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A DESIGN OF FIR FILTERS WITH VARIABLE NOTCHES CONSIDERING REDUCTION METHOD OF POLYNOMIAL COEFFICIENTS FOR REAL-TIME SIGNAL PROCESSING

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Abstract. *In this paper, we propose a design method for finite impulse response (FIR) variable digital filters (VDFs) obtaining equiripple characteristic, even if notch frequencies change. The changes in filter coefficients for changes in notch frequencies are approximated by polynomials using variable parameters. In this method, the minimization problem for the polynomial coefficients with variable notches is formulated as semidefinite programming (SDP) in the frequency domain. As these variable parameters are varied continuous for defined ranges, the amplitude response of the proposed VDF continuously changes. On the other hand, the number of polynomial coefficients increases when variable parameters are a plural number or high order. Therefore, we also propose a method for reducing the number of polynomial coefficients. In addition, we present that the proposed VDF can be implemented using the Farrow structure, which is suitable for real-time signal processing. The usefulness of the proposed VDF is demonstrated through examples.*
Keywords: FIR filters, Variable notches digital filters, SDP method, Farrow structure

1. **Introduction.** Recently, digital signal processing is required in various fields in [13-16]. In this paper, the filter can instantly change the frequency characteristics in digital signal processing which is referred to as variable digital filters (VDFs). The VDFs are important in fields such as communications, measurement, sound, and image processing [2]. Recently, several algorithms for the design of such filters have been proposed [2-6].

In the field of instrument and control, automatically dynamic measurement systems to measure object weights are used in logistics and food industry, etc. The systems are composed of belt conveyor and road cell, and is called checkweigher. Checkweighers have a problem that noises of error factor change according to speed of belt conveyor and installation surrounding. Digital filters are used to reduce these noises with the aim of high accuracy and high speed measurement [5-8]. Digital filters can be classified as finite impulse response (FIR) filters or infinite impulse response (IIR) filters. FIR filters with exactly linear-phase characteristics are important for applications such as waveform transmission and image processing [1]. To perform an exact measurement, we need the filter with high stopband attenuation. FIR filters with high stopband attenuation need high filter's order. Therefore, the delay for the calculation becomes very large. To reduce the delay, we designed a linear-phase FIR filters having a piecewise high attenuation characteristics in the stopband, unlike the overall stopband that achieves high attenuation. The range of the high attenuation in the stopband needs to vary according to differences of the measurement environments and the measurement objects for high speed and high

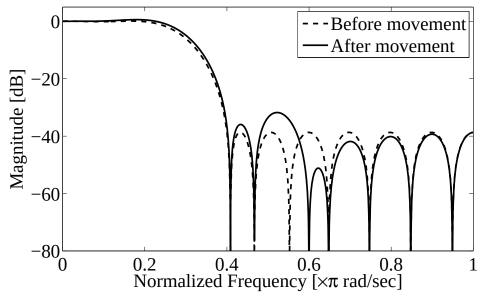

Figure 1. The amplitude response of filter when a notch moved

accuracy measurement. Therefore, we proposed a linear-phase FIR VDF that changes the range of high attenuation in the stopband by using variable parameters [5, 6]. As a result, the filter has the same measurement accuracy as a filter with high attenuation in the overall stopband, and it can be implemented with fewer filter's order. A highly accurate measurement also becomes possible by applying a notch to the principal noise frequency leading to measurement error. In this case, it is necessary to change the notch frequency according to the measurement environments and measurement objects. The method in [8] is used moving average filters and adaptive IIR notch filters to reduce noise. However, moving average filters have very low attenuations in the stopband. Moreover, an adaptive IIR notch filter with only a single notch cannot reduce noises generated by various factors. FIR filters are preferable over IIR notch filters for continuous measurements because IIR notch filters continue transient phenomena. Then, we are considered that notch positions of FIR filters are moved to the principal noise frequency. If the position of the notch in the FIR filter is simply moved as shown in the solid line of Figure 1, the amplitude characteristic in the stopband is undesirable. As a result, it is impossible to measure the same accuracy as the filter before the notch is moved. FIR VDFs have never been proposed which the amplitude characteristics do not change, even if notch frequency changes.

In this paper, we present a transfer function for a linear-phase equiripple FIR VDF which does not change the amplitude characteristics of the filter, even if notch frequencies change. The changes in filter coefficients for changes in notch frequencies are approximated by polynomials using variable parameters. In this method, the minimization problem for filter coefficients with variable notches is formulated as semidefinite programming (SDP) in the frequency domain. These variable parameters have arbitrary real numerical values. Furthermore, as these variable parameters are varied continuous for defined ranges, the amplitude response of the proposed VDF continuously changes. On the other hand, the number of polynomial coefficients increases when the variable parameters are a plural number or high order. Therefore, we also propose a method for reducing the number of polynomial coefficients. The values of the term of high order in the polynomial are very small when the polynomial orders are high. As a result, the polynomial coefficients can be reduced. In addition, we demonstrate that the proposed VDF can be implemented using the Farrow structure, which is suitable for real-time signal processing. We confirm the effectiveness of the proposed method through design examples and a filtering result.

2. **Design Problem.** In general, the transfer function of the FIR digital filter of order $2N$ is

$$H(z) = \sum_{k=0}^{2N} a(k)\, z^{-k}, \tag{1}$$

where $a(\cdot)$ is the real filter coefficient.

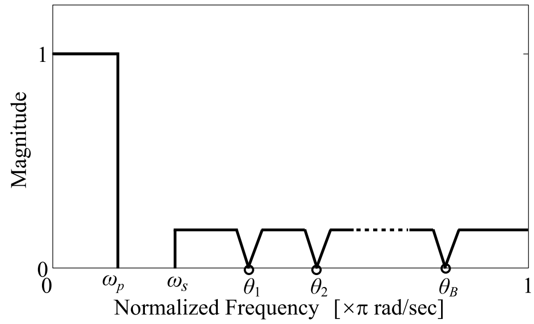

FIGURE 2. Desired characteristics

Now, in order to change some variable notches, Equation (1) is transformed using some variable parameters $(\theta_1, \theta_2, \cdots, \theta_B)$.

$$H(z, \theta_1, \theta_2, \cdots, \theta_B) = \prod_{b=1}^{B} \left\{1 - 2\cos(\theta_b \pi) z^{-1} + z^{-2}\right\} \sum_{k=0}^{2N-2B} \tilde{a}(k, \theta_1, \theta_2, \cdots, \theta_B) z^{-k} \ , \tag{2}$$

where $\tilde{a}(k, \theta_1, \cdots, \theta_B)$ changes when parameter θ_b changes, the ranges of each variable parameter θ_b are

$$\begin{array}{l} \theta_1 \in [\theta_{1,\min}, \theta_{1,\max}] \\ \theta_2 \in [\theta_{2,\min}, \theta_{2,\max}] \\ \quad \vdots \\ \theta_B \in [\theta_{B,\min}, \theta_{B,\max}] \end{array} \ , \tag{3}$$

and each θ_b is limited $0 < \theta_b < 1$. Then, the coefficient of the filter with variable notches $\tilde{a}(k, \theta_1, \cdots, \theta_B)$ is assumed to be a linear combination of some basic functions of the variable parameters θ_b and the polynomial coefficients (sub filter coefficients) $h(k, l_1, l_2, \cdots, l_B)$, as given by

$$\tilde{a}(k, \theta_1, \theta_2, \cdots, \theta_B) = \sum_{l_1=0}^{L_1} \sum_{l_2=0}^{L_2} \cdots \sum_{l_B=0}^{L_B} h(k, l_1, l_2, \cdots, l_B) \theta_1^{l_1} \theta_2^{l_2} \cdots \theta_B^{l_B} \ , \tag{4}$$

where $L_1, L_2, \cdots, L_B$ are order of the polynomial. As these parameters are varied continuous for the defined ranges, the amplitude response of FIR VDF with variable notches also continuously changes. For notational simplicity, we assume that filter is type 1 and B is 2 in the following.

Substituting Equation (4) into Equation (2), the frequency response of the proposed linear-phase FIR VDF with two variable notches is

$$H(\omega, \theta_1, \theta_2) = \prod_{b=1}^{2} 4\left\{\cos(\omega) - \cos(\theta_b \pi)\right\} \sum_{k=0}^{N-2} \sum_{l_1=0}^{L_1} \sum_{l_2=0}^{L_2} g(k, l_1, l_2) \theta_1^{l_1} \theta_2^{l_2} \cos(k\omega) \ , \tag{5}$$

where

$$g(k, l_1, l_2) = \begin{cases} h(N-2, l_1, l_2) & k = 0 \\ 2h(N-2 \pm k, l_1, l_2) & \text{otherwise} \end{cases} . \tag{6}$$

We assume that the desired amplitude response, shown in Figure 2, is denoted by

$$D(\omega) = \begin{cases} 1 & 0 \le \omega \le \omega_p \\ 0 & \omega_s \le \omega \le \pi \end{cases} \ , \tag{7}$$

where ω_p and ω_s are the normalized angular frequencies in the passband edge and the stopband edge, respectively.

Next, we define an error function

$$e(\omega, \theta_1, \theta_2) = D(\omega) - H(\omega, \theta_1, \theta_2). \tag{8}$$

We minimize the error function $e(\omega, \theta_1, \theta_2)$ in the minimax sense. That is

$$\underset{\mathbf{G}}{\text{minimize}} \left[\max_{\omega \in \Omega} |e(\omega, \theta_1, \theta_2)| \right], \tag{9}$$

where

$$\mathbf{G} = [\underbrace{\, g(0,0,0) \quad \cdots \quad g(k, l_1, l_2) \quad \cdots \quad g(N-2, L_1, L_2) \,}_{(N-1)(L_1+1)(L_2+1)}], \tag{10}$$

and Ω is the frequency of interest $\Omega = [0, \omega_p] \cup [\omega_s, \pi]$. To compute the design problem Equation (9) using computers, we digitize the frequency ω and the variable parameter of notch θ_1 and θ_2. That is, the discrete version of error function Equation (8) is defined as

$$e(\omega_m, \theta_{1,q}, \theta_{2,r}) = D(\omega_m) - H(\omega_m, \theta_{1,q}, \theta_{2,r}), \tag{11}$$

where M, Q and R are the number of evaluation points, and $1 \leq m \leq M$, $1 \leq q \leq Q$, $1 \leq r \leq R$.

Therefore, the minimization problem in Equation (9) can be reformulated as

$$\text{minimize} \quad \lambda \tag{12}$$

$$\text{subject to} \quad e(\omega_m, \theta_{1,q}, \theta_{2,r})^2 \leq \lambda \tag{13}$$

where λ is the maximum allowable error.

Using Schur complement, the constraints in Equation (13) are equivalent to

$$\boldsymbol{\Gamma}(\omega_m, \theta_{1,q}, \theta_{2,r}) = \begin{bmatrix} \lambda & e(\omega_m, \theta_{1,q}, \theta_{2,r}) \\ e(\omega_m, \theta_{1,q}, \theta_{2,r}) & 1 \end{bmatrix} \succeq \mathbf{0}. \tag{14}$$

Therefore, a discrete version of minimizing in Equations (12)-(14) is given as follows:

$$\text{minimize } \mathbf{d}\mathbf{x}^T, \tag{15}$$

$$\text{subject to } \mathbf{U}(\mathbf{x}) \succeq \mathbf{0}, \tag{16}$$

where

$$\mathbf{d} = [1 \quad \underbrace{0 \ \cdots \ 0}_{(N-1)(L_1+1)(L_2+1)}], \tag{17}$$

$$\mathbf{U}(\mathbf{x}) = \text{diag}[\ \boldsymbol{\Gamma}(\omega_1, \theta_{1,1}, \theta_{2,1}) \ \cdots \ \boldsymbol{\Gamma}(\omega_m, \theta_{1,q}, \theta_{2,r}) \ \cdots \ \boldsymbol{\Gamma}(\omega_M, \theta_{1,Q}, \theta_{2,R})\ ], \tag{18}$$

$$\mathbf{x} = [\lambda \quad \mathbf{G}]^T. \tag{19}$$

Since matrix $\mathbf{U}(\mathbf{x})$ is affine with respect to $\mathbf{x}$, Equations (15)-(19) are a SDP problem as in [11]. However, the number of polynomial coefficients increases if the number of variable parameters increases or polynomial orders are high. Next, we propose a method for a reducing the number of polynomial coefficients.

3. **Method for Reducing the Number of Polynomial Coefficients.** In this section, we show a method for reducing the number of polynomial coefficients.

The filter coefficient $\tilde{a}(\cdot)$ in Equation (4) is composed of the sum of the product of polynomial coefficients $h(\cdot)$ and power of variable parameters θ_b. Here, the value of each variable parameters θ_b is $0 < \theta_b < 1$. $\theta_b^{l_b}$ becomes very small when l_b in Equation (4) is large. In addition, the value of the product of the many variable parameters in Equation (4), $\theta_1^{l_1}\theta_2^{l_2}\cdots\theta_B^{l_B}$, is also very small. Therefore, in the term of the large power of many variable parameters, because the influence given to the variable coefficient is a small, there is a possibility that the polynomial coefficients can be reduced like a multivariable Taylor expansion.

The filter coefficient $\tilde{a}(\cdot)$ in Equation (4) is redefined as follows:

$$\tilde{a}\left(k,\theta_1,\theta_2,\cdots,\theta_B\right) = \sum_{y_1=0}^{Y_1}\sum_{y_2=0}^{Y_2}\cdots\sum_{y_B=0}^{Y_B} h\left(k,y_1,y_2,\cdots,y_B\right)\theta_1^{y_1}\theta_2^{y_2}\cdots\theta_B^{y_B}, \tag{20}$$

$$Y_b = \begin{cases} K - \sum_{j=1}^{b-1} y_j & \text{if } K - \sum_{j=1}^{b-1} y_j \le L_b \\ L_b & \text{if } K - \sum_{j=1}^{b-1} y_j > L_b \end{cases}, \tag{21}$$

where K is

$$K \le \sum_{b=1}^{B} L_b. \tag{22}$$

If $b-1$ is equal to zero in Equation (21), the Σ terms will also equal to zero. The number of coefficients of the polynomial can be reduced by limiting the value of K to be smaller than the sum of the orders of a polynomial. That is, the total number of polynomial coefficients in Equations (20)-(22) is less than the number of polynomial coefficients in Equation (4).

4. **Farrow Structure.** The frequency response of the proposed VDFs with variable parameters when the number of polynomial coefficients is reduced, that is given by

$$H\left(z,\theta_1,\theta_2\right) = \prod_{b=1}^{2}\left\{1 - 2\cos\left(\theta_b\pi\right)z^{-1} + z^{-2}\right\}\sum_{k=0}^{2N-4}\sum_{y_1=0}^{Y_1}\sum_{y_2=0}^{Y_2} h\left(k,y_1,y_2\right)\theta_1^{y_1}\theta_2^{y_2}z^{-k}. \tag{23}$$

The frequency response has fixed polynomial coefficients $h\left(k,y_1,y_2\right)$ with variable parameters θ_b. Then, Equation (23) can be rearranged as

$$H\left(z,\theta_1,\theta_2\right) = \prod_{b=1}^{2}\left\{1 - 2\cos\left(\theta_b\pi\right)z^{-1} + z^{-2}\right\}\sum_{y_1=0}^{Y_1}\theta_1^{y_1}\sum_{y_2=0}^{Y_2}\theta_2^{y_2}\sum_{k=0}^{2N-4} h\left(k,y_1,y_2\right)z^{-k}. \tag{24}$$

We apply the Farrow structure in [9, 10] to the transfer function given by Equation (23). Figures 3 and 4 show the proposed structure. These figures clearly show that the operation of the Y_bth power of variable parameters θ_b becomes unnecessary for each coefficient $h(k,y_1,y_2)$. The proposed structure has a fixed number of multiplications for filter coefficients of same number of direct structure in Equation (23). However, the proposed structure requires fewer multiplications for variable parameters θ_b than the direct structure. Consequently, the proposed structure requires only a small number of multiplications to obtain a new frequency characteristics, which is particularly suitable

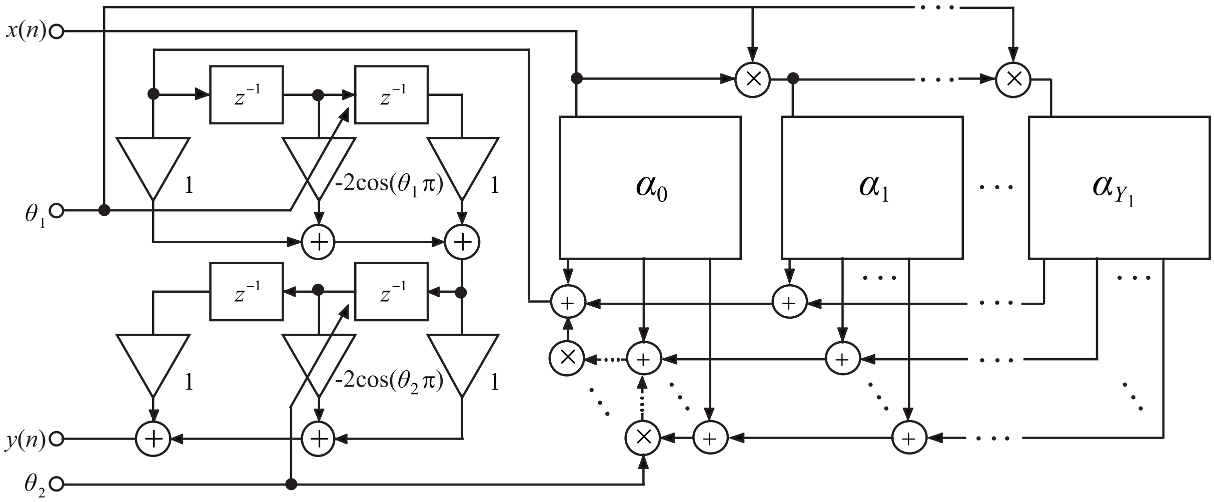

FIGURE 3. The proposed Farrow structure

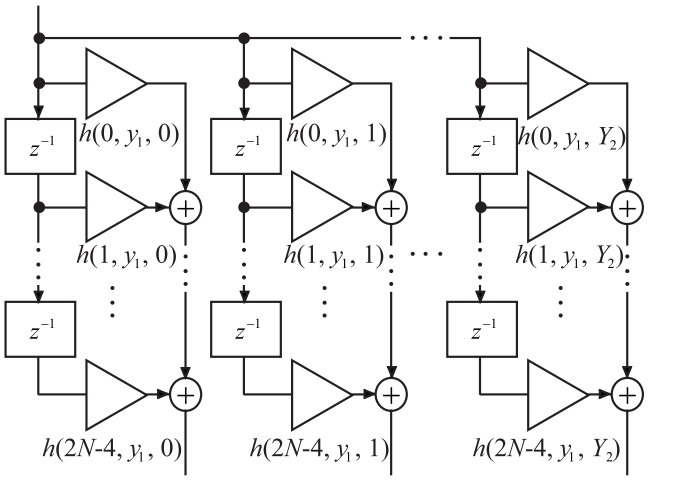

FIGURE 4. Structure of α_{y_1} in Figure 3

for high-speed tuning. Note that, we show the structure with two parameters in this paper, the proposed structure can be applied to more than two parameters.

5. **Design Example.** In this section, we show design examples to demonstrate the efficiency of the proposed VDF with variable notches. These design examples have been designed using a workstation that runs 64-bit Linux (CentOS 5.3) and has a 3.33-GHz Intel Xeon W5590 quad-core processor and a main memory of 48 GB. SeDuMi [12] was used to solve SDP. Moreover, we compare normal version with the proposed method in Section 3. VDF is designed to satisfy the following specifications:

Order of filter: $2N = 70$
Order of polynomial: $L_1 = 3$, $L_2 = 3$
Passband edge: $\omega_p = 0.2\pi$
Stopband edge: $\omega_s = 0.3\pi$
Variable notch range: $\theta_1 = 0.4 \sim 0.5$, $\theta_2 = 0.75 \sim 0.85$

In this example, 450 grid points were used with 100 points in the passband and 350 points in the stopband. Evaluation points of the variable parameters for each moving notches were equally divided into 5. Figures 5 and 6 show the amplitude response of the

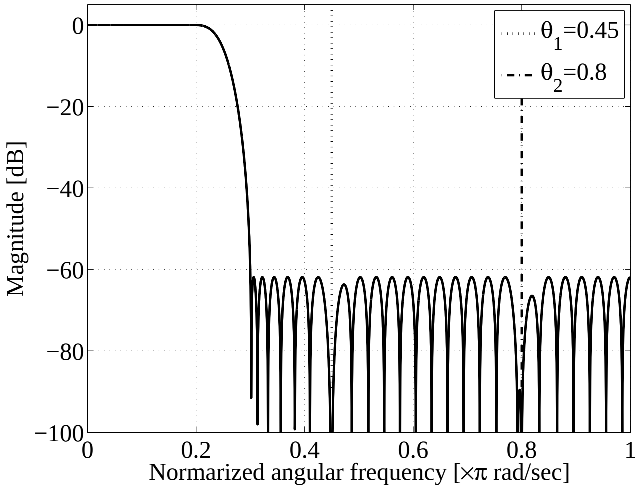

FIGURE 5. Amplitude response of $\theta_1 = 0.45$, $\theta_2 = 0.8$ in design example

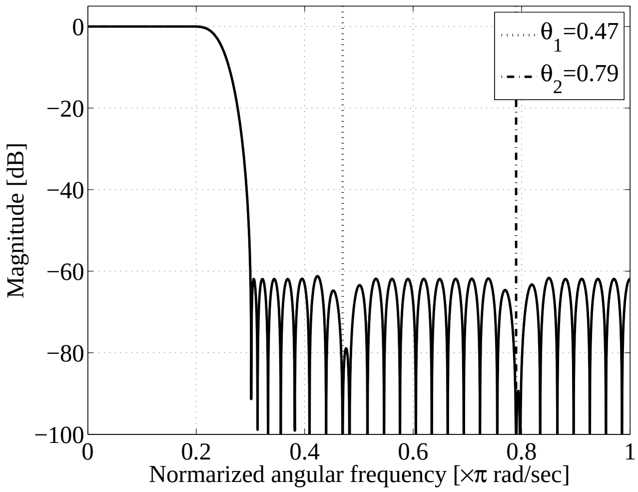

FIGURE 6. Amplitude response of $\theta_1 = 0.47$, $\theta_2 = 0.79$ in design example

proposed VDF. It is clear from Figures 5 and 6, the frequencies of the notches have changed depending on the variable parameters. Moreover, even if notch frequencies change, the amplitude responses have equiripple characteristics. The design time in this example was 23.0 minutes.

Now, we show the design example using a method for reducing the number of polynomial coefficients in Section 3. In this example, we set the limiting value $K = 4$. Figures 7 and 8 show the amplitude response of the proposed VDF. It is clear from Figures 7 and 8, the amplitude responses of the obtained filter using the proposed method for reducing the number of polynomial coefficients have equiripple characteristics. Table 1 shows minimum attenuations in the stopband of obtained using normal version and the proposed method in Section 3. The minimum attenuations in both versions are almost the same values. Now, we compare the number of polynomial coefficients each version. Normal version is 1072 and reducing version is 871. As a result, the proposed reducing method can effectively reduce the number of polynomial coefficients. The design time in this example was 16.3 minutes.

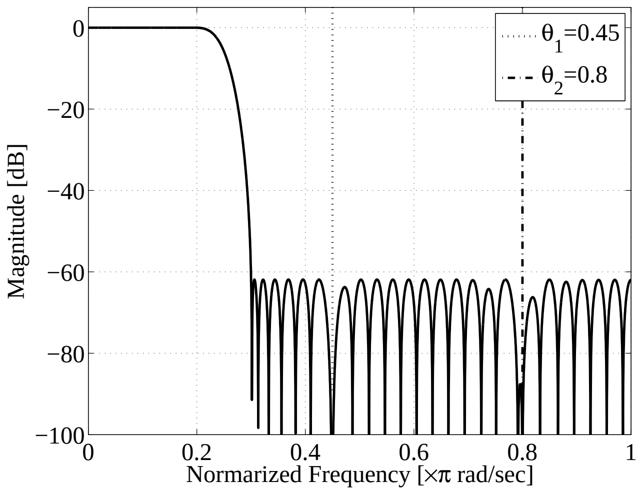

FIGURE 7. Amplitude response of $\theta_1 = 0.45$, $\theta_2 = 0.8$ in design example using a method for reducing the number of polynomial coefficients

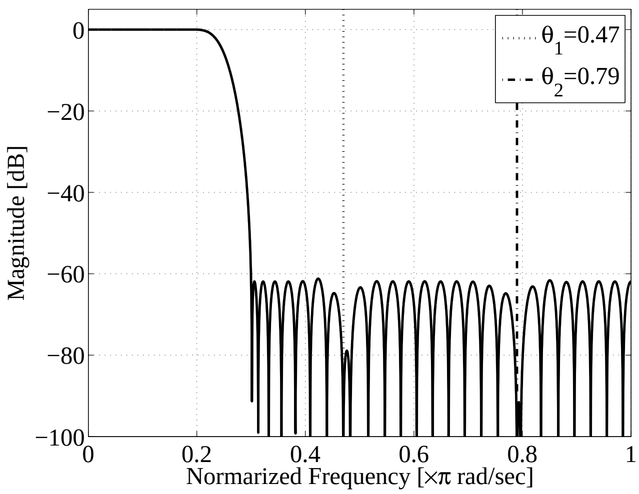

FIGURE 8. Amplitude response of $\theta_1 = 0.47$, $\theta_2 = 0.79$ in design example using a method for reducing the number of polynomial coefficients

In addition, the proposed method has the advantage that the proposed VDF does not need to be redesign only by changing the parameters, even if the demanded filter characteristics changed.

We compare the proposed VDF with the linear-phase FIR filter. Figure 9 shows waveform in the time domain after filtering by the proposed VDF. In addition, Figure 10 shows waveform in the time domain after filtering by the linear-phase FIR filter which is designed by Remez algorithm. In the field of instrument and control, a value of three times of standard deviation (three-sigma) is generally used to evaluate the performance of measurement hardware. Then, we show the three-sigma between the proposed VDF and the linear-phase FIR filter in Table 2. As a result, it is clear from Figures 9, 10 and Table 2 that the proposed VDF effectively reduces the noises compared with the linear-phase FIR filter.

6. **Conclusion.** In this paper, we proposed a design method for linear-phase equiripple FIR VDF which does not change those frequency characteristics, even if notch frequencies

TABLE 1. Comparing minimum attenuations at the stopband in design example

	$\theta_1 = 0.45$, $\theta_2 = 0.8$	$\theta_1 = 0.47$, $\theta_2 = 0.79$
Normal version	-61.8695 dB	-61.2118 dB
Reducing version	-61.8726 dB	-61.1984 dB

TABLE 2. Comparing three-sigma values between the proposed VDF and the linear-phase FIR filter

	Proposed VDF	Linear-phase FIR filter
Three-sigma	0.2785	0.7754

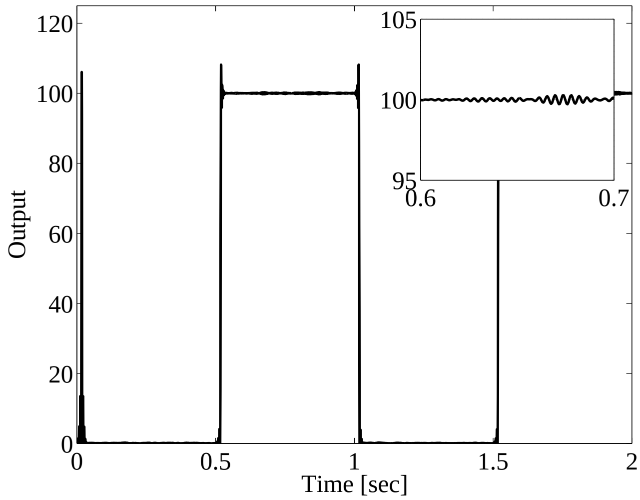

FIGURE 9. Filtering result using the proposed VDF

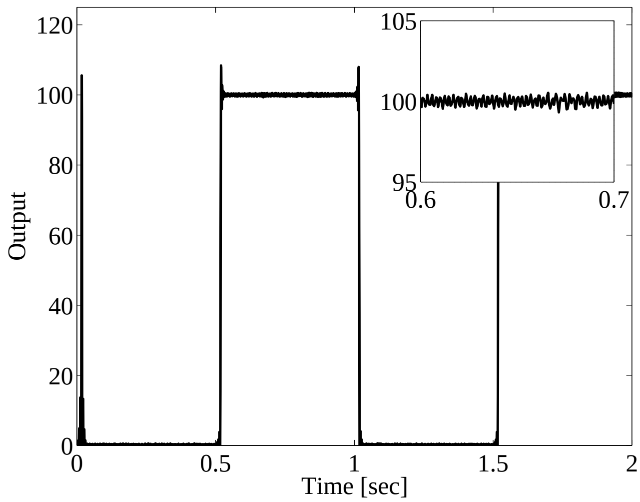

FIGURE 10. Filtering result using the linear-phase FIR filter

change. The changes in filter coefficients for changes in notch frequencies are approximated by polynomials using variable parameters. The variable parameters are varied continuous for defined ranges. In the proposed design method, the minimization problem for the coefficients of the filter with variable notches was formulated as SDP in the frequency domain. Then the amplitude response of the proposed VDF continuously can be changed according to variable parameter values.

Furthermore, we also proposed a method for reducing the number of polynomial coefficients. In addition, we presented a suitable structure for real-time signal processing. We demonstrated the effectiveness of the proposed method through numerical examples.

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