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A DESIGN METHODOLOGY AND ANALYSIS FOR INTERVAL TYPE-2 FUZZY PI/PD CONTROLLERS

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Abstract. *In this paper, a systematical methodology is introduced to construct the rule base of an interval type-2 fuzzy logic controller based on an existing linear PI/PD controller. An easy and rapid generation of the fuzzy rules can be achieved through this technique. In addition, analytical structure of this controller is derived. A closed-form of the fuzzy controller output is achieved under the circumstances that the input type-2 membership functions are diamond-shaped and a closed-form inference engine is used. Consequently, a linear control law is transformed to a nonlinear structure and certain elaborations can be done on the parameters of the evolved closed output structure. Moreover, the designer can benefit from the nonlinear structure of the proposed controller and the extra degree of freedom of type-2 fuzzy sets. It can be concluded from the results that the proposed controller can be more robust to the parameter uncertainties and eliminate the oscillations much better than type-1 fuzzy logic and linear conventional controllers.*
Keywords: Type-2 fuzzy sets, Interval type-2 fuzzy systems, Interval type-2 fuzzy logic controllers, linear PI/PD controller

1. **Introduction.** Fuzzy logic systems (FLSs) have been widely developed and utilized in different branches of science and technology [1-6]. A novel control design for discrete-time Takagi-Sugeno fuzzy systems with time-varying delays is proposed in [1]. The problem of adaptive fuzzy dynamic surface control has been investigated for a class of nonlinear strict-feedback systems with unknown time delays based on fuzzy approximation approach in [2]. A fuzzy controller based on the decomposition of the multivariable rule base into simple rule base has been studied and compared with several PID-type fuzzy logic controllers (FLCs) in [3]. Analytical structure of the fuzzy PID controller and conditions for bounded-input bounded-output stability of fuzzy PID control systems are obtained in [4]. The performance of conventional PID controllers has been compared with type-1 fuzzy logic controller (T1-FLC) through different simulations in [7]. In [8], it has been shown that PID controllers can be realized by fuzzy control and simplified fuzzy reasoning methods. The main difficulty in FLC design is to determine the parameters of the fuzzy logic controllers (e.g., membership functions, rules, scaling factors) for inputs and outputs of a fuzzy system. To ease the FLC design process, the researchers proposed a general methodology to systematically construct a fuzzy logic controller based on the existence of a linear controller in [9]. This methodology guarantees identical performance to an existing linear controller. Since the performances of controllers are identical, it has been advised

to use expert knowledge to improve the performance of fuzzy controller by appropriately changing the rule base. However, changing the rule base through expert knowledge still appears to be a challenging task.

The type-2 fuzzy logic sets (T2FLSs) are the extension of the ordinary type-1 fuzzy sets [10]. The T2FLS may be able to outperform its type-1 (T1) counterpart, and especially, they are better able to cope with nonlinearities and uncertainties because of the additional degrees of freedom provided by the footprint of uncertainty (FOU) in their membership functions [11,12]. Experiments show that the T2FLS may achieve better performance in comparison with type-1 fuzzy set. Nevertheless, the computations of T2FLSs are more complex than type-1 fuzzy sets. Therefore, a special type of type-2 fuzzy logic set called interval type-2 fuzzy set (IT2FS) is proposed in [13]. A review on the design and optimization of interval type-2 fuzzy controllers has been considered in [14]. Interval type-2 fuzzy logic controllers (IT2-FLCs) have attracted much research interest in recent years due to their ability to cope with uncertainties. Several control and engineering applications such as liquid-level process control [15], autonomous mobile robots [16], prediction of air pollutant [17], pH control [18], control and the identification of a real-time servo system [19] and face recognition [20] illustrate the advantages of IT2FS. Studies have been reported in the literature that the IT2-FLCs are generally more robust than T1-FLC [21,22]. Wu [21] has discussed the fundamental differences between interval type-2 and type-1 fuzzy logic controllers. There, it has also been shown that interval type-2 PI controllers have smoother control surfaces than its type-1 counterpart in the region around the zero. In [23], an interval type-2 fuzzy proportional controller with a variable gain has been developed. A method to design interval type-2 Takagi-Sugeno-Kang FLCs, PD-type and PI-type fuzzy controllers to satisfy certain desired transient response, has been proposed in [24]. Derivation and analysis of the two Mamdani interval type-2 fuzzy PI controllers that use the center-of-sets type reducer and the average defuzzifier are studied in [25]. The analytical structure of a special class of interval type-2 fuzzy PI and PD controllers that have symmetrical rule base and symmetrical consequent sets is presented in [26]. Authors compare the IT2-FLC with the corresponding T1-FLC while the potential advantages of using IT2-FLC over T1 fuzzy controller are examined. Analytical structures of interval type-2 fuzzy PI and PD controllers proposed in the studies [25,26] are very complex and therefore, it becomes very difficult to generalize these analyses to IT2-FLC with more than two membership functions for each input.

Motivated by the aforementioned drawbacks, this paper aims to design a systematical methodology to construct an IT2-FLC based on conventional controller and extract the closed-form relation for IT2-FLC. The methodology depends on a nonlinear mapping from an existing conventional linear control law (e.g., PI, PD) to IT2-FLC in which the beneficial sides of a linear controller in terms of simplicity are captured. The proposed nonlinear mapping is done under certain circumstances that input type-2 membership functions are diamond-shaped and the closed-form inference engine given in [27] is used. The diamond-shaped type-2 membership functions provide easiness in the analytical derivation of mathematical closed-form expressions. An important advantage of the proposed methodology is the closed-form relation between input and output of IT2-FLC for any number of type-2 membership functions. This provides a clue about the robustness of IT2-FLC and how they cope with uncertainties. Another beneficial feature of this technique is the ease and rapid generation of the fuzzy rules of the IT2-FLC based on the existing linear controller. When the FOUs of the antecedent membership functions are taken to be zero, the IT2-FLC will be reduced to T1-FLC; thus, an identical mapping is accomplished between conventional linear controller and the proposed controller. If FOU is not equal to zero, then an additional degree of freedom is acquired that provides an uncertainty cloud over the proposed controller. This provides the designer an additional tool to cope with the uncertainties and nonlinearities, which may exist in the system to be controlled. Two special cases of the proposed controller with 2×2 and 2×3 rule bases are mathematically analyzed in detail to show the effect of variable gains that are introduced by the proposed IT2-FLC. Simulations on various processes and a real time application on ball and beam system demonstrate that the proposed controller is more robust and capable to manage the uncertainties much better than conventional linear controller and T1-FLCs. In summary, the main contributions of this paper can be listed as:

i. The closed-form relations between input and output of an IT2-FLC are derived. These relations clarify the unknown internal structure of IT2-FLC and give ability to understand its behavior in comparison with its T1 counterpart.
ii. The proposed analytical derivation method allows the use of any number of input fuzzy sets; whereas, the previous works are limited to two fuzzy input sets.
iii. An easy and rapid generation of the fuzzy rules based on the existing linear controller can be achieved through this technique.

The paper is organized in five sections. In Section 2 the general structure and the components of the proposed IT2-FLC are discussed in detail. In Section 3, the methodology and analytical derivations are done to show the mapping between the proposed control structure and the conventional linear controller. In Section 4, a simulation study has been implemented on first-order plus time delay process. Furthermore, a practical experimental study is performed on a ball and beam system again to demonstrate certain advantages of the proposed IT2-FLC. Finally, discussions and conclusions are presented in Section 5.

2. **The General Structure and the Components of the Proposed IT2-FLC.** In this section, the general structure of the handled IT2-FLC is given. The feedback control structure of IT2-FLC is shown in Figure 1(a). The output of linear PI or PD controller is given by

$$u = k_1 x_1 + k_2 x_2 \tag{1}$$

where x_1 is the error, x_2 is the integral of error or derivative of error.

The configuration for two input-one output PI or PD based IT2-FLC is shown in Figure 1(b). In this structure c_1 and c_2 are the input scaling factors. Without loss of generality, it will be assumed that scaling factors (c_1 and c_2) are equal to 1. In the considered IT2-FLC, the antecedent membership functions are defined with interval type-2 fuzzy sets, while the consequent part is defined with crisp singleton parameters. The rule structure of IT2-FLC is as follows [12]:

$$Rule\ l^{th} : IF\ x_1\ is\ \tilde{F}_1^{j_1}\ and\ x_2\ is\ \tilde{F}_2^{j_2}\ Then\ u^l \tag{2}$$

where x_1 and x_2 are the inputs, while u^l is the consequent crisp set ($l = 1, \ldots, M$), M is the number of rules and $\tilde{F}_i^{j_i}$ denotes the type-2 membership functions for j_i^{th} fuzzy set associated with the i^{th} input ($i = 1, 2\ j_i = 1, \ldots, n$) and n is the number of membership

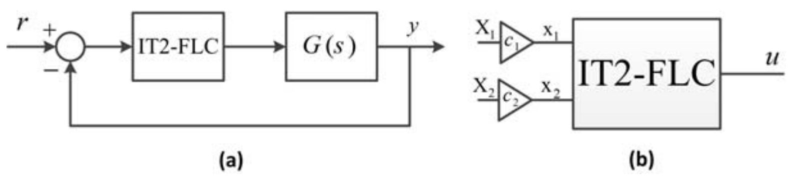

Figure 1. The structure of (a) the feedback control system and (b) the IT2-FLC

functions that cover the universe of discourse of the inputs. The final output of the system can be written as

$$U = \int_{f^1 \in [\underline{f}^1, \overline{f}^1]} \cdots \int_{f^M \in [\underline{f}^M, \overline{f}^M]} 1 \Bigg/ \frac{\sum_{l=1}^{M} f^l u^l}{\sum_{l=1}^{M} f^l} \tag{3}$$

where $\underline{f}^l$ and $\overline{f}^l$ are given by

$$\begin{aligned} \underline{f}^l(x) &= \underline{\mu}_{\tilde{F}_1^{j_1}}(x_1) * \underline{\mu}_{\tilde{F}_2^{j_2}}(x_2) \\ \overline{f}^l(x) &= \overline{\mu}_{\tilde{F}_1^{j_1}}(x_1) * \overline{\mu}_{\tilde{F}_2^{j_2}}(x_2) \end{aligned} \tag{4}$$

and $\overline{\mu}_{\tilde{F}_i^{j_i}}$, $\underline{\mu}_{\tilde{F}_i^{j_i}}$ are the upper and lower membership functions for the l^{th} rule, respectively. Here, the operator $*$ represents the t-norm, which is the product operator. The output of the IT2-FLC is achieved in a closed-form via the inference engine given in [27] as follows:

$$U = \frac{\sum_{l=1}^{M} \underline{f}^l u^l}{\sum_{l=1}^{M} \underline{f}^l + \sum_{l=1}^{M} \overline{f}^l} + \frac{\sum_{l=1}^{M} \overline{f}^l u^l}{\sum_{l=1}^{M} \underline{f}^l + \sum_{l=1}^{M} \overline{f}^l} \tag{5}$$

In this paper, the diamond-shaped type-2 membership function [28] is preferred in the representation of the inputs space. In Figure 2, $\overline{F}_i^{j_i}$ represents the modal of the j_i^{th} fuzzy set associated with the i^{th} input and the parameter Δ defines the uncertainty of the interval type-2 fuzzy set. As it can be clearly seen in Figure 2, the diamond-shaped type-2 membership function gets 0 value at both ends of the support and 1 value at the modal of the membership function. The upper membership functions of the diamond-shaped type-2 fuzzy sets are defined as:

$$\overline{\mu}_{\tilde{F}_i^{j_i}} = \begin{cases} \frac{\left(\overline{F}_i^{j_i+1} - x_i\right) + 2\Delta\left(x_i - \overline{F}_i^{j_i}\right)}{\overline{F}_i^{j_i+1} - \overline{F}_i^{j_i}} & \overline{F}_i^{j_i} \le x_i \le \frac{\overline{F}_i^{j_i} + \overline{F}_i^{j_i+1}}{2} \\ \frac{\left(\overline{F}_i^{j_i+1} - x_i\right)(1+2\Delta)}{\overline{F}_i^{j_i+1} - \overline{F}_i^{j_i}} & \frac{\overline{F}_i^{j_i} + \overline{F}_i^{j_i+1}}{2} \le x_i \le \overline{F}_i^{j_i+1} \end{cases} \tag{6}$$

while the lower membership functions are defined as

$$\underline{\mu}_{\tilde{F}_i^{j_i}} = \begin{cases} \frac{\left(\overline{F}_i^{j_i+1} - x_i\right) - 2\Delta\left(x_i - \overline{F}_i^{j_i}\right)}{\overline{F}_i^{j_i+1} - \overline{F}_i^{j_i}} & \overline{F}_i^{j_i} \le x_i \le \frac{\overline{F}_i^{j_i} + \overline{F}_i^{j_i+1}}{2} \\ \frac{\left(\overline{F}_i^{j_i+1} - x_i\right)(1-2\Delta)}{\overline{F}_i^{j_i+1} - \overline{F}_i^{j_i}} & \frac{\overline{F}_i^{j_i} + \overline{F}_i^{j_i+1}}{2} \le x_i \le \overline{F}_i^{j_i+1} \end{cases} \tag{7}$$

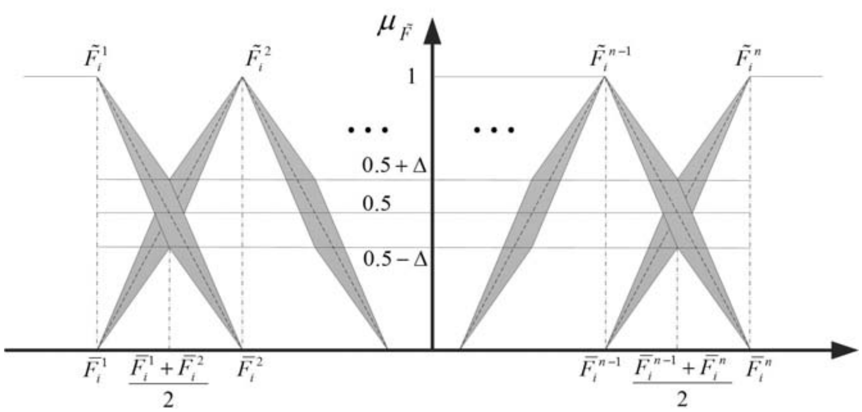

FIGURE 2. The diamond-shaped type-2 membership functions for the proposed IT2-FLC

The membership grades of the $\tilde{F}_i^{j_i+1}$ type-2 fuzzy set satisfy the following property

$$\underline{\mu}_{\tilde{F}_i^{j_i+1}} = 1 - \overline{\mu}_{\tilde{F}_i^{j_i}} \tag{8}$$

$$\overline{\mu}_{\tilde{F}_i^{j_i+1}} = 1 - \underline{\mu}_{\tilde{F}_i^{j_i}} \tag{9}$$

The location of the crisp numbers for the consequent part of IT2-FLC is the key feature of this study. The structure of consequent part of the IT2-FLC is given as

$$u^l = k_1 \overline{F}_1^{j_1} + k_2 \overline{F}_2^{j_2} \qquad j_i \in \{1, \ldots, n\}, \quad i = \{1, 2\} \quad \text{and} \quad l \in \{1, \ldots, M\} \tag{10}$$

where k_1 is the proportional gain, k_2 is the integral or derivative gain, $\overline{F}_i^{j_i}$ represents the modal of the j_i^{th} fuzzy set associated with the i^{th} input ($i = 1, 2$ $j_i = 1, \ldots, n$) and n is the number of membership functions that cover the universe of discourse of inputs [29]. As it is seen from (10), the location of the crisp numbers for the consequent part of IT2-FLC directly incorporates the linear control law.

3. **The Methodology and Analytical Derivation for the Proposed IT2-FLC.** In this section, the analytical derivations for the proposed IT2-FLC based on conventional PI or PD, shown in Figure 1(b), is examined. It is clear from Figure 2 that the membership functions overlap at a membership grade of 0.5. Therefore, only two type-2 membership functions are active for any input set. Consequently, four rules are always activated and considered in computation of the IT2-FLC output ($M = 4$). The IT2-FLC output for the input set (x_1, x_2) can be calculated using (5) as follows:

$$U = \frac{\sum_{l=1}^4 \underline{f}^l u^l + \sum_{l=1}^4 \overline{f}^l u^l}{\sum_{l=1}^4 \underline{f}^l + \sum_{l=1}^4 \overline{f}^l} = \frac{U^N}{U^D} \tag{11}$$

where the consequent part u^l is given as:

$$\begin{aligned} u^1 &= k_1 \overline{F}_1^{j_1} + k_2 \overline{F}_2^{j_2} \\ u^2 &= k_1 \overline{F}_1^{j_1} + k_2 \overline{F}_2^{j_2+1} \\ u^3 &= k_1 \overline{F}_1^{j_1+1} + k_2 \overline{F}_2^{j_2} \\ u^4 &= k_1 \overline{F}_1^{j_1+1} + k_2 \overline{F}_2^{j_2+1} \end{aligned} \tag{12}$$

where $k_1 = k_p$ and k_2 is k_d or k_i. Using the approach described by (10) and (12) the consequent part of active rules for the given inputs are generated as in Table 1.

TABLE 1. IT2-FLC rule base for a system with given inputs

$x_1 \backslash x_2$	$\tilde{F}_2^1$	$\ldots$	$\tilde{F}_2^{j_2}$	$\tilde{F}_2^{j_2+1}$	$\ldots$	$\tilde{F}_2^n$
$\tilde{F}_1^1$			$\vdots$	$\vdots$		
$\vdots$						
$\tilde{F}_1^{j_1}$		$\ldots$	u^1	u^2	$\ldots$	
$\tilde{F}_1^{j_1+1}$		$\ldots$	u^3	u^4	$\ldots$	
$\vdots$			$\vdots$	$\vdots$		
$\tilde{F}_1^n$						

Theorem 3.1. *The output of IT2-FLC in (11) can be formulated as follows:*

$$U = \frac{U^N}{U^D} = \frac{P(x_i, \overline{F}_i^{j_i}, \Delta, K) + 2(k_1 x_1 + k_2 x_2)}{2[Q(x_i, \overline{F}_i^{j_i}, \Delta) + 1]} \tag{13}$$

if and only if

i. The rule base contains all possible combinations of active input sets.
ii. The input interval type-2 fuzzy sets are normal and satisfy (8) and (9) for any given input set.

In (13), x_i is the input, $K = [k_1 \quad k_2]$ is a vector of the conventional controller gains and Δ is the parameter that produces FOU for type-2 membership function. The proof of the theorem is given in two parts via two lemmas. The derivations of the expressions for the denominator U^D and the nominator U^N and related proofs are given in Lemmas 3.1 and 3.2, respectively.

Lemma 3.1. *For any input set, x_1 and x_2, the denominator $U^D = \sum_{l=1}^{4} \underline{f}^l + \sum_{l=1}^{4} \overline{f}^l$ from the IT2-FLC output given in (13) is equal to $U^D = 2\left[Q(x_i, \overline{F}_i^{j_i}, \Delta) + 1\right]$.*

Proof: Using (4) and (11), the following relation for U^D is obtained:

$$\begin{aligned} U^D = & \underline{\mu}_{\tilde{F}_1^{j_1}} \underline{\mu}_{\tilde{F}_2^{j_2}} + \overline{\mu}_{\tilde{F}_1^{j_1}} \overline{\mu}_{\tilde{F}_2^{j_2}} + \underline{\mu}_{\tilde{F}_1^{j_1}} \underline{\mu}_{\tilde{F}_2^{j_2+1}} + \overline{\mu}_{\tilde{F}_1^{j_1}} \overline{\mu}_{\tilde{F}_2^{j_2+1}} \\ & + \underline{\mu}_{\tilde{F}_1^{j_1+1}} \underline{\mu}_{\tilde{F}_2^{j_2}} + \overline{\mu}_{\tilde{F}_1^{j_1+1}} \overline{\mu}_{\tilde{F}_2^{j_2}} + \underline{\mu}_{\tilde{F}_1^{j_1+1}} \underline{\mu}_{\tilde{F}_2^{j_2+1}} + \overline{\mu}_{\tilde{F}_1^{j_1+1}} \overline{\mu}_{\tilde{F}_2^{j_2+1}} \end{aligned} \tag{14}$$

Simplifying (14), one obtains

$$U^D = \left(\underline{\mu}_{\tilde{F}_1^{j_1}} + \underline{\mu}_{\tilde{F}_1^{j_1+1}}\right)\left(\underline{\mu}_{\tilde{F}_2^{j_2}} + \underline{\mu}_{\tilde{F}_2^{j_2+1}}\right) + \left(\overline{\mu}_{\tilde{F}_1^{j_1}} + \overline{\mu}_{\tilde{F}_1^{j_1+1}}\right)\left(\overline{\mu}_{\tilde{F}_2^{j_2}} + \overline{\mu}_{\tilde{F}_2^{j_2+1}}\right) \tag{15}$$

Using (8) and (9), (15) can be rewritten as

$$U^D = 2\left[\left(\overline{\mu}_{\tilde{F}_1^{j_1}} - \underline{\mu}_{\tilde{F}_1^{j_1}}\right)\left(\overline{\mu}_{\tilde{F}_2^{j_2}} - \underline{\mu}_{\tilde{F}_2^{j_2}}\right) + 1\right] = 2\left[Q\left(x_i, \overline{F}_i^{j_i}, \Delta\right) + 1\right] \tag{16}$$

Using (6) and (7), the term $\left(\overline{\mu}_{\tilde{F}_i^{j_i}} - \underline{\mu}_{\tilde{F}_i^{j_i}}\right)$ can be rewritten as

$$\overline{\mu}_{\tilde{F}_i^{j_i}} - \underline{\mu}_{\tilde{F}_i^{j_i}} = \begin{cases} \dfrac{4\Delta\left(x_i - \overline{F}_i^{j_i}\right)}{\overline{F}_i^{j_i+1} - \overline{F}_i^{j_i}} & \overline{F}_i^{j_i} \le x_i \le \dfrac{\overline{F}_i^{j_i} + \overline{F}_i^{j_i+1}}{2} \\ \dfrac{4\Delta\left(\overline{F}_i^{j_i+1} - x_i\right)}{\overline{F}_i^{j_i+1} - \overline{F}_i^{j_i}} & \dfrac{\overline{F}_i^{j_i} + \overline{F}_i^{j_i+1}}{2} \le x_i \le \overline{F}_i^{j_i+1} \end{cases} \tag{17}$$

Substituting (17) in (16), $Q(x_i, \overline{F}_i^{j_i}, \Delta)$ is obtained for four regions in the input domain as tabulated in Table 2.

Table 2. The expressions of $Q(x_i, \overline{F}_i^{j_i}, \Delta)$ for four regions in input domain

$Q(x_i, \overline{F}_i^{j_i}, \Delta)$	$\overline{F}_2^{j_2} \le x_2 \le \dfrac{\overline{F}_2^{j_2} + \overline{F}_2^{j_2+1}}{2}$	$\dfrac{\overline{F}_2^{j_2} + \overline{F}_2^{j_2+1}}{2} \le x_2 \le \overline{F}_2^{j_2+1}$
$\overline{F}_1^{j_1} \le x_1 \le \dfrac{\overline{F}_1^{j_1} + \overline{F}_1^{j_1+1}}{2}$	$\left(\dfrac{4\Delta(\overline{F}_1^{j_1} - x_1)}{\overline{F}_1^{j_1+1} - \overline{F}_1^{j_1}}\right)\left(\dfrac{4\Delta(\overline{F}_2^{j_2} - x_2)}{\overline{F}_2^{j_2+1} - \overline{F}_2^{j_2}}\right)$	$\left(\dfrac{4\Delta(x_1 - \overline{F}_1^{j_1})}{\overline{F}_1^{j_1+1} - \overline{F}_1^{j_1}}\right)\left(\dfrac{4\Delta(\overline{F}_2^{j_2+1} - x_2)}{\overline{F}_2^{j_2+1} - \overline{F}_2^{j_2}}\right)$
$\dfrac{\overline{F}_1^{j_1} + \overline{F}_1^{j_1+1}}{2} \le x_1 \le \overline{F}_1^{j_1+1}$	$\left(\dfrac{4\Delta(\overline{F}_1^{j_1+1} - x_1)}{\overline{F}_1^{j_1+1} - \overline{F}_1^{j_1}}\right)\left(\dfrac{4\Delta(x_2 - \overline{F}_2^{j_2})}{\overline{F}_2^{j_2+1} - \overline{F}_2^{j_2}}\right)$	$\left(\dfrac{4\Delta(\overline{F}_1^{j_1+1} - x_1)}{\overline{F}_1^{j_1+1} - \overline{F}_1^{j_1}}\right)\left(\dfrac{4\Delta(\overline{F}_2^{j_2+1} - x_2)}{\overline{F}_2^{j_2+1} - \overline{F}_2^{j_2}}\right)$

Lemma 3.2. *For any input set, x_1 and x_2, the nominator $U^N = \sum_{l=1}^{4}\left(\underline{f}^l + \overline{f}^l\right) u^l$ from the IT2-FLC output given in (11) is equal to $U^N = P\left(x_i, \overline{F}_i^{j_i}, K, \Delta\right) + 2\left(k_1 x_1 + k_2 x_2\right)$.*

Proof: The nominator U^N of (11) can be written as

$$U^N = \sum_{l=1}^{4}\left(\underline{f}^l + \overline{f}^l\right) u^l \tag{18}$$

Thus, U^N can be subsequently derived as

$$\begin{aligned}U^N =& u^1 \left(\underline{\mu}_{\tilde{F}_1^{j_1}}\underline{\mu}_{\tilde{F}_2^{j_2}} + \overline{\mu}_{\tilde{F}_1^{j_1}}\overline{\mu}_{\tilde{F}_2^{j_2}}\right) + u^2 \left(\underline{\mu}_{\tilde{F}_1^{j_1}}\underline{\mu}_{\tilde{F}_2^{j_2+1}} + \overline{\mu}_{\tilde{F}_1^{j_1}}\overline{\mu}_{\tilde{F}_2^{j_2+1}}\right) \\ &+ u^3 \left(\underline{\mu}_{\tilde{F}_1^{j_1+1}}\underline{\mu}_{\tilde{F}_2^{j_2}} + \overline{\mu}_{\tilde{F}_1^{j_1+1}}\overline{\mu}_{\tilde{F}_2^{j_2}}\right) + u^4 \left(\underline{\mu}_{\tilde{F}_1^{j_1+1}}\underline{\mu}_{\tilde{F}_2^{j_2+1}} + \overline{\mu}_{\tilde{F}_1^{j_1+1}}\overline{\mu}_{\tilde{F}_2^{j_2+1}}\right)\end{aligned} \tag{19}$$

Using the relations (8) and (9), (19) can be reformulated as

$$\begin{aligned}U^N =& (u^1 + u^4) \left(\underline{\mu}_{\tilde{F}_1^{j_1}}\underline{\mu}_{\tilde{F}_2^{j_2}} + \overline{\mu}_{\tilde{F}_1^{j_1}}\overline{\mu}_{\tilde{F}_2^{j_2}}\right) + (u^2 - u^4) \left(\underline{\mu}_{\tilde{F}_1^{j_1}} + \overline{\mu}_{\tilde{F}_1^{j_1}}\right) \\ &- (u^2 + u^3) \left(\underline{\mu}_{\tilde{F}_1^{j_1}}\overline{\mu}_{\tilde{F}_2^{j_2}} + \overline{\mu}_{\tilde{F}_1^{j_1}}\underline{\mu}_{\tilde{F}_2^{j_2}}\right) + (u^3 - u^4) \left(\underline{\mu}_{\tilde{F}_2^{j_2}} + \overline{\mu}_{\tilde{F}_2^{j_2}}\right) + 2u^4\end{aligned} \tag{20}$$

From (12) it is obvious that $u^1 + u^4 = u^2 + u^3$. Thus, (20) can be simplified as

$$\begin{aligned}U^N =& (u^1 + u^4) \left(\underline{\mu}_{\tilde{F}_1^{j_1}}\underline{\mu}_{\tilde{F}_2^{j_2}} + \overline{\mu}_{\tilde{F}_1^{j_1}}\overline{\mu}_{\tilde{F}_2^{j_2}} - \underline{\mu}_{\tilde{F}_1^{j_1}}\overline{\mu}_{\tilde{F}_2^{j_2}} - \overline{\mu}_{\tilde{F}_1^{j_1}}\underline{\mu}_{\tilde{F}_2^{j_2}}\right) \\ &+ (u^2 - u^4) \left(\underline{\mu}_{\tilde{F}_1^{j_1}} + \overline{\mu}_{\tilde{F}_1^{j_1}}\right) + (u^3 - u^4) \left(\underline{\mu}_{\tilde{F}_2^{j_2}} + \overline{\mu}_{\tilde{F}_2^{j_2}}\right) + 2u^4\end{aligned} \tag{21}$$

In order to simplify (21), the following equation is derived from (6) and (7)

$$\underline{\mu}_{\tilde{F}_i^{j_i}} + \overline{\mu}_{\tilde{F}_i^{j_i}} = \frac{2\left(\overline{F}_i^{j_i+1} - x_i\right)}{\overline{F}_i^{j_i+1} - \overline{F}_i^{j_i}} \tag{22}$$

and $G(x_i, \overline{F}_i^{j_i}, \Delta)$ is defined as:

$$G\left(x_i, \overline{F}_i^{j_i}, \Delta\right) = \underline{\mu}_{\tilde{F}_1^{j_1}}\underline{\mu}_{\tilde{F}_2^{j_2}} + \overline{\mu}_{\tilde{F}_1^{j_1}}\overline{\mu}_{\tilde{F}_2^{j_2}} - \underline{\mu}_{\tilde{F}_1^{j_1}}\overline{\mu}_{\tilde{F}_2^{j_2}} - \overline{\mu}_{\tilde{F}_1^{j_1}}\underline{\mu}_{\tilde{F}_2^{j_2}} \tag{23}$$

There are four different cases for $G\left(x_i, \overline{F}_i^{j_i}, \Delta\right)$ according to the range that the inputs are taking place. Thus, by substituting (6) and (7) in (23) and simplifying, $G\left(x_i, \overline{F}_i^{j_i}, \Delta\right)$ is obtained as Table 3. Substituting (22), (23) and (12) into (21), U^N is obtained as

$$U^N = P\left(x_i, \overline{F}_i^{j_i}, K, \Delta\right) + 2(k_1x_1 + k_2x_2) \tag{24}$$

where

$$P\left(x_i, \overline{F}_i^{j_i}, K, \Delta\right) = \left(k_1\left(\overline{F}_1^{j_1} + \overline{F}_1^{j_1+1}\right) + k_2\left(\overline{F}_2^{j_2} + \overline{F}_2^{j_2+1}\right)\right) \times G\left(x_i, \overline{F}_i^{j_i}, \Delta\right) \tag{25}$$

Note that if $\Delta = 0$, it is clear that $Q(x_i, \overline{F}_i^{j_i}, \Delta) = 0$ and $P(x_i, \overline{F}_i^{j_i}, K, \Delta) = 0$. Thus, IT2-FLC reduces to a T1-FLC and has an identical output to the PI or PD control law as follows:

$$U = k_1x_1 + k_2x_2 = k_pe + k_i\int_{t_0}^{t} e\mathrm{d}t \quad \text{(PI) or } U = k_1x_1 + k_2x_2 = k_pe + k_i\dot{e} \quad \text{(PD)} \tag{26}$$

As it can be seen from (13), IT2-FLC produces a nonlinear controller, which has a conventional linear controller part and nonlinear parts $P(x_i, \overline{F}_i^{j_i}, K, \Delta)$ and $Q(x_i, \overline{F}_i^{j_i}, \Delta)$ that vary with its arguments.

In the following two subsections, two special cases with 2×2 and 2×3 rule bases of the proposed controller are mathematically analyzed and the effects of variable gains of the proposed nonlinear PI/PD controller are examined in detail.

TABLE 3. The expressions of $G(x_i, \overline{F}_i^{j_i}, \Delta)$ for four regions in input domain

$G(x_i, \overline{F}_i^{j_i}, \Delta)$	$\overline{F}_2^{j_2} \le x_2 \le \dfrac{\overline{F}_2^{j_2} + \overline{F}_2^{j_2+1}}{2}$	$\dfrac{\overline{F}_2^{j_2} + \overline{F}_2^{j_2+1}}{2} \le x_2 \le \overline{F}_2^{j_2+1}$
$\overline{F}_1^{j_1} \le x_1 \le \dfrac{\overline{F}_1^{j_1} + \overline{F}_1^{j_1+1}}{2}$	$\dfrac{16\Delta^2(\overline{F}_1^{j_1} - x_1)(\overline{F}_2^{j_2} - x_2)}{(\overline{F}_1^{j_1+1} - \overline{F}_1^{j_1})(\overline{F}_2^{j_2+1} - \overline{F}_2^{j_2})}$	$\dfrac{-16\Delta^2(\overline{F}_1^{j_1} - x_1)(\overline{F}_2^{j_2+1} - x_2)}{(\overline{F}_1^{j_1+1} - \overline{F}_1^{j_1})(\overline{F}_2^{j_2+1} - \overline{F}_2^{j_2})}$
$\dfrac{\overline{F}_1^{j_1} + \overline{F}_1^{j_1+1}}{2} \le x_1 \le \overline{F}_1^{j_1+1}$	$\dfrac{-16\Delta^2(\overline{F}_1^{j_1+1} - x_1)(\overline{F}_2^{j_2} - x_2)}{(\overline{F}_1^{j_1+1} - \overline{F}_1^{j_1})(\overline{F}_2^{j_2+1} - \overline{F}_2^{j_2})}$	$\dfrac{16\Delta^2(\overline{F}_1^{j_1+1} - x_1)(\overline{F}_2^{j_2+1} - x_2)}{(\overline{F}_1^{j_1+1} - \overline{F}_1^{j_1})(\overline{F}_2^{j_2+1} - \overline{F}_2^{j_2})}$

3.1. **The analytical formulation for the output of IT2-FLC in case of 2×2 rule base.** In order to show how IT2-FLC affects the control performance, it has been assumed that two interval type-2 fuzzy sets cover the universe of discourse of the input variables which are normalized to $[-1, 1]$. Using (24), (16), Table 2 and Table 3, (13) is simplified as in (27) and it is clear that $P(x_i, \overline{F}_i^{j_i}, K, \Delta) = 0$ and $Q(x_i, \overline{F}_i^{j_i}, \Delta) = 4\Delta^2(1-|x_1|)(1-|x_2|)$. Therefore, the output of the proposed controller can be written as

$$U = \frac{k_1x_1 + k_2x_2}{4\Delta^2(1-|x_1|)(1-|x_2|) + 1} = \gamma(x_1, x_2, \Delta)(k_1x_1 + k_2x_2) \tag{27}$$

where

$$\gamma(x_1, x_2, \Delta) = \frac{1}{4\Delta^2(1-|x_1|)(1-|x_2|) + 1} \tag{28}$$

Note that $\gamma(x_1, x_2, \Delta)$ is a decreasing function of inputs and Δ. If $\Delta = 0$, $\gamma(x_1, x_2, \Delta)$ is always equal to 1 then IT2-FLC naturally reduces to a T1-FLC and possesses an identical control signal output as the conventional controller. On the other hand, as the value of Δ is increased then $\gamma(x_1, x_2, \Delta)$ value decreases and this means variable gains for IT2-FLC. Naturally, these gains soften the control signal and reduce the oscillations in comparison with conventional controller. Consequently, the proposed IT2-FLC controller can manage the uncertainties much better and more robustly than T1-FLC and conventional linear controllers. The variation of $\gamma(x_1, x_2, \Delta)$ will be shown by an example in the illustrative example part of this section.

3.2. **The analytical formulation for the output of IT2-FLC in case of 2×3 rule base.** It can be seen from (13) that $P(x_i, \overline{F}_i^{j_i}, K, \Delta)$ and $Q(x_i, \overline{F}_i^{j_i}, \Delta)$ are also functions of the $\overline{F}_i^{j_i}$ of membership functions. Thus, the output of IT2-FLC is effected by the distance between $\overline{F}_i^{j_i}$'s that makes more nonlinearity for IT2-FLC and may be able to provide better performance. Let us assume that two and three diamond-shaped type-2 membership functions cover the universe of discourse of the x_1 and x_2, respectively, which are normalized to $[-1, 1]$. Thus, the output of IT2-FLC in (13) can be written

$$U = \alpha + \beta(k_1x_1 + k_2x_2) \tag{29}$$

where

$$\begin{aligned} \alpha(x_i, \overline{F}_i^{j_i}, K, \Delta) &= \frac{P(x_i, \overline{F}_i^{j_i}, K, \Delta)}{2[Q(x_i, \overline{F}_i^{j_i}, \Delta) + 1]} \\ \beta(x_i, \overline{F}_i^{j_i}, \Delta) &= \frac{1}{Q(x_i, \overline{F}_i^{j_i}, \Delta) + 1} \end{aligned} \tag{30}$$

One has to calculate $P(x_i, \overline{F}_i^{j_i}, K, \Delta)$ and $Q(x_i, \overline{F}_i^{j_i}, \Delta)$ for eight different regions in the range that the inputs are taking place in order to determine $\alpha(x_i, \overline{F}_i^{j_i}, K, \Delta)$ and $\beta(x_i, \overline{F}_i^{j_i}, \Delta)$ in (30). By substituting the corresponding values and using Table 2 and

Table 4. The expressions of $P(x_i, \overline{F}_i^{j_i}, K, \Delta)$ and $Q(x_i, \overline{F}_i^{j_i}, \Delta)$ for eight regions in the input domain

		$-1 \le x_2 \le -0.5$	$-0.5 \le x_2 \le 0$	$0 \le x_2 \le 0.5$	$0.5 \le x_2 \le 1$
$-1 \le x_1 \le 0$	P(.)	$-8\Delta^2(1+x_1)(1+x_2)k_2$	$-8\Delta^2(-1-x_1)(x_2)k_2$	$8\Delta^2(1+x_1)(x_2)k_2$	$-8\Delta^2(-1-x_1)(1-x_2)k_2$
	Q(.)	$8\Delta^2(-1-x_1)(-1-x_2)$	$-8\Delta^2(1+x_1)(x_2)$	$-8\Delta^2(-1-x_1)(x_2)$	$8\Delta^2(x_1+1)(1-x_2)$
$0 \le x_1 \le 1$	P(.)	$8\Delta^2(1-x_1)(-1-x_2)k_2$	$8\Delta^2(1-x_1)(x_2)k_2$	$8\Delta^2(1-x_1)(x_2)k_2$	$8\Delta^2(1-x_1)(1-x_2)k_2$
	Q(.)	$8\Delta^2(1-x_1)(1+x_2)$	$-8\Delta^2(1-x_1)(x_2)$	$8\Delta^2(1-x_1)(x_2)$	$8\Delta^2(1-x_1)(1-x_2)$

Table 3, $P(x_i, \overline{F}_i^{j_i}, K, \Delta)$ and $Q(x_i, \overline{F}_i^{j_i}, \Delta)$ can be written more explicitly for different regions of the range that the inputs are taking place as given in Table 4. As it can be seen from Table 4 that $P(x_i, \overline{F}_i^{j_i}, K, \Delta)$ and $Q(x_i, \overline{F}_i^{j_i}, \Delta)$ values depend on the input values (x_1 and x_2) and also on the uncertainty interval in the FOU (Δ) and the gains of conventional controller. It is also clear from Table 4 that $Q(x_i, \overline{F}_i^{j_i}, \Delta)$ has always positive sign for all regions and it is equal to zero if $\Delta = 0$. Thus, it can be concluded from (30) that $\beta(x_i, \overline{F}_i^{j_i}, \Delta)$ will get positive values and it will always be equal to or less than one. Increasing the value of Δ, $\alpha(x_i, \overline{F}_i^{j_i}, K, \Delta)$ and $\beta(x_i, \overline{F}_i^{j_i}, \Delta)$ will produce variable gains for IT2-FLC. The output of IT2-FLC with 2×3 rule base may produce a nonlinear control signal that may provide better performance in comparison with IT2-FLC, which has a 2×2 rule base.

In this study, the analytical derivation of the proposed IT2-FLC for two input-one output structure is examined in detail. The variations of $\alpha(x_i, \overline{F}_i^{j_i}, K, \Delta)$ and $\beta(x_i, \overline{F}_i^{j_i}, \Delta)$ will be examined over an example in the following illustrative example.

3.3. **Illustrative example.** In this subsection, the effect of nonlinear and time variable gains (γ, α, β) that are generated by IT2-FLC based on existing conventional controller is examined. Since most of the industrial processes can be approximated by first-order plus time delay (FOPTD) model, the process with the following transfer function is used

$$G_1(s) = \frac{e^{-3.5s}}{4s+1} \tag{31}$$

For the process in (31) a linear PI controller is implemented as $\dot{u} = k_p\dot{e} + k_ie$ where $k_p = 1.03$, $k_i = 0.1$ and $\dot{u}$ is the change of the control signal.

In order to investigate the nonlinearity of the proposed IT2-FLC, two fuzzy logic controllers with different number of type-2 membership functions for inputs are selected to see the effect of the variable gains of the controllers. For the first controller, two membership functions are used for each input and it is named as IT2-FLC22. For the second controller, two membership functions are assigned to the derivative of error input while three membership functions are set to the error input and the second controller is named as IT2-FLC23. The structure of the consequent part is designed as in (10) based on conventional controller with model point of $\overline{F}_i^{j_i} = \{-1, 1\}$ and $\overline{F}_i^{j_i} = \{-1, 0, 1\}$ for inputs with two membership functions and input with three membership functions, respectively. The scaling factors (c_1, c_2) are equal to 1. It is obvious from (13) and (26) that the results of IT2-FLC are identical to T1-FLC and conventional controller if $\Delta = 0$. By increasing the value of Δ in IT2-FLC, the linear conventional controller (which is the same as T1-FLC) is transformed to a nonlinear PI controller as shown in (13). Figure 3 shows the step responses and control signals of the process for the conventional PI and IT2-FLC in the case Δ is 0.2. By increasing the value of Δ to 0.4, the performance of IT2-FLC is improved compared with the conventional PI controller as illustrated in Figure 4. As it has been proven in (27), IT2-FLC22 is a nonlinear PI controller with a time varying parameter $\gamma(x_1, x_2, \Delta)$. Figure 5 shows the variation of $\gamma(x_1, x_2, \Delta)$ for $\Delta = 0.2$ and $\Delta = 0.4$. It can be seen from Figure 5 that $\gamma(x_1, x_2, \Delta)$ is variable that acquires values smaller than 1. By increasing the value of Δ from 0.2 to 0.4, the value of the term $\gamma(x_1, x_2, \Delta)$ will decrease and produce variable gains for IT2-FLC. These gains will cause a smoother control signal which improves the closed loop system performance. The IT2-FLC23 has two time-varying parameters $\alpha(x_i, \overline{F}_i^{j_i}, K, \Delta)$ and $\beta(x_i, \overline{F}_i^{j_i}, \Delta)$ and their variations are shown in Figure 6(a) and Figure 6(b) for Δ equals 0.2 and 0.4, respectively. If $\Delta = 0$, $\alpha(x_i, \overline{F}_i^{j_i}, K, \Delta)$ and $\beta(x_i, \overline{F}_i^{j_i}, \Delta)$ will be 0 and 1, respectively. Then, IT2-FLC reduces to a T1-FLC and has an identical output to the conventional controller. By increasing the Δ it can be seen from Figure 6 and (30) that the parameter $\beta(x_i, \overline{F}_i^{j_i}, \Delta)$ is less than

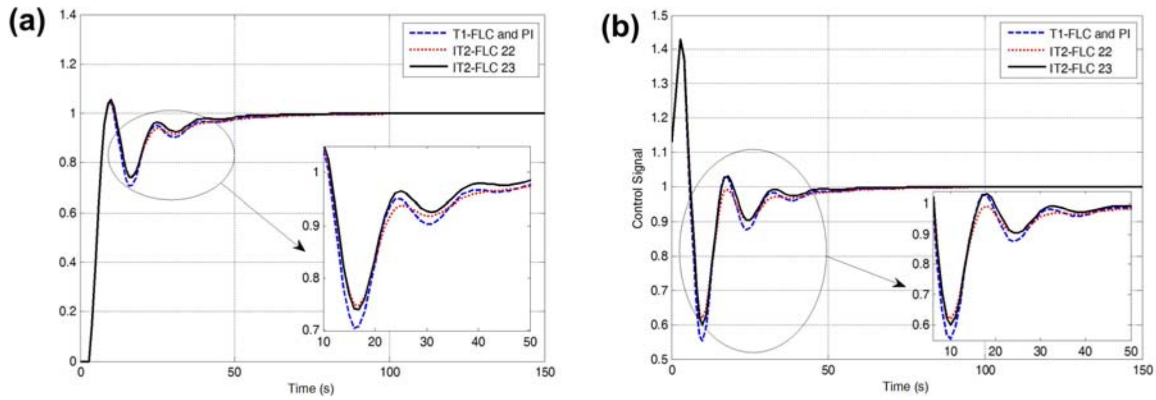

FIGURE 3. Illustration of (a) the step responses and (b) the control signals for $\Delta = 0.2$

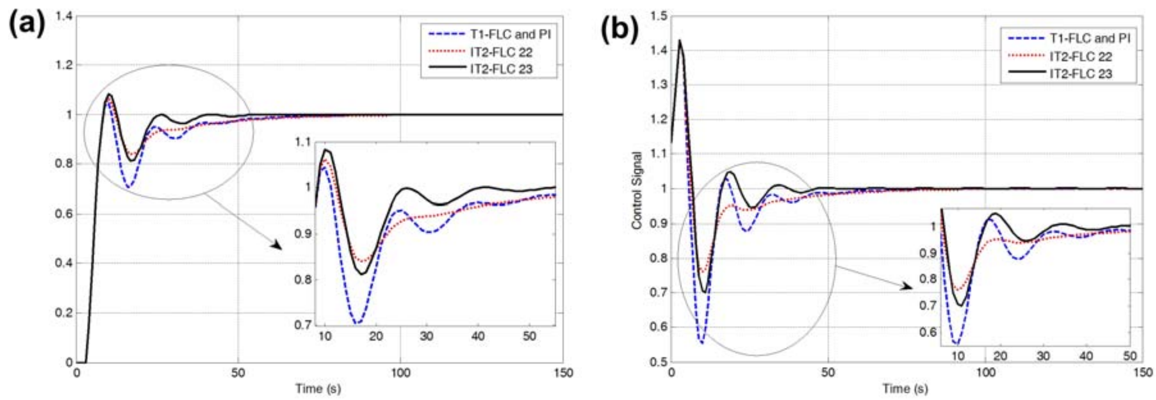

FIGURE 4. Illustration of (a) the step responses and (b) the control signals for $\Delta = 0.4$

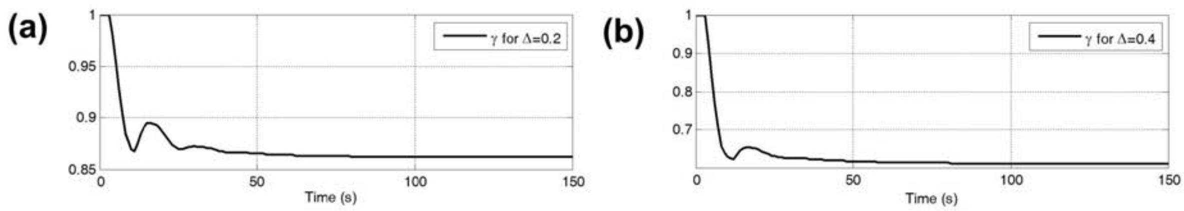

FIGURE 5. Illustration of the term $\gamma(x_1, x_2, \Delta)$ for (a) $\Delta = 0.2$ and (b) $\Delta = 0.4$

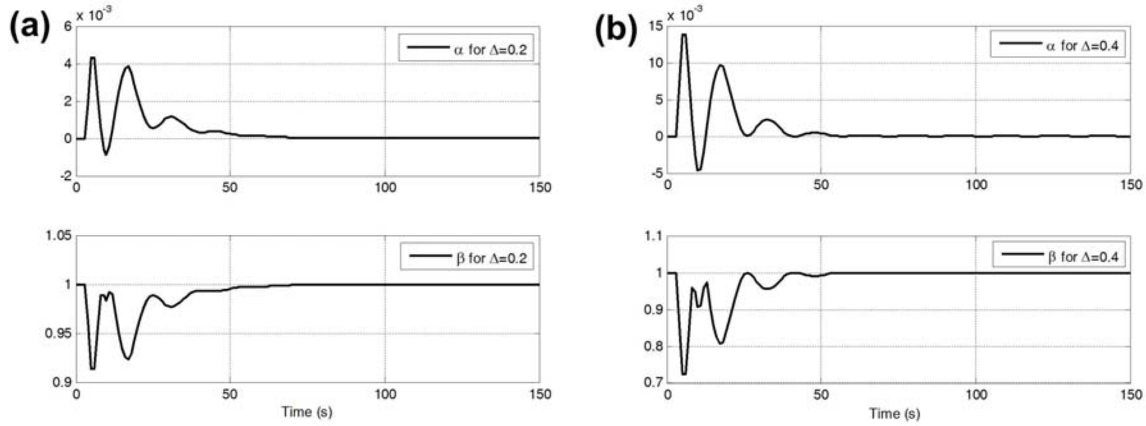

FIGURE 6. Illustration of α and β for (a) $\Delta = 0.2$ and (b) $\Delta = 0.4$

1 and $\alpha(x_i, \overline{F}_i^{j_i}, K, \Delta)$ varies in a small range around zero. The control signals are shown in Figure 3(b) and Figure 4(b).

4. **Simulation Studies.** In this section, the proposed IT2-FLC has been implemented on two benchmark processes. The inputs (x_1, x_2) of IT2-FLC and T1-FLC are already normalized to $[-1, 1]$. The unit step reference signal is applied to the closed-loop system.

4.1. **Robustness of the proposed IT2-FLCs to process parameter variations.** In this subsection, the robustness of the IT2-FLC against the parameter uncertainty in the system is tried to be illustrated on the following uncertain FOPTD process.

$$G_2(s) = \frac{Ke^{-Ls}}{\tau s + 1} \tag{32}$$

The nominal parameters of the process are $K = 1$, $\tau = 10$ and $L = 2.5$. The uncertainty intervals for the parameters are selected as $K = [0.5, 1.5]$, $\tau = [8, 12]$ and $L = [1.5, 4]$ [29]. The nominal parameters are varied to test the robustness of controllers to the parameter uncertainty. The Ziegler-Nichols [30] design method has provided the following PI controller parameters as $k_p = 3.6$ and $k_i = 0.432$. The scaling factors of the fuzzy controllers are set to $[c_1, c_2] = [1, 0.18]$ for the nominal system. Two proposed IT2-FLCs with different number of IT2FSs for the inputs, namely, IT2-FLC22 and IT2-FLC23 are considered for the robustness performance comparison. The step responses and the control signals of the implemented controllers for the nominal plant are shown in Figure 7 and Figure 8 for IT2-FLC22 and IT2-FLC23, respectively. These figures depict that the step response of IT2-FLC23 is more capable of eliminating the oscillation in comparison with the IT2-FLC22 and the PI controller.

Figure 9 shows the step responses of the uncertain system when the static gain (K) of the process changes to 1.5. The system responses when the time delay of the system varies 60% are illustrated in Figure 10.

The performance values $(ITAE, T_s)$ are tabulated in Table 5 for the system parameter variations. According to the results obtained from Table 5, increasing of the value of Δ and changing the number of membership functions do not necessarily improve the performance of the controller. The robustness of the proposed controller to the parameter uncertainty in comparison with linear PI controller can be observed from the results of this subsection. The FOU of the IT2-FLC provides the designer with an additional degree of freedom and provides a tool to design a more robust controller. Results in Table 5 validate this fact.

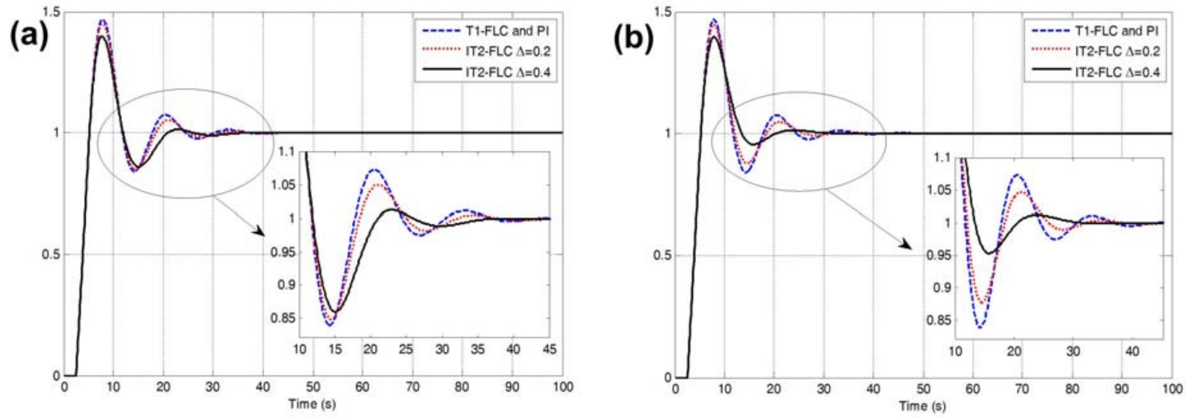

FIGURE 7. Illustration of the step responses (a) IT2-FLC22 and (b) IT2-FLC23

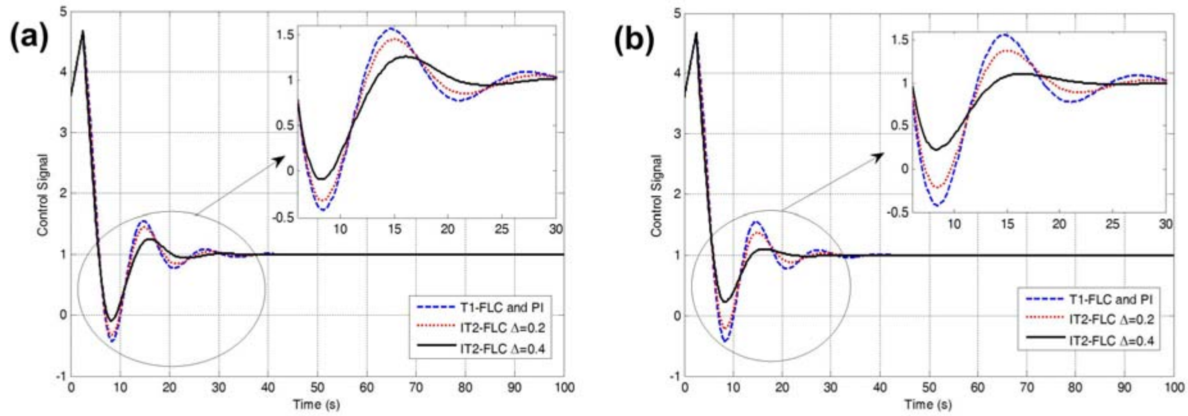

FIGURE 8. Illustration of the control signals (a) IT2-FLC22 and (b) IT2-FLC23

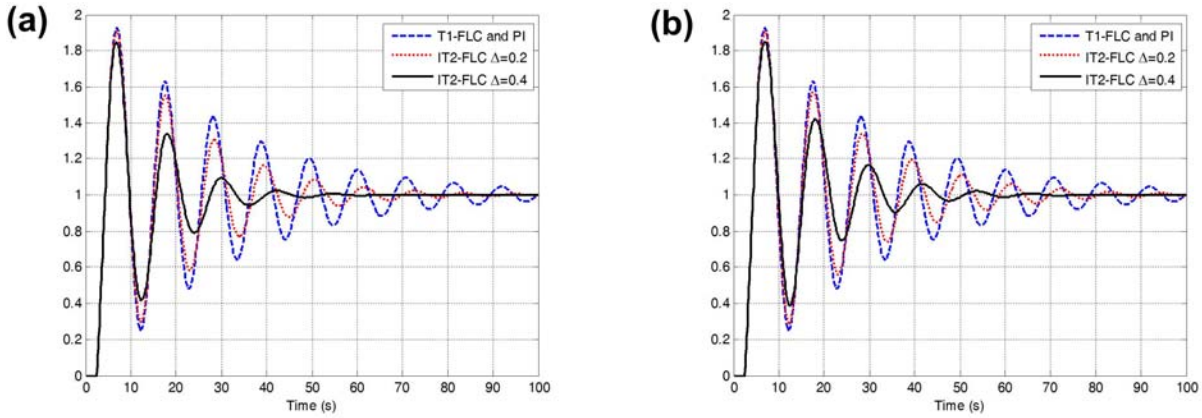

FIGURE 9. Illustration of the step responses (a) IT2-FLC22 and (b) IT2-FLC23 on the plant by varying static gain parameter to $K = 1.5$

4.2. **Experiment.** In this subsection, the IT2-FLC with the proposed structure is implemented on the Quanser ball and beam system. The objective of the ball and beam system is to stabilize the ball to a desired position along the beam. The experimental setup and cascade control structure that will be used for the control of ball and beam

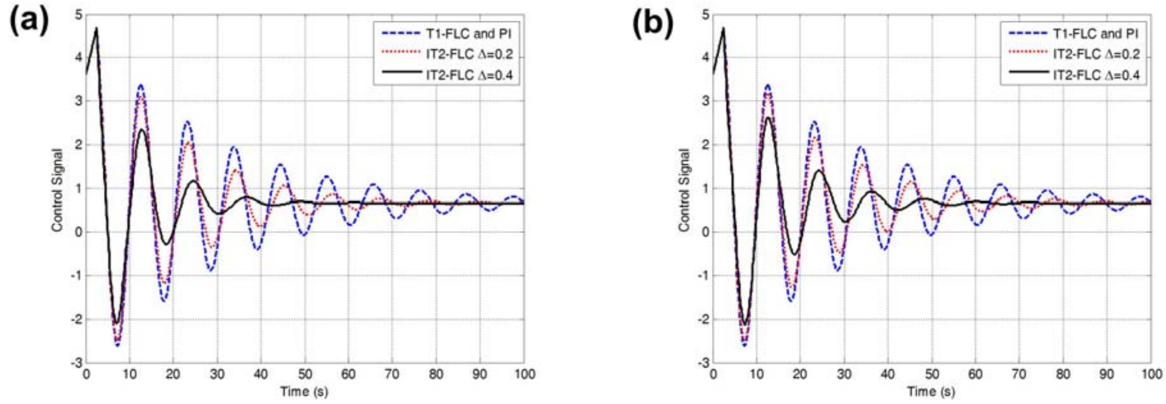

Figure 10. Illustration of the step responses (a) IT2-FLC22 and (b) IT2-FLC23 on the plant by varying the system time delay parameter to $L = 4$

Table 5. The performance values for the $G_2(s)$ with parameter variations

		Nominal		Static gain change				Time constant change				Time delay change			
		$k = 1$		$k = 1.5$		$k = 0.5$		$k = 1$		$k = 1$		$k = 1$		$k = 1$	
		$\tau = 10$		$\tau = 10$		$\tau = 10$		$\tau = 8$		$\tau = 12$		$\tau = 10$		$\tau = 10$	
		$L = 2.5$		$L = 2.5$		$L = 2.5$		$L = 2.5$		$L = 2.5$		$L = 4$		$L = 1.5$	
		ITAE	$T_s(s)$	ITAE	$T_s(s)$	ITAE	$T_s(s)$	ITAE	$T_s(s)$	ITAE	$T_s(s)$	ITAE	$T_s(s)$	ITAE	$T_s(s)$
PI		43.82	22.33	516.5	87.33	30.75	17.83	85.39	30.91	34.12	17.25	8784	429.00	11.31	9.31
$\Delta = 0.2$	IT2-FLC22	39.97	21.42	286.6	57.20	28.52	17.90	70.04	26.37	32.18	17.84	3731	209.30	8.73	8.83
	IT2-FLC23	37.19	17.12	332.8	62.50	29.27	18.15	62.74	26.12	31.71	13.63	2646	166.16	12.65	10.00
$\Delta = 0.4$	IT2-FLC22	36.12	18.83	124.0	37.26	29.89	18.50	52.21	26.36	32.35	20.32	1276	112.81	7.085	4.80
	IT2-FLC23	29.43	12.50	164.8	42.90	31.08	19.40	40.17	21.30	30.51	15.60	797.3	76.82	16.77	13.62

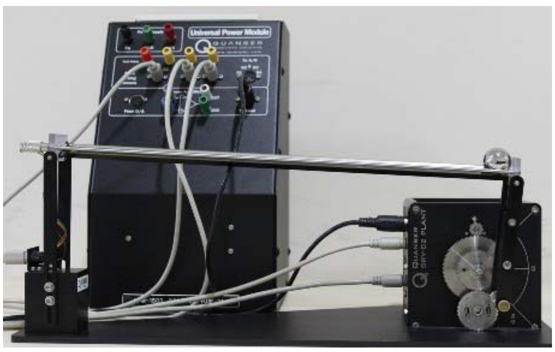

Figure 11. Illustration of the ball and beam experimental system

system are shown in Figure 11 and Figure 12, respectively. As illustrated in Figure 12, this system is comprised of motor and beam plants and both systems are in series.

The complete process transfer function (motor voltage to ball displacement) is expressed as follows [31]:

$$\frac{X(s)}{V_m(s)} = \frac{K_{bb}K}{(\tau s + 1)s^3} \tag{33}$$

where, K_{bb} is the model gain, K is the open-loop steady-state gain, τ is the open-loop time constant, X is the ball position, θ_l is the motor angle and V_m is the motor input voltage. In this example, the parameters of the system are $K_{bb} = 0.418$, $K = 1.76$, $\tau = 0.0285$. Here, the proportional-velocity (PV) controller (motor compensator) gains are $k_{p2} = 13.6$ and

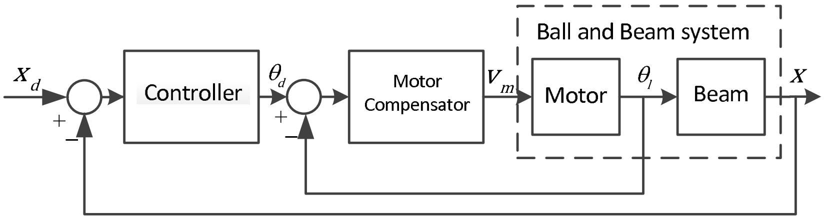

FIGURE 12. Illustration of the cascade control system.

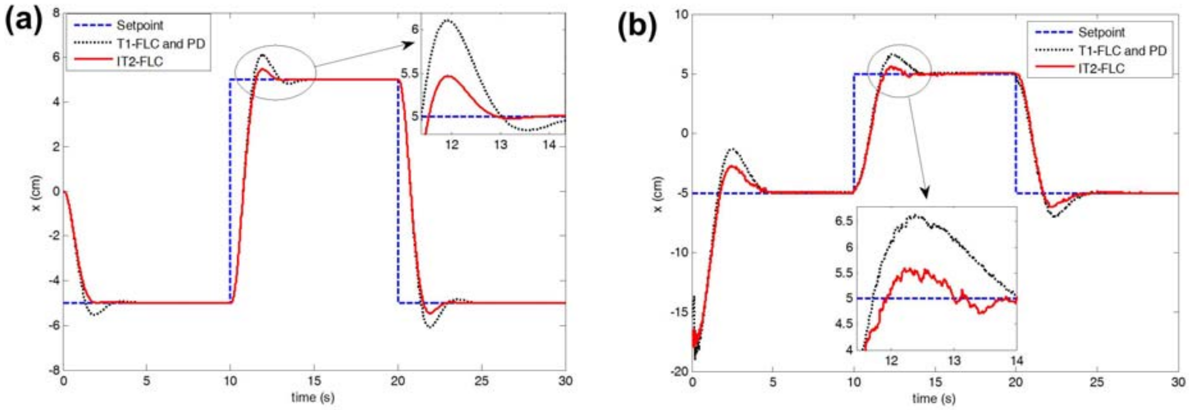

FIGURE 13. Illustration of the ball position responses in case of (a) simulation and (b) experiment

$k_v = 0.0795$, which is the same for IT2-FLC and PD cascade controllers. The proportional (k_{p1}) and derivative (k_d) gains of PD cascade controller are 6.315 and 4.21, respectively. For the IT2-FLC, two diamond-shaped membership functions are assigned to the error input while three membership functions are set to the derivative of error. The scaling factors of the fuzzy controllers are set to $[c_1, c_2] = [10, 5]$ and the value of Δ is 0.3. The simulation and experimental results of the ball position for the T1-FLC (PD) and IT2-FLC are shown in Figure 13(a) and Figure 13(b), respectively. The performance values are tabulated in Table 6 for the ball and beam system for simulation and experiment cases. According to the results obtained from Table 6, IT2-FLC can be characterized by small peak time, settling time and percentage overshoot in comparison with T1-FLC and PD controllers. Nonlinear properties caused by inertia of the ball and error of the position measurement on beam lead to a slight difference within the results of simulation and experiment. The overshoot in the experiment increased by 62.1% in comparison with simulation result when T1-FLC (PD) has been used while the percent increase in the overshoot is about 24.8% for the case of IT2-FLC. Therefore, it can be concluded that the proposed controller is more robust to the possible uncertainties and nonlinearities of the system compared to T1-FLC and linear conventional controllers. The output of controllers and the motor input voltage responses of T1-FLC (PD) and IT2-FLC for experimental test are illustrated in Figure 14(a) and Figure 14(b), respectively.

5. **Conclusion.** In this paper, a systematical methodology to construct an interval type-2 fuzzy logic controller is proposed. The methodology depends on a nonlinear mapping from an existing PI/PD control law to IT2-FLC that eases the generation of rule base. The structure of the proposed controller is achieved under circumstances that input type-2 membership functions are diamond-shaped and a certain closed-form inference engine

TABLE 6. Comparison of the performance of the two controllers

	Simulation		Experiment	
	T1-FLC (PD)	IT2-FLC	T1-FLC (PD)	IT2-FLC
Peak time	1.92	1.93	2.38	2.22
Settling time	2.79	2.45	3.98	2.95
Percentage overshoot	11.1	4.67	18	5.83
Steady-state error	0	0	0.09	0.009

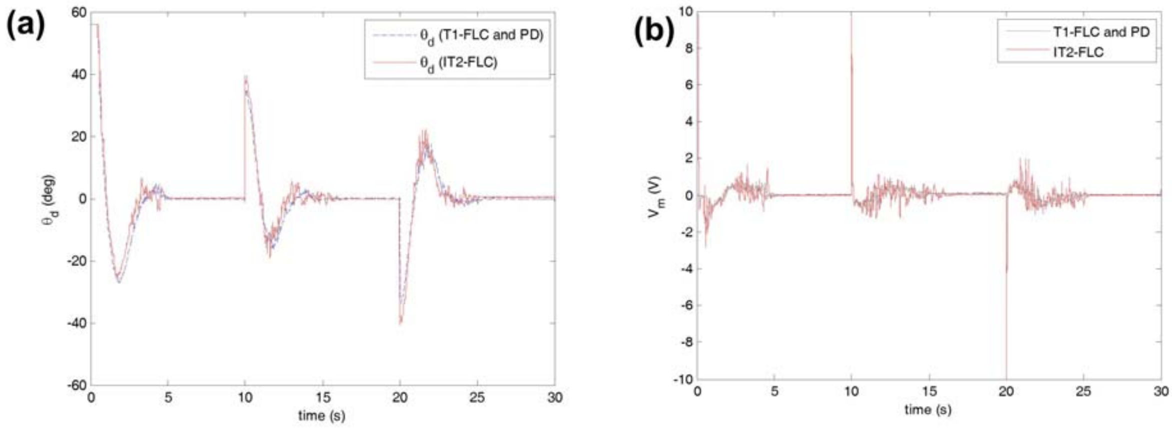

FIGURE 14. Illustration of the (a) output results and (b) motor input voltages of T1-FLC and IT2-FLC for experimental test

is used. It has been proven that these preferences provide a closed-form for the output of IT2-FLC. Then, certain elaborations can be done on the parameters of the evolved closed output form. If the FOU of the IT2-FLC is zero then the obtained control law is identical to the type-1 fuzzy logic and conventional PI/PD controllers. If FOU is not equal to zero, then an additional degree of freedom is acquired and this provides the designer an additional tool to cope with the uncertainties and nonlinearities. Results show that the proposed controller is more robust to the parameter uncertainties and eliminate the oscillations much better than type-1 fuzzy logic and linear conventional controllers. However, it is not so fair and right to draw a general conclusion that the system performance is necessarily guaranteed to be improved for all systems and design methodologies. The analysis for the proposed IT2-FLC can be generalized to different number and type of membership functions for the inputs.

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