

$$\varnothing_{AB} = \varnothing_{BC} = \varnothing_{CD} = 0.1755329892$$

For beam of 3.00m is:

$$\varnothing_{AB} = \varnothing_{BC} = \varnothing_{CD} = 0.4875916368$$

For column of 5.00m is:

$$\varnothing_{AE} = \varnothing_{BF} = \varnothing_{CG} = \varnothing_{DH} = 0.1267991258$$

The fixed-end moments for members due to loads in all the cases are:

$$M_{FAB} = M_{FBC} = M_{FCD} = M_{FBA} = M_{FCB} = M_{FDC} = 0$$

$$M_{FAE} = M_{FEA} = M_{FBF} = M_{FFB} = M_{FCG} = M_{FGC} = M_{FDH} = M_{FHD} = 0$$

Evaluation of "EI" is:

For all beams is:

$$EI = 2241514534\text{kN-cm}^2 = 224151.4534\text{kN-m}^2$$

For all columns is:

$$EI = 1283256360\text{kN-cm}^2 = 128325.636\text{kN-m}^2$$

The stiffness for each one of the members by the proposed method is:

$$K = \left(\frac{4 + \varnothing}{1 + \varnothing}\right) \frac{EI}{L}$$

For beam of 10.00m is:

$$K_{AB} = K_{BC} = K_{CD} = 86833.68658\text{kN-m}$$

For beam of 5.00m is:

$$K_{AB} = K_{BC} = K_{CD} = 159238.7108\text{kN-m}$$

For beam of 3.00m is:

$$K_{AB} = K_{BC} = K_{CD} = 225397.9212\text{kN-m}$$

For column of 5.00m is:

$$K_{AE} = K_{BF} = K_{CG} = K_{DH} = 93996.1898\text{kN-m}$$

The stiffness for each one of the members by the classical method is:

$$K = \frac{4EI}{L}$$

For beam of 10.00m is:

$$K_{AB} = K_{BC} = K_{CD} = 89660.58136\text{kN-m}$$

For beam of 5.00m is:

$$K_{AB} = K_{BC} = K_{CD} = 179321.1627\text{kN-m}$$

For beam of 3.00m is:

$$K_{AB} = K_{BC} = K_{CD} = 298868.6045\text{kN-m}$$

For column of 5.00m is:

$$K_{AE} = K_{BF} = K_{CG} = K_{DH} = 102660.5088\text{kN-m}$$

The distribution factor (FD) for both methods are obtained:

$$FD_{ij} = \frac{K_{ij}}{\sum K_i}$$

The distribution factors are presented in Table 1, for the three different cases.

TABLE 1. The distribution factors in each one of the members

Member	Case 1 L = 10.00m		Case 2 L = 5.00m		Case 3 L = 3.00m	
	PM	CM	PM	CM	PM	CM
AB = DC	0.48020	0.46620	0.62882	0.63593	0.70570	0.74433
AE = DH	0.51980	0.53380	0.37118	0.36407	0.29430	0.25567
BA = BC = CD = CB	0.32441	0.31797	0.38606	0.38873	0.41373	0.42671
BF = CG	0.35117	0.36407	0.22788	0.22254	0.17254	0.14657
EA = FB = GC = HD	0	0	0	0	0	0

PM = proposed method
CM = classical method

The carry-over factor (FC) (proposed method) is:

$$M_{ji} = \left(\frac{2 - \varnothing}{4 + \varnothing}\right) M_{ij}$$

For beam of 10.00m is:

$$M_{ji} = 0.48372 M_{ij}$$

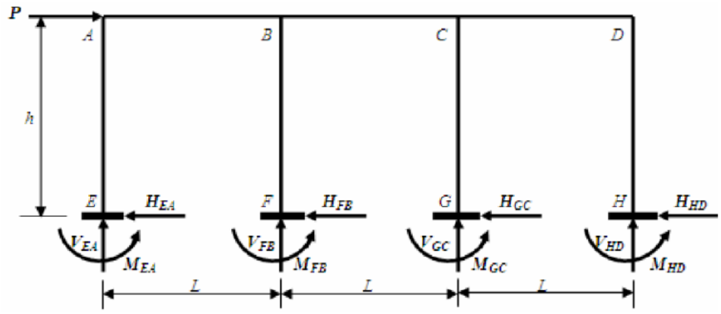

FIGURE 6. Free body diagram of whole frame

For beam of 5.00m is:

$$M_{ji} = 0.43694 M_{ij}$$

For beam of 3.00m is:

$$M_{ji} = 0.33702 M_{ij}$$

For column of 5.00m is:

$$M_{ji} = 0.45391 M_{ij}$$

The carry-over factor (FC) (classical method) for all the members is:

$$M_{ji} = 0.5 M_{ij}$$

The fixed-end moments due to sidesway are:
For proposed method is:

$$M''_{FAE} = M''_{FEA} = M''_{FBF} = M''_{FFB} = M''_{FCG} = M''_{FGC} = M''_{FDH} = M''_{FHD}$$
$$= \frac{6EI_z}{L^2(\varnothing + 1)}\Delta = \frac{6(128325.636)}{(5)^2(0.1267991258 + 1)}\Delta = 27332.42504\Delta$$

For classical method is:

$$M''_{FAE} = M''_{FEA} = M''_{FBF} = M''_{FFB} = M''_{FCG} = M''_{FGC} = M''_{FDH} = M''_{FHD}$$
$$= \frac{6EI_z}{L^2}\Delta = \frac{6(128325.636)}{(5)^2}\Delta = 30798.15264\Delta$$

The distribution of moments due to the application of the loads, because there are no transverse loads, the moments are zero. Only is considered the distribution of moments due to sidesway.

Below are presented in Tables 2, 3, 4, 5, 6 and 7, the results by the method of successive approximations.

Applying Equation (62), for the problem will be:

$$M_{AB} = M'_{AB} + k(M''_{AB}); \quad M_{BA} = M'_{BA} + k(M''_{BA})$$
$$M_{BC} = M'_{BC} + k(M''_{BC}); \quad M_{CB} = M'_{CB} + k(M''_{CB})$$
$$M_{CD} = M'_{CD} + k(M''_{CD}); \quad M_{DC} = M'_{DC} + k(M''_{DC})$$
$$M_{AE} = M'_{AE} + k(M''_{AE}); \quad M_{EA} = M'_{EA} + k(M''_{EA})$$
$$M_{BF} = M'_{BF} + k(M''_{BF}); \quad M_{FB} = M'_{FB} + k(M''_{FB})$$
$$M_{CG} = M'_{CG} + k(M''_{CG}); \quad M_{GC} = M'_{GC} + k(M''_{GC})$$
$$M_{DH} = M'_{DH} + k(M''_{DH}); \quad M_{HD} = M'_{HD} + k(M''_{HD})$$

The equilibrium condition is generated for shear forces at the base of frame, as shown in Figure 6, which is:

$$P - H_{EA} - H_{FB} - H_{GC} - H_{HD} = 0$$

TABLE 2. Classical method for L = 10.00m

Joint		A		B			C			D		E	F	G	H
Member		AB	AE	BA	BC	BF	CB	CD	CG	DC	DH	EA	FB	GC	HD
Stiffness		89661	102661	89661	89661	102661	89661	89661	102661	89661	102661	102661	102661	102661	102661
Distribution Factor		0.46620	0.53380	0.31797	0.31797	0.36407	0.31797	0.31797	0.36407	0.46620	0.53380	0	0	0	0
Carry-Over Factor		0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000
Cycle 1	FEM	0	+30798	0	0	+30798	0	0	+30798	0	+30798	+30798	+30798	+30798	+30798
	Bal.	−14358	−16440	−9793	−9793	−11213	−9793	−9793	−11213	−14358	−16440	0	0	0	0
Cycle 2	CO	−4896	0	−7179	−4896	0	−4896	−7179	0	−4896	0	−8220	−5606	−5606	−8220
	Bal.	+2283	+2613	+3839	+3839	+4396	+3839	+3839	+4396	+2283	+2613	0	0	0	0
Cycle 3	CO	+1920	0	+1142	+1920	0	+1920	+1142	0	+1920	0	+1306	+2198	+2198	+1306
	Bal.	−895	−1025	−974	−974	−1115	−974	−974	−1115	−895	−1025	0	0	0	0
Cycle 4	CO	−487	0	−448	−487	0	−487	−448	0	−487	0	−512	−558	−558	−512
	Bal.	+227	+260	+297	+297	+340	+297	+297	+340	+227	+260	0	0	0	0
Cycle 5	CO	+148	0	+114	+148	0	+148	+114	0	+148	0	+130	+170	+170	+130
	Bal.	−69	−79	−83	−83	−95	−83	−83	−95	−69	−79	0	0	0	0
Cycle 6	CO	−42	0	−34	−42	0	−42	−34	0	−42	0	−40	−48	−48	−40
	Bal.	+20	+22	+24	+24	+28	+24	+24	+28	+20	+22	0	0	0	0
Cycle 7	CO	+12	0	+10	+12	0	+12	+10	0	+12	0	+11	+14	+14	+11
	Bal.	−6	−6	−7	−7	−8	−7	−7	−8	−6	−6	0	0	0	0
Total Moments M''_{ij}		−16143	+16143	−13092	−10042	+23131	−10042	−13092	+23131	−16143	+16143	+23473	+26968	+26968	+23473

TABLE 3. Proposed method for L = 10.00m

Joint		A		B			C			D		E	F	G	H
Member		AB	AE	BA	BC	BF	CB	CD	CG	DC	DH	EA	FB	GC	HD
Stiffness		86834	93996	86834	86834	93996	86834	86834	93996	86834	93996	93996	93996	93996	93996
Distribution Factor		0.48020	0.51980	0.32441	0.32441	0.35117	0.32441	0.32441	0.35117	0.48020	0.51980	0	0	0	0
Carry-Over Factor		0.48372	0.45391	0.48372	0.48372	0.45391	0.48372	0.48372	0.45391	0.48372	0.45391	0.45391	0.45391	0.45391	0.45391
Cycle 1	FEM	0	+27332	0	0	+27332	0	0	+27332	0	+27332	+27332	+27332	+27332	+27332
	Bal.	−13125	−14207	−8867	−8867	−9598	−8867	−8867	−9598	−13125	−14207	0	0	0	0
Cycle 2	CO	−4289	0	−6349	−4289	0	−4289	−6349	0	−4289	0	−6449	−4357	−4357	−6449
	Bal.	+2060	+2229	+3451	+3451	+3736	+3451	+3451	+3736	+2060	+2229	0	0	0	0
Cycle 3	CO	+1669	0	+996	+1669	0	+1669	+996	0	+1669	0	+1012	+1696	+1696	+1012
	Bal.	−801	−868	−865	−865	−936	−865	−865	−936	−801	−868	0	0	0	0
Cycle 4	CO	−418	0	−387	−418	0	−418	−387	0	−418	0	−394	−425	−425	−394
	Bal.	+201	+217	+261	+261	+283	+261	+261	+283	+201	+217	0	0	0	0
Cycle 5	CO	+126	0	+97	+126	0	+126	+97	0	+126	0	+98	+128	+128	+98
	Bal.	−61	−65	−72	−72	−78	−72	−72	−78	−61	−65	0	0	0	0
Cycle 6	CO	−35	0	−30	−35	0	−35	−30	0	−35	0	−30	−48	−48	−30
	Bal.	+17	+18	+21	+21	+23	+21	+21	+23	+17	+18	0	0	0	0
Cycle 7	CO	+10	0	+8	+10	0	+10	+8	0	+10	0	+8	+10	+10	+8
	Bal.	−5	−5	−6	−6	−6	−6	−6	−6	−5	−5	0	0	0	0
Total Moments M''_{ij}		−14651	+14651	−11742	−9014	+20756	−9014	−11742	+20756	−14651	+14651	+21577	+24336	+24336	+21577

TABLE 4. Classical method for L = 5.00m

Joint		A		B			C			D		E	F	G	H
Member		AB	AE	BA	BC	BF	CB	CD	CG	DC	DH	EA	FB	GC	HD
Stiffness		179321	102661	179321	179321	102661	179321	179321	102661	179321	102661	102661	102661	102661	102661
Distribution Factor		0.63593	0.36407	0.38873	0.38873	0.22254	0.38873	0.38873	0.22254	0.63593	0.36407	0	0	0	0
Carry-Over Factor		0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000
Cycle 1	FEM	0	+30798	0	0	+30798	0	0	+30798	0	+30798	+30798	+30798	+30798	+30798
	Bal.	−19585	−11213	−11972	−11972	−6854	−11972	−11972	−6854	−19585	−11213	0	0	0	0
Cycle 2	CO	−5986	0	−9792	−5986	0	−5986	−9792	0	−5986	0	−5606	−3427	−3427	−5606
	Bal.	+3807	+2179	+6133	+6133	+3511	+6133	+6133	+3511	+3807	+2179	0	0	0	0
Cycle 3	CO	+3066	0	+1904	+3066	0	+3066	+1904	0	+3066	0	+1090	+1756	+1756	+1090
	Bal.	−1950	−1116	−1932	−1932	−1106	−1932	−1932	−1106	−1950	−1116	0	0	0	0
Cycle 4	CO	−966	0	−975	−966	0	−966	−975	0	−966	0	−558	−553	−553	−558
	Bal.	+614	+352	+755	+755	+432	+755	+755	+432	+614	+352	0	0	0	0
Cycle 5	CO	+378	0	+307	+378	0	+378	+307	0	+378	0	+176	+216	+216	+176
	Bal.	−240	−138	−266	−266	−152	−266	−266	−152	−240	−138	0	0	0	0
Cycle 6	CO	−133	0	−120	−133	0	−133	−120	0	−133	0	−69	−76	−76	−69
	Bal.	+85	+48	+98	+98	+56	+98	+98	+56	+85	+48	0	0	0	0
Cycle 7	CO	+49	0	+42	+49	0	+49	+42	0	+49	0	+24	+28	+28	+24
	Bal.	−31	−18	−35	−35	−20	−35	−35	−20	−31	−18	0	0	0	0
Total Moments M''_{ij}		−20892	+20892	−15853	−10811	+26665	−10811	−15853	+26665	−20892	+20892	+25855	+28742	+28742	+25855

TABLE 5. Proposed method for L = 5.00m

Joint		A		B			C			D		E	F	G	H
Member		AB	AE	BA	BC	BF	CB	CD	CG	DC	DH	EA	FB	GC	HD
Stiffness		159239	93996	159239	159239	93996	159239	159239	93996	159239	93996	93996	93996	93996	93996
Distribution Factor		0.62882	0.37118	0.38606	0.38606	0.22788	0.38606	0.38606	0.22788	0.62882	0.37118	0	0	0	0
Carry-Over Factor		0.43694	0.45391	0.43694	0.43694	0.45391	0.43694	0.43694	0.45391	0.43694	0.45391	0.45391	0.45391	0.45391	0.45391
Cycle 1	FEM	0	+27332	0	0	+27332	0	0	+27332	0	+27332	+27332	+27332	+27332	+27332
	Bal.	−17187	−10145	−10552	−10552	−6228	−10552	−10552	−6228	−17187	−10145	0	0	0	0
Cycle 2	CO	−4611	0	−7510	−4611	0	−4611	−7510	0	−4611	0	−4605	−2827	−2827	−4605
	Bal.	+2899	+1712	+4679	+4679	+2762	+4679	+4679	+2762	+2899	+1712	0	0	0	0
Cycle 3	CO	+2044	0	+1267	+2044	0	+2044	+1267	0	+2044	0	+777	+1254	+1254	+777
	Bal.	−1285	−759	−1278	−1278	−755	−1278	−1278	−755	−1285	−759	0	0	0	0
Cycle 4	CO	−558	0	−561	−558	0	−558	−561	0	−558	0	−345	−343	−343	−345
	Bal.	+351	+207	+432	+432	+255	+432	+432	+255	+351	+207	0	0	0	0
Cycle 5	CO	+189	0	+153	+189	0	+189	+153	0	+189	0	+94	+116	+116	+94
	Bal.	−119	−70	−132	−132	−78	−132	−132	−78	−119	−70	0	0	0	0
Cycle 6	CO	−58	0	−52	−58	0	−58	−52	0	−58	0	−32	−35	−35	−32
	Bal.	+36	+22	+42	+42	+25	+42	+42	+25	+36	+22	0	0	0	0
Cycle 7	CO	+18	0	+16	+18	0	+18	+16	0	+18	0	+10	+11	+11	+10
	Bal.	−11	−7	−13	−13	−8	−13	−13	−8	−11	−7	0	0	0	0
Total Moments M''_{ij}		−18292	+18292	−13509	−9798	+23305	−9798	−13509	+23305	−18292	+18292	+23231	+25508	+25508	+23231

TABLE 6. Classical method for L = 3.00m

Joint		A		B			C			D		E	F	G	H
Member		AB	AE	BA	BC	BF	CB	CD	CG	DC	DH	EA	FB	GC	HD
Stiffness		298869	102661	298869	298869	102661	298869	298869	102661	298869	102661	102661	102661	102661	102661
Distribution Factor		0.74433	0.25567	0.42671	0.42671	0.14657	0.42671	0.42671	0.14657	0.74433	0.25567	0	0	0	0
Carry-Over Factor		0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000	0.50000
Cycle 1	FEM	0	+30798	0	0	+30798	0	0	+30798	0	+30798	+30798	+30798	+30798	+30798
	Bal.	−22924	−7874	−13142	−13142	−4514	−13142	−13142	−4514	−22924	−7874	0	0	0	0
Cycle 2	CO	−6571	0	−11462	−6571	0	−6571	−11462	0	−6571	0	−3937	−2257	−2257	−3937
	Bal.	+4891	+1680	+7695	+7695	+2643	+7695	+7695	+2643	+4891	+1680	0	0	0	0
Cycle 3	CO	+3848	0	+2446	+3848	0	+3848	+2446	0	+3848	0	+840	+1223	+1223	+840
	Bal.	−2864	−984	−2686	−2686	−923	−2686	−2686	−923	−2864	−984	0	0	0	0
Cycle 4	CO	−1343	0	−1432	−1343	0	−1343	−1432	0	−1343	0	−492	−462	−462	−492
	Bal.	+1000	+343	+1184	+1184	+407	+1184	+1184	+407	+1000	+343	0	0	0	0
Cycle 5	CO	+592	0	+500	+592	0	+592	+500	0	+592	0	+172	+204	+204	+172
	Bal.	−441	−151	−466	−466	−160	−466	−466	−160	−441	−151	0	0	0	0
Cycle 6	CO	−233	0	−220	−233	0	−233	−220	0	−233	0	−76	−80	−80	−76
	Bal.	+173	+60	+193	+193	+66	+193	+193	+66	+173	+60	0	0	0	0
Cycle 7	CO	+96	0	+86	+96	0	+96	+86	0	+96	0	+30	+33	+33	+30
	Bal.	−71	−25	−78	−78	−27	−78	−78	−27	−71	−25	0	0	0	0
Total Moments M''_{ij}		−23847	+23847	−17382	−10911	+28290	−10911	−17382	+28290	−23847	+23847	+27335	+29459	+29459	+27335

TABLE 7. Proposed method for L = 3.00m

Joint		A		B			C			D		E	F	G	H
Member		AB	AE	BA	BC	BF	CB	CD	CG	DC	DH	EA	FB	GC	HD
Stiffness		225398	93996	225398	225398	93996	225398	225398	93996	225398	93996	93996	93996	93996	93996
Distribution Factor		0.70570	0.29430	0.41373	0.41373	0.17254	0.41373	0.41373	0.17254	0.70570	0.29430	0	0	0	0
Carry-Over Factor		0.33702	0.45391	0.33702	0.33702	0.45391	0.33702	0.33702	0.45391	0.33702	0.45391	0.45391	0.45391	0.45391	0.45391
Cycle 1	FEM	0	+27332	0	0	+27332	0	0	+27332	0	+27332	+27332	+27332	+27332	+27332
	Bal.	−19288	−8044	−11308	−11308	−4716	−11308	−11308	−4716	−19288	−8044	0	0	0	0
Cycle 2	CO	−3811	0	−6500	−3811	0	−3811	−6500	0	−3811	0	−3651	−2141	−2141	−3651
	Bal.	+2689	+1122	+4266	+4266	+1779	+4266	+4266	+1779	+2689	+1122	0	0	0	0
Cycle 3	CO	+1438	0	+906	+1438	0	+1438	+906	0	+1438	0	+509	+808	+808	+509
	Bal.	−1015	−423	−970	−970	−404	−970	−970	−404	−1015	−423	0	0	0	0
Cycle 4	CO	−327	0	−342	−327	0	−327	−342	0	−327	0	−192	−183	−183	−192
	Bal.	+231	+96	+277	+277	+115	+277	+277	+115	+231	+96	0	0	0	0
Cycle 5	CO	+93	0	+78	+93	0	+93	+78	0	+93	0	+44	+52	+52	+44
	Bal.	−66	−27	−71	−71	−30	−71	−71	−30	−66	−27	0	0	0	0
Cycle 6	CO	−24	0	−22	−24	0	−24	−22	0	−24	0	−12	−14	−14	−12
	Bal.	+17	+7	+19	+19	+8	+19	+19	+8	+17	+7	0	0	0	0
Cycle 7	CO	+6	0	+6	+6	0	+6	+6	0	+6	0	+3	+4	+4	+3
	Bal.	−4	−2	−5	−5	−2	−5	−5	−2	−4	−2	0	0	0	0
Total Moments M''_{ij}		−20061	+20061	−13666	−10417	+24082	−10417	−13666	+24082	−20061	+20061	+24033	+25858	+25858	+24033

Shear forces on the base of frame are expressed in terms of the final moments, as shown in Figure 7, which are:

$$H_{EA} = \frac{M_{AE} + M_{EA}}{h}; \quad H_{FB} = \frac{M_{BF} + M_{FB}}{h};$$
$$H_{GC} = \frac{M_{CG} + M_{GC}}{h}; \quad H_{HD} = \frac{M_{DH} + M_{HD}}{h}$$

The condition of shear is converted as is shown:

$$\frac{M_{AE} + M_{EA}}{h} + \frac{M_{BF} + M_{FB}}{h} + \frac{M_{CG} + M_{GC}}{h} + \frac{M_{DH} + M_{HD}}{h} = P$$

Substituting:

$$\frac{M'_{AE} + k(M''_{AE}) + M'_{EA} + k(M''_{EA})}{h} + \frac{M'_{BF} + k(M''_{BF}) + M'_{FB} + k(M''_{FB})}{h}$$
$$+ \frac{M'_{CG} + k(M''_{CG}) + M'_{GC} + k(M''_{GC})}{h} + \frac{M'_{DH} + k(M''_{DH}) + M'_{HD} + k(M''_{HD})}{h} = P$$

Or

$$M'_{AE} + M'_{EA} + M'_{BF} + M'_{FB} + M'_{CG} + M'_{GC} + M'_{DH} + M'_{HD}$$
$$+ k(M''_{AE} + M''_{EA} + M''_{BF} + M''_{FB} + M''_{CG} + M''_{GC} + M''_{DH} + M''_{HD}) = Ph$$

For this case is:

$$M'_{AE} + M'_{EA} + M'_{BF} + M'_{FB} + M'_{CG} + M'_{GC} + M'_{DH} + M'_{HD} = 0$$

Substituting into the above equation and solving for the ratio k, in Table 8 shows the values for the problems developed.

TABLE 8. Values k for the six cases

Ratio k	Case 1 L = 10.00m		Case 2 L = 5.00m		Case 3 L = 3.00m	
	PM	CM	PM	CM	PM	CM
	0.001367	0.001508	0.001200	0.001357	0.001126	0.001304

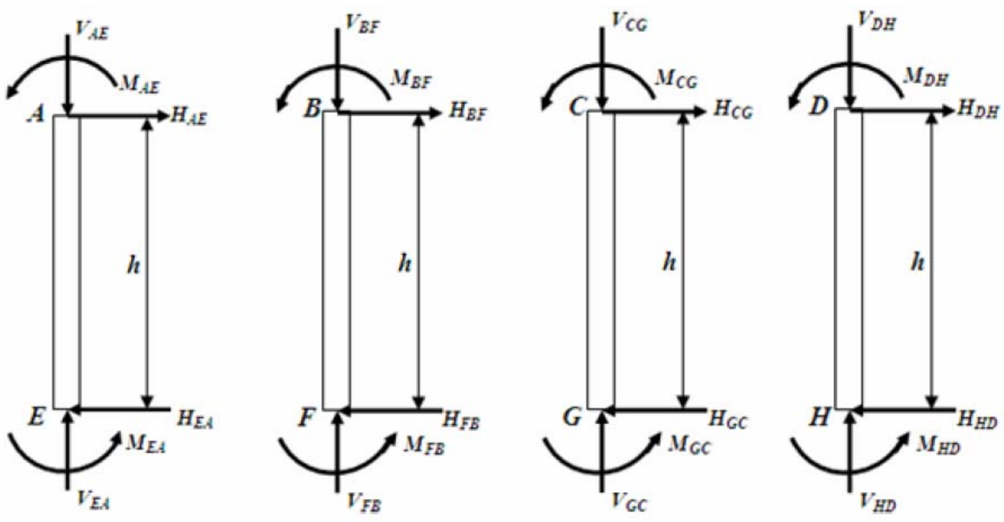

FIGURE 7. Free body diagram of each column

Once that are found the values of k were subsequently substituted into the equations corresponding to localize the final moments at the ends of the members. Now by static equilibrium, shear forces are obtained for each member. Then, it is obtains the diagrams shear forces, axial forces and moments.

Below, the results are presented in Tables 9, 10 and 11, for the three cases.

4. **Results.** With regard to Table 9, this shows the axial forces of the members between both methods. According to the results, there are differences, for L = 10.00m is lower 1% in the members CD, BF and CG, in absolute value for the classical method with respect to the proposed method, and for L = 3.00m is higher to a 30% in the member BF and CG, in absolute value for the classical method with respect to the proposed method. In Figure 8 is presented the diagram of axial forces in general form for the three problems different.

In Table 10 which present the shear forces in the ends of the members for the two methods. For example, for L = 3.00m is lower to a 10% in the member BC, in absolute value for the classical method with respect to the proposed method, and for L = 3.00m is greater to a 6% in the member AE, in absolute value for the classical method with respect to

TABLE 9. The axial forces of each member in kN

Axial Force	Case 1 L = 10.00m			Case 2 L = 5.00m			Case 3 L = 3.00m		
	CM	PM	$\underline{\text{CM}}$ PM	CM	PM	$\underline{\text{CM}}$ PM	CM	PM	$\underline{\text{CM}}$ PM
N_{AB}	+38.22	+38.12	1.00	+37.83	+37.78	1.00	+37.53	+37.55	1.00
N_{BC}	+24.52	+24.53	1.00	+24.52	+24.52	1.00	+24.52	+24.52	1.00
N_{CD}	+10.83	+10.93	0.99	+11.22	+11.27	1.00	+11.52	+11.50	1.00
N_{AE}	−4.00	−3.98	1.00	−8.82	−8.63	1.02	−15.47	−14.66	1.06
N_{BF}	+1.25	+1.26	0.99	+3.63	+3.31	1.10	+7.28	+5.60	1.30
N_{CG}	−1.25	−1.26	0.99	−3.63	−3.31	1.10	−7.28	−5.60	1.30
N_{DH}	+4.00	+3.98	1.00	+8.82	+8.63	1.02	+15.47	+14.66	1.06

N_{ij} = axial force the member "ij"
Nomenclature for the members:
+ Force compression
− Force tension

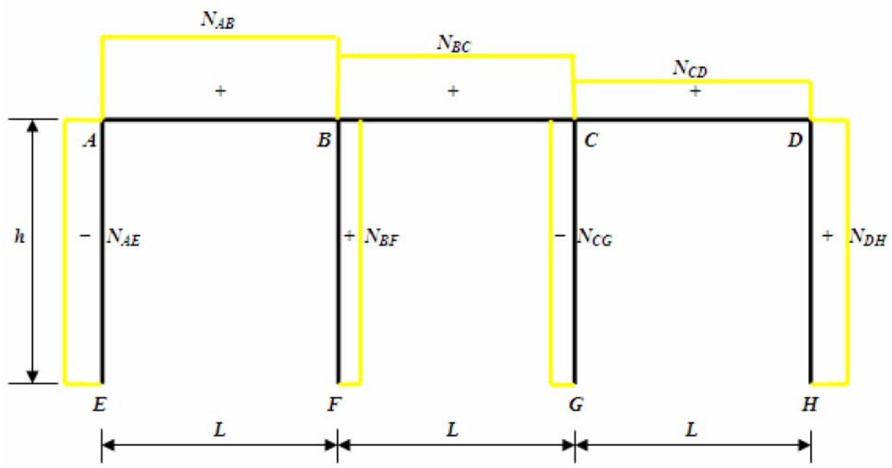

FIGURE 8. Diagram of axial forces

Table 10. The shear forces of each member in kN

Shear Force	Case 1 L = 10.00m			Case 2 L = 5.00m			Case 3 L = 3.00m		
	CM	PM	$\frac{CM}{PM}$	CM	PM	$\frac{CM}{PM}$	CM	PM	$\frac{CM}{PM}$
V_{AB}	−4.00	−3.98	1.00	−8.82	−8.63	1.02	−15.47	−14.66	1.06
V_{BC}	−2.75	−2.72	1.01	−5.19	−5.32	0.98	−8.19	−9.06	0.90
V_{CD}	−4.00	−3.98	1.00	−8.82	−8.63	1.02	−15.47	−14.66	1.06
V_{AE}	−10.83	−10.93	0.99	−11.22	−11.27	1.00	−11.52	−11.50	1.00
V_{BF}	−13.70	−13.60	1.01	−13.30	−13.25	1.00	−13.00	−13.02	1.00
V_{CG}	−13.70	−13.60	1.01	−13.30	−13.25	1.00	−13.00	−13.02	1.00
V_{DH}	−10.83	−10.93	0.99	−11.22	−11.27	1.00	−11.52	−11.50	1.00

V_{ij} = shear force the member "ij", in the joint "i"
Nomenclature for beams:
+ Force shear upstairs of the axis of reference
− Force shear down of the axis of reference
Nomenclature for columns
+ Force shear to the right of the axis of reference
− Force shear to the left of the axis of reference
Being: $V_{AB} = -V_{BA}$; $V_{BC} = -V_{CB}$; $V_{CD} = -V_{DC}$; $V_{AE} = -V_{EA}$; $V_{BF} = -V_{FB}$; $V_{CG} = -V_{GC}$ and $V_{DH} = -V_{HD}$

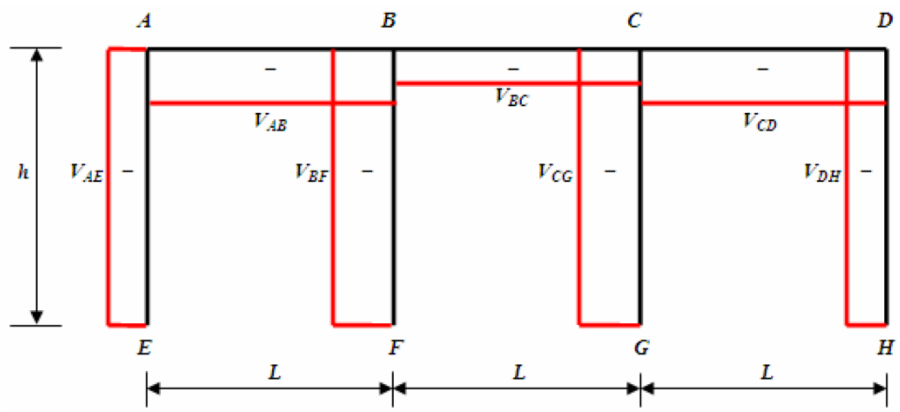

Figure 9. Diagram of shear forces

to the proposed method. In Figure 9 is showed the diagram of shear forces in general form for the three problems different.

With respect to Table 11, this illustrates the negative moments and positive for both methods. As soon as to the results, for L = 3.00m is greater in a 10% in the members *BA* and *CD*, in absolute value for the classical method with respect to the proposed method, and for L = 3.00m is lower in a 10% in the members *BC* and *CB*, in absolute value for the classical method with respect to the proposed method. In Figure 10 is obtained the diagram of moments in general form for the three problems different.

5. **Conclusions.** The results of the problem considered, through the application of two different techniques: classical method (considering deformations by flexure) and the proposed method (considering the deformations by flexure and shear), allowed to conclude that:

According to the axial forces, shear forces and moments acting on the members. These mechanical elements are those which govern the design of a structure. The results show

TABLE 11. The moments of each member in kN-m

Moment	Case 1 L = 10.00m			Case 2 L = 5.00m			Case 3 L = 3.00m		
	CM	PM	CM PM	CM	PM	CM PM	CM	PM	CM PM
M_{AB}	+22.06	+22.09	1.00	+25.08	+24.83	1.01	+26.84	+26.16	1.03
M_{BA}	−17.89	−17.71	1.01	−19.03	−18.34	1.04	−19.57	−17.82	1.10
M_{BC}	+13.73	+13.59	1.01	+12.98	+13.30	0.98	+12.28	+13.58	0.90
M_{CB}	−13.73	−13.59	1.01	−12.98	−13.30	0.98	−12.28	−13.58	0.90
M_{CD}	+17.89	+17.71	1.01	+19.03	+18.34	1.04	+19.57	+17.82	1.10
M_{DC}	−22.06	−22.09	1.00	−25.08	−24.83	1.01	−26.84	−26.16	1.03
M_{AE}	−22.06	−22.09	1.00	−25.08	−24.83	1.01	−26.84	−26.16	1.03
M_{EA}	+32.08	+32.54	0.99	+31.04	+31.53	0.98	+30.77	+31.34	0.98
M_{BF}	−31.62	−31.30	1.01	−32.01	−31.63	1.01	−31.85	−31.40	1.01
M_{FB}	+36.86	+36.70	1.00	+34.50	+34.63	1.00	+33.16	+33.72	0.98
M_{CG}	−31.62	−31.30	1.01	−32.01	−31.63	1.01	−31.85	−31.40	1.01
M_{GC}	+36.86	+36.70	1.00	+34.50	+34.63	1.00	+33.16	+33.72	0.98
M_{DH}	−22.06	−22.09	1.00	−25.08	−24.83	1.01	−26.84	−26.16	1.03
M_{HD}	+32.08	+32.54	0.99	+31.04	+31.53	0.98	+30.77	+31.34	0.98

M_{ij} = moment of the member "ij", in the joint "i"
Nomenclature for moments:

- For horizontal members:

+ Moment upstairs of axis of reference (compression in superior fibers and tension in inferior fibers)
− Moment down of axis of reference (tension in superior fibers and compression in inferior fibers)

- For vertical members:

+ Moment to the right of the axis of reference (compression in the right side fibers and tension in the left side fibers)
− Moment to the left of the axis of reference (tension in the right side fibers and compression in the left side fibers)

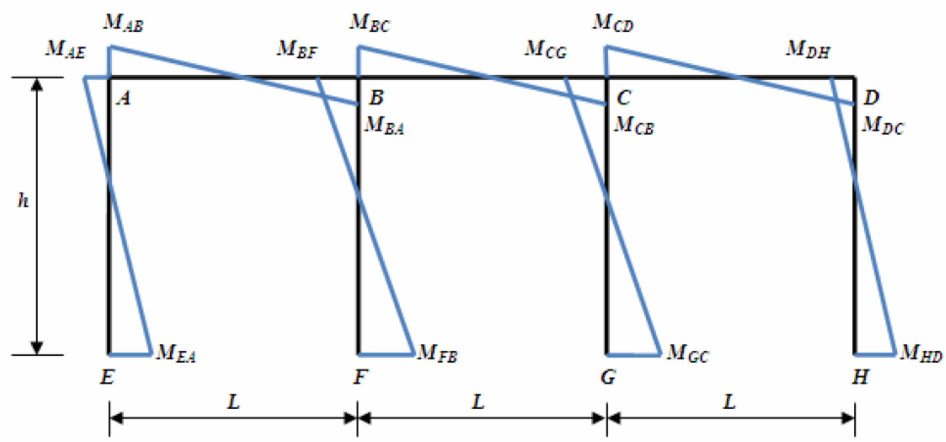

FIGURE 10. Diagram of moments

that differences exist between the two methods, both on the conservative side as of insecure side with respect to the classical method. This means that, it is this designing wrongly, because for a side some members are more than enough in their cross section dimensions and in another situation does not comply with the minimum conditions according to building regulations for that a is satisfactory structure. Since that the principle in civil engineering, with regard to the structural conditions is that have to be safe and economical.

Therefore, the usual practice of using the classical method (method of successive approximations considering deformations by flexure) is not a recommended solution.

According to the above, the proposed method of successive approximations (considering the deformations by flexure and shear), happens to be the more appropriate method for structural analysis of statically indeterminate rigid frames and also is more become attached to the real conditions.

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