

## STUDY ON FAST TERMINAL SLIDING MODE CONTROL FOR A HELICOPTER VIA QUANTUM INFORMATION TECHNIQUE AND NONLINEAR FAULT OBSERVER

FUYANG CHEN<sup>1</sup>, RUI HOU<sup>1</sup>, BIN JIANG<sup>1</sup> AND GANG TAO<sup>2</sup>

<sup>1</sup>College of Automation Engineering  
Nanjing University of Aeronautics and Astronautics  
No. 29, Yudao Street, Baixia District, Nanjing 210016, P. R. China  
{ chenfuyang; binjiang }@nuaa.edu.cn; ranger\_hr@163.com

<sup>2</sup>Department of Electrical and Computer Engineering  
University of Virginia  
Charlottesville, VA 22903, USA  
gt9s@class6.ee.Virginia.edu

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**ABSTRACT.** *In this paper, a novel direct self-repairing controller via quantum information technique and nonlinear fault observer is designed, which addresses tracking control and convergence rate problem of a helicopter. First, a nonlinear fault observer is designed to diagnose the helicopter's fault. Next, in order to improve the convergence rate and stabilization of the helicopter system, an exponential and symbolic function is introduced to design the novel fast terminal sliding mode surface. Then, a novel method based on the newly FTSM surface is designed. An adaptive approach is adopted to estimate upper bound of the fault. The higher tracking performance and faster convergence rate are achieved by the proposed method. In addition, quantum information technique is used to develop the direct self-repairing control law, which can increase the self-repairing control accuracy of the helicopter in faulty case and improve its strong anti-interference ability. Finally, several numerical simulation results have shown the satisfactory control performance.*

**Keywords:** Direct self-repairing controller, Fast terminal sliding mode, Helicopter, Nonlinear fault observer, Quantum information technique

**1. Introduction.** Helicopter is a nonlinear, strong coupling, time-varying complex system. As the helicopter's flight parameters change in the rapidity [1,2], the stability and reliability cannot be realized by conventional control [3,4]. The actuator faults of the helicopter may lead to catastrophic events. In order to maintain the specified performance of the helicopter, a fault tolerant control approach has been proposed.

Sliding mode control (SMC) has robustness for disturbance and model uncertainty [5,6]. Therefore, it is widely applied to solve control problems of the motor and the robotic arm [7,8]. Because the conventional SMC adopts the linear sliding mode surface, the convergence rate of system states is slow. When the systematic biases become large, the control results based on SMC are not satisfactory [9]. By introducing terminal attractors of neural networks, a nonlinear terminal sliding mode (TSM) surface has been designed in reference [10]. The states of system can converge to the equilibrium point from any initial position. The fast terminal sliding mode control (FTSMC) has been introduced in references [11,12]. Fast convergence rate of the system states can be retained when the initial state is far from the equilibrium point. Obviously, it is of theoretical significance to design the FTSMC.

Considering a strong anti-interference, a quantum control technology is introduced in the paper. The study of quantum control technology [13,14] has been a hot research topic increasingly, in which the applied research on quantum information technique is very wide [15,16]. In this paper, the quantum information technique is used to develop the direct self-repairing control law, which can increase the self-repairing control accuracy of the helicopter in faulty case and improve the anti-interference ability. Therefore, the novel FTSM surface and the new DSRC are designed in this paper. First, the nonlinear fault observer (NFO) is introduced to observe the fault of the system. By selecting the design parameters, the observed error can converge exponentially. Second, the exponential and symbolic functions are introduced to design the novel FTSM surface which is applied to design DSRC. In addition, an adaptive approach is used to estimate upper bound of the uncertain fault. The quantum information technique is adopted to develop the direct self-repairing control law simultaneously. The global stability of DSRC system is achieved in a finite time. Compared with the traditional TSM controller, faster convergence rate and higher tracking performance are realized by the proposed method. The effectiveness of the method is verified by several numerical simulations. The method proposed by the paper not only can be applied to the tracking control of helicopters, but also can be used to other nonlinear systems.

**2. Problem Statement for the Helicopter.** A helicopter can have pitch, yaw and roll movement. In this section, the pitch movement of the helicopter is considered as the control object. The model of pitch movement can be expressed as

$$\dot{x}_1 = x_2 \quad (1)$$

$$\dot{x}_2 = g(X)u + f(X) \quad (2)$$

$$y = x_1 \quad (3)$$

where  $X = [x_1, x_2]^T \in R^2$  is state vector;  $u(t) \in R$  is control input vector.  $y \in R$  is output vector.  $g(X)$  and  $f(X)$  are known nonlinear function.

In fact, the faults of the helicopter always occur in actual operation. Therefore, the fault model should be established when the faults happen [17]. The corresponding control law needs to be designed. When the faults are considered, the fault model of the helicopter can be expressed as

$$\dot{x}_1 = x_2 \quad (4)$$

$$\dot{x}_2 = g(X)u + f(X) + d(X) \quad (5)$$

$$y = x_1 \quad (6)$$

In Equation (5),  $d(X)$  is the sum of failure factor and unknown external disturbance.

**Remark 2.1.** In Equation (5),  $d(X)$  has its upper bound which satisfies  $\|d(X)\| \leq \eta$ .  $\eta$  is positive constant.

The control objective is to obtain a controller and achieve fast convergence rate and tracking precisely.

**3. Quantum Control Module.** In quantum computation,  $|0\rangle$  and  $|1\rangle$  denote the two basic states of micro-particles, which are named as quantum bits (qubits). Arbitrary single-qubit state can be expressed as the linear combination of two basic states. The state of qubit is not only  $|0\rangle$  and  $|1\rangle$ , but also a linear combination of the state, which is usually called as superposition state, namely,

$$|\varphi\rangle = \alpha \cdot |0\rangle + \beta \cdot |1\rangle \quad (7)$$

where,  $\alpha$  and  $\beta$  are a pair of complex, called as the probability amplitude of quantum state, namely, the measurement result in quantum state  $|\varphi\rangle$  collapsing  $|0\rangle$  with a probability of  $|\alpha|^2$  or collapsing  $|1\rangle$  with a probability of  $|\beta|^2$ , and satisfying

$$|\alpha|^2 + |\beta|^2 = 1 \tag{8}$$

Therefore, quantum state can be also denoted by the probability amplitude of quantum state in the form of  $|\varphi\rangle = [\alpha, \beta]^T$ . Obviously, when  $\alpha = 1, \beta = 0$  in (7),  $|\varphi\rangle$  is the basic state  $|0\rangle$ , which can be described by  $|\varphi\rangle = [1, 0]^T$ . Otherwise, when  $\alpha = 0, \beta = 1$  in (7),  $|\varphi\rangle$  is the basic state  $|1\rangle$ , which can be described by  $|\varphi\rangle = [0, 1]^T$ . Generally speaking, quantum state is the unit vector of two-dimensional complex vector space.

As the collapse of quantum states is caused by observation, the quantum bits can be seen as a continuous state between  $|0\rangle$  and  $|1\rangle$ , until it has been observed. The existence of continuous state qubit and behavior have been confirmed by a large number of experiments. And there are many different physical systems which can be used to realize quantum bits.

Similar to the single-qubit state, double-quantum-bit state can be expressed as

$$|\varphi\rangle = \alpha_{00} \cdot |00\rangle + \alpha_{01} \cdot |01\rangle + \alpha_{10} \cdot |10\rangle + \alpha_{11} \cdot |11\rangle \tag{9}$$

with the probability amplitude satisfying

$$|\alpha_{00}|^2 + |\alpha_{01}|^2 + |\alpha_{10}|^2 + |\alpha_{11}|^2 = 1 \tag{10}$$

Similarly, three-qubit state can be expressed as

$$\begin{aligned} |\varphi\rangle = & \alpha_{000} \cdot |000\rangle + \alpha_{001} \cdot |001\rangle + \alpha_{010} \cdot |010\rangle + \alpha_{011} \cdot |011\rangle \\ & + \alpha_{100} \cdot |100\rangle + \alpha_{101} \cdot |101\rangle + \alpha_{110} \cdot |110\rangle + \alpha_{111} \cdot |111\rangle \end{aligned} \tag{11}$$

and the probability amplitude satisfying

$$|\alpha_{000}|^2 + |\alpha_{001}|^2 + |\alpha_{010}|^2 + |\alpha_{011}|^2 + |\alpha_{100}|^2 + |\alpha_{101}|^2 + |\alpha_{110}|^2 + |\alpha_{111}|^2 = 1 \tag{12}$$

Helicopter’s uncertainties can be divided into failure factor  $f_1$  and unknown external disturbance  $d_1$ . In this study, the quantum control module is designed for helicopter’s uncertainties using the quantum bits state of quantum control technique. The quantum control module in Figure 1 realizes the three quantum bits state description and control. The specific description of three quantum bits probability amplitude for the helicopter quantum control module can be seen in Table 1.

TABLE 1. Three quantum bit probability amplitude for quantum control module

Probability amplitude	Input	Fault	Disturbance
	(Yes/No)	(Yes/No)	(Yes/No)
$d$	$u$	$f_1$	$d_1$
$\alpha_{000}$	No	No	No
$\alpha_{001}$	No	No	Yes
$\alpha_{010}$	No	Yes	No
$\alpha_{011}$	No	Yes	Yes
$\alpha_{100}$	Yes	No	No
$\alpha_{101}$	Yes	No	Yes
$\alpha_{110}$	Yes	Yes	No
$\alpha_{111}$	Yes	Yes	Yes
$ \alpha_{000} ^2 +  \alpha_{001} ^2 +  \alpha_{010} ^2 +  \alpha_{011} ^2 +  \alpha_{100} ^2 +  \alpha_{101} ^2 +  \alpha_{110} ^2 +  \alpha_{111} ^2 = 1$			

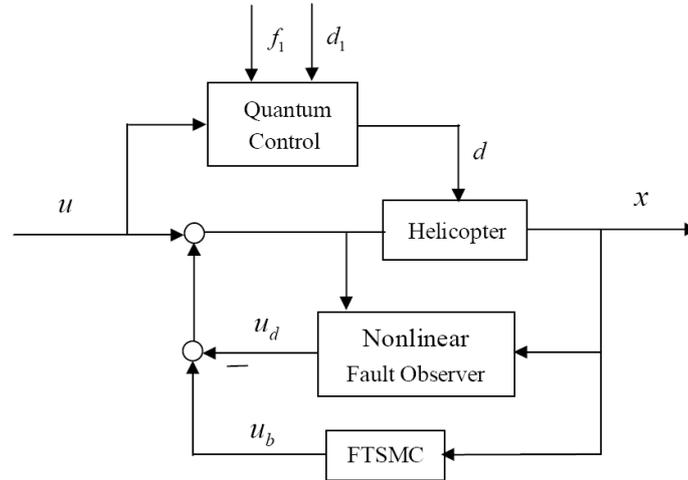


FIGURE 1. The chart of FTSM control for a helicopter via quantum control technique and NFO

4. **Nonlinear Fault Observer.** For the control system of the helicopter, the external interference and the change of load will affect the system’s performance. The practical way is to design a disturbance observer. By measuring the error between output of the actual system and output of nominal model, the system’s disturbance is estimated [18-21]. The structure of the system can be expressed as Figure 1.

The NFO module is constructed as

$$\begin{cases} \dot{\hat{d}} = z + p(X) \\ \dot{z} = -L(X)z + L(X)(-p(X) - f - gu) \end{cases} \tag{13}$$

where  $p(X)$  is a nonlinear function which should be designed.  $L(X)$  is the gain of NFO, which should satisfy Equation (14).

$$L(X)\dot{x}_2 = dp(X)/dt \tag{14}$$

The NFO’s observed error is defined as

$$\tilde{d} = d - \hat{d} \tag{15}$$

The parameter  $\hat{d}$  is the estimator of  $d$ .

As the change rate of the fault is slower than that of the dynamic characteristic of the observer, the equation can be obtained as

$$\dot{\hat{d}} = 0 \tag{16}$$

$$\begin{aligned} \dot{\tilde{d}} &= \dot{d} - \dot{\hat{d}} = -\dot{z} - \dot{p} = L(z + p) - L(\dot{x} - f - gu) \\ &= L\hat{d} - Ld = -L\tilde{d} \end{aligned} \tag{17}$$

By selecting positive  $L(X)$ , the observed error can converge exponentially. Select  $L(X) = b, b > 0, p(X)$  can be constructed as

$$p(X) = bx_2 \tag{18}$$

The NFO’s output is brought to the gained adjusting model, which converts the observed fault into the corresponding input. From Equations (4)-(6), the state  $\dot{x}_2$  can be expressed as

$$\dot{x}_2 = f + g(u + \hat{d}/g) \tag{19}$$

$$u_d = g^{-1}\hat{d} \tag{20}$$

After the NFO is adopted, the system can be expressed as

$$\begin{aligned}\dot{x}_2 &= gu + f + d = g(u_b - u_d) + f + d \\ &= gu_b + f + \tilde{d}\end{aligned}\quad (21)$$

**Remark 4.1.** *NFO is adopted to observe the uncertainties and faults of the system. By constructing the reasonable fault observer, the inference and fault will be estimated all the time. By selecting the design parameters, the observed error can converge exponentially. In addition, the NFO not only reduces the chattering, but also reduces the burden on the controller. The unobserved interference can be compensated by fast terminal sliding mode control.*

**5. Fast Terminal Sliding Mode Control.** In fact, the NFO's dynamic performance is similar to a low-pass filter. Its bandwidth cannot be set too wide. In addition, the NFO also has the error itself. Therefore, in order to compensate the NFO's error, the SMC is used in the paper, which compensates the rest of the error.

For the system  $\dot{x} = f(x)$ , the surface of the basic TSM can be expressed as

$$s = \dot{x} + b|x|^\alpha \operatorname{sgn}(x) = 0 \quad (22)$$

with  $s$  representing sliding mode surface, where  $x \in R$ ,  $b > 0$ ,  $\alpha \in (0, 1)$ . By designing the corresponding control law, the system's stability can be realized in a finite time [22-24]. However, when the initial state of the system is far from the equilibrium point, the system's performance and convergence rate will be influenced. To solve these problems, the FTSMC is proposed [25-28]. The surface of FTSM can be expressed as

$$s = \dot{x} + ax + b|x|^\alpha \operatorname{sgn}(x) = 0, \quad a > 0, \quad \alpha \in (0, 1) \quad (23)$$

The sliding mode surface (23) could force the state of the system to the equilibrium point at a fast convergence rate. Compared with the traditional SMC, the reliability is improved by FTSM.

Therefore, according to the FTSMC and Equation (23), the new surface of FTSM could be designed as

$$s = x_1 + k_1|x_1|^\gamma \operatorname{sgn} x_1 + k_2|x_2|^\lambda \operatorname{sgn} x_2 \quad (24)$$

In Equation (24), the scale factor  $k_1, k_2 > 0$ . The parameter of control law  $\gamma > 1$ ,  $1 < \lambda < 2$ .

**Remark 5.1.** *An exponential function of the system is introduced to accelerate the convergence when the states are far away from the equilibrium point. A symbolic function is brought in to ensure the stabilization of the system. Then, a novel sliding mode control law is deduced based on the newly proposed manifold, and is composed of the equivalent control law and the shift control law.*

**Theorem 5.1.** *According to the nonlinear unknown system Equations (4)-(6); if the new surface of FTSM Equation (24) and the control laws (25)-(27) could be adopted, the state factor  $X$  will converge to the equilibrium point in a finite time.*

$$u = u_e + u_d \quad (25)$$

$$u_e = -g^{-1} \left( f + \frac{k_1\gamma}{k_2\lambda} |x_1|^{\gamma-1} |x_2|^{2-\lambda} \operatorname{sgn} x_2 + \frac{1}{k_2\lambda} |x_2|^{2-\lambda} \operatorname{sgn} x_2 \right) \quad (26)$$

$$u_d = -g^{-1}(\eta + m) \operatorname{sgn} s \quad (27)$$

In Equations (25)-(27),  $u_e$  is equivalent control input;  $u_d$  is switching control input. The design parameter  $m > 0$ .

**Proof:** Firstly, the Lyapunov function can be chosen as

$$V = s^2/2 \geq 0 \tag{28}$$

Then, putting Equations (25)-(27) to Equation (2)

$$\dot{x}_2 = d(X) - \frac{k_1\gamma}{k_2\lambda} |x_1|^{\gamma-1} |x_2|^{2-\lambda} \operatorname{sgn} x_2 - \frac{1}{k_2\lambda} |x_2|^{2-\lambda} \operatorname{sgn} x_2 - (\eta + m) \operatorname{sgn} s \tag{29}$$

Now, after putting Equations (24) and (29) into derivative of  $V$ , derivative of  $V$  can be expressed as

$$\begin{aligned} \dot{V} &= s\dot{s} = s \left( \dot{x}_1 + k_1\gamma |x_1|^{\gamma-1} \dot{x}_1 + k_2\lambda |x_2|^{\lambda-1} \dot{x}_2 \right) \\ &= s \left[ x_2 + k_1\gamma |x_1|^{\gamma-1} x_2 + k_2\lambda |x_2|^{\lambda-1} \left( d(X) \right. \right. \\ &\quad \left. \left. - \frac{k_1\gamma}{k_2\lambda} |x_1|^{\gamma-1} |x_2|^{2-\lambda} \operatorname{sgn} x_2 - \frac{1}{k_2\lambda} |x_2|^{2-\lambda} \operatorname{sgn} x_2 - (\eta + m) \operatorname{sgn} s \right) \right] \\ &= sk_2\lambda |x_2|^{\lambda-1} [d(X) - (\eta + m) \operatorname{sgn} s] \end{aligned} \tag{30}$$

When  $s \neq 0$  and  $x_2 \neq 0$ , according to Remark 2.1,

$$\dot{V} = s\dot{s} \leq -m |s| < 0 \tag{31}$$

It means that the states of the system will converge to the equilibrium point in a finite time. Therefore, a uniformly asymptotically stability is realized by using the proposed control law.

**6. Direct Self-Repairing Controller.** In order to achieve the control objective, the design of controller can be divided into two steps. First, by designing the DSRC, the states of the system could converge to the equilibrium point [29]. Second, after satisfying the sliding mode surface  $s = 0$ , the error of the output will be eliminated in a finite time.

Defining the tracking error of  $x_2$ ,  $e = [ e_1 \ e_2 ]^T$ .

$$e_1 = x_1 - y_m(t) \tag{32}$$

$$e_2 = \dot{e}_1 = x_2 - \dot{y}_m(t) \tag{33}$$

In Equation (32),  $y_m(t)$  is a reference tracking command. Then, the equations of the tracking error can be expressed as

$$\left. \begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= f(X) + \tilde{d} + gu_b - \ddot{y}_m(t) \end{aligned} \right\} \tag{34}$$

**Remark 6.1.** In Equation (32), the reference tracking command  $y_m(t)$  and its second derivative are existing and bounded.

In order to design the tracking control of  $x_1$ , the novel surface  $\delta$  of FTSM can be chosen as

$$\delta = e_1 + k_1 |e_1|^\gamma \operatorname{sgn} e_1 + k_2 |\dot{e}_1|^\lambda \operatorname{sgn} \dot{e}_1 \tag{35}$$

In Equation (35), the scale factor  $k_1, k_2 > 0$ . The control law parameter  $\gamma > 1, 1 < \lambda < 2$ .

**Theorem 6.1.** In order to solve convergence rate and tracking control problem of helicopter (4)-(6) with disturbance and fault, the DRSC (36)-(38) have been designed. If the above conditions are satisfied, the proposed controller could make the closed-loop system stable. The tracking error  $e_1$  will converge to the zero in a finite time.

$$u_b = u_{be} + u_{bd} \tag{36}$$

$$u_{be} = -g^{-1}(X)(f(X) - \ddot{y}_m(t) + \frac{1}{k_2\lambda} |e_2|^{2-\lambda} \operatorname{sgn} e_2 + \frac{k_1\gamma |e_1|^{\gamma-1}}{k_2\lambda} |e_2|^{2-\lambda} \operatorname{sgn} e_2) \quad (37)$$

$$u_{bd} = -g^{-1}(X)(\eta + \xi) \operatorname{sgn} \delta \quad (38)$$

In Equations (36)-(38),  $u_{be}$  is the equivalent control input of system;  $u_{bd}$  is switching control input of the system.

**Remark 6.2.** The parameter  $\xi$  should be noted in Theorem 6.1. It can be defined as

$$\xi = \hat{\eta} \frac{1}{k_2\lambda} |e_2|^{1-\lambda} \quad (39)$$

The parameter  $\hat{\eta}$  is the estimator of  $\eta$ . The estimation error  $\tilde{\eta}$  can be defined as

$$\tilde{\eta} = \eta - \hat{\eta} \quad (40)$$

Then, the parameter adaptive law  $\dot{\hat{\eta}}$  can be defined as

$$\dot{\hat{\eta}} = \beta |\delta| \quad (41)$$

In Equation (41), the parameter  $\beta$  is needed to be designed, satisfying  $\beta > 0$ .

**Proof:** According to the closed-loop system, the Lyapunov function can be chosen as

$$V = \delta^2/2 + \tilde{\eta}^2/2\beta + \tilde{d}^2/2 \geq 0 \quad (42)$$

$$\dot{V} = \delta\dot{\delta} + \tilde{\eta}\dot{\tilde{\eta}}/\beta + \tilde{d}\dot{\tilde{d}} = \delta \left( e_2 + k_1\gamma |e_1|^{\gamma-1} e_2 + k_2\lambda |e_2|^{\lambda-1} \dot{e}_2 \right) + \tilde{\eta}\dot{\tilde{\eta}}/\beta + \tilde{d}\dot{\tilde{d}} \quad (43)$$

From Equations (36)-(38), it can be obtained that

$$\tilde{d}\dot{\tilde{d}} = \tilde{d}(-L\tilde{d}) = -L\tilde{d}^2 \quad (44)$$

Since  $L(X)$  is positive definite,  $\tilde{d}\dot{\tilde{d}}$  is negative definite.

At the same time,  $\delta\dot{\delta} + \tilde{\eta}\dot{\tilde{\eta}}/\beta$  can be expressed as

$$\begin{aligned} \delta\dot{\delta} + \tilde{\eta}\dot{\tilde{\eta}}/\beta &= \delta [e_2 + k_1\gamma |e_1|^{\gamma-1} e_2 + k_2\lambda |e_2|^{\lambda-1} \\ &\quad \cdot (f + \tilde{d} + g(u_{be} + u_{bd})) - \ddot{y}_m(t)] + \tilde{\eta}\dot{\tilde{\eta}}/\beta \\ &= \delta k_2\lambda |e_2|^{\lambda-1} (\tilde{d} + gu_{bd}) - \tilde{\eta}\dot{\tilde{\eta}}/\beta \end{aligned} \quad (45)$$

Then, Equations (38), (39) and (41) are put into Equation (45). It can be expressed as

$$\begin{aligned} \delta\dot{\delta} + \tilde{\eta}\dot{\tilde{\eta}}/\beta &= \delta k_2\lambda |e_2|^{\lambda-1} \left[ \tilde{d} - \left( \eta + \hat{\eta} \frac{1}{k_2\lambda} |e_2|^{1-\lambda} \right) \operatorname{sgn} \delta \right] - \tilde{\eta} |\delta| \\ &\leq \delta k_2\lambda |e_2|^{\lambda-1} (\tilde{d} - \eta) - \hat{\eta} |\delta| - \tilde{\eta} |\delta| \\ &\leq \delta k_2\lambda |e_2|^{\lambda-1} (\tilde{d} - \eta) + \eta |\delta| - \hat{\eta} |\delta| - \tilde{\eta} |\delta| \\ &\leq \delta k_2\lambda |e_2|^{\lambda-1} (\tilde{d} - \eta) \leq 0 \end{aligned} \quad (46)$$

According to Remarks 2.1 and 6.1, Equation (43) can be presented as

$$\dot{V} = \delta\dot{\delta} + \tilde{\eta}\dot{\tilde{\eta}}/\beta + \tilde{d}\dot{\tilde{d}} \leq \delta k_2\lambda |e_2|^{\lambda-1} (\tilde{d} - \eta) - L\tilde{d}^2 < 0 \quad (47)$$

It can be ensured that the states of the system will converge to the equilibrium point in a finite time. When  $\delta = 0$ , the tracking error  $e_1$  will converge to zero in a finite time. An adaptive approach is adopted to estimate upper bound of the fault in the design of the control law. Compared with the conventional TSM control, the FTSMC with the parameter adaptive law has a strong adaptive capacity. By using the proposed method, a better tracking performance and robustness can be achieved.

**7. Simulation Analysis.** The flight control system of the helicopter is very difficult. To simplify, only the vertical flight is investigated in this simulation. The dynamic models of the helicopter in vertical flight systems have been developed in [30]. The state assignments  $x_1 = h$ ,  $x_2 = \dot{h}$ ,  $x_3 = \omega$ ,  $x_4 = \theta_c$ , and  $x_5 = \dot{\theta}_c$ , then the fault dynamic model can be expressed as

$$\left. \begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= K_1 C_T(x_4)x_3^2 - g - K_2x_2 - K_3x_2^2 - K_4 \\ \dot{x}_3 &= -K_5x_3 - K_6x_3^2 - K_7x_3^2 \sin x_4 + K_8u_{th} + K_9 \\ \dot{x}_4 &= x_5 \\ \dot{x}_5 &= K_{10}(-0.0003175u_{\theta_c} + 0.5436 - x_4) - K_{11}x_5 - K_{12}x_3^2 \sin x_4 + d(x_4, x_5, t) \end{aligned} \right\} \quad (48)$$

where  $C_T(x_4) = \left(-K_{c1} + \sqrt{K_{c1}^2 + K_{c2}x_4}\right)^2$ ,  $d(x_4, x_5, t)$  is the unknown fault;  $h$  is the helicopter altitude above the ground and  $\theta_c$  is the collective angle of blade,  $\omega$  is the rotational speed of the rotor blades,  $g$  is the gravitational acceleration;  $K_{c1}$ ,  $K_{c2}$  and  $K_1$  to  $K_{12}$  are the system parameters which are given from the document [30]. The input of the throttle is  $u_{th}$  and the input of the collective servomechanism is  $u_{\theta_c}$ .

The control objective is that the altitude and the collective angle of the blades of the helicopter can track the desired values. Here the desired values are chosen as  $x_{1m} = 2(\text{m})$ , and  $x_{4m} = 0.3(\text{rad})$ . The system and control parameters are shown in Table 2.

In order to illustrate the effectiveness of controller proposed in the paper, the anti-interference and self-repairing capability of control system will be shown. The interference and fault are supposed to be  $d = 5$ . The upper bound of the fault is estimated as  $\hat{\eta} = 4$ . The result of altitude and altitude tracking error are shown in Figure 2 and Figure 3.

The NFO's control performance is depicted in Figure 2. The curve 1 represents the output of DSRC. The curve 2 represents the output of DSRC with NFO. The NFO's effectiveness is shown by comparing the curves. After the interference and fault have been put into the system, the DSRC based on NFO can suppress the interference and fault more effectively than the method without NFO.

Then, the control performance of DSRC is depicted in Figure 3. The curve 1 represents the desired outputs. The curve 2 represents the output of DSRC based on the traditional TSM. The curve 3 represents the output of FTSMC via quantum information technique and NFO. After the interference and fault have been put into the system, the effectiveness of the proposed method is shown by comparing two cases:

(1) The convergence rate by using FTSMC via quantum information technique and NFO is faster than the conventional TSM controller in altitude and altitude tracking error.

TABLE 2. The system parameters of the helicopter

Parameter	Value	Parameter	Value
$K_1$	$0.25m$	$K_2$	$0.1s^{-1}$
$K_3$	$0.1m^{-1}$	$K_4$	$7.86m$
$K_5$	$0.7s^{-1}$	$K_6$	$0.0028$
$K_7$	$0.005$	$K_8$	$0.1088s^{-2}$
$K_9$	$-13.92s^{-1}$	$K_{10}$	$800s^{-2}$
$K_{11}$	$65s^{-1}$	$K_{12}$	$0.1$
$K_{c1}$	$0.032592$	$K_{c2}$	$0.06145$

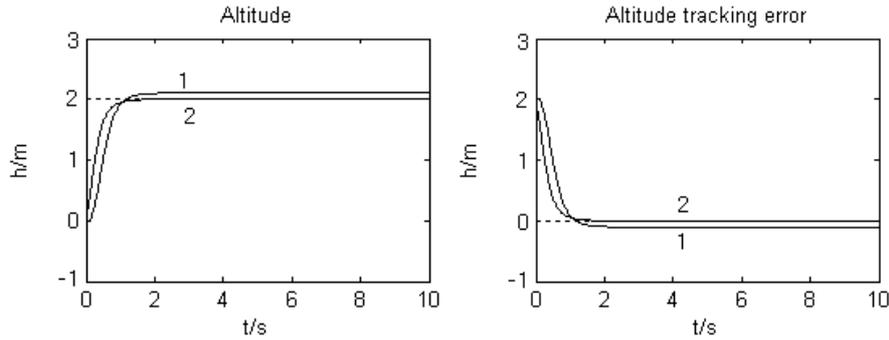


FIGURE 2. The simulation results of NFO

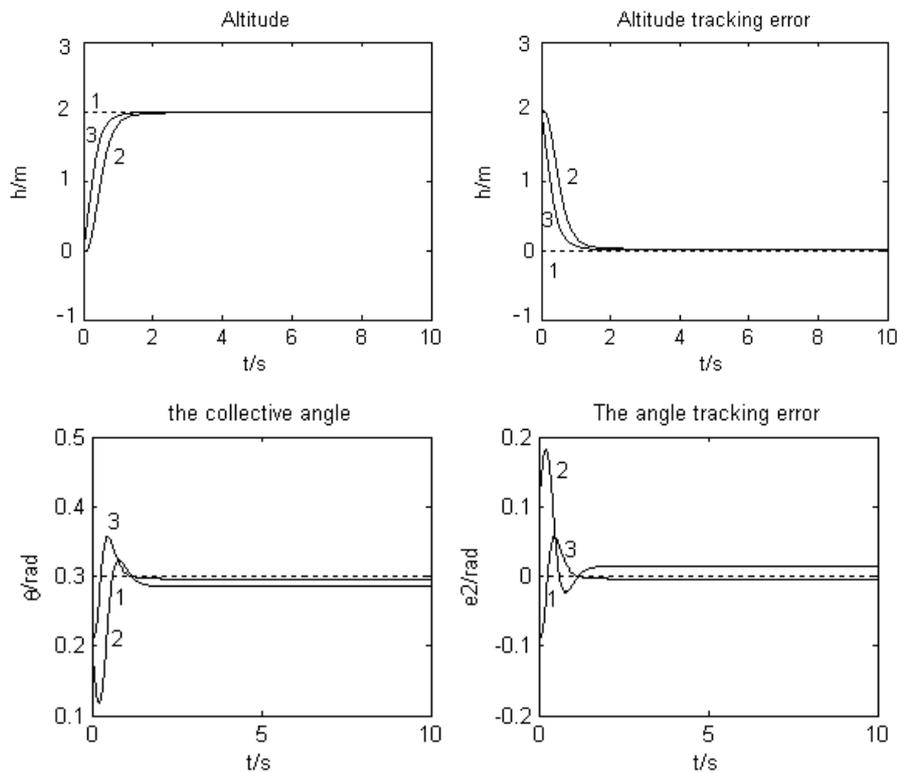


FIGURE 3. The simulation results of the helicopter with interference and fault

(2) The tracking accuracy by using FTSMC via quantum information technique and NFO is better than the conventional TSM controller. Therefore, the simulation results demonstrate that the proposed method has faster convergence rate and better tracking performance.

In order to verify the actual effect of the proposed method, the method needs to be applied to semi-physical simulation platform of the actual flight control system. The 3-DOF Helicopter plant is depicted in Figure 4. Two DC motors are mounted at the two ends of a rectangular frame and drive two propellers. The motors axes are parallel and the thrust vector is normal to the frame. The helicopter frame is suspended at the end of a long arm and is free to pitch about its centre.

After the interference and fault have been put into the system, the 3-DOF helicopter can run safely by using the proposed method. The proposed method can be applied to solve the actual system, which has convergence rate or tracking control problem.



FIGURE 4. The results of 3-DOF Helicopter when running with interference and fault

8. **Conclusion.** The new DRSC via quantum information technique and NFO is designed in this paper. Different from the traditional TSM, the faster convergence rate and higher tracking effect can be obtained by the proposed method. The globally stability of the DSRC system is achieved. The method's effectiveness has been verified by the simulations.

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