International Journal of Innovative Computing, Information and Control
Volume **9**, Number **8**, August **2013**
 ISSN 1349-4198
pp. 3403–3413

ADAPTIVE AND FAULT-TOLERANT REACTIVE POWER COMPENSATION IN POWER SYSTEMS VIA MULTILEVEL STATCOMS

Yongduan Song[1,2], Xinghua Li[2] and Wenchuan Cai[2]

[1]School of Automation
Chongqing University
No. 174, Shazhengjie, Shapingba, Chongqing 400044, P. R. China

[2]Center for Intelligent Systems and Renewable Energy
School of Electronic and Information Engineering
Beijing Jiaotong University
No. 3, Shangyuan Cun, Haidian District, Beijing 100044, P. R. China
ydsong@bjtu.edu.cn

Received July 2012; revised November 2012

Abstract. *Rapid and smooth reactive power compensation is crucial to maintain safe and reliable operation of power systems. This paper presents an adaptive and fault-tolerant control (FTC) approach for reactive power compensation via a three-level cascaded STATCOM (static synchronous compensator). Two sets of FTC algorithms are derived to concurrently accommodate element/actuation faults, modeling uncertainties, system nonlinearities, as well as external disturbances. The resultant control scheme is fault-tolerant and adaptive, yet structurally simple and computationally inexpensive in that little information on system dynamics/parameters is needed for control setup and implementation and there is no need for complicated and costly process for fault-detection and identification. Such user-friendly features are deemed favorable in practical applications.*
Keywords: Static synchronous compensator (STATCOM), Reactive power, Robust adaptive control, Fault-tolerant control, Voltage stability

1. Introduction.

The use of FACTS (flexible ac transmission systems) devices at strategic locations with well-designed controllers can help in improving the operational efficiency of power systems, and static synchronous compensator (STATCOM) is one of such powerful units that has been widely used to increase the transmission capability, enhance the voltage stability, and improve the transient performance of power system by injecting/absorbing the reactive power to/from the power system.

The essential role of STATCOM is to provide voltage compensation by injecting reactive current into the power line according to a line voltage control scheme. Note that the reactive current injected by a STATCOM is proportional to the voltage difference between the STATCOM and the line. Therefore, the performance of the STATCOM (controlled through its reactive current adjustment) heavily depends on the behavior of the capacitor voltage, which is relatively slow, the large capacitor needs to reduce dc voltage ripple, and highly coupled to the STATCOM currents. This makes implementation of fast control strategies difficult. Furthermore, the STATCOM dynamics are essentially nonlinear, the performance of a controller, typically designed on the basis of a linear approximation around an operation condition, is valid only in a small region. This limits the STATCOM performance outside the optimal region and therefore, a form of exact nonlinear cancellation (compensation) has been investigated [1-4]. Some works make use of the feedback linearization approach to design nonlinear controllers for power converters, the nonlinearities are hidden in the transformation and a feedback control law thus was designed applying linear techniques [5-8]. However, as full linearization is not always possible for such system, partial linearization was used in some cases under the assumption that the internal zero dynamics exhibit a proper transient performance. In [5], the control of the reactive current is achieved using partial linearization where a compensation term is added in order to improve the overall dynamics. However, since the complete model of STATCOM is essentially nonlinear with significant uncertainties, the linear approach does not lead to better dynamic decoupling [9-13].

As mentioned above, the basic function of the STATCOM is to sustain line voltage stability, which is implemented by an ac voltage controller in the STATCOM through regulating the reactive power exchange between the STATCOM and power system. There normally installed a dc voltage controller, which regulates the dc voltage across the dc capacitor of the STATCOM. The PI (proportional integral) control scheme has been traditionally utilized for both voltage regulators [15-17].

To maintain reliable operation of power systems, it is important to restore the normal operation under fault conditions because the failed operation of STATCOM could cause tremendous losses for the power grid, particularly when STATCOM is injecting reactive power to the grid to support the voltage. As higher level multilevel converters are required in the higher power rating application, a large number of power switching devices are used. Each of these devices is a potential failure point, which will dramatically reduce the reliability of the system. It is therefore of theoretical and practical importance to design a fault-tolerant STATCOM system to enhance the system reliability [18-20].

In this paper, we present an approach to construct computationally inexpensive control algorithms for controlling the STACOM unit in power systems. We began with the introduction of an improved dynamic model reflecting the coupling effects of the cascaded multiple level STATCOMs. Inspired by the recent work on using core information for control design [20], a simple yet effective robust adaptive control scheme is developed. The salient feature of the resultant control scheme lies in its simplicity in structure and effectiveness in dealing with unpredictable lumped disturbances and unexpected faults. Two sets of FTC algorithms are derived to concurrently accommodate actuation faults, modeling uncertainties, system nonlinearities, as well as external disturbances. The resultant control scheme is fault-tolerant and FDD (fault detection and diagnosis)-independent, yet structurally simple and computationally inexpensive in that little information on system dynamics/parameters is needed for control setup and implementation and there is no need for complicated and costly process for fault-detection and identification. Such user-friendly features are deemed favorable in practical applications.

2. **Problem Statement and Preliminaries.**

The STATCOM is a voltage or current source inverter (VSI-CSI)-based Custom Power device connected in shunt with power system. It provides voltage regulation, power factor correction, flicker and harmonic compensation in distribution systems [8]. Exchange of reactive power between the grid and STATCOM is achieved by adjusting amplitude of the inverter output voltage V_i. If amplitude of inverter output voltage is greater than grid voltage, then STATCOM generates capacitive reactive power. Otherwise, STATCOM absorbs inductive reactive power. If the amplitude of inverter output voltage is equal to grid voltage, there will be no exchange of reactive power between the STATCOM and grid. The apparent power exchanged between grid and STATCOM can be determined by following equation:

$$S = \dot{V}\dot{I}^* = \left(\dot{V}_i - \dot{V}_s\right)\left(\frac{\dot{V}_i - \dot{V}_s}{X}\right)^* = \left(\dot{V}_i - \dot{V}_s\right)\left(\frac{\dot{V}_i^*}{X^*} - \frac{\dot{V}_s^*}{X^*}\right)$$

$$
\begin{aligned}
&= \frac{\dot{V}_i\dot{V}_i^*}{X^*} + \frac{\dot{V}_s\dot{V}_s^*}{X^*} - \frac{\dot{V}_i\dot{V}_s^*}{X^*} - \frac{\dot{V}_s\dot{V}_i^*}{X^*} \\
&= \frac{V_i^2}{\|X\|}\angle\alpha + \frac{V_s^2}{\|X\|}\angle\alpha - \frac{V_iV_s}{\|X\|}\angle(\alpha+\delta) - \frac{V_iV_s}{\|X\|}\angle(\alpha-\delta) \\
&= P + jQ
\end{aligned} \tag{1}
$$

where $X = R + jwL = \|X\|\angle\alpha$, $\|X\| = \sqrt{R^2 + (wL)^2}$ is reactance of coupling inductance, $\dot{V}_s = V_s\angle\theta$ and $\dot{V}_i = V_i\angle\beta$ denote the grid voltage and output voltage of the inverter, respectively, $\delta = \beta - \theta$ is phase angle between AC grid voltage and the STATCOM fundamental voltage. Reactive power generated/absorbed by STATCOM can be calculated from (1):

$$
\begin{aligned}
P &= \frac{V_i^2 + V_s^2}{\|X\|}\cos\alpha - \frac{V_iV_s}{\|X\|}[\cos(\alpha+\delta) + \cos(\alpha-\delta)] \\
&= \frac{V_i^2 + V_s^2}{\|X\|}\cos\alpha - \frac{2V_iV_s}{\|X\|}\cos\alpha\cos\delta
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
Q &= \frac{V_i^2 + V_s^2}{\|X\|}\sin\alpha - \frac{V_iV_s}{\|X\|}[\sin(\alpha+\delta) + \sin(\alpha-\delta)] \\
&= \frac{V_i^2 + V_s^2}{\|X\|}\sin\alpha - \frac{2V_iV_s}{\|X\|}\sin\alpha\cos\delta
\end{aligned} \tag{3}
$$

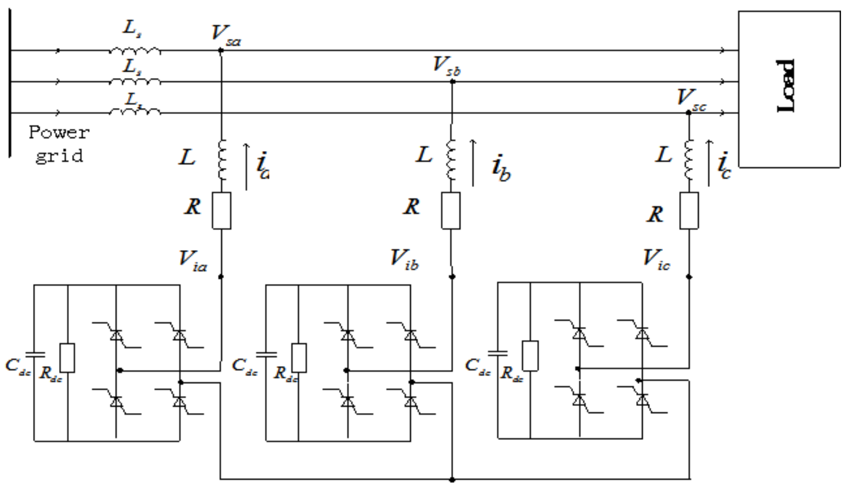

FIGURE 1. Equivalent circuit of three-level cascaded inverter based STATCOM

The STATCOM is modeled by voltage source connected to the power system through coupling inductance. The nonlinear STATCOM state equations for the equivalent circuit model as shown in Figure 1 in the reference frame are given by [15]

$$
\begin{aligned}
L\frac{di_a}{dt} &= -Ri_a + (V_{ia} - V_{sa}) + \Delta V_a(.) \\
L\frac{di_b}{dt} &= -Ri_b + (V_{ib} - V_{sb}) + \Delta V_b(.) \\
L\frac{di_c}{dt} &= -Ri_c + (V_{ic} - V_{sc}) + \Delta V_c(.)
\end{aligned} \tag{4}
$$

The output of STATCOM is given by

$$
V_{ia} = kV_{dc}\cos(wt + \beta) \tag{5}
$$

where i_a, i_b, i_c are the three-phase inverter currents, V_{dc} is the voltage across dc capacitor, V_{iabc} is the inverter output voltage, V_{sabc} is grid voltage, L is coupling inductance, and R is equivalent resistance of coupling inductance, k is the modulation gain, β is the injected voltage phase angle, w reflects the frequency of power grid. Note that $\Delta V(\cdot)$ is added to account for the unpredictable disturbance voltage drop arisen from loose (on-off) connection, short circuit, and others. By using the Park Transform

$$T = \frac{2}{3}\begin{bmatrix} \cos w_s t & \cos\left(w_s t - \frac{2\pi}{3}\right) & \cos\left(w_s t + \frac{2\pi}{3}\right) \\ -\sin w_s t & -\sin\left(w_s t - \frac{2\pi}{3}\right) & -\sin\left(w_s t + \frac{2\pi}{3}\right) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \tag{6}$$

We can transfer the three phase currents into the following dq coordinates

$$\frac{d}{dt}\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \frac{w_s V_{dc}}{L(.)}\begin{bmatrix} u_d \\ u_q \end{bmatrix} - \frac{w_s}{L(.)}\begin{bmatrix} V_{sd} \\ V_{sq} \end{bmatrix} - \begin{bmatrix} \frac{R(.)w_s}{L(.)} & -w_s \\ w_s & \frac{R(.)w_s}{L(.)} \end{bmatrix}\begin{bmatrix} i_d \\ i_q \end{bmatrix} + D(.) \tag{7}$$

with

$$u_d = k\cos(\delta + \theta) \quad u_q = k\sin(\delta + \theta) \tag{8}$$

and

$$V_{sd} = V_s \cos\theta \quad V_{sq} = V_s \sin\theta \tag{9}$$

where w_s is the dq coordinate rotation speed; k and δ represent the modulation ratio and phase shift, respectively. The circuit equation for DC side can be written as follows:

$$\frac{dV_{dc}}{dt} = -\frac{3w_s V_{dc}}{R_{dc}C_{dc}} - \frac{w_s}{3C_{dc}}\left[\vec{u}_{abc}\right]^T i_{abc} \tag{10}$$

which can be transformed into dq coordinates as:

$$\frac{dV_{dc}}{dt} = -\frac{3w_s V_{dc}}{R_{dc}C_{dc}} - \frac{w_s}{3C_{dc}}\begin{bmatrix} u_d & u_q \end{bmatrix}\begin{bmatrix} i_d \\ i_q \end{bmatrix} \tag{11}$$

It should be noted that STATCOM control consists of "internal", "external" and "gate" control. An internal controller generates a fundamental output voltage waveform with a desired magnitude and phase angle in synchronism with the AC system. This fundamental signal is then used to drive the gating signals through an appropriate pulse width modulation scheme. An external controller responds to system conditions and determines how much reactive current the STATCOM should generate or absorb to meet the requirements of the system.

In this work, we focus on the internal controller as it is the most fundamental and crucial phase for STATCOM. The reactive current generated/absorbed by STATCOM can be adjusted by controlling the output voltage of the inverter, which can be done by means of different control techniques such as phase angle control, constant DC link voltage control scheme, direct and indirect current control methods. Here, we develop a robust adaptive and fault-tolerant control scheme to achieve direct current control, and at the same time, counteract modeling uncertainties, unexpected disturbances, coupling effects, as well as actuator (inverter) failures.

While STATCOM can be used to improve the stability condition of the system, the efficiency of the STATCOM depends largely on the proper functioning of the controller utilized. In next section, we present a control scheme that is adaptive to unknown system parameters and is able to accommodate unpredictable faults.

3. **Adaptive and Fault-Tolerant Control Scheme.** For later control design, we define the reactive and active current tracking errors as follows:

$$e_1 = i_d - i_d^*, \quad e_2 = i_q - i_q^* \tag{12}$$

where i_d^* and i_q^* are desired values of reactive current and active current, respectively, which is specified according to system operation requirement. For practical system, these currents and their first derivatives are smooth and bounded. The objective is to design u_d and u_q so that the reactive current and active current tracking error $e_1 \to 0$ and $e_2 \to 0$ as $t \to \infty$. To this end, we rewrite (7) in terms of the derivative of e_1 and e_2 as follows:

$$\dot{e}_1 = f_1 + gu_d, \quad \dot{e}_2 = f_2 + gu_q \tag{13}$$

with

$$\begin{aligned} f_1(\cdot) &= -\frac{Rw_s}{L} i_d + w_s i_q - \frac{w_s}{L} V_{sd} - \dot{i}_d^* + D_1(.) \\ g(\cdot) &= \frac{w_s V_{dc}}{L} \\ f_2(\cdot) &= -w_s i_d - \frac{Rw_s}{L} i_q - \frac{w_s}{L} V_{sq} - \dot{i}_q^* + D_2(.) \end{aligned} \tag{14}$$

or expressed in a compact form

$$\dot{y} = F(\cdot) + B(\cdot)u \tag{15}$$

where $y = [e_1 \quad e_2]^T$ and

$$F(\cdot) = \begin{bmatrix} f_1(\cdot) \\ f_2(\cdot) \end{bmatrix} + D(.), \quad B(\cdot) = \begin{bmatrix} g(\cdot) & 0 \\ 0 & g(\cdot) \end{bmatrix}, \quad u = \begin{bmatrix} u_d \\ u_q \end{bmatrix} \tag{16}$$

3.1. **Robust adaptive tracking control.** In light of (12) and (15), it is seen that the control objective is achieved if y is controlled to converge to zero or to the small neighborhood of zero as $t \to \infty$. It is trivial to solve this problem if $F(\cdot)$, $B(\cdot)$ are known precisely. However, the underlying problem is quite challenging as the precise information regarding these quantities are practically unavailable. We will develop a control scheme without the need for any such information. The following observation is useful for later development.

Observation 1: *Let $\varphi(\cdot)$ be defined by*

$$\varphi(\cdot) = |i_d| + |i_q| + 1 \tag{17}$$

Then there always exists some unknown constant $a > 0$ such that:

$$\|F(\cdot)\| \le a\varphi(\cdot) \tag{18}$$

Proof: By (14), it can be shown that

$$\begin{aligned} |f_1(\cdot)| &\le \frac{Rw_s}{L} |i_d| + \left|\dot{i}_d^*\right| + w_s |i_q| + \left|\frac{w_s}{L} V_{sd}\right| + |D_1(.)| \\ &\le m_1 |i_d| + m_2 + m_3 |i_q| + m_4 \\ &\le \mu_1(|i_d| + |i_q| + 1) \le \mu_1 \varphi(\cdot) \end{aligned} \tag{19}$$

where m_1, m_2, m_3 and m_4 are some unknown and non-negative constants, and $\mu_1 = \max\{m_1, m_2 + m_4, m_3\}$. Similarly $|f_2(\cdot)| \le \mu_2(|i_q| + |i_d| + 1) \le \mu_2\varphi(\cdot)$ and then (18) holds in which $a = \mu_1 + \mu_2$.

Remark 3.1. *It is worth noting that (18) is independent of any physical parameters or operating conditions of the STATCOM under consideration. This fact is useful for the development of the highly robust adaptive control scheme with simple structure and inexpensive online computations, as detailed in what follows.*

Theorem 3.1. *Consider the error dynamics governed by (15) where all the parameters in $F(\cdot)$ are unknown and some of them are even time-varying. Let the control signal u be generated by:*

$$u = -\frac{\hat{a}\varphi(\cdot)y}{\|y\|} - k_0 y \tag{20}$$

where $k_0 > 0$ is a free control parameter; $\hat{a}$ is the estimation of a. If the following adaptive algorithm is implemented,

$$\dot{\hat{a}} = \xi \|y\| \varphi(\cdot) \tag{21}$$

where $\xi > 0$ is a positive constant for adaptation rate, then asymptotically stable path tracking is achieved in that $y \to 0$ as $t \to \infty$.

Proof: Based on (15) and (20), we get

$$\dot{y} = F(\cdot) - \frac{B(\cdot)\hat{a}\varphi(\cdot)y}{\|y\|} - k_0 B(\cdot)y \tag{22}$$

Consider the Lyapunov function candidate

$$V(\cdot) = \frac{1}{2}y^T y + \frac{1}{2\xi b_m}(b_m\hat{a} - a)^2 \tag{23}$$

where $0 < b_m \leq \min\{g\}$ is some unknown constant. With the adaptive algorithm given in (21) and in light of Observation 1, its derivative with respect to time becomes:

$$\begin{aligned}
\dot{V}(\cdot) =& y^T\dot{y} + \frac{1}{\xi}\dot{\hat{a}}(b_m\hat{a} - a) \\
=& -\frac{y^T B(\cdot)\hat{a}\varphi(\cdot)y}{\|y\|} - k_0 y^T B(\cdot)y + y^T F(\cdot) + \frac{1}{\xi}\dot{\hat{a}}(b_m\hat{a} - a) \\
\leq& -b_m \|y\| \hat{a}\varphi(\cdot) - k_0 b_m \|y\|^2 + \|y\| \varphi(\cdot)a + \frac{1}{\xi}\dot{\hat{a}}(b_m\hat{a} - a) \\
=& -k_0 b_m \|y\|^2 \leq 0
\end{aligned} \tag{24}$$

from which we get $V \in \ell_\infty$, implying that $\hat{a} \in \ell_\infty$ and $y \in \ell_\infty$, thus $u \in \ell_\infty$. Furthermore, from (22) we have $\dot{y} \in \ell_\infty$, y is uniformly continuous. Also since $\int_0^t y^2 dt \leq \frac{V(0)}{k_0 b_m} < \infty$, it is then established that $y \in \ell_2 \cap \ell_\infty$, by Barbalat Lemma, it is concluded that $\lim\limits_{t\to\infty} y = 0$; therefore, $e_1 \to 0$, $\dot{e}_1 \to 0$, and $e_2 \to 0$, $\dot{e}_2 \to 0$ as $t \to \infty$ by the definition of y.

Remark 3.2. *The proposed control scheme (20) and (21) are model independent that can robust against parameter uncertainties and unexpected disturbances. Although the parameter b_m is used in stability analysis, analytical estimate of such parameter is not needed because the proposed control algorithms do not involve such parameter. Furthermore, only simple functions and trivial on-line computations are involved in the scheme, much fewer onboard computing resources are needed compared with most existing methods.*

As actuator faults might occur during STATCOM operation, a fault-tolerant control design is highly desirable, which is addressed in next subsection.

3.2. **Fault-tolerant output tracking control.** Reactive power compensation device primarily consists of four units: detection unit, control unit, execution unit and compensation unit. The actuator unit receives modulation signal from the controller (here it refers to k and δ), as shown in Figure 2, the pulse width modulation (PWM) circuit generates the corresponding pulses to trigger the switching devices of the bridge circuit (inverter), then the regulation of reactive power exchange is achieved by changing the inverter output. The voltage source (PWM inverter) circuit works as an actuator in STATCOM

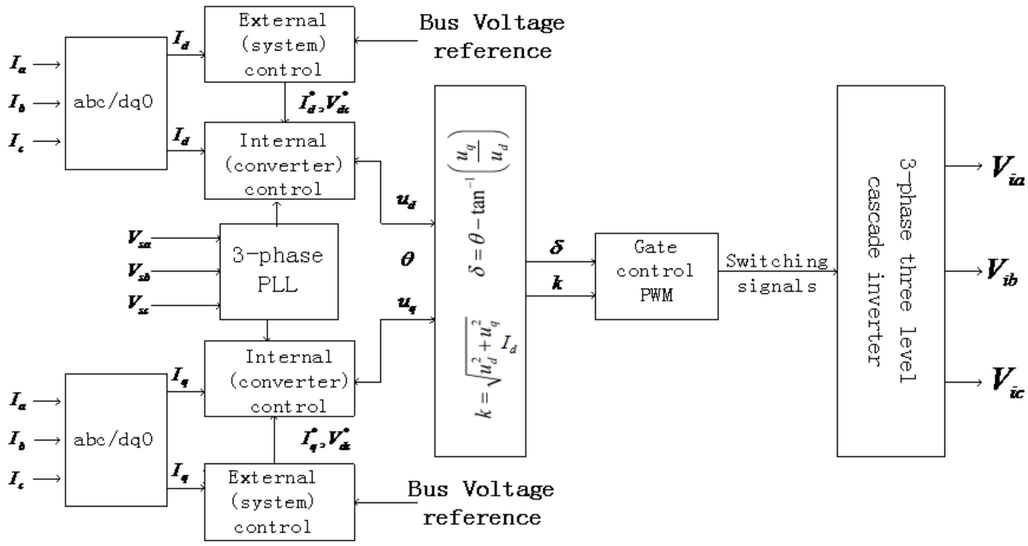

FIGURE 2. STATCOM control logic

system, which might suffer from actuation failure due to the various malfunctions of the elements/devices or component failuresduring system operation (i.e., base drive circuit failure, short-circuit, intermittent misfiring, inverter output phase failure, capacitor partially or completely breakdown, leak explosion, saturated inductance, resistance device blown).

It is therefore important to develop a fault-tolerant STATCOM to maintain satisfactory reactive power compensation operation under a failure event. To address this issue, we need to include the failure mode into the dynamic equation to get

$$\dot{y} = F(\cdot) + B(\cdot)u_a \tag{25a}$$

$$u_a(t) = p(t)u(t) + E(t) \tag{25b}$$

With $P(t) = diag\left\{\rho_d(t), \rho_q(t)\right\}$, $E(t) = [E_d(t),\ E_q(t)]^T$, here $P(t)$ is a time-varying and diagonal matrix related to actuator efficiency or "health indicator", $E(t)$ denotes a vector function corresponding to the portion of the control action produced by the actuator that is completely out of control, note that $E(t)$ might be time-varying. It is assumed that there exists some constant $a_E > 0$ such that $\|E(t)\| \leq a_E < \infty$. The actuator health indicators $\rho_d(t) \in (0, 1]$ and $\rho_q(t) \in (0, 1]$ are considered, with "0" indicating the total power loss, "1" corresponding to the healthy (no fault) actuation, and the value between 0 and 1 representing partial loss of power.

To derive the fault-tolerant control scheme, it is important to make the following observation.

Observation 2: *Let $\zeta(\cdot) = F(\cdot) + B(\cdot)E(\cdot)$. Since the matrix $B(\cdot)$ is diagonal and $B(\cdot)P(t) = diag\left\{B_{11}(\cdot)\rho_d(t), B_{22}(\cdot)\rho_q(t)\right\}$ is also diagonal, then*

i) *for all aforementioned actuator faults and disturbances, there exists some constant $\lambda > 0$ such that $0 < \lambda \leq \min\{g(\cdot)\rho_d(t), g(\cdot)\rho_q(t)\}$, and*
ii) *there exists some unknown constant $b > 0$ such that*

$$\|\zeta(\cdot)\| \leq b\varphi(\cdot) \tag{26}$$

where $\varphi(\cdot)$ is defined as before.

Proof: i) It can be shown easily. ii) Under the assumption on $E(t)$, it is straightforward to show that

$$\|\zeta(\cdot)\| \leq \|F(\cdot)\| + \|B(\cdot)E(t)\| \leq a\varphi(\cdot) + b_M a_E \leq b\varphi(\cdot) \tag{27}$$

where $b = a + b_M a_E$ and $b_M = \max\{g(\cdot)\}$. It is interesting to note that without altering any part of the previous robust adaptive control (20) and (21), asymptotically stable path tracking is still maintained in the presence of actuator faults, as formally stated in the following theorem.

Theorem 3.2. *Consider the inverter with actuator failures as imposed in (25). Let the control scheme be the same as in (20) and (21), then it is ensured that $y \to 0$ as $t \to \infty$.*

Proof: Inserting (16) into (25), one has

$$\begin{aligned} \dot{y} &= F(\cdot) + B(\cdot)u_a(t) \\ &= F(\cdot) + B(\cdot)E(t) - \frac{B(\cdot)P(t)\hat{b}\varphi(\cdot)y}{\|y\|} - B(\cdot)P(t)k_0 y \\ &= \xi(\cdot) - B(\cdot)P(t)k_0 y - \frac{B(\cdot)P(t)\hat{b}\varphi(\cdot)y}{\|y\|} \end{aligned} \tag{28}$$

Choose the Lyapunov function candidate

$$V(\cdot) = \frac{1}{2}y^T y + \frac{1}{2\xi\lambda}(\lambda\hat{b} - b)^2 \tag{29}$$

where $\lambda > 0$ is some unknown constant defined in Observation 2. With the adaptive algorithm given in (17) and using Observation 2, it can be shown that

$$\begin{aligned} \dot{V}(\cdot) &= y^T\dot{y} + \frac{1}{\xi}\dot{\hat{b}}(\lambda\hat{b} - b) \\ &= -y^T k_0 B(\cdot)P(t)y - \frac{y^T B(\cdot)P(t)y\varphi(\cdot)\hat{b}}{\|y\|} + y^T\xi(\cdot) + \frac{1}{\xi}\dot{\hat{b}}(\lambda\hat{b} - b) \\ &\leq -k_0\lambda\|y\|^2 - \hat{b}\varphi(\cdot)\frac{\lambda\|y\|^2}{\|y\|} + b\varphi(\cdot)\|y\| + \frac{1}{\xi}\dot{\hat{b}}(\lambda\hat{b} - b) \\ &= -k_0\lambda\|y\|^2 + (\lambda\hat{b} - b)\left(\frac{1}{\xi}\dot{\hat{b}} - \varphi(\cdot)\|y\|\right) = -k_0\lambda\|y\|^2 \leq 0 \end{aligned} \tag{30}$$

Then the result is established using the same argument as in the proof of Theorem 3.1.

Once u_d and u_q are generated by the proposed strategy, the modulation index k and phase of the modulation signals δ are then computed by

$$\delta = \theta - \tan^{-1}\left(\frac{u_q}{u_d}\right) k = \sqrt{u_d^2 + u_q^2} \tag{31}$$

Figure 2 illustrates the control logic and the analytical relation between the modulation index (k) and the phase angle of the modulation signals (δ) versus u_d and u_q.

Remark 3.3. *Note that the proposed control scheme contains a component of the form $\frac{y}{\|y\|}$, which might cause discontinuous in control action as y gets closer to zero. In order to ensure smooth and bounded control, a simple and useful modification is to replace $\frac{y}{\|y\|}$ with $\frac{y}{\|y\|+\varepsilon}$ where ε is a small constant. Meanwhile, in order to prevent estimate parameters drifting, the following updating algorithm can be used*

$$\dot{\hat{b}} = -\xi_1\hat{b} + \xi_2\frac{\|y\|^2\varphi(\cdot)^2}{\|y\|\varphi(\cdot) + \varepsilon}$$

where $\xi_1 > 0$ *and* $\xi_2 > 0$ *are design parameters chosen by the designer. The first term* $-\xi_1 \hat{b}$ *is to make suitable correction to prevent parameter drift. In this case uniformly ultimately bounded tracking is ensured.*

4. **Simulation Verification.** To verify and visualize the efficacy of the developed control scheme, numerical simulations under healthy and faulty actuators are conducted. Parameters related to operating conditions of STATCOM are given in Table 1, which is taken from [8].

TABLE 1. Circuit parameters for numerical simulation of STATCOM

Line voltage and frequency of grid	$200\sqrt{3}$V, 50Hz
Coupling inductance	2.89mH
Charging resistor	1kΩ/100w
DC link capacitance	3.3mF/400V
Discharging resistor	23.5kΩ
Carrier frequency	1.25kHz
Dead time for each power switch	4.5us
Reference DC voltage	340.8V

The desired d-q currents are set as

$$i_d^* = 0.4\cos(4\pi t) \quad i_q^* = 0.8\cos(4\pi t).$$

The control parameters are selected quite arbitrarily as $\beta = 0.15$ and $k_0 = 20$. Both healthy and faulty actuators are considered. The tracking performance under healthy actuators is shown in Figure 3 and the tracking performance under faulty actuators is shown in Figure 4. One can observe that satisfactory trajectory tracking is achieved with the proposed control scheme.

It is observed that the DC link voltage is maintained nearby the ideal value even in the case of time-varying exchanging reactive current. Thus, balanced reactive power for all phases is generated by the adaptively controlled STATCOM.

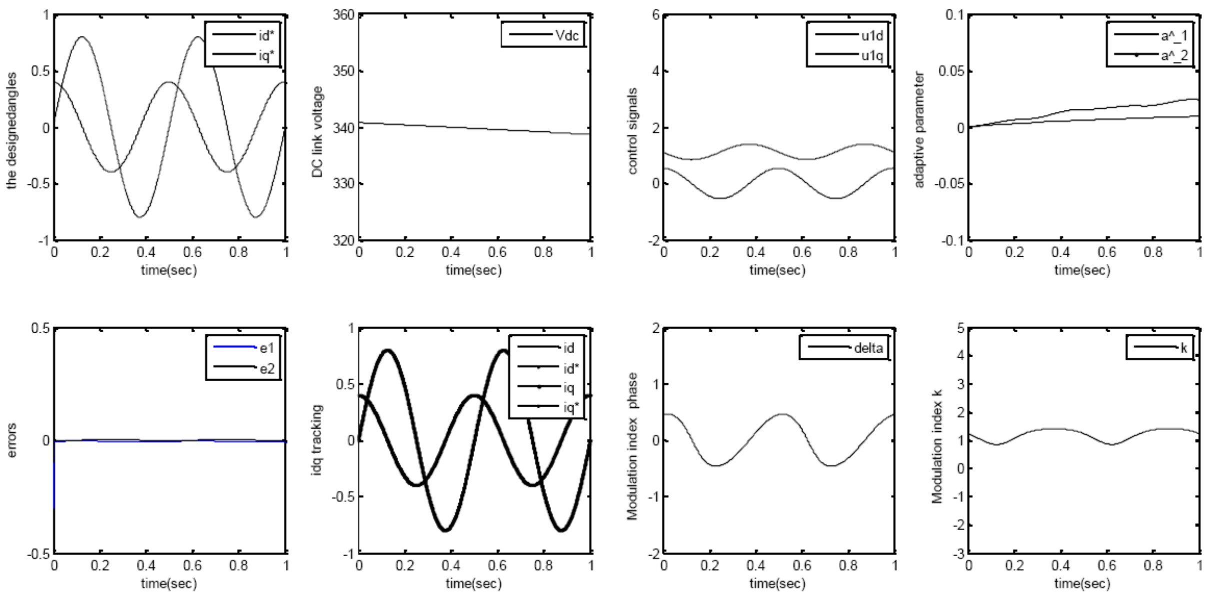

FIGURE 3. AC current tracking with healthy actuators

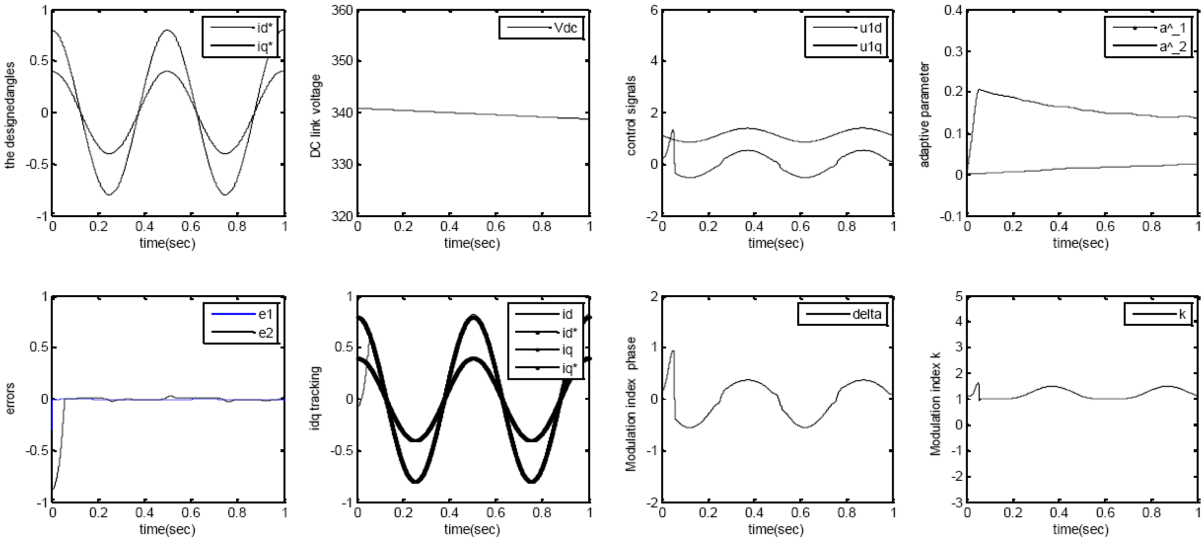

Figure 4. AC current tracking with actuator failures

5. **Conclusions.** The problem of reliable reactive power control via multilevel cascaded inverter based STATCOM for power system is investigated in this work. Adaptive and fault-tolerant control algorithms for STATCOM are derived without requiring any precise information of the system parameters. It is shown that the proposed control scheme is able to maintain high precision current tracking and is capable of supplying demanded reactive current for reactive power compensation in a short period of time, regardless of parameter uncertainties and actuator failures. Both theoretical analysis and numerical simulation demonstrate the benefits and effectiveness of the proposed approach.

Acknowledgment. This work is partially supported by the Major State Basic Research Development Program 973 (No. 2012CB215202) and the National Natural Science Foundation of China (No. 60974052, 61203124 and 61134001).

REFERENCES

[1] K. Wang and M. L. Crow, Power system voltage regulation via STATCOM internal nonlinear control, *IEEE Transactions on Power Systems*, vol.26, no.3, pp.1252-1262, 2011.

[2] L. Dong, M. L. Crow, Z. Yang, C. Shen, L. Zhang and S. Atcitty, A reconfigurable FACTS system for university laboratories, *IEEE Transactions on Power Systems*, vol.19, no.1, pp.120-128, 2004.

[3] M. Hagiwara, R. Maeda and H. Akagi, Negative-sequence reactive-power control by a PWM STATCOM based on a modular multilevel cascade converter (MMCC-SDBC), *IEEE Transactions on Industry Applications*, vol.48, no.2, pp.3728-3735, 2012.

[4] M. Kenan, Döşoğlu and A. Öztürk, Investigation of different load changes in wind farm by using FACTS devices, *Advances in Engineering Software*, vol.45, no.1, pp.292-300, 2012.

[5] P. Peticlair, S. Bahoha and J. P. Ferrieux, Optimized linearization via feedback control law for a STATCOM, *Industry Application Conference*, vol.2, pp.880-885, 1997.

[6] J. Jung, S. Lim and K. Nam, A feedback linearzing control scheme for a PWM converter-inverter having a small DC-link capacitor, *IEEE Transactions on Industry Applications*, vol.35, no.5, pp.1124-1131, 1999.

[7] D. Soto and R. Peña, Nonlinear control strategies for cascaded multilevel STATCOMs, *IEEE Transactions on Power Delivery*, vol.19, no.4, pp.1919-1927, 2004.

[8] R. Coteli, E. Deniz, S. Tuncer and B. Dandil, Three-level cascaded inverter based D-STATCOM using decoupled indirect current control, *IEEE Journal of Research*, vol.57, no.3, pp.207-214, 2011.

[9] W.-L. Chen, W.-G. Liang and H.-S. Gau, Design of a mode decoupling STATCOM for voltage control of wind-driven induction generator systems, *IEEE Transactions on Power Delivery*, vol.25, no.3, pp.1758-1767, 2010.

[10] A. Jain, K. Joshi, A. Behal and N. Mohan, Voltage regulation with STATCOMs: Modeling, control and results, *IEEE Transactions on Power Delivery*, vol.21, no.2, pp.726-735, 2006.

[11] R. Coteli, E. Deniz, B. Dandil, S. Tuncer and F. Ata, Phase angle control of three level inverter based D-STATCOM using Neuro-fuzzy controller, *Advances in Electrical and Computer Engineering*, vol.12, no.1, pp.77-84, 2012.

[12] M. J. Hossain, H. R. Pota and R. A. Ramos, Robust STATCOM control for the stabilisation of fixed-speed wind turbines during low voltages, *Renewable Energy*, vol.36, no.11, pp.2897-2905, 2011.

[13] L. Ding and Y. Miao, Research of control strategy of STATCOM in AC/DC hybrid power system, *Proc. of Energy*, pp.437-442, 2011.

[14] C. S. de Araujo and J. C. Castro, Vectors analysis and control of advanced static VAR compensators, *IEE Proc. of C Generations, Transmission and Distribution*, vol.140, no.4, pp.299-306, 1993.

[15] A. H. Norouzi and A. M. Sharaf, Two control scheme to enhance the dynamic performance of the STATCOM and SSSC, *IEEE Transactions on Power Delivery*, vol.20, no.1, pp.435-442, 2005.

[16] R. Mienski, R. Pawelek and I. Wasiak, Shunt compensation for power quality improvement using a STATCOM controller: Modeling and simulation, *IEE Proc. of C Generations, Transmission and Distribution*, vol.151, no.2, pp.274-280, 2004.

[17] F. Z. Peng and J.-S. Lai, Dynamic performance and control of a static var generator using cascade multilevel inverters, *IEEE Transactions on Industrial Applications*, vol.33, no.3, pp.748-755, 1997.

[18] W. Song and A. Q. Huang, Fault-tolerant design and control strategy for cascaded H-bridge multilevel converter-based STATCOM, *IEEE Transactions on Industrial Electronics*, vol.57, no.8, pp.2700-2708, 2010.

[19] L. L. Fan and Y. D. Song, Fault-tolerant control and disturbance attenuation of a class of nonlinear systems with actuator and component failures, *Acta Automatic Sinica*, vol.37, no.5, pp.623-628, 2011.

[20] Y. Song, H. Chen and D. Li, Virtual point based fault-tolerant lateral and longitudinal control of 4W-steering vehicles, *IEEE Transactions on Intelligent Transportation Systems*, vol.12, no.4, pp.1343-1351, 2011.