International Journal of Innovative Computing, Information and Control
Volume **9**, Number **10**, October **2013**

ICIC International ©2013 ISSN 1349-4198
pp. 3991–4000

DECENTRALIZED ROBUST CONTROL FOR UNCERTAIN NONLINEAR DESCRIPTOR LARGE-SCALE COMPOSITE SYSTEMS WITH INPUT SATURATION

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Received October 2012; revised March 2013

Abstract. *This paper investigated the problem of decentralized robust stabilization for a class of uncertain nonlinear descriptor similar large-scale composite systems with input saturation. Based on decomposing theory of large-scale systems, Lyapunov stable theory and matrix theory, a kind of decentralized robust stabilization controller is designed which ensures asymptotic stability of the closed-loop systems for these systems and the closed-loop systems for their isolated subsystems. Since the similar structures of the controllers, all controllers can be obtained by only one controller and similar parameters; thus, they are easy to perform in engineering practice. Further, a numerical example is given to demonstrate the effectiveness of the proposed methods.*
Keywords: Robust stabilization, Similar structure, Input saturating actuation, Descriptor composite system, Decentralized control

1. **Introduction.** Decentralized control has drawn more and more attention in recent decades, and many results about it have been obtained (see, for example, [1-7] and the references therein). On the other hand, in practical control systems, input with saturation is a common phenomenon, such as, the communicative satellite attitude regulation systems [8]. Without consideration of input saturation in the design controllers, the stability of closed-loop system cannot be ensured. In the past several years, the analysis and design for normal system with input saturation have been extensively investigated [9-13]. Decentralized control for nonlinear lager-scale interconnected systems with input saturation is reported in [9]. The control synthesis problem for a class of linear time-delay systems with actuator saturation is investigated in [10]. A novel decentralized adaptive neural control scheme is proposed for a class of interconnected large-scale uncertain nonlinear time-delay systems with input saturation in [11]. In [12], a novel and elegant approach to solving the problem of global stabilization for a chain of integrators in the presence of input saturation and disturbances is brought up. The attitude tracking control problem for rigid spacecraft with actuator saturations, inertia uncertainties and external disturbances is considered in [13]. The decentralized control for composite systems with input saturation is studied in [14].

In recent years, the researchers discussed the stabilization problem for descriptor linear system with input saturation [15-19]. For example, the simultaneous semi-global L_p-stabilization and asymptotic stabilization in both semi-global and global cases were considered for continuous-time linear singular systems subject to input saturation in [17]. In [18], a linear feedback control law is designed for the step tracking control problem of linear singular systems subject to input saturation. Then, based on this linear feedback gain, a CNF control law is constructed to improve the transient performance of the closed-loop system. In [19], a sufficient condition is obtained which guarantees that the discrete-time singular Markov jump system with actuators saturation is regular, causal, bounded state stable, and satisfies the H_∞ performance. However, little attention has been paid to the study of stabilization with input saturation for the nonlinear descriptor system, especially for uncertain nonlinear composite descriptor system. Similar composite system has been applied in the fields of communicative systems, manual nerve cell networks, electric power system and so on [20-22]. Thus, the study of decentralized control for nonlinear descriptor similar composite large-scale system with input saturation has important theoretical and practical meaning.

Due to these, in the paper, we consider a class of uncertain nonlinear descriptor similar composite large-scale system with input saturation. A sufficient condition for asymptotic stability of the closed-loop systems for these systems and the closed-loop systems for their isolated subsystems is obtained by using decomposing theory of large-scale systems, Lyapunov stable theory and matrix theory. By similar structure, a concise design method for decentralized robust stabilization controller is given. The design results show that this design is easy to calculate and the controllers have similar structure, so it is easy to realize project. Without consideration of the nonlinearity, the interconnection is considered in [14]; neither the nonlinearity nor the interconnection is considered in [15,16]. However, in this paper, the uncertainty, the nonlinearity, and the interconnection are all considered, so the application of this paper is more widely than them of reference [14,15,16].

Introduce the following marks: $\| \bullet \|$ shows spectral norm. $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ show the maximum and minimum eigenvalues of symmetric matrix A. In this paper, the system (4) possesses the PS similar structure and $S(i,1) = (T_i, S_i, K_i)$, $1 \le i \le N$.

2. Problem Statement and Assumptions.

Consider the following descriptor large-scale systems composed of N regular subsystems:

$$E_i\dot{x}_i = A_i x_i + \Delta f_i(x_i,t) + B_i u_i + \sum_{j=1, j\neq i}^{N} \Delta H_{i,j}(x_j), \quad 1 \le i \le N \tag{1}$$

where $x_i \in R^n$, $u_i \in R^m$ are state and input for the i-th subsystem, respectively; $E_i, A_i \in R^{n\times n}, B_i \in R^{n\times m}$ are constant value matrixes, respectively; $rankE_i < n$, $det(sE_i - A) \neq 0$; $\Delta f_i(x_i,t)$ shows the structure uncertain term of the i-th subsystem; $\sum_{j=1,j\neq i}^{N} \Delta H_{i,j}(x_j)$, $1 \le i \le N$ shows the uncertain unmatched interconnection of the i-th subsystem. We suppose that every descriptor subsystem is regular and $\Delta f_i(0,t) = \Delta H_{ij}(0) = 0$, $1 \le i$, $j \le N$, $j \neq i$.

The system:

$$E_i\dot{x}_i = A_i x_i + B_i u_i \tag{2}$$

is said to be the nominal system of the i-th subsystem and simply denoted as (E_i, A_i, B_i), $1 \le i \le N$.

Definition 2.1. [22] *To the nominal system (E_i, A_i, B_i) and (E_j, A_j, B_j) of the i-th subsystem and the j-th subsystem for the system (1), if there exists a constant matrix $K \in R^{m\times n}$ and nonsingular matrixes $T, S \in R^{n\times n}$, such that*

$$TE_iS = E_j, \;\; T(A_i + B_iK)S = A_j, \;\; TB_i = B_j \tag{3}$$

then the nominal systems of the i-th subsystem and the j-th subsystem are said to be proportional state feedback similar. Simply denote it as $(E_i, A_i, B_i)\overset{PS}{\longrightarrow}(E_j, A_j, B_j)$, and (T, S, K) is said to be similar parameter.

Definition 2.2. [22] *If there exists* j, $1 \le j \le N$, *such that* $(E_i, A_i, B_i) \underline{PS} (E_j, A_j, B_j)$, *and similar parameters are* T_i, S_i, K_i, $1 \le i \le N$, $i \ne j$, *then the system (1) is said to be* PS *similar structure, and* $S(i,j) = (T_i, S_i, K_i)$, $1 \le i$, $j \le N$, $i \ne j$ *is said to be similar index.*

Then we have the following results [22]: the similar relation "$\underline{PS}$" is equivalent relation; if the system (1) with PS similar structure has the concurrence of the stable, R-stable and impulse controllable nominal systems, then all of the nominal systems (E_i, A_i, B_i) are stable, R-stable and impulse controllable.

Consider the following nonlinear descriptor composite large-scale system:

$$\sum_i : E_i\dot{x}_i = A_i x_i + \Delta f_i(x_i,t) + B_i\sigma_i(u_i) + \sum_{j=1, j\ne i}^{N} \Delta H_{ij}(x_j), \quad 1 \le i \le N \tag{4}$$

where $\sigma_i(u_i)$ is saturating function, $1 \le i$, $j \le N$, we suppose that every descriptor subsystem is regular.

The system (2) is said to be the reference system of the i-th subsystem for the system (4).

Our problem is: The system (4) should satisfy what condition, can we find state feedback:

$$u_i = K_i x_i, \quad i = 1, 2, \cdots, N$$

such that

Question 1: For the i-th closed-loop subsystem

$$E_i\dot{x}_i = A_i x_i + \Delta f_i(x_i,t) + B_i\sigma_i(K_i x_i) \tag{5}$$

is asymptotic stable about $x_i = 0$, $1 \le i \le N$.

Question 2: For the absolute closed-loop systems

$$E_i\dot{x}_i = A_i x_i + \Delta f_i(x_i,t) + B_i\sigma_i(K_i x_i) + \sum_{j=1, j\ne i}^{N} \Delta H_{ij}(x_j), \quad 1 \le i \le N, \tag{6}$$

is asymptotic stable about $x = 0$.

Lemma 2.1. [9] *If* σ *is saturating actuators, then*

$$\left\| \frac{1}{2}S - \sigma(S) \right\| \le \frac{1}{2}\|S\|, \quad \forall S \in R^m.$$

Assumption 2.1. (E_1, A_1, B_1) *is stable and impulse controllable.*

By Assumption 2.1, there exists matrix $K \in R^{m\times n}$ and nonsingular matrixes $T, S \in R^{n\times n}$, such that

$$TE_1S = \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}, \quad T(A_1 + B_1K)S = \begin{pmatrix} A_{(1)} & 0 \\ 0 & I_{n-r} \end{pmatrix} \tag{7}$$

where $r = rankE_1$, $A_{(1)}$ is Hurwitz stable matrix, then to arbitrary positive definite matrix $Q \in R^{r\times r}$, Lyapunov equation

$$A_{(1)}^T P + PA_{(1)} = -Q \tag{8}$$

has unique positive definite solution P. Denote as

$$T = \begin{pmatrix} T_{(1)} \\ T_{(2)} \end{pmatrix}, \quad T^{-1} = \begin{pmatrix} T_{[1]} & T_{[2]} \end{pmatrix}, \quad S^{-1} = \begin{pmatrix} S_{(1)} \\ S_{(2)} \end{pmatrix}. \tag{9}$$

$T_{(1)}$, $S_{(1)} \in R^{r\times n}$; $S_{(2)} \in R^{(n-r)\times n}$; $T_{[1]} \in R^{n\times r}$, $T_{[2]} \in R^{n\times (n-r)}$.

Assumption 2.2.

$$\|\Delta f_i(x_i,t)\| \le \beta_i \|E_i x_i\|, \ \ 1 \le i \le N;$$
$$\|\Delta H_{ij}(x_j)\| \le \alpha_{ij}\|E_j x_j\|, \ \ 1 \le i, \ \ j \le N, \ \ i \ne j;$$
$$r = \max_{1 \le i \le N} r_i, \ \ r < 1$$

where $r_i = \|T_{(2)}B_1\|\|KS_i^{-1} + K_i\|\|SS_i\|$.

Assumption 2.3. *Matrix* $(W^T + W)$ $(W = (W_{ij})_{N\times N})$ *is positive definite. That is*

$$W_{ij} = \begin{cases} \lambda_{\min}(Q) - 2\beta_i\|PT_{(1)}T_i\|\|T_i^{-1}T_{[2]}\| - 2\|PT_{(1)}B_1\|\|KS_i^{-1}+K_i\|\|SS_i\| \\ \times \left(1 + \frac{\beta_i\|T_{(2)}T_i\|\|T_i^{-1}T_{[1]}\|+r_i}{1-r_i}\right), \ \ i = j; \\ -2\alpha_{ij}\left(\|PT_{(1)}T_i\|\|T_j^{-1}T_{[1]}\| + \frac{\|PT_{(1)}B_1\|\|KS_i^{-1}+K_i\|\|SS_i\|}{1-r_i}\|T_{(2)}T_i\|\|T_j^{-1}T_{[1]}\|\right), \ \ i \ne j \end{cases}$$

where $S_1 = T_1 = I_n$, $K_1 = 0$.

We design the following decentralized controller to the system (4):

$$u_1(x_1) = 2Kx_1 \tag{10}$$

by both Equation (10) and the similar parameter (T_i, S_i, K_i), we construct decentralized controller as follows:

$$u_i(x_i) = 2(KS_i^{-1} + K_i)x_i, \ \ 2 \le i \le N. \tag{11}$$

3. Main Results.

Theorem 3.1. *If Assumption 2.1, Assumption 2.2 and Assumption 2.3 hold, then the system (4) has decentralized stabilization controllers:*

$$u_i(x_i) = 2(KS_i^{-1} + K_i)x_i, \ \ 1 \le i \le N.$$

Proof: For convenience, denote

$$T_1 = S_1 = I_n, \ \ K_1 = 0.$$

By Assumptions 2.1, Definition 2.1 and its equivalent properties we know:

$$T_iE_iS_i = E_1, \ \ T_i(A_i + B_iK_i)S_i = A_1, \ \ T_iB_i = B_1, \tag{12}$$

$$TT_iE_iS_iS = \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix},$$

$$TT_i(A_i + B_iK_i + B_iKS_i^{-1})S_iS = \begin{pmatrix} A_{(1)} & 0 \\ 0 & I_{n-r} \end{pmatrix}, \ \ 1 \le i \le N. \tag{13}$$

The closed-loop system composed of the i-th subsystem (4) and controller u_i is

$$E_i\dot{x}_i = (A_i + B_iK_i + B_iKS_i^{-1})x_i + B_i\left(\sigma_i(u_i) - \frac{1}{2}u_i\right) + \Delta f_i(x_i), \ \ 1 \le i \le N. \tag{14}$$

Make nonsingular transform

$$\begin{pmatrix} z_{(1)} \\ z_{(2)} \end{pmatrix} = S^{-1}S_i^{-1}x_i = \begin{pmatrix} S_{(1)} \\ S_{(2)} \end{pmatrix} S_i^{-1}x_i, \ \ 1 \le i \le N \tag{15}$$

and make multiplication by TT_i at the both sides of Equation (14), then we get

$$\begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \dot{z}_{i(1)} \\ \dot{z}_{i(2)} \end{pmatrix} = \begin{pmatrix} A_{(1)} & 0 \\ 0 & I_{n-r} \end{pmatrix}\begin{pmatrix} z_{i(1)} \\ z_{i(2)} \end{pmatrix} + TT_i\Delta f_i(x_i,t) + TB_1\left(\sigma_i(u_i) - \frac{1}{2}u_i\right) \tag{16}$$

where $T = \begin{pmatrix} T_{(1)} \\ T_{(2)} \end{pmatrix}$.

Equation (16) is equivalent to

$$\dot{z}_{i(1)} = A_{(1)} z_{i(1)} + T_{(1)} T_i \Delta f_i(x_i, t) + T_{(1)} B_1 \left(\sigma_i(u_i) - \frac{1}{2} u_i \right) \tag{17}$$

$$0 = z_{i(2)} + T_{(2)} T_i \Delta f_i(x_i, t) + T_{(2)} B_1 \left(\sigma_i(u_i) - \frac{1}{2} u_i \right) \tag{18}$$

$$x_i = S_i S \begin{pmatrix} z_{i(1)} \\ z_{i(2)} \end{pmatrix}$$

$$\begin{aligned} E_i x_i &= T_i^{-1} T^{-1} T T_i E_i S_i S^{-1} S_i^{-1} x_i \\ &= T_i^{-1} \begin{pmatrix} T_{[1]} & T_{[2]} \end{pmatrix} \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} z_{i(1)} \\ z_{i(2)} \end{pmatrix} \\ &= T_i^{-1} T_{[1]} z_{i(1)} \end{aligned} \tag{19}$$

According to Lemma 2.1, we can get

$$\begin{aligned} \left\| \sigma_i(u_i) - \frac{1}{2} u_i \right\| \le \frac{1}{2} \|u_i\| &= \|K S_i^{-1} + K_i\| \|x_i\| \\ &\le \|S S_i\| \|K S_i^{-1} + K_i\| (\|z_{i(1)}\| + \|z_{i(2)}\|) \end{aligned} \tag{20}$$

By Equation (18), we can get

$$-z_{i(2)} = T_{(2)} T_i \Delta f_i(x_i, t) + T_{(2)} B_1 \left(\sigma_i(u_i) - \frac{1}{2} u_i \right).$$

By Assumption 2.2 and Equation (20), we can get

$$\|z_{i(2)}\| \le \|T_{(2)} T_i\| \|T_i^{-1} T_{[1]}\| \beta_i \|z_{i(1)}\| + r_i (\|z_{i(1)}\| + \|z_{i(2)}\|)$$

where $r_i < 1$.

So

$$\|z_{i(2)}\| \le \frac{\beta_i \|T_{(2)} T_i\| \|T_i^{-1} T_{[1]}\| + r_i}{1 - r_i} \|z_{i(1)}\|. \tag{21}$$

We easily know: in the case when r_i <1, the i-th isolated subsystem is impulse free.

For the system (17) we construct positive definite function

$$V(z_{i(1)}) = z_{i(1)}^T P z_{i(1)}$$

differentiate V along trail of system (17); by Equation (20) and Equation (21), we have

$$\begin{aligned} \dot{V} &= z_{i(1)}^T (A_{(1)}^T P + P A_{(1)}) z_{i(1)} + 2 z_{i(1)}^T P T_{(1)} B_1 \left(\sigma_i(u_i) - \frac{1}{2} u_i \right) + 2 z_{i(1)}^T P T_{(1)} T_i \Delta f_i(x_i, t) \\ &= -z_{i(1)}^T Q z_{i(1)} + 2 z_{i(1)}^T P T_{(1)} B_1 \left(\sigma_i(u_i) - \frac{1}{2} u_i \right) + 2 z_{i(1)}^T P T_{(1)} T_i \Delta f_i(x_i, t) \\ \dot{V} &\le -\lambda_{\min}(Q) \|z_{i(1)}\|^2 + 2 \|P T_{(1)} B_1\| \|K S_i^{-1} + K_i\| \|S S_i\| (\|z_{i(1)}\| + \|z_{i(2)}\|) \|z_{i(1)}\| \\ &\quad + 2 \beta_i \|P T_{(1)} T_i\| \|T_i^{-1} T_{[1]}\| \|z_{i(1)}\|^2 \\ &\le -\left[\lambda_{\min}(Q) - 2 \beta_i \|P T_{(1)} T_i\| \|T_i^{-1} T_{[1]}\| - 2 \|P T_{(1)} B_1\| \|K S_i^{-1} + K_i\| \|S S_i\| \right. \\ &\quad \left. \times \left(1 + \frac{\beta_i \|T_{(2)} T_i\| \|T_i^{-1} T_{[1]}\| + r_i}{1 - r_i} \right) \right] \|z_{i(1)}\|^2 \end{aligned}$$

According to Assumption 2.3, since $W^T + W$ is positive definite, $W_{ii} > 0$, $1 \le i \le N$ and $\dot{V}$ is negative definite, then Question 1 can be solved.

The closed-loop composed of system (4) and controller u_i is:

$$E_i\dot{x}_i = (A_i + B_iK_i + B_iKS_i^{-1})x_i + B_i\left(\sigma_i(u_i) - \frac{1}{2}u_i\right) + \Delta f_i(x_i) + \sum_{j=1,j\neq i}^{N} \Delta H_{ij}(x_j) \quad (22)$$

Make a nonsingular transform

$$\begin{pmatrix} z_{i(1)} \\ z_{i(2)} \end{pmatrix} = S^{-1}S_i^{-1}x_i = \begin{pmatrix} S_{(1)} \\ S_{(2)} \end{pmatrix} S_i^{-1}x_i, \quad 1 \le i \le N$$

and multiplication by TT_i at the both sides of Equation (22), then we get

$$\begin{aligned} \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \dot{z}_{i(1)} \\ \dot{z}_{i(2)} \end{pmatrix} &= \begin{pmatrix} A_{(1)} & 0 \\ 0 & I_{n-r} \end{pmatrix}\begin{pmatrix} z_{i(1)} \\ z_{i(2)} \end{pmatrix} \\ &\quad + TT_i\Delta f_i(x_i,t) + TB_1\left(\sigma_i(u_i) - \frac{1}{2}u_i\right) + TT_i\sum_{j=1,j\neq i}^{N} \Delta H_{ij}(x_j) \end{aligned} \quad (23)$$

where $T = \begin{pmatrix} T_{(1)} \\ T_{(2)} \end{pmatrix}$.

Equation (23) is equivalent to

$$\dot{z}_{i(1)} = A_{(1)}z_{i(1)} + T_{(1)}T_i\Delta f_i(x_i,t) + T_{(1)}B_1\left(\sigma_i(u_i) - \frac{1}{2}u_i\right) + T_{(1)}T_i\sum_{j=1,j\neq i}^{N} \Delta H_{ij}(x_j) \quad (24)$$

$$\begin{aligned} 0 = z_{i(2)} + T_{(2)}T_i\Delta f_i(x_i,t) + T_{(2)}B_1\left(\sigma_i(u_i) - \frac{1}{2}u_i\right) + T_{(2)}T_i\sum_{j=1,j\neq i}^{N} \Delta H_{ij}(x_j), \\ i = 1, 2, \cdots, N. \end{aligned} \quad (25)$$

By Equation (25), we can get

$$-z_{i(2)} = T_{(2)}T_i\Delta f_i(x_i,t) + T_{(2)}B_1\left(\sigma_i(u_i) - \frac{1}{2}u_i\right) + T_{(2)}T_i\sum_{j=1,j\neq i}^{N} \Delta H_{ij}(x_j), \quad i = 1, 2, \cdots, N \quad (26)$$

By Assumption 2.2 and Equation (20), we have

$$\begin{aligned} \|z_{i(2)}\| \le{}& \beta_i\|T_{(2)}T_i\|\|T_i^{-1}T_{[1]}\|\|z_{i(1)}\| + r_i\left(\|z_{i(1)}\| + \|z_{i(2)}\right)\| \\ &+ \sum_{j=1,j\neq i}^{N} \|T_j^{-1}T_{[1]}\|\|T_{(2)}T_i\|\alpha_{ij}\|z_{j(1)}\| \end{aligned}$$

where $r_i < 1$.

So

$$\|z_{i(2)}\| \le \frac{\beta_i\|T_{(2)}T_i\|\|T_i^{-1}T_{[1]}\| + r_i}{1 - r_i}\|z_{i(1)}\| + \sum_{j=1,j\neq i}^{N} \frac{\alpha_{ij}\|T_{(2)}T_i\|\|T_j^{-1}T_{[1]}\|}{1 - r_i}\|z_{j(1)}\|. \quad (27)$$

For the system (24) we construct positive definite function

$$V(z_{1(1)}, z_{2(1)}, \cdots, z_{N(1)}) = \sum_{i=1}^{N} z_{i(1)}^T P z_{i(1)}$$

differentiate V along trail of system (24); by Equation (20) and Equation (27), we can get

$$\begin{aligned}
\dot{V} =& -\sum_{i=1}^{N} z_{i(1)}^T Q z_{i(1)} + \sum_{i=1}^{N} 2z_{i(1)}^T PT_{(1)}B_1\left(\sigma_i(u_i) - \frac{1}{2}u_i\right) + \sum_{i=1}^{N} 2z_{i(1)}^T PT_{(1)}T_i \Delta f_i(x_i,t) \\
&+ \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} 2z_{i(1)}^T PT_{(1)}T_i \Delta H_{ij}(x_j) \\
\leq& \sum_{i=1}^{N} -\lambda_{\min}(Q)\|z_{i(1)}\|^2 + \sum_{i=1}^{N} 2\|PT_{(1)}B_1\|\|KS_i^{-1}+K_i\|\|SS_i\|(\|z_{i(1)}\|+\|z_{i(2)}\|)\|z_{i(1)}\| \\
&+ \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} 2\alpha_{ij}\|PT_{(1)}T_i\|\|T_j^{-1}T_{[1]}\|\|z_{i(1)}\|\|z_{j(1)}\| \\
&+ \sum_{i=1}^{N} 2\beta_i\|PT_{(1)}T_i\|\|T_i^{-1}T_{[1]}\|\|z_{i(1)}\|^2 \\
\leq& -\sum_{i=1}^{N}\left[\lambda_{\min}(Q) - 2\beta_i\|PT_{(1)}T_i\|\|T_i^{-1}T_{[1]}\| - 2\|PT_{(1)}B_1\|\|SS_i\|\|KS_i^{-1}+K_i\|\right. \\
&\left.\times\left(1+\frac{\beta_i\|T_{(2)}T_i\|\|T_i^{-1}T_{[1]}\|+r_i}{1-r_i}\right)\right]\|z_{i(1)}\|^2 + \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} 2\alpha_{ij}\Bigg(\|PT_{(1)}T_i\|\|T_j^{-1}T_{[1]}\| \\
&+ \|PT_{(1)}B_1\|\|SS_i\|\|KS_i^{-1}+K_i\| \times \frac{\|T_{(2)}T_i\|\|T_j^{-1}T_{[1]}\|}{1-r_i}\Bigg)\|z_{i(1)}\|\|z_{j(1)}\| \\
=& -Y^TWY \\
=& -\frac{1}{2}Y^T(W^T+W)Y
\end{aligned}$$

where

$$Y = (\|z_{1(1)}\|, \|z_{2(1)}\|, \cdots, \|z_{N(1)}\|)^T.$$

By Assumption 2.3, W^T+W is positive definite, so $\dot{V}$ is negative definite, then Question 2 can be solved.

Corollary 3.1. *If Assumption 2.1 and Assumption 2.2 hold, then the i-th subsystem for the system (4) is impulse free.*

The design process of decentralized robust stabilization controller:
1) According to Assumption 2.1, we choose k, then get T and S.
2) We choose Q, get the positive solution of the Lyapunov Equation (8).
3) Design the stabilization controller Equation (11) ($1 \leq i \leq N$).

Remark 3.1. *Compared with reference* [15,16]*, the uncertainty, nonlinearity and interconnection of the system are considered in this paper, so it is of more importance.*

Remark 3.2. *In the system (4) of this paper, let $E_i = I$, $A_i = A_1$, $B_i = B_1$, $\Delta H_{ij}(x_j) = A_{12}x_j$, $\Delta f_i(x_i,t) = 0$, then the system (4) could be reduced to the system (5) in reference* [14]*, so the Theorem 2 in reference* [14] *is an exception of Theorem 3.1 in this paper.*

4. **Numerical Example.**

Example 4.1. *Consider following uncertain descriptor large-scale composite systems composed of two subsystems:*

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \dot{x}_1 = \begin{pmatrix} -0.9 & 0.1 \\ 0 & 1 \end{pmatrix} x_1 + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sigma_1(u_1) + \Delta f_1(x_1, t) + \Delta H_{12}(x_2) \tag{28}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \dot{x}_2 = \begin{pmatrix} -0.9 & 0.1 \\ 0 & 1 \end{pmatrix} x_2 + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sigma_2(u_2) + \Delta f_2(x_2, t) + \Delta H_{12}(x_1) \tag{29}$$

where

$$\|\Delta f_1(x_1,t)\| \leq \frac{1}{4}\|Ex_1\|, \quad \|\Delta f_2(x_2,t)\| \leq \frac{1}{4}\|Ex_2\|;$$
$$\|\Delta H_{12}(x_2)\| \leq \frac{1}{4}\|Ex_2\|, \quad \|\Delta H_{21}(x_1)\| \leq \frac{1}{4}\|Ex_1\|.$$

The system (28) is similar with the system (29), similar parameter is $(T, S, K) = (I_2, I_2, 0)$.

1) $rank(sE - A, B) = 2$, $rank \begin{pmatrix} E & 0 & 0 \\ A & E & B \end{pmatrix} = 2 + rankE$.

The system (28) and system (29) are R-stable and impulse controllable.

Let $K = (-0.1 - 0.1)$, $A + BK = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$, $A_{(1)} = -1$. *Let* $Q = 4$, *solve Lyapunov equation*

$$(-1)^T P + P(-1) = -4, \quad P = 2, \quad T = S = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = T^{-1} = S^{-1}.$$

$$S_{(1)} = T_{(1)} = \begin{pmatrix} 1 & 0 \end{pmatrix}, \quad S_{(2)} = T_{(2)} = \begin{pmatrix} 1 & 0 \end{pmatrix}, \quad T_{[1]} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad T_{[2]} = \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$
$$T_1 = T_2 = S_1 = S_2 = I_2.$$

2) $T_{(2)}B_1 = 0$, *so* $r_i = 0$, $r_i < 1$, $i = 1, 2$.

3) $W = \begin{pmatrix} 2.2929 & -1.1414 \\ -1.1414 & 2.2929 \end{pmatrix}$ *is positive definite.*

So

$$u_1 = \begin{pmatrix} -0.2 & -0.2 \end{pmatrix} x_1,$$
$$u_2 = \begin{pmatrix} -0.2 & -0.2 \end{pmatrix} x_2,$$

is decentralized robust controller of this system.

The result of the numerical example is obtained by Theorem 3.1; however, it could not be solved by the reference [14-16].

5. **Conclusions.**

In this paper, a class of descriptor similar large-scale composite systems with input saturation is considered. By using decomposing theory of descriptor systems, Lyapunov stable theory and matrix theory, a design method for a kind of decentralized robust stabilization controller is given. Finally, a numerical example is given to demonstrate the effectiveness of the results obtained in this paper.

Acknowledgment. This paper was supported by the National Natural Science Foundation of China, under Grant number 61273004. The authors would like to thank the helpful suggestions of reviewers which have improved the presentation and quality of the paper.

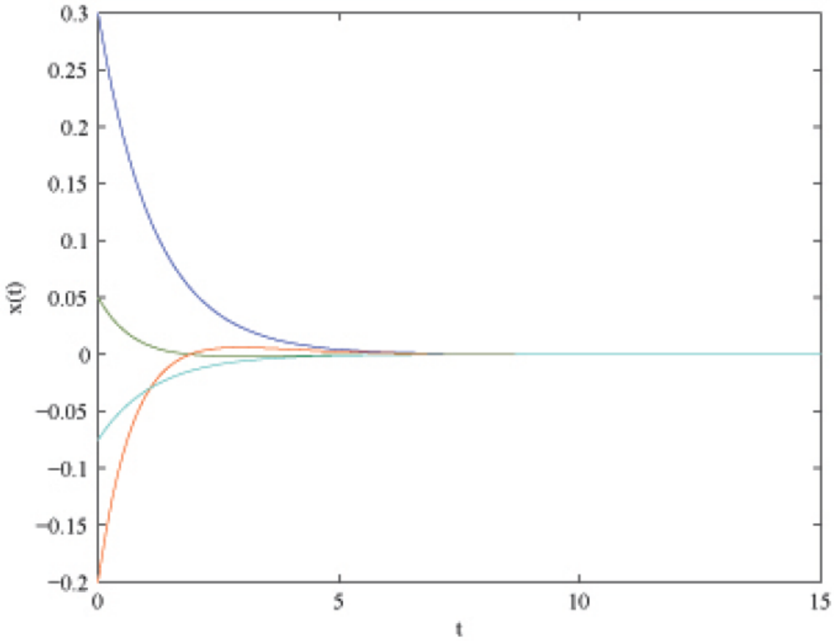

Figure 1. The state feedback response for the closed-loop system

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