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MATHEMATICAL MODEL AND ALGORITHM OF SPLIT LOAD VEHICLE ROUTING PROBLEM WITH SIMULTANEOUS DELIVERY AND PICKUP

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Abstract. *This paper aims to study the split load vehicle routing problem in order to find a solution which would decrease logistics' cost by implementing simultaneous delivery and pickup which is part of the multi-attribute vehicle routing problem. A model was established based on certain preconditions and assumptions considering some specific characteristics in this simultaneous delivery and pickup procedure. A tabu search algorithm was applied to obtain the satisfactory solution. In this algorithm, a good initial solution was acquired by using the nearest neighborhood insertion method with the nearest neighbor structured by four operation methods including relocation operation, exchange operation, 2-opt, and the split point reposition operation. A parameter of tabu length was determined by a randomly selecting method, and a penalty function was designed through an automatic adjustment parameter. Data derived from case studies from a subsidiary company of China Railway Express were used to testify to the effectiveness and feasibility of the model and algorithm studied in this paper by comparing characteristics of solution convergence, solution quality and computing time with those of present analogous studies.*

Keywords: Vehicle routing problem, Split load, Simultaneous delivery and pickup, Tabu search

1. Introduction.

As a multi-attribute vehicle routing problem, two constraints are merged into the Simultaneous Delivery and Pick up Vehicle Routing Problem with Split Loads (SDPVRPSL). One constraint is simultaneous delivery and pickup, and the other constraint is split-load. For the split-load, this problem belongs to the relaxation of the Vehicle Routing Problem (VRP), but it could not make this problem easily resolved. It is a more complicated NP-hard problem.

1.1. Relative studies.

Simultaneous Delivery and Pick up Vehicle Routing Problem with Split loads (SDPVRPSL) has both constraint of delivery and pickup and constraint of the split-load, which is a multi-attribute vehicle routing problem. This problem is relaxation of the VRP problem considering split-load, but this could not make the problem become simpler, and it is a more complicated NP-hard problem. This problem well reflects the objective conditions and is widespread in the transport system, logistic distribution system, and express delivery system. It can provide an effective decision support strategy for the sub-system design of vehicle scheduling decisions. This study hence shows more practical value in the transport application.

The Vehicle Routing Problem with Simultaneous Delivery and Pickup was originally put forward by Min [1] in the late 1980s, and the approximate algorithm was the leading research method inclusive of insertion heuristic [2,3], genetic algorithm [4,5,8], simulated annealing algorithm [6-10], ant colony optimization [11], tabu search [12-14] and particle swarm optimization [15], etc. Vehicle Routing Problem with Split-Load (VRPSL) and Vehicle Routing Problem with Split Delivery (VRPSD) were originally put forward by Dror in 1989 and resolved by heuristic algorithm. After that an integer programming model was established by him in 1990. Meanwhile, he also improved the heuristic algorithm resolving this model. In 1994 some special inequality constraints were developed for decomposing the feasible region in order to obtain the optimal solution [16-18]; hence, the heuristic algorithm is the primary method at present in the existing research work [19-42].

Based on the aggregate analysis for studies at home and abroad, we can find that although some disadvantages of present studies for VRPSL or VRPSD have been overcome by the researchers, the present algorithms still mainly focused on the random search procedure, and the customer demands are generally less than the vehicle maximum load capacity. There are few synthetic studies for both simultaneous delivery and pick up and split-load, and the multi-attribute vehicle routing problem will still become the emphasis of the future VRP study. It has more practical functions in the actual logistics operation [43].

1.2. **Objective, scope and significance.** This paper aims to study split-load vehicle routing procedures for the logistic distribution operation in order to obtain minimum vehicle number and minimum transport cost for the logistic enterprise under the conditions of simultaneous delivery and pick up. In this procedure, customer demands can be larger than the maximum vehicle load capacity. The customer with this characteristic is defined as a bulk-customer and the whole vehicle service will be executed for the bulk-customer. Remaining demands will be inputted into a tabu search with an overall consideration of the general customer. In the tabu search algorithm, a good initial solution is obtained based on the nearest neighbor insertion method and the nearest neighbor is structured by four operation methods. Tabu length is randomly selected and the penalty function is designed through automatically adjusting the parameter. It is this tabu search algorithm utilized for resolving the SDPVRPSL problem that embodies the value of this study.

The research objectives of this paper are described as follows: (1) to introduce a mathematical model solving the Simultaneous Delivery and Pick Up Vehicle Routing Problem with Split Loads; (2) to seek a heuristic solution for the problem obtaining minimum vehicle number and transportation cost for the logistic enterprise; and (3) to obtain an effective split load vehicle route with simultaneous delivery and pickup.

There are four parts in this paper. The first part is the problem model which introduces the preconditions, assumptions and model structure. The second part is the solution algorithm which introduces initial solution generating method, neighborhood construction, solution evaluation rules and tabu rules. Experiments and result analysis are in the third part which shows the effectiveness of the model and algorithm by comparing two simulation procedures. The major findings of the study are summarized finally in the conclusion.

2. **Mathematical Model of SDPVRPSL Problem.** There are some pre-conditions assumed for the model being established. First, there is only one distribution center. Second, there is an identical maximum travel distance for all the vehicles. Third, there is a known customer demand used for the distribution. Fourth, there is a linear relationship between transportation cost and transportation distance. Fifth, there is an identical restriction for any vehicle's maximum load capacity. In addition, we assume that all the vehicles depart from the distribution center at the beginning of distribution procedure, and respectively return to the center at the end of each delivery procedure. Hence, it is a vehicle routing problem with a closed loop. Based on the above assumptions, the mathematical model of the Vehicle Routing Problem with Simultaneous Delivery and Pick Up is established as follows.

$$\min\ F = \sum_{i\in V}\sum_{j\in V}\sum_{k\in K} c_{ij}d_{ij}x_{ijk} \tag{1}$$

Subject to:

$$1 \le \sum_{i\in V}\sum_{j\in V\backslash 0} x_{ijk} \le 2, \quad i,j \notin C, \quad k \in K \tag{2}$$

$$\sum_{i\in V} x_{ihk} = \sum_{j\in V} x_{hjk}, \quad h \in V\backslash\{0\}, \quad k \in K \tag{3}$$

$$\sum_{i\in V\backslash\{0\}} f_{ik}^{d} = D_i, \quad k \in K \tag{4}$$

$$\sum_{i\in V\backslash\{0\}} f_{ik}^{p} = P_i, \quad k \in K \tag{5}$$

$$0 \le D_j - \mu_j Q < Q, \quad j \in C, \quad \mu = 0,1,2,\dots,N \tag{6}$$

$$0 \le P_j - \mu_j Q < Q, \quad j \in C, \quad \mu = 0,1,2,\dots,N \tag{7}$$

$$\gamma_k = \sum_{i\in V}\sum_{j\in V\backslash\{0\}} D_j x_{ijk}, \quad k \in K \tag{8}$$

$$\gamma_k \le Q, \quad k \in K \tag{9}$$

$$\sum_{i\in V}\sum_{j\in V\backslash\{0\}} d_{ijk}x_{ijk} \le L, \quad k \in K \tag{10}$$

$$\max\ d_{0jk} \le \frac{L}{2}, \quad j \in V\backslash\{0\}, \quad k \in K \tag{11}$$

$$\gamma_j = \gamma_{j-1} + (P_j - D_j), \quad j \in V\backslash\{0\} \tag{12}$$

$$0 \le \gamma_j \le Q - \max\{0, \Delta s\}, \quad j \in V\backslash\{0\} \tag{13}$$

$$\Delta s = \sum_{i\in V}\sum_{j\in V\backslash\{0\}} (P_j - D_j)x_{ijk} \tag{14}$$

where,

V: Set of all the sites;
C: Set of all the bulk customers;
K: Set of all the vehicles;
Q: Maximum vehicle load capacity;
L: Maximum vehicle travel distance;
c_{ij}: Unit transportation cost from site node i to site node j, $i \in V$, $j \in V$;
d_{ij}: Distance between node i and node j, ($i \in V$, $j \in V$, $i \neq j$; $d_{ii} = \infty$, $i \in V\backslash\{0\}$, $d_{\infty} = 0$);
D_j: Delivery demand of distribution site node j, $j \in V\backslash\{0\}$;

P_j: Pick up demand of distribution site node j, $j \in V\backslash\{0\}$;
δ_j: $\delta_j = P_j - D_j$, $j \in V\backslash\{0\}$;
Δs: $\Delta s = \sum_{j\in S} \delta_j$;
n: Number of all the distribution sites;
γ_k: Load weight of vehicle k departing from distribution center, $k \in K$;
γ_j: Load weight of vehicle k after serving the distribution site j;
μ_j: Number of vehicles serving the bulk customer with a full load;
f_{ik}^d: Customer i demand delivered by vehicle k;
f_{ik}^p: Customer i demand picked up by vehicle k.

Equation (1) is the objective function acquiring minimum total vehicle transport cost. Inequality (2) indicates that for each customer's demand, it can only be split up one time to achieve the customer demand, but bulk-customer demand is the exception. Equation (3) ensures that each vehicle arriving at a certain delivery site node will depart from the same site node. Equations (4) and (5) ensure that each customer's demand can be achieved, namely, the total load capacity of all the vehicles serving the customer i should achieve this customer's demand. Inequality (6) and (7) indicate that the remaining transportation weight of distribution site is less than the maximum vehicle load capacity after fully loaded delivery and pick up. Equation (8) gives an initial vehicle load weight. Inequality (9) assures that initial vehicle load weight cannot exceed the maximum vehicle load capacity. Inequality (10) ensures that any vehicle's travel distance cannot exceed maximum travel distance. Inequality (11) achieves demand of the farthest site. Equation (12) indicates vehicle load weight departing from distribution site. Inequality (13) and Equation (14) indicate that demand remaining after fulfilling delivery and pick up satisfies requirement for maximum vehicle load capacity. In the model, symbol x_{ijk} represents 0-1 variable depending on situation of distribution site visited by vehicle k. Value $x_{ijk} = 1$ indicates that distribution site j is visited by vehicle k after the distribution site i, and $x_{ijk} = 0$ otherwise.

3. Algorithm of SDPVRPSL Problem.

Considering the main characteristics of the SDPVRPSL problem with the simultaneous delivery and pick up based on present study results, tabu search is utilized to acquire a reasonable solution. Since it strongly depends on the quality of the initial solution, searching for an initial solution as good as possible is the key step of this algorithm.

3.1. Initial solution and nearest neighbor.

Whether an algorithm executes well or not depends much on the quality of the initial solution. A good initial solution can accelerate calculation procedure and improve the final solution. In this paper, Nearest Neighbor Heuristic (NNH) method is utilized to construct the initial solution [34]. The initial solution generation is shown in Figure 1. The basic idea of NNH is that a set of paths is generated in succession, and for each path, the original node is the customer site with shortest distance from distribution center. Then a customer site accepted with the shortest distance from this customer node is added to the path. As customer node j is added, if parameter value $\gamma_j = Q$, this route successfully comes into being. However, if parameter value $\gamma_j \succ Q$, customer node j is split based on full load criterion for this path, and the transportation weight remaining is regarded as a new customer node and placed in the unvisited node set for the sake of reconstructing another path. The procedure will not stop until all customers have been added.

For constructing nearest neighbor, four operations are used randomly including relocation operation, exchange operation, 2-opt and split-point reposition operation.

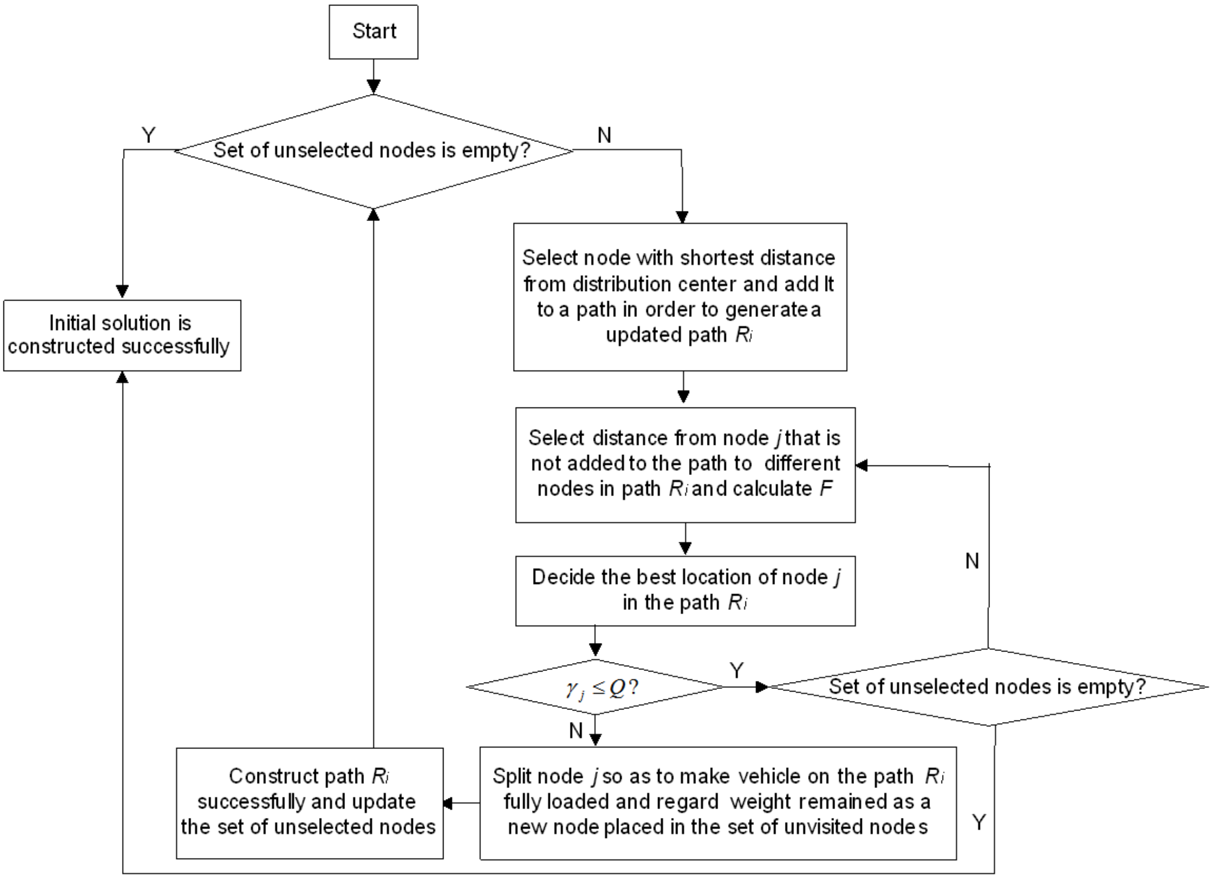

FIGURE 1. Initial solution generation procedure

3.2. **Solution evaluation.** An exchange operation generating an unfeasible solution is accepted in the algorithm in order to sufficiently search in a solution space, and a penalty function is introduced so as to evaluate solution quality. Calculation method is shown as Equation (15).

$$Z = \sum_{r=1}^{R} [f(r) + pE(r)] \tag{15}$$

where,

Z: Objective value of evaluation function;
R: Total number of distribution path;
$f(r)$: Minimum vehicle travel cost on a distribution path;
$E(r)$: Transportation weight exceeding minimum vehicle load capacity;
p: Penalty factor.

Transportation weight exceeding vehicle load capacity $E(r)$ is 0 if a solution is feasible. Value scope of penalty factor is $p \in [0.001, 1024]$, which is weighted by a self-adjusting parameter. Evaluation runs after every ten iterations. If all these previous ten solutions are feasible, the penalty factor is divided by 2 and if they are unfeasible, multiplied by 2. This mechanism can generate a combination of feasible solutions and unfeasible solutions and decrease the possibility of acquiring a local optimal solution [44].

3.3. **Tabu characteristics and other rules.** Tabu objects represent local optimal solutions in a tabu table. In this paper, the best solution derived from each iteration procedure is put into a tabu table and regarded as a tabu object. After that, the tabu cycle is modified and the tabu list is updated. A method of randomly selected tabu length is utilized here in order to find a better solution than that obtained by a fixed selection method or a dynamic selection method. A candidate set is generated by randomly selecting several neighbors from a neighborhood of current solutions.

A typical amnesty rule is executed, namely, if a solution obtained by a neighborhood exchange is better than that found, tabu status of this exchange is ignored and this exchange can be executed. Achieving maximum iteration steps is a termination criterion.

3.4. **Description of algorithm flow.** Symbol $\mathrm{int}er$ represents present iteration steps and symbol $\max\_\mathrm{int}er$ represents maximum iteration steps, then algorithm flow is described as follows:

Step 0: Obtain an initial solution by using method described in 3.2 and let this solution become the current best solution. Let $\mathrm{int}er$ be $\mathrm{int}er$: $\mathrm{int}er = 0$ and tabu table be $S : S = \{\Phi\}$;

While ($\mathrm{int}er \leq \max\_\mathrm{int}er$)

Begin

Step 1: Randomly select an exchange method of neighborhood among four exchange methods and use it for the current solution, adding the generated solution to candidate set;

Step 2: Let $\mathrm{int}er = \mathrm{int}er + 1$. Select a non-tabu best solution from the candidate set, or if an existing tabu candidate solution is better than the current best solution, remove tabu for this solution and set as an updated current best solution;

Step 3: Select a number by randomly $[1, m+1]$ and symbol m represents an alternative vehicle number;

Step 4: If a best feasible solution is not improved after 100 consecutive iterations, $\mathrm{int}er = \max\_\mathrm{int}er$, or else $\mathrm{int}er = \mathrm{int}er + 1$;

end

In the flow, tabu length is selected in Step 3. In Step 4, if a best feasible solution is not improved after 100 consecutive iterations, a desired search result has been obtained and current solution is the optimum solution. This is good for reducing circle times and saving calculation time. This can also assure solution quality.

4. **Case Studies.** Data derived from cases studied from a subsidiary company of China Railway Express were used to testify to the effectiveness and feasibility of the model and algorithm studied in this paper by comparing characteristics of solution convergence, solution quality and computing time with those of present analogous studies.

4.1. **Case 1 study.** There are ten distribution vehicles in a subsidiary company of China Railway Express. The maximum vehicle load capacity is five tons and maximum vehicle travel distance is sixty km. There are a total of twenty-four luggage and package distribution sites that need to be served. The location coordinate of the distribution center is (0, 0) and the location coordinate of each distribution site is shown as Table 1.

Delivery demands and pick-up demands at a certain time for these twenty-four distribution sites are randomly generated, and the vehicles need to be reasonably arranged to fulfill the distribution procedure in order to acquire a minimum distribution distance.

(1) Parameter setting

Parameter setting is that unit transportation cost is $c_{ij} = 1$ and maximum iteration steps are $\max\_\mathrm{int}er = 500$. Maximum candidate solutions number is $\max\_c$ and_list $= 500$ and maximum consecutive iteration steps are $\max\_cons\_\mathrm{int}er = 200$.

(2) Data processing before calculation operation

Table 1. Location coordinate and transportation weight of luggage and package distribution site

Number	1	2	3	4	5	6	7	8	9	10	11	12
x-coordinate /km	15	5	12	14	7	13	16	8	3	3	2	7
y-coordinate /km	10	2	3	5	5	7	9	8	8	12	15	12
Delivery demand /t	2.0	0.2	1.0	0.8	1.0	0.3	0.7	0.8	0	0	1.8	0.5
Pick up demand /t	1.0	0.3	0.7	2.4	0.8	1.4	0.9	1.0	2	0.5	0	1.0
Number	13	14	15	16	17	18	19	20	21	22	23	24
x-coordinate /km	12	18	15	13	10	7	10	20	0	18	6	5
y-coordinate /km	13	12	15	18	16	18	10	2	10	4	16	11
Delivery demand /t	1.3	0.7	0.2	1.0	0.5	1.0	1.9	0.5	1.6	7.5	0	11
Pick up demand /t	0.5	0.6	0.3	0.8	0.5	1.0	1.5	0	1.4	10.6	5.3	0

Let bulk customers demand fully loaded vehicles for delivery or pick up and obtain path of 0-22-0, 0-24-22-0 and 0-24-23-0, with an accumulated distance being 116.4352 km. Transportation weight remaining is regarded as general customer demand and executed by tabu search.

(3) Calculation result of tabu search

MATLAB R2009b is utilized to execute algorithm running. The algorithm is executed ten times and one best result is selected as an ultimate solution. Calculation results are shown as follows:

Path 1: 0—20(0.5, 0)—22(2.5, 0.6)—4(0.8, 2.4)—3(1.0, 0.7)—0;

Path 2: 0—21(1.6, 1.4)—11(1.8, 0)—10(0, 0.5)—24(1.00)—9(0, 2.0)—0;

Path 3: 0—8(0.8, 1.0)—9(1.9, 1.5)—0;

Path 4: 0—23(0, 0.3)—18(1.0, 1.0)—17(0.5, 0.5)—13(1.3, 0.5)—15(0.2, 0.3)—16(1.0, 0.8)—12(0.5, 1.0)—0;

Path 5: 0—5(1.0, 0.8)—7(0.7, 0.9)—14(0.7, 0.6)—1(2.0, 1.0)—6(0.3, 1.4)—2(0.2, 0.3)–0.

In the path, number "0" represents the distribution center and the number before brackets represents the distribution site. In the brackets, the front number is delivery demand and the back number is pick-up demand.

Total length of these five paths is calculated as:

$$42.2490 + 32.9331 + 28.2843 + 56.1240 + 44.0870 = 203.6774 \text{ km}.$$

(4) Total length

Adding the length of three paths generated for all the bulk customers, the total length of these eight paths is calculated as: 116.4352 + 203.6774 = 320.1126 km. For each running, calculating time is a value of 4∼6 seconds.

(5) Convergence process of solution

Convergence process of solution is shown as Figure 2.

The total paths' length converges after executing thirty-five iterations, and is kept at a stable status with total paths' length being 203.6774 km.

4.2. **Case 2 study.** Six distribution sites and fifteen vehicles need to be added as a result of increasing distribution demand, with the same other conditions as Case 1. All the specific data are listed in Table 2.

(1) Parameter setting and data processing methods needed before calculation operation are the same as those in Case 1. Let bulk customers demand fully loaded for delivery or pick up to obtain path of 0-22-0, 0-24-22-0, 0-24-23-0 and 0-25-0, with the accumulated

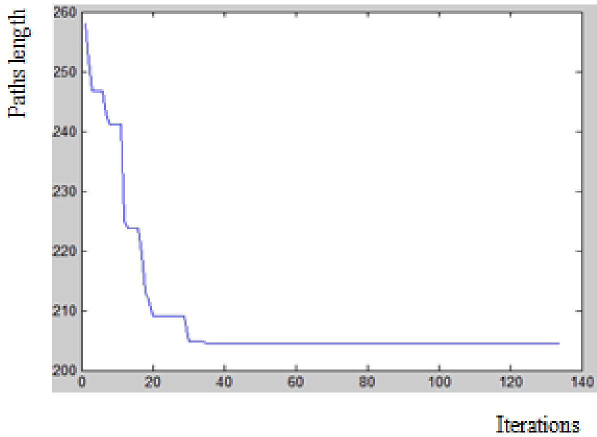

FIGURE 2. Convergence process of solution

TABLE 2. Location coordinate and transportation weight of luggage and package distribution site

Number	1	2	3	4	5	6	7	8	9	10
x-coordinate /km	15	5	12	14	7	13	16	8	3	3
y-coordinate /km	10	2	3	5	5	7	9	8	8	12
Delivery demand /t	2.0	0.2	1.0	0.8	1.0	0.3	0.7	0.8	0	0
Pick up demand /t	1.0	0.3	0.7	2.4	0.8	1.4	0.9	1.0	2	0.5
Number	11	12	13	14	15	16	17	18	19	20
x-coordinate /km	2	7	12	18	15	13	10	7	10	20
y-coordinate /km	15	12	13	12	15	18	16	18	10	2
Delivery demand /t	1.8	0.5	1.3	0.7	0.2	1.0	0.5	1.0	1.9	0.5
Pick up demand /t	0	1.0	0.5	0.6	0.3	0.8	0.5	1.0	1.5	0
Number	21	22	23	24	25	26	27	28	29	30
x-coordinate /km	0	18	6	5	1	4	6	8	10	12
y-coordinate /km	10	4	16	11	17	14	16	3	9	11
Delivery demand /t	1.6	7.5	0	11	8	4	2	1	2	3
Pick up demand /t	1.4	10.6	5.3	0	2	1	0	3	1	2

distance being 133.4645 km. Transportation weight remaining is regarded as general customer demand and executed by tabu search.

(2) Calculation result of tabu search

Similar to Case 1, the algorithm is executed ten times and one best result is selected as an ultimate solution. Eight paths are obtained with the total length being 303.2880 km.

(3) Total length

Adding four paths' length generated for all the bulk customers, the total length of these twelve paths is calculated as: $133.4645 + 303.2880 = 436.7525$ km. For each running, calculating time is a value of 11$\sim$16 seconds.

(4) Convergence process of the solution

Convergence process of solution is shown as Figure 3.

The total length of all paths converges after executing sixty iterations, keeping a stable status with total paths' length being 203.6774 km.

4.3. **Comparative analysis of two cases.** Comparative analysis is not only carried out between the calculated results obtained from the two cases in order to testify to the

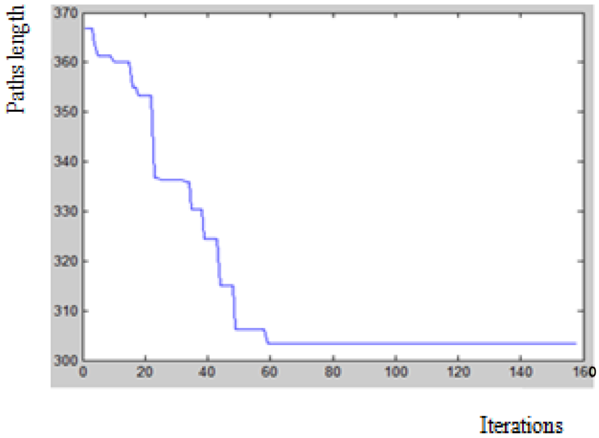

FIGURE 3. Convergence process of solution

TABLE 3. Total length of paths obtained by algorithm

Calculating time	Total length of paths (not including the bulk customers)	
	Example 1	Example 2
1	**203.6774**	311.2672
2	206.0083	313.2929
3	204.0996	316.8096
4	212.3811	309.6790
5	209.2467	306.9640
6	210.5528	**303.2880**
7	204.5190	310.5015
8	203.6774	316.8096
9	207.3277	307.8068
10	206.6337	304.7826
Mean	206.8124	310.1201

feasibility of the model and the algorithm studied in this paper, but also is carried out between these calculation results and present research results in order to testify to the validity of the model and algorithm. The calculation process is executed by MATLAB R2009b on computer with Inter (R) Core (TM) 2 Duo CPU 1.80 GHz.

4.3.1. *Feasibility analysis of algorithm.* Feasibility analysis is carried out by analyzing the solution convergence and the quality obtained from these two cases.

(1) Solution convergence analysis

As shown in Figure 1 and Figure 2, the algorithm indicates good convergence in both of these two cases with a better convergence speed. Using the reasonable tabu strategy makes the algorithm avoid acquiring local optimal solution. An optimum solution is obtained after thirty-five iterations in Case 1 and sixty iterations in Case 2 respectively.

(2) Comparative analysis of solution quality

In order to testify to solution feasibility, the algorithm runs ten times in each case to obtain results shown in Table 3.

Calculating results listed in Table 3 show that solutions in both of these two examples are relatively stable.

A fluctuation coefficient of the solution is used here to evaluate solution quality, which is calculated by equation: fluctuation coefficient = (maximum value-minimum value)/

average value∗100%. Fluctuation coefficient of Case 1 is 4.21% and 4.36% of Case 2. Both values are less than 5%, showing that solutions in these two cases are stable.

The solutions' convergence and quality obtained from these two cases indicate that the algorithm studied herein is feasible and robust.

4.3.2. *Algorithm validity analysis.* Algorithm validity is shown by comparing calculation time with results acquired by an analogous study.

In Case 1, calculation time in each running is 4∼6 seconds, and in Case 2 it is 11∼16 seconds. Compared with a calculation time of 20 seconds obtained in reference [33] with fifty customers and split-loads, it indicates that the calculation time decreases by decreasing the problem scale. This is reasonable and valid. The authors also tried to utilize a genetic algorithm to resolve Case 2, but because of complicated coding characteristics, its calculation efficiency is less than this, and these results are embodied in reference [42]. For the Simultaneous Delivery and Pick up Vehicle Routing Problem with Split Loads, the algorithm studied in this paper has advantages in calculating time when it resolves a problem with a certain scale. It also explains a common problem in the vehicle routing problem. Calculating time tends to increase exponentially with increasing customer scale.

5. **Conclusions.** This paper studies the Simultaneous Delivery and Pick-up Vehicle Routing Problem with Split Loads based on current research and establishes a mathematical model. Transportation weight remaining after serving a bulk customer by full load vehicles is executed by tabu search. It not only ensures service quality for bulk customers, but also can decrease logistics cost. A tabu search algorithm is designed to acquire a good initial solution. Tabu rules are also designed effectively in this algorithm.

Case study results indicate that the algorithm studied here can resolve this problem effectively and obtain an ultimate robust solution. Compared analysis between these results and analogous studies shows that the model and algorithm studied in this paper are valid and feasible. This study can supply theoretical references for resolving a multi-attribute VRP problem with split loads, and in the practice, it also can be applied widely in the area of distribution vehicle scheduling, especially when total delivery demands are larger than the total loads.

However, we also notice that the solution difference is more than 4% and it still needs to improve in order to acquire a better solution. So studying a more effective algorithm and improving the solution quality by a more effective tabu search algorithm are still the main issues to be resolved. In addition, how to obtain an integrated solution considering traffic information and customer information is another focus for future research work.

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