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ROBUST MULTIVARIABLE GENERALIZED PREDICTIVE CONTROL DESIGN

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Abstract. *The paper addresses the problem of design stable robust model predictive controller with hard input constraints in the frequency domain. Numerical results are given to illustrate the effectiveness of the proposed method.*
Keywords: Predictive control, Multivariable GPC control, Equivalent subsystem method, PI controller, GPC

1. **Introduction.** The model predictive control (MPC) has attracted notable attention in the control of dynamic systems and played an important role in control practice. The ideas of MPC can be summarized as follows [3,16]: 1) Predict the future behaviour of the process state/output over a finite time horizon – prediction horizon. 2) Compute the present and future input signals online at each step by minimizing the cost function under inequality constraints on the same process variables. 3) Apply only the first of vector control variable (calculated for time t) on the controlled plant and repeat the previous step with new measured data.

Since the end of the 1970's many structures of MPC have been proposed. One of the most popular MPC is Generalized Predictive Control (GPC) [5,6]. Applications of GPC to a cement mill, a spray drying tower and compliant robot arm are described in [4]. GPC for systems with constrained input and output signals are presented in [2]. Continuous-time GPC has been proposed in [8].

In [13] the authors consider an extension of the GPC algorithm to a multivariable case by designing several single-input-single-output controllers and compensation for cross-coupling interactions. The idea of robustness of GPC for multivariable systems can be found in [7]. Authors analyse closed loop robustness in terms of "tuning knobs" and give a guideline as regards the final choice of "tuning knobs" with prevention of robust performance.

Authors of [9] present the stability problem of GPC when it is applied to a multivariable system with unstable transmission zeros. Feed-forward control of multivariable GPC is presented in [10] and decoupling multivariable GPC is presented in [1]. Other approaches of GPC design can be found in [15].

From the above survey it is clear that the basic formulation of GPC for SISO and MIMO systems does not guarantee stability. In the references, two methods for ensuring stability of GPC are shown: end-point constraints and end-point state weighting [11]. In some cases GPC closed-loop stability can be reached by using the effect of weighting of the control and control error or changing the output and input horizons.

In this paper a modified version of multivariable (or SISO) GPC is developed. The design procedure of MGPC guarantees the closed-loop system robust stability, performance and hard input constraints.

The success of MPC depends on the plant model precision. In practice, modelling of real plants inherently includes uncertainties that have to be considered in control system design. In the paper the bounded unstructured uncertainties are considered. The design of multivariable GPC is based on a novel approach, equivalent subsystem method [12]. The full design procedure of multivariable GPC is partially open and is under research.

The paper is organized as follows. Section 2 brings preliminaries and problem formulation. Theoretical results are presented in Section 3. In Section 4, numerical example illustrates the effectiveness of the proposed approach. Section 5 concludes the paper.

2. **Problem Statement and Preliminaries.** Consider a multivariable system with transfer function matrix

$$y(t) = G_u(z^{-1})u(t), \tag{1}$$

where $G_u(z^{-1}) \in R^{m\times m}$ and z^{-1} is the shift operator.

When deriving the MPC controller, a major source of difficulty is the plant model inaccuracy. To deal with it, the uncertainty model is used. So instead of a single model the behaviour of a class of models is considered. Let $G(z^{-1})$ be any member of a set of possible plants Π and $G_0(z^{-1}) \in \Pi$ be the nominal model of the plant. In the sequel the bounded unstructured uncertainty in three most common types is considered: additive, multiplicative input and multiplicative output uncertainty model for stable systems and their inverse part for unstable ones. More details can be found [17].

Simultaneously with (1), we consider the nominal model of system (1) in the form

$$y(t) = G_0(z^{-1})u(t), \tag{2}$$

where $G_0(z^{-1}) \in R^{m\times m} \in \Pi$ is any constant transfer function matrix from set Π.

The nominal model (2) will be used for output prediction while (1) is considered as a real plant description providing the plant output. Therefore, in the robust controller design we assume that for time t, output $y(t)$ is obtained from the uncertain model (1) and predicted outputs for times $t+1,\dots,t+N_y$ will be obtained from model prediction, where the nominal model (2) is used.

Let modified GPC algorithm for time t, $u(t)$ be given as

$$u(t) = F_{00}(z^{-1})(w(t) - y(t)) + \sum_{i=1}^{N_y} F_{0i}(z^{-1})(w(t+i) - y(t+i)) \tag{3}$$

and for time $t+k$, $k>0$

$$u(t+k) = F_{kk}(z^{-1})(w(t+k) - y(t+k)), \quad k = 1,2,\dots,N_y, \tag{4}$$

where

$F_{kk}(z^{-1}) \in R^{m\times m}$ is the controller transfer function matrix for $k = 0,1,\dots,N_y$

$F_{0i}(z^{-1}) \in R^{m\times m}$ is the controller transfer function matrix $i = 1,2,\dots,N_y$

$w(t+k)$ is demanded output variable value for time $t+k$, $k = 0,1,2,\dots,N_y$ predicted at time t.

The proposed modified GPC control structure is given in Figure 1.

In Figure 1, $K_u = diag\{k_{wi}\}_{m\times m}$ is an auxiliary gain matrix (7) (see later).

Consider the cost function associated with the system in Figure 1 in the following form:

$$J = \sum_{i=0}^{N_y} (w(t+i) - y(t+i))^T Q_i (w(t+i) - y(t+i)) + u(t+i)^T R_i u(t+i), \tag{5}$$

where Q_i, R_i, $i = 1, 2, \ldots, N_y$ are positive (semidefinite) definite matrices of corresponding dimensions.

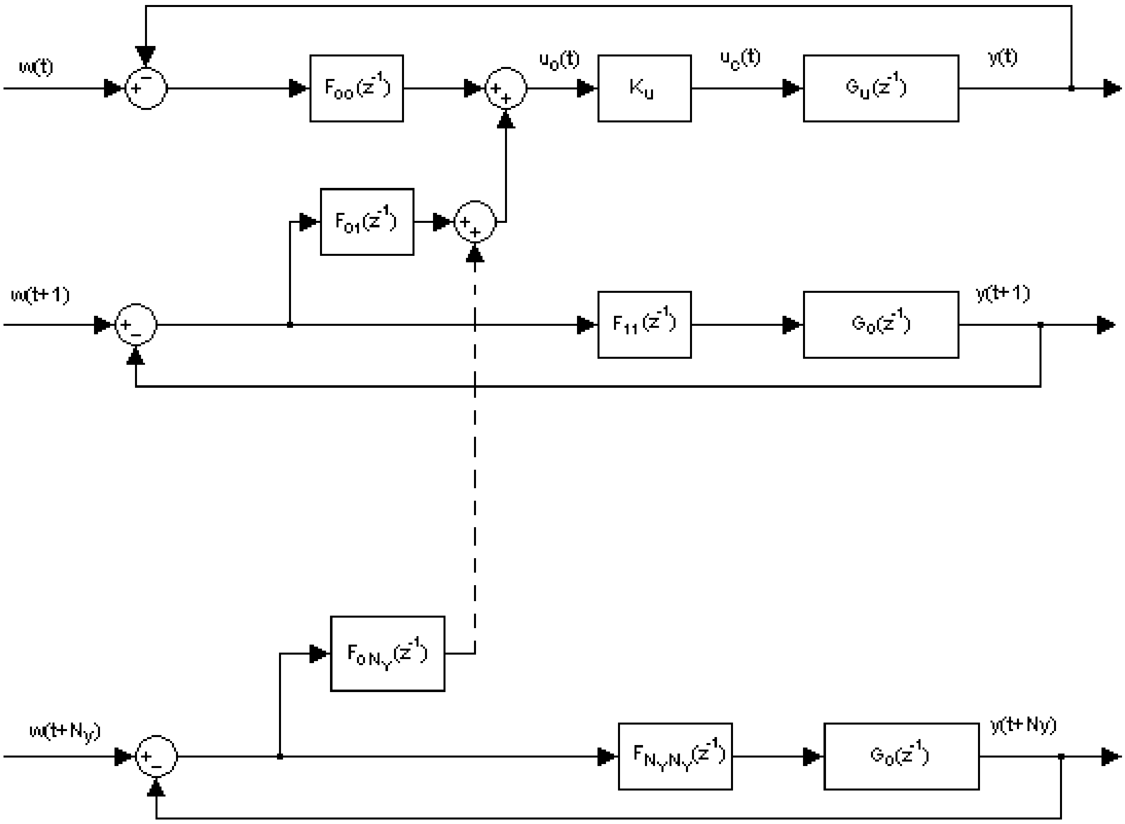

Figure 1. GPC control structure

Consider the system in Figure 1, where the control algorithm $u_c(t)$ is constrained to evolve in the following set

$$\Gamma = \{u_c(t) \in R^m : |u_{ci}(t)| \le u_i, i = 1, 2, \ldots, m\}, \tag{6}$$

where (see Figure 1)

$$u_c(t) = K_u u(t). \tag{7}$$

The problem studied in this paper can be summarized as follows. Design the robust model predictive controller with an output feedback control algorithm (3) and (4), input constraints (6) such that for a given prediction horizon N_y it guarantees robust closed-loop system stability, and optimal performance (5).

3. **Main Results.** On substituting the control algorithm (4) to the nominal model (2) and some manipulation one obtains the transfer matrix for $y(t + k)$, $k = 1, 2, \ldots, N_y$

$$y(t + k) = (I + G_0(z^{-1})F_{kk}(z^{-1}))^{-1}G_0(z^{-1})F_{kk}(z^{-1})w(t + k). \tag{8}$$

For control algorithm (3) using (8) one obtains

$$u(t) = F_{00}(z^{-1})(w(t) - y(t)) + \\ + \sum_{i=1}^{N_y} F_{0i}(z^{-1})(I + G_0(z^{-1})F_{ii}(z^{-1}))^{-1}w(t + i)$$

or if $w(t+i) = V_i z^i w(t)$

$$u(t) = F_{00}(z^{-1})(w(t) - y(t)) + + \sum_{i=1}^{N_y} F_{0i}(z^{-1})(I + G_0(z^{-1})F_{ii}(z^{-1}))^{-1} V_i z^i w(t) \tag{9}$$

where $V_i = diag\{v_{ij}\}_{m\times m}$ and v_{ij} is the j-th setpoint value for i-th prediction horizon.

With (9), (7) and (1) we can obtain the closed-loop transfer function of GPC in the following form:

$$y(t) = (I + G_u(z^{-1})F_{00}^u(z^{-1}))^{-1} G_u(z^{-1})[F_{00}^u(z^{-1}) + + \sum_{i=0}^{N_y} F_{0i}^u(z^{-1})(I + G_0 F_{ii})^{-1} V_i z^i] w(t) \tag{10}$$

where $F_{00}^u(z^{-1}) = K_u F_{00}(z^{-1})$ and $F_{0i}^u(z^{-1}) = K_u F_{0i}(z^{-1})$.

Equation (10) implies the following robust stability conditions for the proposed GPC:

a)

$$G_{cl}(z^{-1}) = (I + G_u(z^{-1})F_{00}^u(z^{-1}))^{-1} G_u(z^{-1}) F_{00}^u(z^{-1}) \tag{11}$$

The closed loop system (11) needs to be robust stable. Controller $F_{00}(z^{-1})$ and matrix K_u guarantee robust stability for the whole set Π.

b)

$$(I + G_0(z^{-1})F_{ii}(z^{-1}))^{-1}, \quad i = 1, 2, \ldots, N_y \tag{12}$$

The closed-loop with transfer matrix (12) needs to be stable. Controller $F_{ii}(z^{-1})$ need to be designed in such a way that it guarantees stability of (12).

c)

$$G_u(z^{-1})F_{0i}^u(z^{-1}), \quad i = 1, 2, \ldots, N_y \tag{13}$$

The open-loop system has to be stable for all plants transfer function matrices from set Π.

Note: Controllers $F_{00}(z^{-1}), F_{ii}(z^{-1})$, $i = 1, 2, \ldots, N_y$ guarantee closed-loop system robust stability and controllers $F_{0i}(z^{-1})$, $i = 1, 2, \ldots, N_y$ are used to minimize the cost function (5).

The robust stability condition of uncertain models set Π can be formulated using a M-delta structure depending on the uncertainty type. For input multiplicative uncertainty, used also in the studied case, has robust stability condition assumes the following form:

$$\sigma_M((I + G_u(z^{-1})F_{00}^u(z^{-1}))^{-1} G_u(z^{-1})F_{00}^u(z^{-1}) \leq \frac{1}{l_i(\omega)}, \quad \forall\omega, \tag{14}$$

where

$$l_i(\omega) = \max_k \sigma_M[G_0(j\omega)^{-1}(G_0(j\omega) - G_u(j\omega))] \tag{15}$$

Let us introduce a heuristic method for input constraints. For the obtained value of $u(t)$ (3), the control algorithm with input constraints which guarantee robust stability is constructed as in (7):

$$u_c(t) = K_u u(t) \tag{16}$$

where $K_u = diag\{k_{ui}\}_{m\times m}$ and k_{ui}, $i = 1, 2, \ldots, m$ is defined as follows

$$k_{ui} = \left\langle \begin{array}{l} 1 \rightarrow \text{ if } |u_i(t)| \leq U_{Mi} > 0 \\ \frac{U_{Mi}}{|u_i(t)|} \rightarrow \text{ if } |u_i(t)| > U_{Mi} \end{array} \right| i = 1, 2, \ldots, m \tag{17}$$

Equation (17) implies

$$K_{i\min} \leq k_{ui} \leq 1, \quad i = 1, 2, \ldots, N_y \tag{18}$$

$K_{i\min}$ is the minimal value of gain when robust stability (11) and (13) is guaranteed. The minimal value of K_u can be reached substituting $K_{i\min}$ to the entries of matrix K_u.

The maximal value of K_u is $K_u = I$. Using both marginal values of K_u one obtains two different closed-loop systems (19) which are stable in the set Π.

$$G_u(z^{-1}) = \left\langle \begin{array}{c} G_u^1(z^{-1}) \to K_u = I \\ G_u^2(z^{-1}) \to K_u = diag\{k_{i\,\min}\}_{m\times m} \end{array} \right| \qquad (19)$$

For a given structure, controller parameters $F_{00}(z^{-1})$, $F_{ii}(z^{-1})$, $i = 1, 2, \ldots, N_y$ and minimal value of K_u entries are calculated using the small gain theorem, under the same model of uncertainty (additive, multiplicative input and output), in such a way that the closed loop system is stable for any value k_{ui}, $i = 1, 2, \ldots, N_y$ in (18).

Robust GPC controller design using Equivalent Subsystem Method

In this part the independent controller design method will be used. At first, we introduce the equivalent subsystem method (ESM) proposed in [12] which will be used in GPC controller design. In this paper, ESM in the basics proposed design procedure for robust decentralized GPC controller design.

Consider the transfer function matrix of nominal model $G_0(s) \in R^{m\times m}$, $i = 1, 2, \ldots, 2^p$ with m subsystems that can be split into diagonal and off-diagonal parts describing respective models of decoupled subsystems $G_d(s)$ and interactions $G_m(s)$ (i is omitted):

$$G_0(s) = G_d(s) + G_m(s), \qquad (20)$$

where

$$G_d(s) = diag\{G_{di}(s)\}_{i=1,\ldots,m} \qquad (21)$$

with

$$\det(G_d(s)) \neq 0 \quad \forall s \in D. \qquad (22)$$

Here, the Nyquist D-contour, $s \in D$ comprises the imaginary axis and an infinite semicircle into the right-half plane avoiding locations, where open-loop transfer function has $j\omega$-axis poles by small indentations around them. Factorization of the determinant of the return-difference matrix under decentralized controller in terms of the correspondingly partitioned system for the i-th vertices yields

$$\begin{array}{l} \det F(s) = \det\{I + G_0(s)R(s)\} = \\ \det\{I + [G_d(s) + G_m(s)]R(s)\} = \\ \det[R(s)^{-1} + G_d(s) + G_m(s)] \det R(s) \end{array} \qquad (23)$$

The existence of $R^{-1}(s)$ is implied by the assumption

$$\det(R(s)) \neq 0 \qquad (24)$$

Denote

$$\det F_1(s) = \det[R^{-1}(s) + G_d(s) + G_m(s)] \qquad (25)$$

Using Nyquist stability conditions, [17], the necessary and sufficient conditions of closed-loop system stability can be determined as follows:

Corollary 3.1. *Closed loop system will be stable if controller $R(s)$ is stable and*

a)

$$\det F_1(s) \neq 0 \qquad (26)$$

b)

$$N[0, \det F_1(s)] + N[0, \det[R(s)] = n_q \qquad (27)$$

where n_q is number of unstable poles of $R(s)G_0(s)$.

Both matrices $R^{-1}(s)$ and $G_d(s)$ are diagonal so $P(s)$ will be diagonal too.

$$P(s) = R(s)^{-1} + G_d(s) = diag\{p_i(s)\}_{m\times m} \tag{28}$$

Simple manipulation of (21) yields

$$I + R(s)(G_d(s) - P(s)) = 0 \tag{29}$$

$$I + R(s)G^{eq}(s) = 0 \tag{30}$$

$G^{eq}(s)$ is a diagonal matrix of equivalent subsystems.

$$G^{eq}(s) = G_d(s) - P(s) \tag{31}$$

Characteristic equations for individual subsystems are

$$I + R_i(s)G_i^{eq}(s) = 0 \quad i = 1, 2, \ldots, m \tag{32}$$

Substituting (28) into (25)

$$\det F_1(s) = \det[P(s) + G_m(s)] \tag{33}$$

According to the independent design philosophy $p_i(s)$, $i = 1, 2, \ldots, m$ can actually represent the bounds for local controller designs. To be able to guarantee closed-loop stability of the full system they have to be chosen so as to appropriately consider the interaction term $G_m(s)$. The problem of creating the diagonal equivalent subsystem model $G^{eq}(s)$ allowing to design an independent controller without increasing conservatism is reduced to finding the diagonal matrix $P(s) = diag\{p_i(s)\}_{i=1,2,\ldots,m}$. A general method for choosing $P(s)$ is not available but interesting results have been obtained for the case of [12] choosing $P(s)$ with identical entries.

$$P(s) = p_i(s)I \tag{34}$$

The entries of diagonal matrix $P(s)$ are chosen so as to appropriately take into account interactions between subsystems given by the transfer function $G_m(s)$.

Substituting $P(s) = diag\{p_i(s)\}$ to (30) and equating to zero yields

$$\det[p_i(s)I + G_m(s)] = 0, \quad i = 1, \ldots, m \tag{35}$$

which actually defines m characteristic functions $g_i(s)$, $i = 1, \ldots, m$ of the matrix $[-G_m(s)]$. Moreover, if $p(s) = p_i(s)$ is taken to be any of the characteristic functions of $[-G_m(s)]$ then for fixed $l \in \{1, \ldots, m\}$ and $p(s) = -g_l(s)$ we obtain

$$P(s) = -g_l(s)I \tag{36}$$

and

$$\begin{aligned} \det(P(s) + G_m(s)) &= \prod_{i=1}^{m} [p(s) + g_i(s)] = \\ &= \prod_{i=1}^{m} [-g_l(s) + g_i(s)] = 0 \end{aligned} \tag{37}$$

For $P(s) = diag\{p_i(s)\}_{i=1,2,\ldots,m}$ the closed-loop system has some poles on the imaginary axis and no poles in the right half-plane, i.e., it is at the limit of instability.

Note: Characteristic functions of matrix $[-G_m(s)]$ can be found as functions of frequency. This is the reason why the equivalent subsystem model also depends on frequency and a graphical SISO frequency domain method has to be applied to stabilize equivalent subsystems using local controllers.

4. **Control Design.** Consider a heating system as a multivariable system where the temperature in two channels is controlled. Due to introducing 25% uncertainty operating points no. 2 and no. 3 are obtained as $Gu_2 = 0.75Gu_1$ and $Gu_3 = 1.25Gu_1$.

$$Gu_1(z^{-1}) = \begin{bmatrix} \frac{0.3231z^2+0.2658z+0.01559}{z^3-0.1448z^2-0.01064z-0.001308} & \frac{-0.1094z^2-0.05881z+0.0003448}{z^3-0.2542z^2+0.0761z-0.0004704} \\ \frac{0.2675z^2+0.03886z+0.0007296}{z^3+0.06892z^2+0.001236z-0.00000765} & \frac{0.2609z^2+0.2846z+0.007013}{z^3-0.6561z^2+0.1399z-0.000026} \end{bmatrix}$$

$$Gu_2(z^{-1}) = \begin{bmatrix} \frac{0.2424z^2+0.1993z+0.01169}{z^3-0.1448z^2-0.01064z-0.001308} & \frac{-0.08208z^2-0.04411z+0.0002586}{z^3-0.2542z^2+0.0761z-0.0004704} \\ \frac{0.2007z^2+0.02915z+0.0005472}{z^3+0.06892z^2+0.001236z-0.00000765} & \frac{0.1956z^2+0.2135z+0.005259}{z^3-0.6561z^2+0.1399z-0.00002.64} \end{bmatrix} \tag{38}$$

$$Gu_3(z^{-1}) = \begin{bmatrix} \frac{0.4039z^2+0.3322z+0.01948}{z^3-0.1448z^2-0.01064z-0.001308} & \frac{-0.1368z^2-0.07351z+0.000431}{z^3-0.2542z^2+0.0761z-0.0004704} \\ \frac{0.3344z^2+0.04858z+0.000912}{z^3+0.06892z^2+0.001236z-0.00000765} & \frac{0.3261z^2+0.3558z+0.008766}{z^3-0.6561z^2+0.1399z-0.0000264} \end{bmatrix}$$

According to GPC control structure from Figure 1 we will start with designing a controller for the first loop. A standard robust controller with performance specified by its phase margin will be designed.

First, the nominal model $G_0(z^{-1})$ is calculated as an average value of previous three matrices. According to the way how operating points no. 2 and no. 3 were calculated, the nominal model will be equal to the first operating point, so $G_0(z^{-1}) = Gu_1(z^{-1})$.

Controller $F_{00}(z^{-1})$ will be calculated by the equivalent subsystem method (ESM) as a robust controller from the nominal model $G_0(z^{-1})$ and it will be used for the whole set of operating points.

Off-diagonal elements of $G_0(z^{-1})$ were used to calculate the characteristic functions.

Equivalent subsystem models were calculated choosing $P(s) = p(s)I$ with identical entries and $p(s) = -g_2(s)$. Equivalent subsystems were calculated according to (31) and plotted as bode characteristics (Figures 3 and 4).

SISO PI controller with a phase margin of 55° which should ensure overshoot less than 20% for nominal model (nominal performance) was designed for each subsystem.

Designed controller for subsystem no. 1:

$$r_{11}(z^{-1}) = \frac{0.913 - 0.457z^{-1}}{1 - z^{-1}} \tag{39}$$

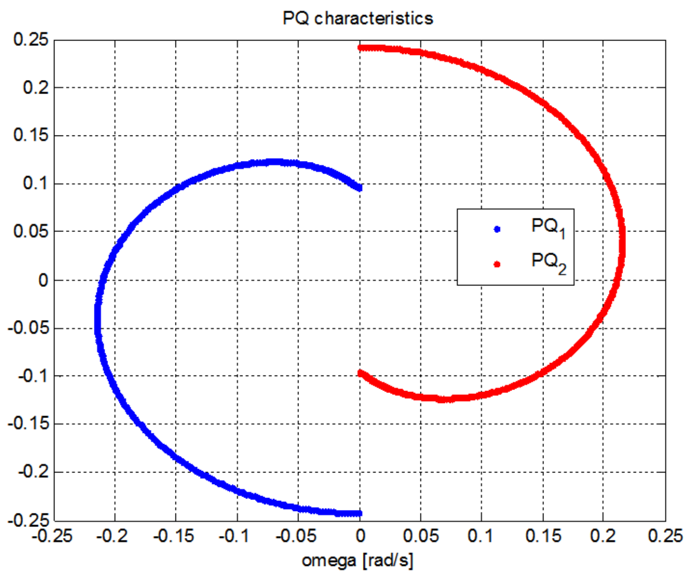

FIGURE 2. Characteristic functions of nominal model

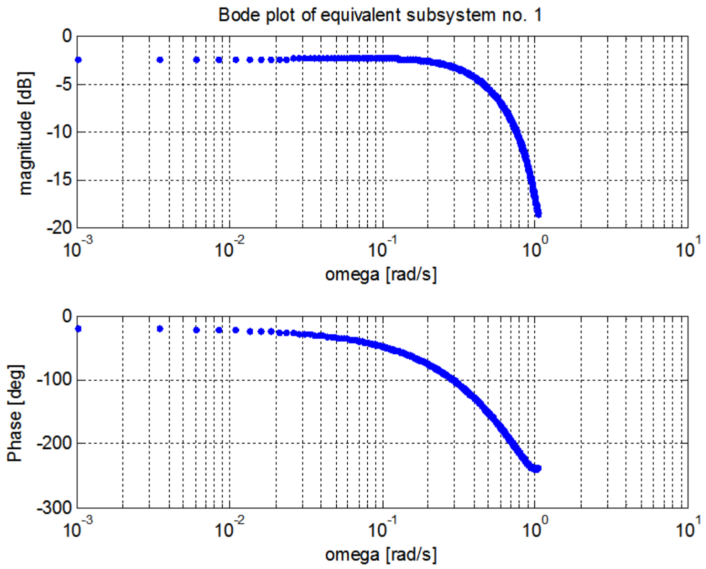

FIGURE 3. Equivalent subsystem no. 1

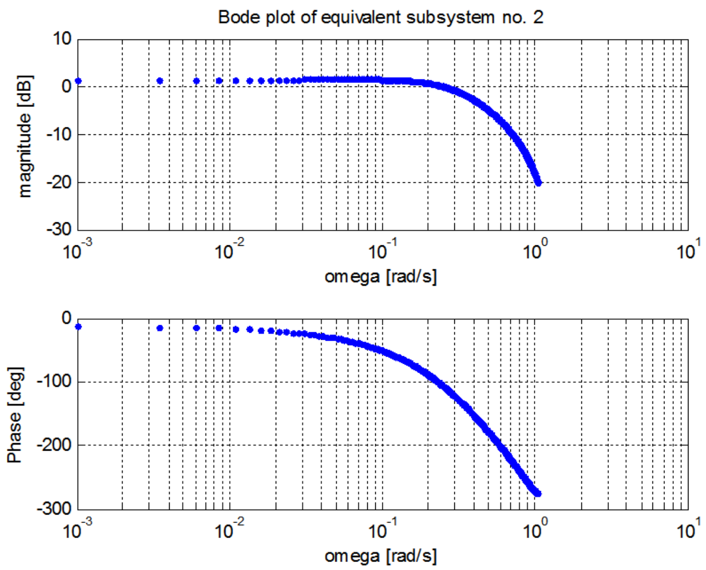

FIGURE 4. Equivalent subsystem no. 2

Designed controller for subsystem no. 2:

$$r_{22}(z^{-1}) = \frac{0.6 - 0.295z^{-1}}{1 - z^{-1}} \tag{40}$$

The final decentralized controller $F_{00}(z^{-1})$ is in the form $F_{00}(z^{-1}) = \begin{bmatrix} r_{11}(z^{-1}) & 0 \\ 0 & r_{22}(z^{-1}) \end{bmatrix}$ and guarantee stability of (11), see Figure 6 and nominal performance see Figure 5. The minimum value of K_u guaranteeing robust stability is $K_{u\_\min} = 0.75$.

Since the decentralized controller $F_{00}(z^{-1})$ was designed as robust, it can be used also in predictive loops (Figure 1, loops $2, 3, \ldots, N_y$) for local control of nominal models.

$$F_{ii}(z^{-1}) = F_{00}(z^{-1}), \quad i = 1, 2, \ldots, N_y \tag{41}$$

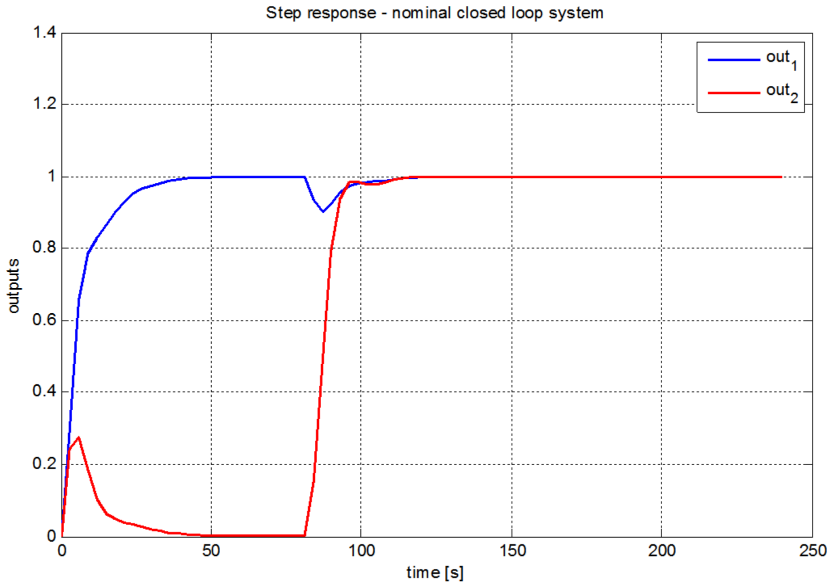

FIGURE 5. Step response of nominal model with designed feedback controller

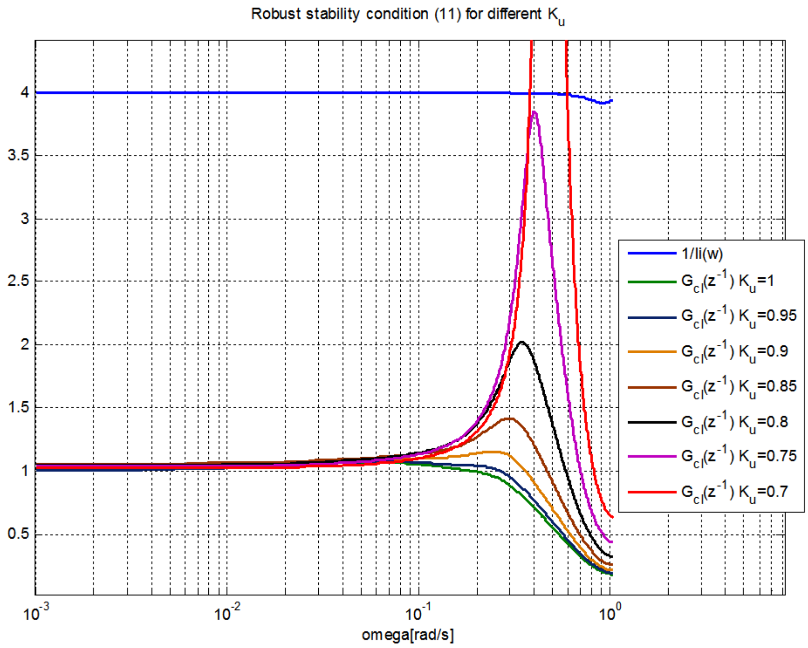

FIGURE 6. Robust stability condition for different K_u

As a final step of GPC controller design, $F_{0i}(z^{-1})$ have to be designed in order to minimize the slightly modified criteria function (5).

$$J = \sum_{k=0}^{N_k} (w(kT_s) - y(kT_s))^T Q (w(kT_s) - y(kT_s)) + u(kT_s)^T R u(kT_s), \tag{42}$$

where

T_s is the sampling time (in this case $T_s = 3$s),

Q is the matrix of weights for system outputs ($Q = qI$),

I is a unity matrix appropriate dimension,
R is the matrix of weights for controller outputs ($R = rI$),
N_k is the number of samples in simulation.

To minimize this criteria function, $F_{0i}(z^{-1})$ controllers were chosen also with a PI structure and its parameters were found using a simple genetic algorithm.

Finally, the heating system model was controlled in all operating points using a standard robust decentralized controller and also with a GPC controller, which is in our case a robust controller extended by prediction loops.

Simulation was made for $q = 1$, $r = 0.1$ and prediction horizon $N_y = 5$.

$$F_{01}(z^{-1}) = \begin{bmatrix} \frac{-0.71+1.94z^{-1}}{1-z^{-1}} & 0 \\ 0 & \frac{-2.85+7.1z^{-1}}{1-z^{-1}} \end{bmatrix}, \quad F_{02}(z^{-1}) = \begin{bmatrix} \frac{-0.8+1.7z^{-1}}{1-z^{-1}} & 0 \\ 0 & \frac{2.44-5.2z^{-1}}{1-z^{-1}} \end{bmatrix}$$

$$F_{03}(z^{-1}) = \begin{bmatrix} \frac{0.75-2.8z^{-1}}{1-z^{-1}} & 0 \\ 0 & \frac{1.1-0.24z^{-1}}{1-z^{-1}} \end{bmatrix}, \quad F_{04}(z^{-1}) = \begin{bmatrix} \frac{3.25+10.65z^{-1}}{1-z^{-1}} & 0 \\ 0 & \frac{3.74.-0.86z^{-1}}{1-z^{-1}} \end{bmatrix}, \quad (43)$$

$$F_{05}(z^{-1}) = \begin{bmatrix} \frac{-0.37-0.49z^{-1}}{1-z^{-1}} & 0 \\ 0 & \frac{-0.57+0.15z^{-1}}{1-z^{-1}} \end{bmatrix}$$

Comparison of the system control with a robust decentralized controller and GPC controller is in Figure 7.

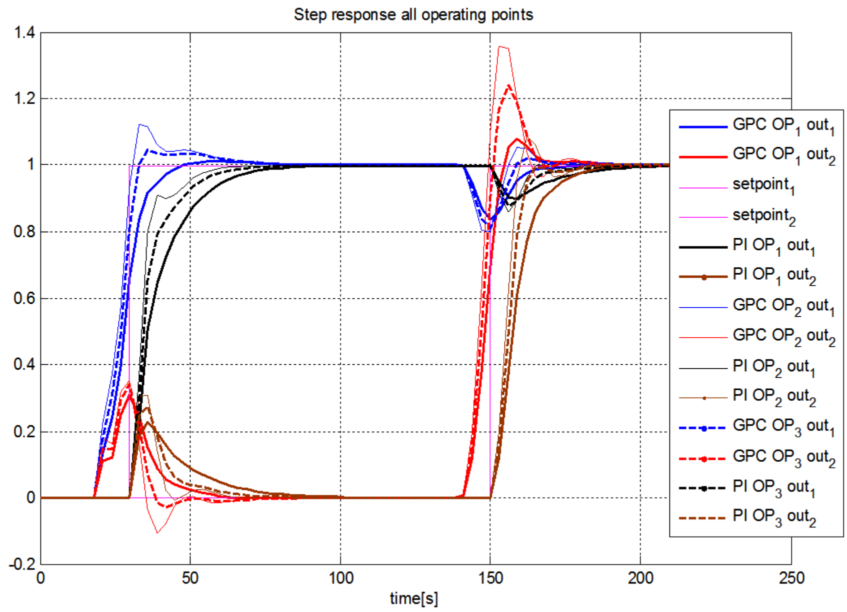

FIGURE 7. Comparison of robust decentralized controller and GPC controller in all operating points

TABLE 1. Cost functions comparison

	Cost function GPC	Cost function PI
Operating point 1	26.61	28.2
Operating point 2	11.47	12.43
Operating point 3	16.10	17.56

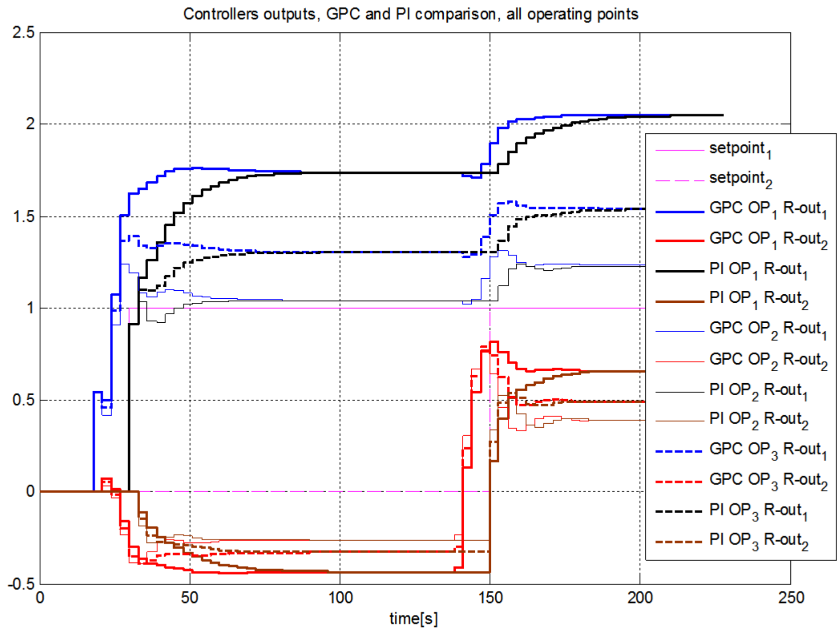

FIGURE 8. Comparison of GPC and PI controller outputs

The operation points have different gains; hence, the controller outputs for each operating point have different settling values.

One of several advantages of using GPC controller is the possibility to add constraints for the controller output into the cost function. Consider controller output limitation where in the second loop the maximum value 0.81 will be limited to 0.66. Then the controller output in the second loop will be limited to 81.5% and the whole system will stay stable because $0.815 > 0.75 = K_{u\,\min}$.

The cost function will be changed so that if the output is higher than 0.66 in second loop controller, a value of 100 will be added to the final value of the cost function.

New parameters of $F_{0i}(z^{-1})$ controllers have been found again using a genetic algorithm.

$$F_{01}(z^{-1}) = \begin{bmatrix} \frac{5.83-3.2z^{-1}}{1-z^{-1}} & 0 \\ 0 & \frac{-3+0.99z^{-1}}{1-z^{-1}} \end{bmatrix}, \quad F_{02}(z^{-1}) = \begin{bmatrix} \frac{-5.34+8.05z^{-1}}{1-z^{-1}} & 0 \\ 0 & \frac{0.54-0.12z^{-1}}{1-z^{-1}} \end{bmatrix}$$

$$F_{03}(z^{-1}) = \begin{bmatrix} \frac{3.82-5.79z^{-1}}{1-z^{-1}} & 0 \\ 0 & \frac{6.51-0.7z^{-1}}{1-z^{-1}} \end{bmatrix}, \quad F_{04}(z^{-1}) = \begin{bmatrix} \frac{2.84-5.42z^{-1}}{1-z^{-1}} & 0 \\ 0 & \frac{-5.3+2.25z^{-1}}{1-z^{-1}} \end{bmatrix}, \quad (44)$$

$$F_{05}(z^{-1}) = \begin{bmatrix} \frac{-0.49+0.25z^{-1}}{1-z^{-1}} & 0 \\ 0 & \frac{-0.2+0.045z^{-1}}{1-z^{-1}} \end{bmatrix}$$

Due to the new parameters of $F_{0i}(z^{-1})$ controllers, system outputs and controller outputs changed a little.

The final controller output in the second loop was successfully limited to 0.66 (Figures 10 and 11) by finding new parameters for $F_{0i}(z^{-1})$ controllers. The cost function of the constrained system has worse values than without constraint but still better than in a classical PI controller.

5. **Conclusions.** In this paper, a modified version of multivariable GPC is developed. The design procedure of MGPC guarantees closed-loop system robust stability performance and input constraints. As a robust controller design procedure, the equivalent

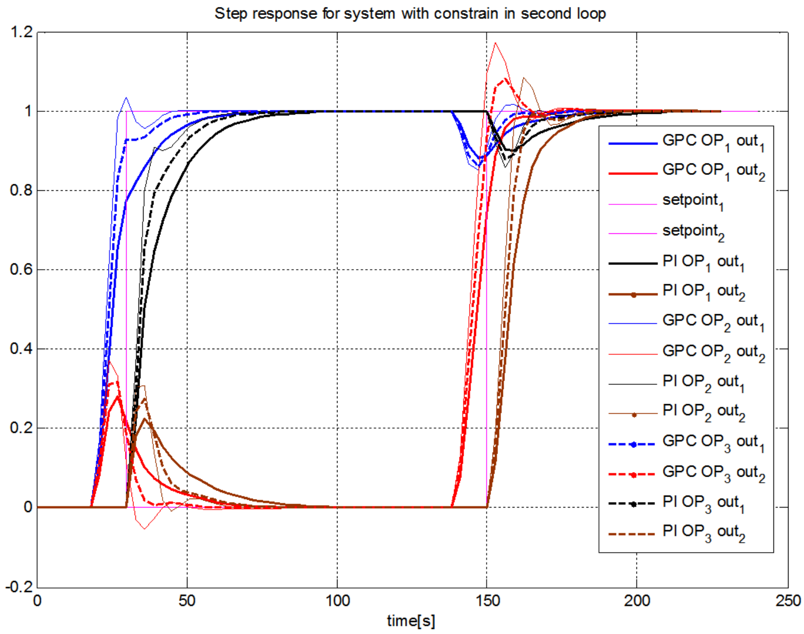

FIGURE 9. Step response for system with controller output constraint, all operating points

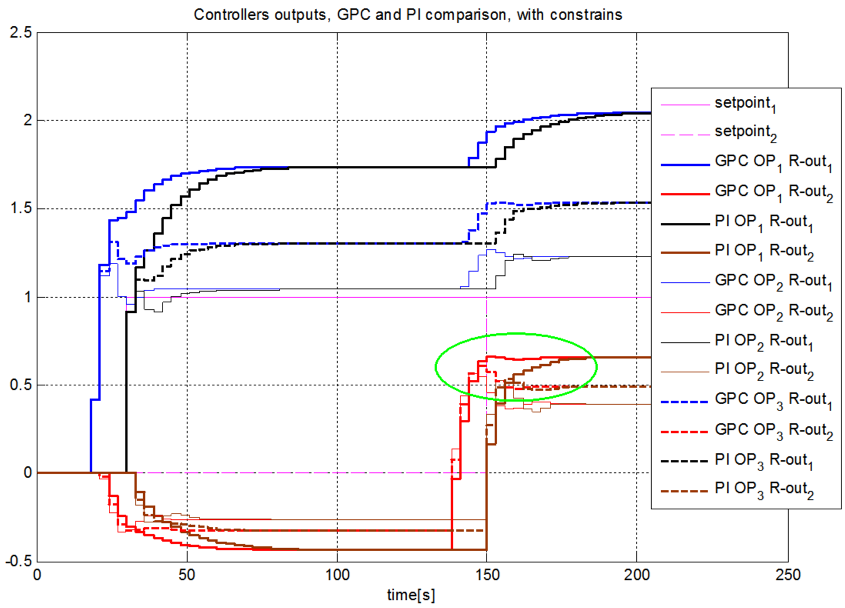

FIGURE 10. Controller outputs with constraint in second loop

TABLE 2. Comparison of cost functions GPC, GPC with constraints and PI controller

	Cost function GPC with constraint	Cost function PI	Cost function GPC without constraint
Operating point 1	26.81	28.2	26.61
Operating point 2	12.34	12.43	11.47
Operating point 3	16.65	17.56	16.10

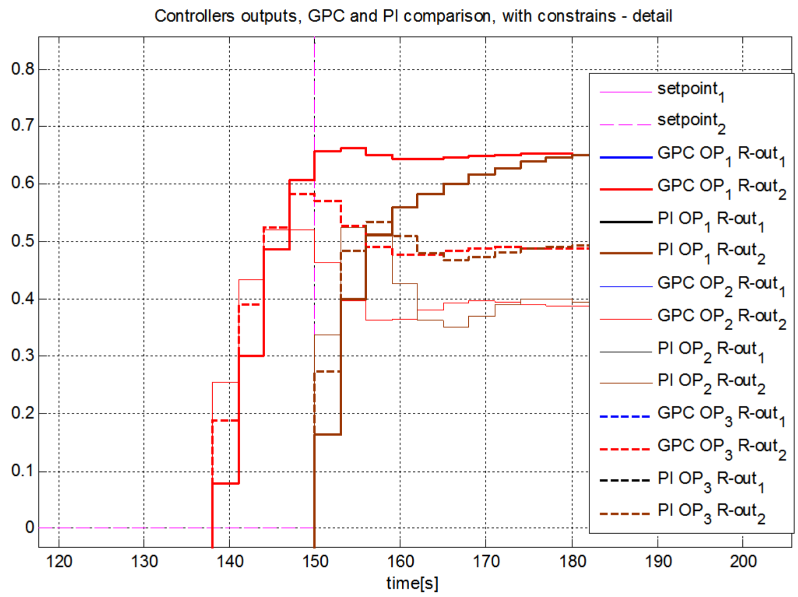

FIGURE 11. Controller outputs with constraint in second loop – detail (green ellipse)

subsystems method was used. The parameters of the predictive controller were found using a genetic algorithm due to missing analytical rules for its design. The advantage of GPC controller is the possibility of designing a criterion function J according to performance needs. System output constraints like no overshoot or controller output constraints can be simply integrated into the criteria function J with no effect upon stability. The full design procedure of multivariable GPC is partially open and is under research.

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