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CONSTRAINED MODEL PREDICTIVE CONTROL ON CONVEX POLYHEDRON STOCHASTIC LINEAR PARAMETER VARYING SYSTEMS

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Abstract. *The problem of constrained model predictive control on a class of stochastic linear parameter varying systems is discussed. First, constant coefficient matrices are obtained at each vertex in the interior of system, and then, by considering semi-definite programming constraints, weight coefficients between each vertex are calculated, and the equal coefficient matrices for the time variant system are obtained. Second, in the given receding horizon, for each mode sequence of the stochastic system, the optimal control input sequences are designed in order to make the states into a terminal invariant set. Outside of the receding horizon, stability of the system is guaranteed by searching a state feedback control law. Finally, constraints on both inputs and outputs are considered for such system and predictive controller is designed in terms of linear matrix inequality. Simulation example shows the validity of this method.*
Keywords: Constrained predictive control, Convex polyhedron, Linear parameter varying systems, Markov jump parameters

1. **Introduction.** The last several decades have witnessed rapid growing of research interests on Markov jump systems (MJSs) since the pioneering work in [1]. The reasons are given twofold: (1) MJSs have great application in modeling parameter-variation or structure-variation in many practical systems, which are caused by component failures or repairs, sudden environmental disturbance, or change of operation points. (2) The dynamical behaviors of MJSs have been found in many fields: such as aerospace industry, manufacturing systems, economic systems and electrical systems. Under the assumption that the parameters in MJSs are time-invariant, analysis and control of MJSs have received much attention, for example, the problems of worst case control [2], slide mode control [3], sampled-data control [4], guaranteed cost control [5], fault detection and filtering [6, 7, 8, 9].

However, the assumption that the jump parameters are time invariant is not realistic in many practical situations. One typical example is in chemical system, the chemical reaction is not fixed, such that the parameters cannot be time-invariant, and another example is the VOTL (vertical take-off landing) helicopter system, the multiple airspeeds are varying when the surrounding environment changes. Therefore, it is full of practical meaning to study MJSs with time-varying parameters. Although the jumping parameters of the Markov process is not known and fixed, but one can evaluate some values in some working points, so we can model these time-varying parameters by a polytope, which belongs to a convex set. This motivated us to apply this kind of set to time-varying MJSs.

On another research front line, model predictive control (MPC) is an effective control algorithm to deal with multi-variable control problems in various fields, such as in chemical process, which also has great potential to deal with input and output constraints. The model information of dynamic process is used to predict the future behavior of the plant over the prediction horizon, so as to compute control inputs. Normally more than one input is computed at the current sampling time, however, only the first controller input will be implemented to the plant. At the next sequential sampling time, these actions will be repeated, that is why MPC is also called receding horizon control (MPC). More precisely, for a constrained finite input horizon MPC, a standard MPC approach can be formulated as a compact quadratic program (QP) that is online calculated at each sampling time [10]. Some work on model predictive control has been done, in [11], a disturbance attenuation problem is proposed for discrete systems using receding horizon control technique, and there is also a great deal of research work has been done focusing on predictive control for discrete MJSs [12, 13, 14], in which a nonlinear control sequence is obtained by solving a finite horizon optimal control problem, and there are also some work on predictive control for linear parameter varying systems [15, 16], however, there is little work done on predictive problems for MJSs with time varying parameters, not to mention the multistep receding horizon control for such systems. Thus, it is full of application meaning to focus on the problem of multistep receding horizon control problem for discrete time stochastic linear time varying systems.

Motivated by the aforementioned points, in this paper, we focus on the design of a predictive controller for a class of MJSs with time-varying parameters. The rest of the paper is organized as follows. Problem statement and preliminaries of this paper are given in Section 2. In Section 3, model predictive control problem is given. In Section 4, constrained predictive controller is designed here. A numerical example is given to illustrate the effectiveness of our approach in Section 5. Finally, some concluding remarks are given in Section 6.

In the sequel, the notation R^n stands for an n-dimensional Euclidean space, the transpose of a matrix A is denoted by A^{T}; $E\{\cdot\}$ denotes the mathematical statistical expectation; a positive-definite matrix is denoted by $P > 0$; I is the unit matrix with appropriate dimension, and $*$ means the symmetric term in a symmetric matrix.

2. **Problem Statement and Preliminaries.**

Consider a class of stochastic system with time-varying parameters:

$$\begin{aligned} x_{k+1} &= A_{r_k} x_k + B_{r_k} u_k \\ y_k &= C_{r_k} x_k \end{aligned} \tag{1}$$

where $x_k \in R^n$ is the state vector of the system, $u_k \in R^m$ is the input vector of the system, $y_k \in R^p$ is the output vector and $\{r_k, k \geq 0\}$ is the concerned time-discrete Markov stochastic chain which takes values in a finite state set $\Lambda = \{1, 2, 3, \ldots, N\}$, and r_0 represents the initial mode, the transition probability matrix is defined as $\Pi(k) = \{\pi_{ij}(k)\}$, i, $j \in \Lambda$, $\pi_{ij}(k) = P(r_{k+1} = j | r_k = i)$ is the transition probability from mode i at time k to mode j at time $k+1$, which satisfies $\pi_{ij}(k) \geq 0$ and $\sum\limits_{j=1}^{N} \pi_{ij}(k) = 1$, $A_{r_k} = \sum\limits_{l=1}^{m} b_l A_{r_k}(w_l)$, $B_{r_k} = \sum\limits_{l=1}^{m} b_l B_{r_k}(w_l)$, $C_{r_k} = \sum\limits_{l=1}^{m} b_l C_{r_k}(w_l)$, where $A_{r_k}(w_l)$, $B_{r_k}(w_l)$ and $C_{r_k}(w_l)$ are coefficient matrices of each vertex for the stochastic polyhedron linear parameter time-varying (LPV) system, $l \in [1, m]$, where $0 \le b_l \le 1$ and $\sum_{l=1}^{m} b_l = 1$.

Definition 2.1. *The stochastic LPV system (1) (setting $u_k = 0$) is said to be stochastically stable, if for any initial state x_0 and mode r_0, then*

$$\lim_{T\to\infty} E\left\{\sum_{k=0}^{T} x_k^{\mathrm{T}} x_k | x_0, r_0\right\} < \infty \tag{2}$$

In this paper, we discuss the problem of model predictive control for LPV system (1), as the predictive model relies not only on the state equations of system (1), but also relies on the Markov jump modes, so multistep model predictive traces of such system are defined in Definition 2.2.

Definition 2.2. *Suppose that multistep mode trace set as $M = \{r_k, r_{k+1}, \cdots, r_{k+N-1} | r_k \in \Lambda\}$, and transition probability at step N is*

$$p_\lambda = p(r_k, r_{k+1}) p(r_{k+1}, r_{k+2}) \cdots p(r_{k+N-2}, r_{k+N-1}), \quad \lambda \in M$$

Then, the N steps predictive model of the states can be shown as:

$$\begin{bmatrix} x(k+1|\,k) \\ x(k+2|\,k) \\ \vdots \\ x(k+N|\,k) \end{bmatrix} = \begin{bmatrix} A_{r_k} \\ A_{r_{k+1}} A_{r_k} \\ \vdots \\ A_{r_{k+N-1}} A_{r_{k+N-2}} \cdots A_{r_k} \end{bmatrix} x(k|\,k) +$$
$$\begin{bmatrix} B_{r_k} & 0 & \cdots & 0 \\ A_{r_{k+1}} B_{r_k} & B_{r_{k+1}} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ A_{r_{k+N-1}} \cdots A_{r_{k+1}} B_{r_k} & A_{r_{k+N-1}} \cdots A_{r_{k+2}} B_{r_{k+1}} & \cdots & B_{r_{k+N-1}} \end{bmatrix} \cdot \begin{bmatrix} u(k|\,k) \\ u(k+1|\,k) \\ \vdots \\ u(k+N-1|\,k) \end{bmatrix} \tag{3}$$

which equals to:

$$\begin{bmatrix} \hat{x}(k) \\ x(k+N|\,k) \end{bmatrix} = \begin{bmatrix} \hat{A} \\ A_{r_{k+N-1}} A_{r_{k+N-2}} \cdots A_{r_k} \end{bmatrix} x(k|\,k) + \begin{bmatrix} \hat{B} \\ \hat{B}_{r_{k+N-1}} \end{bmatrix} \hat{u}(k) \tag{4}$$

or, equivalently,

$$\widetilde{x}(k+1) = \widetilde{A} x(k|k) + \widetilde{B} \hat{u}(k) \tag{5}$$

Remark 2.1. *Beyond the N steps predictive horizon, state feedback controller given below is applied to system (4)*

$$u(k+j|k) = K x(k+j|k), \quad \forall j \ge N \tag{6}$$

And then, dynamic equation on $k+N+1$ step is

$$x(k+N+1|k) = \sum_{i\in\Lambda} p(r_{k+N}, i) P_i^\lambda (A_{r_{k+N}} + B_{r_{k+N}} K) x(k+N|k) \tag{7}$$

where P_i^λ is a positive definite matrix in the trace of λ.

Lemma 2.1. *Suppose there exists a trace of λ, stochastic LPV system $\widetilde{x}(k+1) = \widetilde{A} x(k|k) + \widetilde{B} \hat{u}(k)$ is stochastically stable after N steps, if there exists a set of matrices $P_i^\lambda > 0$ such that*

$$P_{r_{k+N}}^\lambda - (A_{r_{k+N}} + B_{r_{k+N}} K)^{\mathrm{T}} \sum_{i\in\Lambda} p(r_{k+N}, i) P_i^\lambda (A_{r_{k+N}} + B_{r_{k+N}} K) \ge Q + K^{\mathrm{T}} R K \tag{8}$$

Proof: Given N steps predictive horizon and the mode trace $\lambda = r_k, r_{k+1}, \cdots, r_{k+N-1}$, then there exists the following state equation:

$$x(k+N|k) = A_{r_{k+N-1}}, \cdots, A_{r_k} x(k|k) + \hat{B}_{r_{k+N-1}} \hat{u}(k) \tag{9}$$

Then, $x(k+N|k)$ can be obtained at the instant k following the mode trace λ, so the dynamic equation at instant $k+N+1$ is

$$x(k+N+1|k) = \sum_{i\in\Lambda} p(r_{k+N}, i) P_i^{\lambda} (A_{r_{k+N}} + B_{r_{k+N}} K) x(k+N|k) \tag{10}$$

It is known from [17] that the system is stochastically stable in λ, if and only if there exists a set of symmetric matrices $P_i^{\lambda} > 0$, such that

$$P_{r_{k+N}}^{\lambda} - (A_{r_{k+N}} + B_{r_{k+N}} K)^{\mathrm{T}} \sum_{i\in\Lambda} p(r_{k+N}, i) P_i^{\lambda} (A_{r_{k+N}} + B_{r_{k+N}} K) > 0 \tag{11}$$

The proof is thus completed.

Remark 2.2. *It can be seen from Lemma 2.1 that, the conditions are given which guarantee the stability of the system (5) at the instant $k+N$ after N steps predictive horizon.*

3. **Multistep Predictive Control Problem.** Quadratic performance function for stochastic LPV system (5) is defined:

$$\min_{\Lambda,\hat{u}(k),K,P_i^{\lambda}} \ \max_{\widetilde{A},\widetilde{B},p(i,j),x(k|k)} \ J(x(k|k), \hat{u}(k), \lambda, K, P_i^{\lambda}, k) \tag{12}$$

where

$$\begin{aligned} &J(x(k|k), \hat{u}(k), \lambda, K, P_i^{\lambda}, k) \\ &= E\left\{ \sum_{n=0}^{\infty} [x^{\mathrm{T}}(k+n|k) Q x(k+n|k) + u^{\mathrm{T}}(k+n|k) R u(k+n|k)] \right\} \end{aligned} \tag{13}$$

Q and R are selected weighting coefficient matrices.

Definition 3.1. *For given matrices $Q > 0$ and $R > 0$ for system (5), if there exists a set of matrices $P_i^{\lambda} > 0$ such that*

$$P_{r_{k+N}}^{\lambda} - (A_{r_{k+N}} + B_{r_{k+N}} K)^{\mathrm{T}} \sum_{i\in\Lambda} p(r_{k+N}, i) P_i^{\lambda} (A_{r_{k+N}} + B_{r_{k+N}} K) \geq Q + K^{\mathrm{T}} R K \tag{14}$$

then, the above min-max optimal problem can be cast into

$$\min_{\Lambda,\hat{u}(k),K,P_i^{\lambda}} \ \max_{\widetilde{A},\widetilde{B},p(i,j),x(k|k)} \ J_1(x(k), \lambda, \hat{u}(k), P_i^{\lambda}, k) + J_2(x(k), \lambda, \hat{u}(k), k) \tag{15}$$

where

$$\begin{aligned} &J_1(x(k), \lambda, \hat{u}(k), P_i^{\lambda}, k) \\ &= E\left\{ \sum_{n=0}^{N-1} [x^{\mathrm{T}}(k+n|k) Q x(k+n|k) + u^{\mathrm{T}}(k+n|k) R u(k+n|k)] \right\} \end{aligned} \tag{16}$$

$$J_2(x(k), \lambda, \hat{u}(k), k) = E\{x^{\mathrm{T}}(k+N|k) P_{r_{k+N}}^{\lambda} x(k+N|k)\} \tag{17}$$

Lemma 3.1. *The multistep predictive controller or sequence is obtained for system (5), if and only if $\hat{u}^*(k)$, K and $P_{r_{k+N}}^{\lambda} x(k+N|k)$ are the optimal solutions for the following SDP problem*

$$\min_{\lambda,\hat{u}(k),K,P_i^{\lambda}} \ \rho_1 + \rho_2 \tag{18}$$

s.t.

$$\max_{\widetilde{A},\widetilde{B},p(i,j),x(k|k)} J_1(x(k),\lambda,\hat{u}(k),P_i^{\lambda},k) \leq \rho_1 \tag{19}$$

$$\max_{\widetilde{A},\widetilde{B},p(i,j),x(k|k)} J_2(x(k),\lambda,\hat{u}(k),k) \leq \rho_2 \tag{20}$$

$$P_{r_{k+N}}^{\lambda} - (A_{r_{k+N}} + B_{r_{k+N}}K)^{\mathrm{T}} \sum_{i\in\Lambda} p(r_{k+N},i)P_i^{\lambda}(A_{r_{k+N}} + B_{r_{k+N}}K) \geq Q + K^{\mathrm{T}}RK \tag{21}$$

Proof: First, by condition (14), we have

$$\begin{aligned} &x^{\mathrm{T}}(k+j|k)P_{r_{k+N}}^{\lambda}x(k+j|k) - x^{\mathrm{T}}(k+j+1|k)\sum_{i\in\Lambda} p(r_{k+N},i)P_i^{\lambda}x(k+j+1|k) \\ &\geq x^{\mathrm{T}}(k+j|k)Qx(k+j|k) + u^{\mathrm{T}}(k+j|k)Ru(k+j|k), \quad j\in[0,\infty] \end{aligned} \tag{22}$$

and then, it follows

$$\begin{aligned} &E\{x^{\mathrm{T}}(k+N|k)P_{r_{k+N}}^{\lambda}x(k+N|k)\} \\ &\geq E\left\{\sum_{n=0}^{N-1}[x^{\mathrm{T}}(k+n|k)Qx(k+n|k) + u^{\mathrm{T}}(k+n|k)Ru(k+n|k)]\right\} \end{aligned} \tag{23}$$

Second, suppose $\rho_{\min}$ is the optimal solution of the SDP problem, and then $\hat{u}(k)$ can be expressed as $\hat{u}^*(k)$, and

$$J_{\min} = \min_{\Lambda,\hat{u}(k),K,P_i^{\lambda}} \max_{\widetilde{A},\widetilde{B},p(i,j),x(k|k)} J(x(k|k),\hat{u}(k),\lambda,K,P_i^{\lambda},k) \tag{24}$$

If

$$\max_{\widetilde{A},\widetilde{B},p(i,j),x(k|k)} J(x(k|k),\lambda',\hat{u}'(k),K',P_i^{\lambda'},k) < \max_{\widetilde{A},\widetilde{B},p(i,j),x(k|k)} J(x(k|k),\lambda^*,\hat{u}^*(k),K^*,P_i^{\lambda^*},k) \tag{25}$$

then

$$\begin{aligned} &\max_{\widetilde{A},\widetilde{B},p(i,j),x(k|k)} J(x(k|k),\lambda',\hat{u}'(k),K',P_i^{\lambda'},k) \\ &\leq \rho' < \max_{\widetilde{A},\widetilde{B},p(i,j),x(k|k)} J(x(k|k),\lambda^*,\hat{u}^*(k),K^*,P_i^{\lambda^*},k) = \rho_{\min} \end{aligned} \tag{26}$$

Obviously, $\rho' < \rho_{\min}$ opposites with the fact that $\rho_{\min}$ is the optimal solution of the semi-definite programming problem, so $\rho_{\min} \leq J_{\min}$.

On the other hand,

$$\begin{aligned} &\max_{\widetilde{A},\widetilde{B},p(i,j),x(k|k)} J(x(k|k),\hat{u}(k),\lambda,K,P_i^{\lambda},k) \\ &\leq \max_{\widetilde{A},\widetilde{B},p(i,j),x(k|k)} J_1(x(k),\lambda,\hat{u}(k),P_i^{\lambda},k) + \max_{\widetilde{A},\widetilde{B},p(i,j),x(k|k)} J_2(x(k),\lambda,\hat{u}(k),k) = \rho_{\min} \end{aligned} \tag{27}$$

and then $J_{\min} \leq \rho$, so $J_{\min} = \rho$.

Definition 3.2. *A set is defined as follows:*

$$W := \left\{x(k+N|k)\in R^n | x^{\mathrm{T}}(k+N|k)S_{r_{k+N}}^{-1}x(k+N|k) \leq 1, S_{r_{k+N}} > 0\right\}$$

then, the system states belong to such a set after $k+N$ steps.

Corollary 3.1. *Define $P_{r_{k+N}}^{\lambda} = \rho_2 S_{r_{k+N}}^{-1}$, the states of system (5) belong to W through λ, if there exists a set of symmetric matrices $S_{r_{k+N}} > 0$, such that*

$$\begin{bmatrix} 1 & x^{\mathrm{T}}(k+N|k) \\ x(k+N|k) & S_{r_{k+N}} \end{bmatrix} \geq 0. \tag{28}$$

Theorem 3.1. *System (5) is stochastically stable under the predictive controller (29), if the above semi-definite programming problem is feasible under the input sequence* $\hat{u}(k)$ *at the instance* k

$$\overline{u}(k) = \sum_{\lambda \in M} p_\lambda \hat{u}(k) \tag{29}$$

Proof: One can obtain the sequence $[\rho_1^*, \rho_2^*, \hat{u}^*(k)]$, which makes the min-max performance to be the optimal one, that is J_k^*, and the above semi-definite programming problem is feasible by the input sequence $\hat{u}^*(k)$ at the instance k, meanwhile, state optimal sequence $[x^*(k|k), x^*(k+1|k), \cdots, x^*(k+N-1|k)]$ will be obtained. We define $Y^*_{r_{k+N}}(S^*_{r_{k+N}})^{-1} = K^*$, and

$$u(k+i|k+1) = u^*(k+i|k), \quad i = 1, \cdots, N-1 \tag{30}$$

$$u(k+i|k+1) = Y^*_{r_{k+N}}(S^*_{r_{k+N}})^{-1} x^*(k+j|k+1), \quad j \geq N \tag{31}$$

Suppose J_k^* is the optimal performance at the instance k, J_{k+1}^* is the optimal performance at the instance $k+1$, under $\hat{u}^*(k)$, J_{k+1} is the performance function at instant $k+1$ under $\hat{u}^*(k)$, and all these follows the trace λ.

It is obvious that $J_{k+1} \geq J_{k+1}^*$, then

$$\begin{aligned}
& E\{J_k^* - J_{k+1}^*\} \geq E\{J_k^* - J_{k+1}\} \\
= & \sum_{n=0}^{N-1} \left[x^*(k+n|k)^{\mathrm{T}} Q x^*(k+n|k) + u^*(k+n|k)^{\mathrm{T}} R u^*(k+n|k)\right] \\
& - \sum_{n=1}^{N-1} \left[x^{\mathrm{T}}(k+n|k+1) Q x(k+n|k+1) + u^{\mathrm{T}}(k+n|k+1) R u(k+n|k+1)\right] \\
& + x^*(k+n|k)^{\mathrm{T}} P^{\lambda *}_{r_{k+N}} x^*(k+n|k) \\
& - x^{\mathrm{T}}(k+n|k+1) Q x(k+n|k+1) \\
& - u^{\mathrm{T}}(k+n|k+1) R u(k+n|k+1) \\
& - x^{\mathrm{T}}(k+N+1|k+1) \overline{P}^{\lambda *}_{r_{k+N}} x(k+N+1|k+1)
\end{aligned}$$

where $\overline{P}_i^\lambda = \sum\limits_{j=1}^{l} p_{ij} P_j^\lambda$.

Next, the following condition is derived from conditions (30) and (31)

$$\begin{aligned}
& E\{J_{k+1}^* - J_k^*\} \leq E\{J_{k+1}\} - J_k^* \\
= & \sum_{n=1}^{N-1} [x^*(k+n|k)^{\mathrm{T}} Q x^*(k+n|k) + u^*(k+n|k)^{\mathrm{T}} R u^*(k+n|k)] \\
& + u^*(k+N|k)^{\mathrm{T}} R u^*(k+N|k) \\
& + x^{\mathrm{T}}(k+N+1|k+1) \overline{P}^{\lambda *}_{r_{k+N}} x(k+N+1|k+1) \\
& + x^{\mathrm{T}}(k+N|k) Q x(k+N|k) \\
& - x^*(k+N|k)^{\mathrm{T}} P^{\lambda *}_{r_{k+N}} x^*(k+N|k) \\
& - \sum_{n=0}^{N-1} [x^*(k+n|k)^{\mathrm{T}} Q x^*(k+n|k) + u^*(k+n|k)^{\mathrm{T}} R u^*(k+n|k)]
\end{aligned}$$

Based on Lemma 2.1, it shows that

$$\begin{aligned}&(S^*_{r_{k+N}})^{-1} - (A_{r_{k+N}} + B_{r_{k+N}}Y^*_{r_{k+N}}(S^*_{r_{k+N}})^{-1})^{\mathrm{T}}\\ &\sum_{i\in\Lambda} p(r_{k+N},i)P_i^{\lambda}(A_{r_{k+N}} + B_{r_{k+N}}Y^*_{r_{k+N}}(S^*_{r_{k+N}})^{-1})\\ &- Q - Y^*_{r_{k+N}}((S^*_{r_{k+N}})^{-1})^{\mathrm{T}}R(S^*_{r_{k+N}})^{-1} \geq 0\end{aligned} \tag{32}$$

Multiply $x^*(k+N|k)^{\mathrm{T}}$ on the left hand side of Equation (32) and multiply $x^*(k+N|k)$ on the right hand side, respectively, then

$$\begin{aligned}&x(k+N+1|k+1)^{\mathrm{T}}(S^*_{r_{k+N}})^{-1}x(k+N+1|k+1)\\ \leq\, &x^*(k+N|k)^{\mathrm{T}}(S^*_{r_{k+N}})^{-1}x^*(k+N|k)\end{aligned} \tag{33}$$

Derived from conditions (32) and (33), we have

$$E\{J^*_{k+1} - J^*_k\} \leq E\{J_{k+1}\} - J^*_k \leq -x^*(k|k)^{\mathrm{T}}Qx^*(k|k) - u^*(k|k)^{\mathrm{T}}Ru^*(k|k)$$

and then, $E\left\{\sum_{k=0}^{\infty} x^*(k|k)^{\mathrm{T}}Qx^*(k|k) + u^*(k|k)^{\mathrm{T}}Ru^*(k|k)\right\} \leq J^*_0 - J^*_\infty \leq J^*_0$

Since J^*_0 is a finite constant parameter, and then, we can obtain $\lim_{k\to\infty} E\{x^*(k|k)\} = 0$; therefore, the system is stochastically stable.

Remark 3.1. *By Schur complement, (33) equals to (34), which means the states of system (5) belong to the following ellipsoid invariant set after $k+N$ steps through λ*

$$\begin{bmatrix} 1 & x^{\mathrm{T}}(k+N+1|k+1) \\ x(k+N+1|k+1) & S^*_{r_{k+N}} \end{bmatrix} \geq 0 \tag{34}$$

4. **Constrained Predictive Controller Design.**

Theorem 4.1. *For a given instant k and a state $x(k|k)$, suppose that there exists a set of positive definite symmetric matrices $S_i \in R^{n\times n}$, $Y_i \in R^{n\times n}$ and a set of vectors ρ_2, $b_1, \cdots, b_{m-1} \in R$, $i \in \Lambda$, $\lambda \in M$, which optimize the above semi-definite programming problem, then at instance $k+i$, the coefficient matrices of such convex polyhedral LPV systems are described as follows:*

$$A_{r_{k+i}} = b_1A_{r_{k+i}}(w_1) + b_2A_{r_{k+i}}(w_2) + \cdots + b_{m-1}A_{r_{k+i}}(w_{m-1}) + \left(1 - \sum_{l=1}^{m-1} b_l\right)A_{r_{k+i}}(w_m) \tag{35}$$

$$\begin{bmatrix} S_{r_{k+N}} & * & * & * & * & * \\ \sqrt{p((r_{k+N}),1)}(\widetilde{M}_{r_{k+N}}) & S_1 & \cdots & * & * & * \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \sqrt{p((r_{k+N}),h)}(\widetilde{M}_{r_{k+N}}) & 0 & \cdots & S_h & * & * \\ S_{r_{k+N}} & 0 & \cdots & \cdots & \rho_2Q^{-1} & * \\ Y_{r_{k+N}} & 0 & \cdots & \cdots & \ldots & \rho_2R^{-1} \end{bmatrix} \geq 0 \tag{36}$$

where $\widetilde{M}_{r_{k+N}} = M_1(A_{r_{k+N}}(w_1) - (A_{r_{k+N}}(w_m)) + M_2(A_{r_{k+N}}(w_2) - (A_{r_{k+N}}(w_m)) + \ldots + M_{m-1}(A_{r_{k+N}}(w_{m-1}) - (A_{r_{k+N}}(w_m)) + A_{r_{k+N}}(w_m)S_{r_{k+N}} + B_{r_{k+N}}(w_m)Y_{r_{k+N}}$, $M_l = b_lS_{r_{k+N}}$.

Proof: First, we consider the constraint problem of SDP

$$P^{\lambda}_{r_{k+N}} - (A_{r_{k+N}} + B_{r_{k+N}}K)^{\mathrm{T}}\sum_{i\in\Lambda} p(r_{k+N},i)P_i^{\lambda}(A_{r_{k+N}} + B_{r_{k+N}}K) \geq Q + K^{\mathrm{T}}RK \tag{37}$$

By Schur complement, we obtain (38)

$$\begin{bmatrix} P^{\lambda}_{r_{k+N}} & * & * & * & * & * \\ \sqrt{p((r_{k+N}),1)}(A_{r_{k+N}}+B_{r_{k+N}}K) & (P^{\lambda}_1)^{-1} & \cdots & * & * & * \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \sqrt{p((r_{k+N}),h)}A_{r_{k+N}}+B_{r_{k+N}}K & 0 & \cdots & (P^{\lambda}_h)^{-1} & * & * \\ 1 & 0 & \cdots & \cdots & Q^{-1} & * \\ K & 0 & \cdots & \cdots & \ldots & R^{-1} \end{bmatrix} \geq 0 \quad (38)$$

By multiplying $diag\{\rho_2(P^{\lambda}_{r_{k+N}})^{-1}, \rho_2 I, \cdots, \rho_2 I\}$ on both sides of Equation (38), and define $S_{r_{k+N}} = \rho_2(P^{\lambda}_{r_{k+N}})^{-1}$, $Y_{r_{k+N}} = KS_{r_{k+N}}$,

$$A_{r_{k+i}} = b_1 A_{r_{k+i}}(w_1) + b_2 A_{r_{k+i}}(w_2) + \cdots + b_{m-1}A_{r_{k+i}}(w_{m-1}) + \left(1 - \sum_{l=1}^{m-1} b_l\right) A_{r_{k+i}}(w_m)$$

One can obtain the conditions (35) and (36). This completes the proof.

Theorem 4.2. *For a given instant k and a state $x(k|k)$, consider the input constraints $|u_k| \leq u_{\max}$ and output constraints $|y_k| \leq y_{\max}$, suppose that there exist feasible solutions of $b_1, b_2, \cdots, b_{m-1} \in R$ in Theorem 4.1, then, if there exist a set of positive definite symmetric matrices $S_i \in R^{n\times n}$, $Y_i \in R^{m\times n}$, and vectors $\hat{u}(k) \in R^{N\times 1}$, K, ρ_1, ρ_2, $i \in \Lambda$, $\lambda \in M$, which optimize the above SDP problem, then under controller $\hat{u}(k)$, the system is stochastically stable, and $\hat{u}_{opt}(k) = \sum p_\lambda \hat{u}(k)$ is the optimal predictive input after N steps*

$$\min\left\{\sum_{\lambda\in M} p_\lambda(\rho_1+\rho_2)\right\} \quad (39)$$

s.t.

$$\begin{bmatrix} S_{r_{k+N}} & * & * & * & * & * \\ \sqrt{p((r_{k+N}),1)}(\widetilde{M}_{r_{k+N}}) & S_1 & \cdots & * & * & * \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \sqrt{p((r_{k+N}),h)}(\widetilde{M}_{r_{k+N}}) & 0 & \cdots & S_h & * & * \\ S_{r_{k+N}} & 0 & \cdots & \cdots & \rho_2 Q^{-1} & * \\ Y_{r_{k+N}} & 0 & \cdots & \cdots & \ldots & \rho_2 R^{-1} \end{bmatrix} \geq 0 \quad (40)$$

$$\begin{bmatrix} \rho_1 & * & * \\ \hat{A}x(k|k)+\hat{B}\hat{u}(k) & \widetilde{Q}^{-1} & * \\ \hat{u}(k) & 0 & \widetilde{R}^{-1} \end{bmatrix} \geq 0 \quad (41)$$

$$\begin{bmatrix} 1 & * \\ x(k+N|k) & S_{r_{k+N}} \end{bmatrix} \geq 0 \quad (42)$$

$$\begin{bmatrix} u^2_{\max} & * \\ M(\alpha_1)\hat{u}(k) & I \end{bmatrix} \geq 0 \quad (43)$$

$$\begin{bmatrix} u^2_{\max} & * \\ Y^{\mathrm{T}}_{r_{k+N}} & S_{r_{k+N}} \end{bmatrix} \geq 0 \quad (44)$$

$$\begin{bmatrix} y^2_{\max} & * \\ M(\alpha_2)\hat{C}(\hat{A}x(k|k)+\hat{B}\hat{u}(k)) & I \end{bmatrix} \geq 0 \quad (45)$$

$$\begin{bmatrix} y^2_{\max} & C_{r_{k+N}}(A_{r_{k+N}}S_{r_{k+N}}+B_{r_{k+N}}Y_{r_{k+N}}) \\ * & S_{r_{k+N}} \end{bmatrix} \geq 0 \quad (46)$$

where $\alpha_1 = 1, \cdots, N, \alpha_2 = 1, \cdots, N-1$

$$A_{r_{k+i}} = b_1 A_{r_{k+i}}(w_1) + b_2 A_{r_{k+i}}(w_2) + \cdots + b_{m-1} A_{r_{k+i}}(w_{m-1}) + \left(1 - \sum_{l=1}^{m-1} b_l\right) A_{r_{k+i}}(w_m)$$

$$\begin{aligned} A_{r_{k+N}} S_{r_{k+N}} =& M_1(A_{r_{k+N}}(w_1) - (A_{r_{k+N}}(w_m)) + M_2(A_{r_{k+N}}(w_2) - (A_{r_{k+N}}(w_m)) + \dots \\ &+ M_{m-1}(A_{r_{k+N}}(w_{m-1}) - (A_{r_{k+N}}(w_m)) + A_{r_{k+N}}(w_m) S_{r_{k+N}} \end{aligned}$$

$$M_l = b_l S_{r_{k+N}},\ \widetilde{Q} = QI,\ \widetilde{R} = RI,\ M_\alpha = [\overbrace{\underbrace{0, \cdots, 0, I}_{\alpha}, 0, \cdots, 0}^{N}],\ \hat{C} = [C_{r_{k+N}}, \cdots, C_{r_{k+N-1}}].$$

Proof: First, the proof of (40) is the same with Theorem 4.1. Next, we consider the function:

$$J_1(x(k), \lambda, \hat{u}(k), P_i^\lambda, k) = \sum_{\lambda \in M} p_\lambda \left\{ \sum_{n=0}^{N-1} \left[x^{\mathrm{T}}(k+n|k) Q x(k+n|k) + u^{\mathrm{T}}(k+n|k) R u(k+n|k) \right] \right\} \leq \widetilde{\rho}_1 \tag{47}$$

It equals to

$$\sum_{\lambda \in M} p_\lambda \{ x^{\mathrm{T}}(k|k) Q x(k|k) + (\hat{A}x(k|k) + \hat{B}\hat{u}(k))^{\mathrm{T}} \widetilde{Q} (\hat{A}x(k|k) + \hat{B}\hat{u}(k)) + \hat{u}^{\mathrm{T}}(k) \widetilde{R} \hat{u}(k) \} \leq \widetilde{\rho}_1 \tag{48}$$

As $x^{\mathrm{T}}(k|k)Qx(k|k)$ is a constant value, we suppose that $\widetilde{\rho}_1' = \widetilde{\rho}_1 - x^{\mathrm{T}}(k|k)Qx(k|k)$, then

$$\widetilde{\rho}_1 - \sum_{\lambda \in M} p_\lambda \{ (\hat{A}x(k|k) + \hat{B}\hat{u}(k))^{\mathrm{T}} \widetilde{Q} (\hat{A}x(k|k) + \hat{B}\hat{u}(k)) + \hat{u}^{\mathrm{T}}(k) \widetilde{R} \hat{u}(k) \} \geq 0 \tag{49}$$

If we suppose $\widetilde{\rho}_1' = \sum_{\lambda \in M} p_\lambda \rho_1$, then, (49) is described as

$$\rho_1 - \sum_{\lambda \in M} p_\lambda \{ (\hat{A}x(k|k) + \hat{B}\hat{u}(k))^{\mathrm{T}} \widetilde{Q} (\hat{A}x(k|k) + \hat{B}\hat{u}(k)) + \hat{u}^{\mathrm{T}}(k) \widetilde{R} \hat{u}(k) \} \geq 0 \tag{50}$$

By Schur complement, (41) is obtained.

Then, we consider the function:

$$J_2(x(k), \lambda, \hat{u}(k), k) = E\{ x^{\mathrm{T}}(k+N|k) P_{r_{k+N}}^\lambda x(k+N|k) \} \leq \rho_2 \tag{51}$$

That is

$$\rho_2 - x^{\mathrm{T}}(k+N|k) P_{r_{k+N}}^\lambda x(k+N|k) \geq 0 \tag{52}$$

Define $P_{r_{k+N}}^\lambda = \rho_2 S_{r_{k+N}}^{-1}$, by Schur complement, (42) is obtained.

Consider the input constraints:

$$|u_\alpha| \leq u_{\max}, \quad \alpha \in 1, \cdots, N-1 \tag{53}$$

That is

$$u_\alpha^{\mathrm{T}} u_\alpha \leq u_{\max}^2 \tag{54}$$

Define $u_\alpha = M(\alpha)\hat{u}(k)$, by using Schur complement, condition (43) is obtained. For the terminal instance $k+N$

$$|u(k+N|k)| \leq u_{\max} \tag{55}$$

That is

$$|Kx(k+N|k)| \leq u_{\max} \tag{56}$$

From reference [18], inequality (57) is derived from (56).

$$K^{\mathrm{T}} \rho_2 (P_{r_{k+N}}^\lambda)^{-1} K \leq u_{\max}^2 \tag{57}$$

By Schur complement inequality, (58) is obtained from (57)

$$\begin{bmatrix} u_{\max}^2 & K \\ K^{\mathrm{T}} & \rho_2^{-1}(P_{r_{k+N}}^{\lambda}) \end{bmatrix} \geq 0. \tag{58}$$

Multiply $diag\{I, \rho_2(P_{r_{k+N}}^{\lambda})^{-1}\}$ on the left hand side and right hand side, respectively, and define $S_{r_{k+N}} = \rho_2(P_{r_{k+N}}^{\lambda})^{-1}$, $Y_{r_{k+N}} = KS_{r_{k+N}}$, condition (44) is obtained.

Consider the output constraints:

$$|y(t)| \leq y_{\max}, \quad t \in k, \cdots, k+N-1 \tag{59}$$

That is

$$y^{\mathrm{T}}(t)y(t) \leq y_{\max}^2 \tag{60}$$

One can define $y(t) = M(t)\hat{C}(\hat{A}x(k|k) + \hat{B}\hat{u}(k))$, by using Schur complement to (60), for the N step, we obtain:

$$|y(k+N|k)| \leq y_{\max} \tag{61}$$

That is

$$|C_{r_{k+N}}(A_{r_{k+N}} + B_{r_{k+N}}K)x(k+N|k)| \leq y_{\max} \tag{62}$$

Using the terminal invariant set, we can obtain

$$x(k+N|k)^{\mathrm{T}}P_{r_{k+N}}^{\lambda}x(k+N|k)| \leq 1 \tag{63}$$

From reference [18], Equation (64) equals to (63):

$$(C_{r_{k+N}}(A_{r_{k+N}} + B_{r_{k+N}}K))^{\mathrm{T}}\rho_2(P_{r_{k+N}}^{\lambda})^{-1}(C_{r_{k+N}}(A_{r_{k+N}} + B_{r_{k+N}}K)) \leq 1 \tag{64}$$

By Schur complement to (64), inequality (65) is obtained:

$$\begin{bmatrix} y_{\max}^2 & C_{r_{k+N}}(A_{r_{k+N}} + B_{r_{k+N}}K) \\ * & \rho_2^{-1}P_{r_{k+N}}^{\lambda} \end{bmatrix} \geq 0 \tag{65}$$

And by multiply $diag\{I, \rho_2(P_{r_{k+N}}^{\lambda})^{-1}\}$ on both sides of Equation (65), and define $S_{r_{k+N}} = \rho_2(P_{r_{k+N}}^{\lambda})^{-1}$, $Y_{r_{k+N}} = KS_{r_{k+N}}$, inequality (46) can be obtained. This completes the proof.

Remark 4.1. *As well known, in practice, almost all actuators and outputs have their limited working region, if the input of system exceeds the maximal capacity or lower than the minimal capacity, then, it will lead to some damages, so the constraints in our paper are full of practical meaning. It is worth mentioning that, in many real systems, especially in chemical reaction systems, the parameters are time-varying, and we set a numerical example to illustrate the effectiveness. In our future work, we will try to do some predictive work on networked control systems, time delay and nonlinear systems* [19, 20, 21].

5. **Simulation Example.** The matrices of Markov jump LPV systems are given below:

$$A_1 = \begin{bmatrix} 0 & -\sin(w) \\ \sin(w) & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & -2\sin(w) \\ 2\sin(w) & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1 & 2 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Transition probability matrix is described as

$$\Pi = \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}$$

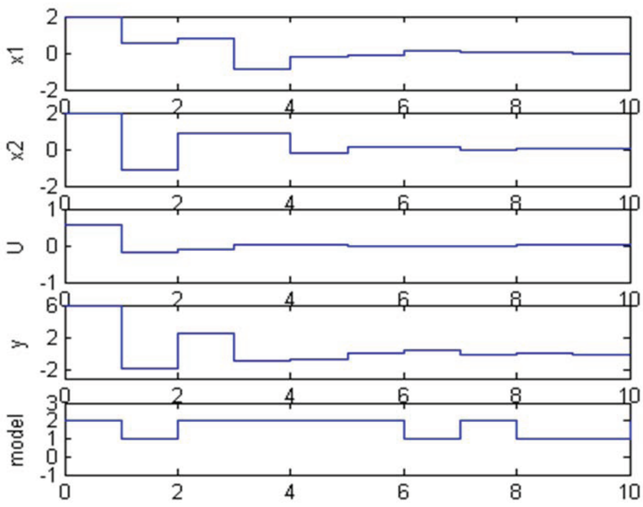

FIGURE 1. The trajectory of states x_1, x_2, input u, output y and jump modes

One can select w_l as $w_l = \begin{bmatrix} \frac{\pi}{6} & \frac{\pi}{3} & \frac{\pi}{2} \end{bmatrix}$, from Theorem 4.1, we have $b_1 = 0.3249$, $b_2 = 0.017$, $b_3 = 0.658$. The parameters and initial condition are given as $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $R = [1]$, $x_0 = \begin{bmatrix} 2 & 2 \end{bmatrix}$, $N = 2$, $|u_{\max}| = 1$, $|y_{\max}| = 6$, then, the trajectory of states x_1, x_2, input u, output y and jump modes are given in Figure 1.

6. **Conclusions.** In this paper, multistep predictive controller is designed for a class of Markov jump convex polyhedron LPV systems with both constraints on inputs and outputs. First, the stochastic LPV system is expressed by some linear time-invariant systems at different selected working points, next, in the given receding horizon, the optimal control inputs are designed in order to make the states into a terminal invariant set. Outside of the receding horizon, state feedback controller is designed to guarantee the stability of the system. Finally, constraints on both inputs and outputs are considered for such system and receding horizon predictive controller is designed in terms of linear matrix inequality. The simulation shows the effectiveness of our method.

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