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DESIGN OF ISOLATED FOOTINGS OF RECTANGULAR FORM USING A NEW MODEL

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Abstract. *In the design of reinforced concrete rectangular footings subject to axial load and flexure in two directions, there are different pressures in the four corners, these are exercised by soil. In this paper, a mathematical model is developed to take into account the real pressure of soil acting on the contact surface of the footings, these pressures are presented in terms of the mechanical elements (axial load, around moment the axis "X" and around moment the axis "Y"), when applying the load that must support said structural member. The classical model takes into account only the maximum pressure of the soil for design of footings and it is considered uniform at all points of contact area of footing, i.e., that all the contact surface has the same pressure. Also a comparison is developed between the two models as shown in the results table. The data show that the classical model is larger than the model proposed. Therefore, normal practice to use the classic model will not be a recommended solution. Then the proposed model is the most appropriate, since it is more economic and also is adjusted to real conditions.*
Keywords: Rectangular footings, Real pressures, Contact surface, Resultant force, Center of gravity, Moments, Shear force by flexure, Shear force by penetration

1. **Introduction.** The foundation is part of the structure which transmits the loads to the soil. Each building demands the need to solve a problem of foundation. The foundations are classified into superficial and deep, which have important differences: in terms of geometry, the behavior of the soil, its structural functionality and its constructive systems [1-4].

A superficial foundation is a structural member whose cross section is of large dimensions with respect to height and whose function is to transfer the loads of a building at depths relatively short, less than 4 m approximately with respect to the level of the surface of natural ground [1-4].

Superficial foundations, whose constructive systems generally do not present major difficulties, may be of various types according to their function: isolated footing, combined footing, strip footing, or mat foundation [1-4].

The structural design of foundations, by itself, represents the union and the frontier of structural design and soil mechanics [1-7]. As such, shared the hypothesis and models of both disciplines, which do not always coincide, the high degree of specialization with which are being designed today makes that structural engineers and engineers of soil mechanics will have different approaches, which in some way affects the final product that will find in these two disciplines: foundation design.

Indeed, for normal working, structural analysis is usually done with the hypothesis that the building structure is embedment in the ground, i.e., it is supported by an undeformable material [1-4].

On the other hand, the engineer of soil mechanics, for calculating the conditions of service by soil settlement, despises the structure, whose model are only forces as resulting from the reactions.

The reality is that neither the soil is undeformable, neither the structure is as flexible as for that its effects are not interrelated. After all, the system soil-structure is a continuous element whose deformations of one depend on the other.

In the design of superficial foundations, the specific case of isolated footings are of three types in terms of the application of loads: 1) The footings subject to concentric axial load; 2) The footings subject to axial load and moment in one direction (unidirectional flexure); 3) The footings subject to axial load and moment in two directions (bidirectional flexure) [1,4-8]. The hypothesis used in the classical model is to consider the pressures uniforms for the design, i.e., the same pressure at all points of contact in the foundation with the soil, the design pressure is the maximum that occurs of at the four corners the footings rectangular.

The classical model for dimensioning of footings rectangular is developed by trial and error, i.e., it is proposed a dimension and using the expression of the bidirectional flexure to obtain the stresses acting on the four corners of the rectangular footing, which must meet with the following conditions: 1) The minimum stress should be equal to or greater than zero, because the soil is not capable of withstand tensile stresses; 2) The maximum stress must be equal or less than the allowable capacity that can withstand the soil.

A direct method of proportioning a rectangular footing area subjected to biaxial flexure is proposed as an alternative to the trial and error method of solution. Formulas for the dimensions of the footing area are derived using the ordinary flexure formula and the limiting conditions that the maximum and minimum pressures are developed at the critical corners which are diagonally opposite each other. In addition, the maximum pressure is equated to the allowable bearing capacity of the soil while the minimum pressure is equated to zero. The analysis yielded the basic relationship of the footing area dimensions as 12 times the eccentricities of the total vertical load about the centroidal axes while the minimum area is controlled by the allowable soil bearing capacity [9].

A comparative study of different integration methods of stresses (both analytical and numerical) for concrete sections subjected to axial loads and biaxial flexure, such methods are applied to circular and rectangular sections. The comparison was performed with regard to the accuracy and the computational speed of each method. The objective of the paper is to determine which of the integration methods compared is more efficient in computing the interaction surfaces for rectangular and circular sections [10].

A simple design chart is also provided to determine the minimum dimensions of a rigid rectangular footing resting on elastic mass subjected to the combination of biaxial flexure in both axes and vertical column load [11].

Luévanos-Rojas developed a mathematical model to obtain the dimensions most economic for rectangular footings subjected to axial load and moment in two directions (bidirectional flexure), which must meet with the two conditions mentioned previously [12].

Luévanos-Rojas developed a mathematical model to take into account the real pressure of soil acting on the contact surface of the rectangular footings when applying the load that must support said structural member, this model is presented in function of the pressures, for obtain the moments acting on the rectangular footings [13].

This paper develops a full mathematical model for design of rectangular footings for obtain: 1) The around moment of a axis a'-a' that is parallel to axis "X-X" and around a axis b'-b' that is parallel to axis "Y-Y"; 2) The shear forces by flexure (unidirectional shear force); 3) The shear forces by penetration (bidirectional shear force) for footings that are supporting to a rectangular column or a circular column, for footings subject to axial load and moment in two directions (bidirectional flexure), where pressures are different in the four corners, these pressures are presented in terms of the mechanical elements (axial load, around moment the axis "X-X" and around moment the axis "Y-Y"), when the load is applied to said structural member, having a along linear variation all its contact area, which is as it presents the really pressure. Also, a comparison is developed in terms of materials that are used (steel and concrete) between the traditional model and the proposed model to observe the differences.

2. **Mathematical Development of Model New.**

The general equation for any type of footings subjected to bidirectional flexure [12-14]:

$$\sigma = \frac{P}{A} \pm \frac{M_x C_y}{I_x} \pm \frac{M_y C_x}{I_y} \tag{1}$$

where σ is the stress exerted by the soil on the footing (soil pressure), A is the contact area of the footing, P is the axial load applied at the center of gravity of the footing, M_x is the around moment the axis "X", M_y is the around moment the axis "Y", C_x is the distance in the direction "X" measured from the axis "Y" up the farthest end, C_y is the distance in direction "Y" measured from the axis "X" up the farthest end, I_y is the moment of around inertia the axis "Y" and I_x is the moment of around inertia the axis "X".

Figure 1 shows the pressures diagram for rectangular footings subject to axial load and moment in two directions (bidirectional flexure), where pressures are presented differently in the four corners and along linearly varying the entire contact surface.

Figure 2 are presented the stresses in any point of the contact surface of a rectangular footing due to the pressure exerted by the soil.

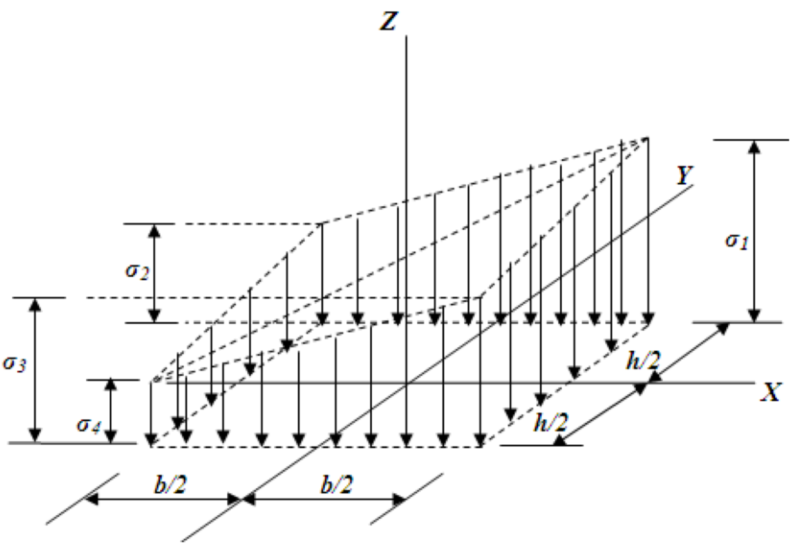

FIGURE 1. Pressures soil on the foundation

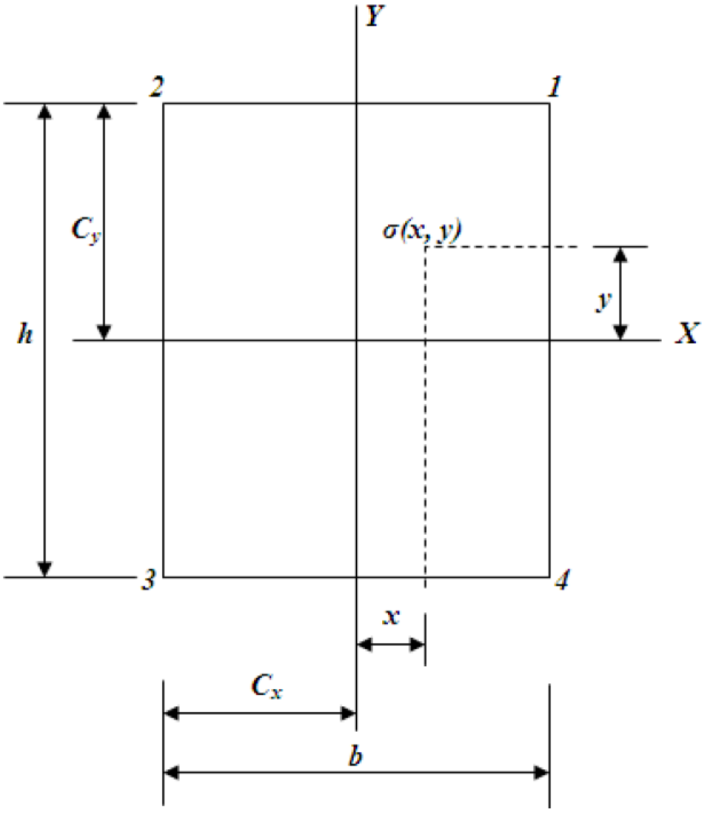

FIGURE 2. Typical rectangular footing

The stresses are found by Equation (1) at any point on a rectangular footing subjected bidirectional flexure, it shows:

$$\sigma(x,y) = \frac{P}{bh} + \frac{12M_x y}{bh^3} + \frac{12M_y x}{hb^3} \tag{2}$$

where h is the side of the parallel footing to axis "Y", b is the side of the parallel footing to the axis "X".

Equation (2) is used to find the stresses in each corner of the footing as follows:

$$\sigma_1 = \frac{P}{bh} + \frac{6M_x}{bh^2} + \frac{6M_y}{hb^2} \tag{3}$$

$$\sigma_2 = \frac{P}{bh} + \frac{6M_x}{bh^2} - \frac{6M_y}{hb^2} \tag{4}$$

$$\sigma_3 = \frac{P}{bh} - \frac{6M_x}{bh^2} + \frac{6M_y}{hb^2} \tag{5}$$

$$\sigma_4 = \frac{P}{bh} - \frac{6M_x}{bh^2} - \frac{6M_y}{hb^2} \tag{6}$$

where σ_1 is the maximum stress and σ_4 is the minimum stress.

2.1. **Model to obtain the moments.** Critical sections for moments are presented in section a'-a' and b'-b', as shown in Figure 3.

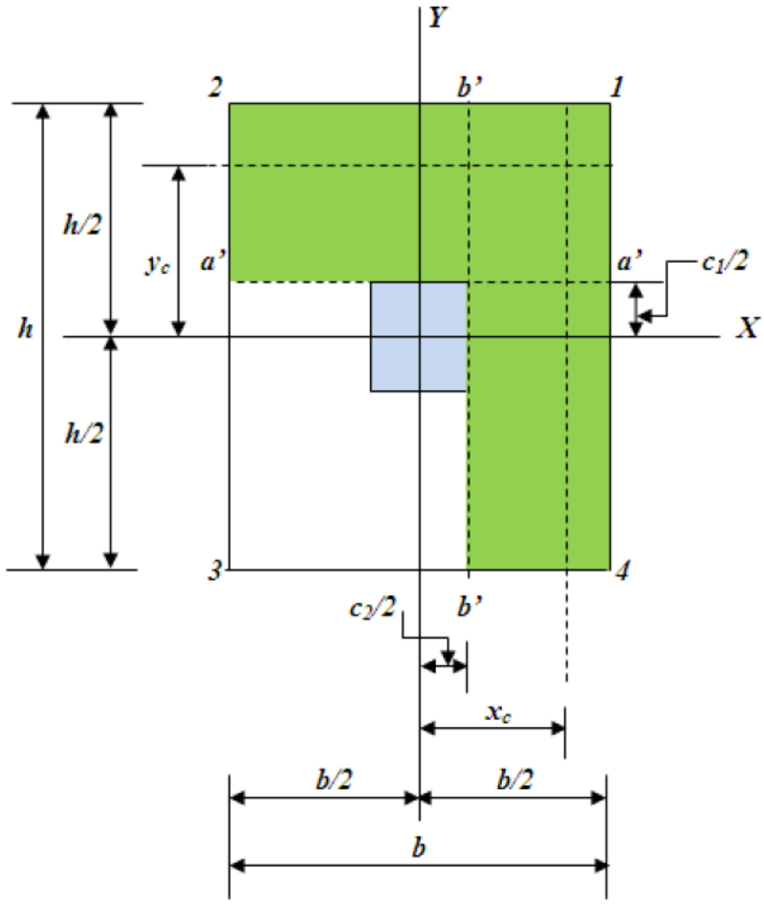

Figure 3. Critical sections for moments

2.1.1. *Around moment of axis a'-a'.* The resultant force "F_{R1}" is obtained through the volume of pressure of the area formed by the axis a'-a' and the corners 1 and 2 of the footing, it is presented [15-17]:

$$F_{R1} = \int_{c_1/2}^{h/2} \int_{-b/2}^{b/2} \sigma(x, y) dxdy \tag{7}$$

Equation (2) is substituted into Equation (7), we obtain:

$$F_{R1} = \int_{c_1/2}^{h/2} \int_{-b/2}^{b/2} \left[\frac{P}{bh} + \frac{12M_x y}{bh^3} + \frac{12M_y x}{hb^3} \right] dxdy \tag{8}$$

where c_1 is the dimension of the parallel column to the axis "Y", c_2 is the dimension of the parallel column to the axis "X".

From Equation (8) is developed the integration double and boundary conditions are substituted; it is shown:

$$F_{R1} = \frac{P(h - c_1)}{2h} + \frac{3M_x(h^2 - c_1^2)}{2h^3} \tag{9}$$

Now, the integral is developed to obtain the center of gravity "y_c" of the soil pressures:

$$y_c = \frac{\int_{c_1/2}^{h/2} \int_{-b/2}^{b/2} y\sigma(x, y) dydx}{\int_{c_1/2}^{h/2} \int_{-b/2}^{b/2} \sigma(x, y) dydx} \tag{10}$$

Equation (2) is substituted into Equation (10), we obtain:

$$y_c = \frac{\int_{c_1/2}^{h/2} \int_{-b/2}^{b/2} \left[\frac{P}{bh} + \frac{12M_x y}{bh^3} + \frac{12M_y x}{hb^3} \right] y dx dy}{\int_{c_1/2}^{h/2} \int_{-b/2}^{b/2} \left[\frac{P}{bh} + \frac{12M_x y}{bh^3} + \frac{12M_y x}{hb^3} \right] dx dy} \tag{11}$$

From Equation (11) is developed the integration double and boundary conditions are substituted; it is shown:

$$y_c = \frac{Ph^2(h^2 - c_1^2) + 4M_x(h^3 - c_1^3)}{4Ph^2(h - c_1) + 12M_x(h^2 - c_1^2)} \tag{12}$$

The around moment the axis a'-a' is found by the equation shown as follows:

$$M_{a'\text{-}a'} = F_{R1}(y_c - c_1/2) \tag{13}$$

Equations (9) and (12) are substituted into Equation (13), we obtain:

$$M_{a'\text{-}a'} = \left[\frac{P(h - c_1)}{2h} + \frac{3M_x(h^2 - c_1^2)}{2h^3} \right] \left[\frac{Ph^2(h^2 - c_1^2) + 4M_x(h^3 - c_1^3)}{4Ph^2(h - c_1) + 12M_x(h^2 - c_1^2)} - \frac{c_1}{2} \right] \tag{14}$$

2.1.2. *Around moment of axis b'-b'.* The resultant force "F_{R2}" is obtained through the volume of pressure of the area formed by the axis b'-b' and corners 1 and 4 of the footing, it is presented [15-17]:

$$F_{R2} = \int_{-h/2}^{h/2} \int_{c_2/2}^{b/2} \sigma(x, y) dx dy \tag{15}$$

Equation (2) is substituted into Equation (15), we obtain:

$$F_{R2} = \int_{-h/2}^{h/2} \int_{c_2/2}^{b/2} \left[\frac{P}{bh} + \frac{12M_x y}{bh^3} + \frac{12M_y x}{hb^3} \right] dx dy \tag{16}$$

From Equation (16) is developed the integration double and boundary conditions are substituted; it is shown:

$$F_{R2} = \frac{P(b - c_2)}{2b} + \frac{3M_y(b^2 - c_2^2)}{2b^3} \tag{17}$$

Now, the integral is developed to obtain the center of gravity "x_c" of the soil pressures:

$$x_c = \frac{\int_{-h/2}^{h/2} \int_{c_2/2}^{b/2} x\sigma(x, y) dy dx}{\int_{-h/2}^{h/2} \int_{c_2/2}^{b/2} \sigma(x, y) dy dx} \tag{18}$$

Equation (2) is substituted into Equation (18), we obtain:

$$x_c = \frac{\int_{-h/2}^{h/2} \int_{c_2/2}^{b/2} \left[\frac{P}{bh} + \frac{12M_x y}{bh^3} + \frac{12M_y x}{hb^3} \right] x dx dy}{\int_{-h/2}^{h/2} \int_{c_2/2}^{b/2} \left[\frac{P}{bh} + \frac{12M_x y}{bh^3} + \frac{12M_y x}{hb^3} \right] dx dy} \tag{19}$$

From Equation (19) is developed the integration double and boundary conditions are substituted; it is shown:

$$x_c = \frac{Pb^2(b^2 - c_2^2) + 4M_y(b^3 - c_2^3)}{4Pb^2(b - c_2) + 12M_y(b^2 - c_2^2)} \tag{20}$$

The around moment the axis b'-b' is found by the equation following:

$$M_{b'\text{-}b'} = F_{R2}(x_c - c_2/2) \tag{21}$$

Equations (17) and (20) are substituted into Equation (21), we obtain:

$$M_{b'\text{-}b'} = \left[\frac{P(b-c_2)}{2b} + \frac{3M_y(b^2-c_2^2)}{2b^3}\right]\left[\frac{Pb^2(b^2-c_2^2)+4M_y(b^3-c_2^3)}{4Pb^2(b-c_2)+12M_y(b^2-c_2^2)} - \frac{c_2}{2}\right] \tag{22}$$

2.2. **Model to obtain shear force by flexure (unidirectional shear force).** The critical section for shear force by flexure is obtained at a distance "d" to from the junction of the column with the footing as shown in Figure 4, it is presented in section c'-c'.

Shear force by flexure acting on the footing "V_f" is obtained through the volume of pressure of the area formed by the axis c'-c' and corners 1 and 2 of the footing, it is presented as follows [15-17]:

$$V_f = \int_{c_1/2+d}^{h/2}\int_{-b/2}^{b/2} \sigma(x,y)dxdy \tag{23}$$

where "d" is the distance measured vertically from extreme compression fiber to the centroid of the longitudinal reinforcement steel of the footing.

Equation (2) is substituted into Equation (23), we obtain:

$$V_f = \int_{c_1/2+d}^{h/2}\int_{-b/2}^{b/2}\left[\frac{P}{bh} + \frac{12M_xy}{bh^3} + \frac{12M_yx}{hb^3}\right]dxdy \tag{24}$$

From Equation (24) is developed the integration double and boundary conditions are substituted; it is shown:

$$V_f = \frac{P(h-c_1-2d)}{2h} + \frac{3M_x(h^2-c_1^2-4c_1d-4d^2)}{2h^3} \tag{25}$$

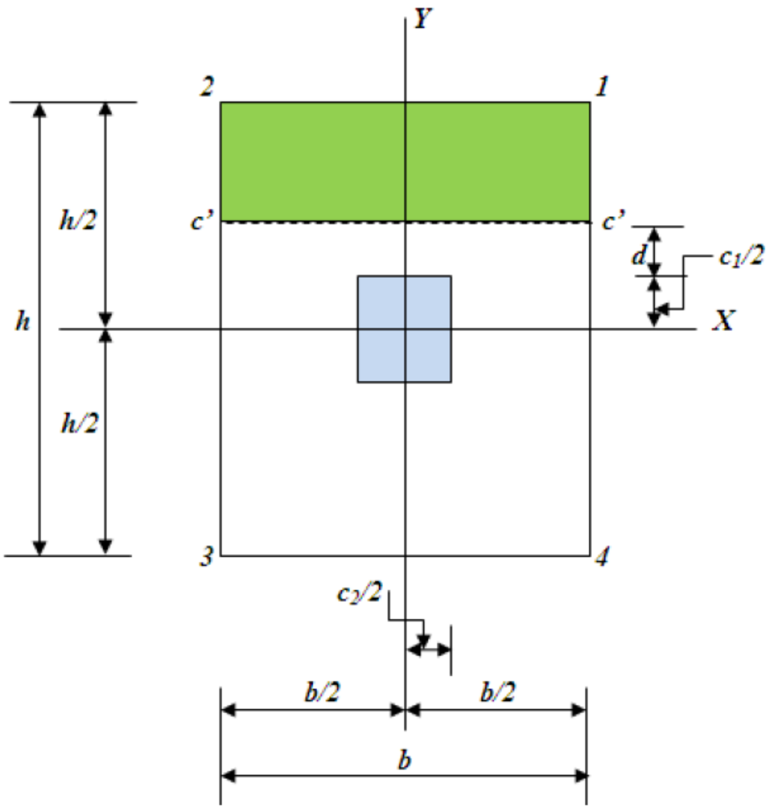

FIGURE 4. Critical sections for shear force by flexure

2.3. **Model to obtain shear force by penetration (bidirectional shear force).** The critical section for shear force by penetration appears at a distance "$d/2$" to from the junction of the column with the footing in the two directions.

2.3.1. *Shear force by penetration that supports a rectangular column.* The critical section for shear force by penetration occurs in the rectangular section formed by points 5, 6, 7 and 8, as shown in Figure 5.

Shear force by penetration acting on the footing "V_p" is obtained through the volume of pressure of the rectangular area formed by points 1, 2, 3 and 4 less than the rectangular area formed by points 5, 6, 7 and 8.

The force generated on rectangular area formed by points 1, 2, 3 and 4 "F_{1234}" of the footing is as follows [15-17]:

$$F_{1234} = \int_{-h/2}^{h/2} \int_{-b/2}^{b/2} \sigma(x, y) dxdy \tag{26}$$

Equation (2) is substituted into Equation (26), we obtain:

$$F_{1234} = \int_{-h/2}^{h/2} \int_{-b/2}^{b/2} \left[\frac{P}{bh} + \frac{12M_x y}{bh^3} + \frac{12M_y x}{hb^3} \right] dxdy \tag{27}$$

From Equation (27) is developed the integration double and boundary conditions are substituted; it is shown:

$$F_{1234} = P \tag{28}$$

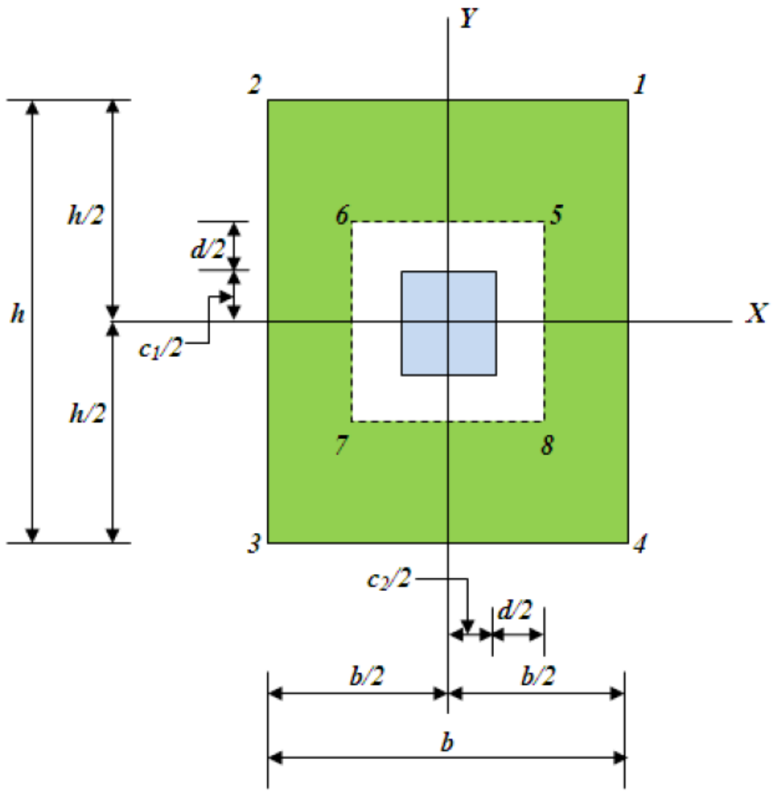

FIGURE 5. Critical sections for shear force by penetration that supports a rectangular column

The force generated on rectangular area formed by points 5, 6, 7 and 8 "F_{5678}" of the footing is as follows [15-17]:

$$F_{5678} = \int_{-c_1-d/2}^{c_1+d/2} \int_{-c_2-d/2}^{c_2+d/2} \sigma(x, y) dxdy \tag{29}$$

Equation (2) is substituted into Equation (29), we obtain:

$$F_{5678} = \int_{-c_1/2-d/2}^{c_1/2+d/2} \int_{-c_2/2-d/2}^{c_2/2+d/2} \left[\frac{P}{bh} + \frac{12M_x y}{bh^3} + \frac{12M_y x}{hb^3} \right] dxdy \tag{30}$$

From Equation (30) is developed the integration double and boundary conditions are substituted; it is shown:

$$F_{5678} = \frac{P(c_1 + d)(c_2 + d)}{bh} \tag{31}$$

Now, shear force by penetration "V_p" is as follows:

$$V_p = F_{1234} - F_{5678} \tag{32}$$

Equations (28) and (31) are substituted into Equation (32), we obtain:

$$V_p = P - \frac{P(c_1 + d)(c_2 + d)}{bh} \tag{33}$$

2.3.2. *Shear force by penetration that supports a circular column.* The critical section for shear force by penetration occurs in the circular section formed by points 5, 6, 7 and 8, as shown in Figure 6.

Shear force by penetration acting on the footing "V_p" is obtained through the volume of pressure of the rectangular area formed by points 1, 2, 3 and 4 less the circular area formed by points 5, 6, 7 and 8.

The force generated on rectangular area formed by points 1, 2, 3 and 4 "F_{1234}" of the footing is as follows [15-17]:

$$F_{1234} = \int_{-h/2}^{h/2} \int_{-b/2}^{b/2} \sigma(x, y) dxdy \tag{34}$$

Equation (2) is substituted into Equation (34), we obtain:

$$F_{1234} = \int_{-h/2}^{h/2} \int_{-b/2}^{b/2} \left[\frac{P}{bh} + \frac{12M_x y}{bh^3} + \frac{12M_y x}{hb^3} \right] dxdy \tag{35}$$

From Equation (35) is developed double integration and boundary conditions are substituted; it is shown:

$$F_{1234} = P \tag{36}$$

The force generated on circular area formed by points 5, 6, 7 and 8 "F_{5678}" of the footing is as follows [15-17]:

$$F_{5678} = \int_{-r-d/2}^{r+d/2} \int_{-\sqrt{(r+d/2)^2-y^2}}^{\sqrt{(r+d/2)^2-y^2}} \sigma(x, y) dxdy \tag{37}$$

where r is radius of the circular column.

Equation (2) is substituted into Equation (37), we obtain:

$$F_{5678} = \int_{-r-d/2}^{r+d/2} \int_{-\sqrt{(r+d/2)^2-y^2}}^{\sqrt{(r+d/2)^2-y^2}} \left[\frac{P}{bh} + \frac{12M_x y}{bh^3} + \frac{12M_y x}{hb^3} \right] dxdy \tag{38}$$

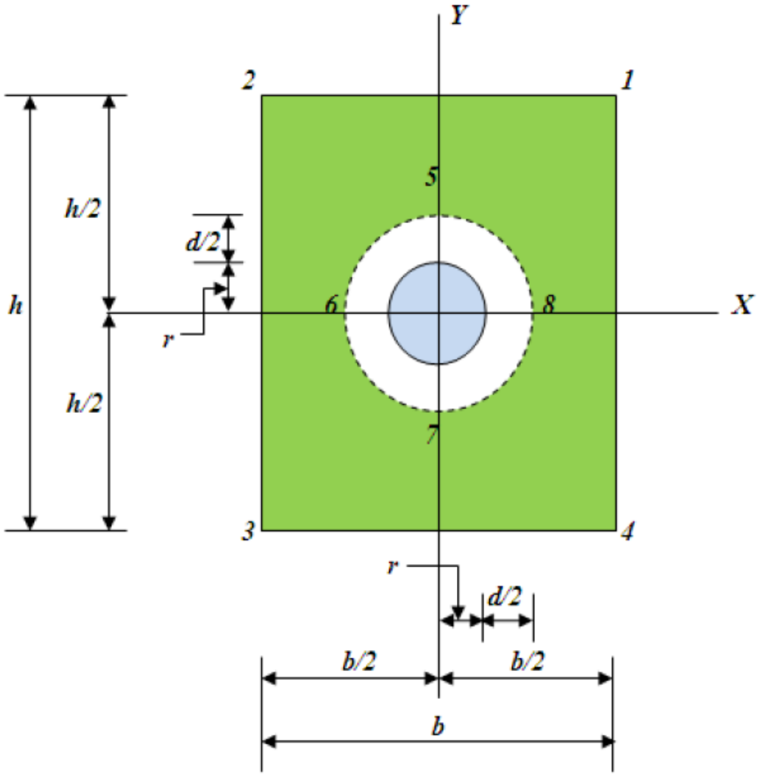

FIGURE 6. Critical sections for shear force by penetration that supports a circular column

From Equation (38) is developed the integration double and boundary conditions are substituted; it is shown:

$$F_{5678} = \frac{P\pi(r + d/2)^2}{bh} \tag{39}$$

Now, shear force by penetration "V_p" is as follows:

$$V_p = F_{1234} - F_{5678} \tag{40}$$

Equations (36) and (39) are substituted into Equation (40), we obtain:

$$V_p = P - \frac{P\pi(r + d/2)^2}{bh} \tag{41}$$

2.4. **Procedure of design.**

Step 1: The mechanical elements (P, M_x, M_y) acting on the footing is obtained by the sum of: the dead loads, live loads and accidental loads (wind or earthquake) from each of these effects [18-23].

Step 2: The available load capacity of the soil "$\sigma_{\max}$" is [18-23]:

$$\sigma_{\max} = q_a - \gamma_{ppz} - \gamma_{pps} \tag{42}$$

where q_a is the allowable load capacity of the soil, γ_{ppz} is the self weight of the footing, γ_{pps} is the self weight of soil fill.

Step 3: The value of "h" is selected according to the following equations [12]:

$$h = \frac{2M_x}{P} \tag{43}$$

$$\sigma_{\max} M_y h^3 - PM_x h - 12M_x^2 = 0 \tag{44}$$

where the value of "h" obtained from Equation (43) is when the soil pressure is zero and the value of "h" found in Equation (44) is when the pressure of the soil is load capacity available "$\sigma_{\max}$", of these two values is taken the greater to meet the two conditions, because the pressure generated by footing must greater than zero and less than the load capacity available of the soil [12]. Note: if the combinations are included the wind and/or the earthquake, the load capacity of the soil can be increased by 33% [24].

Step 4: The value of "b" is found through the following equation [12]:

$$b = \frac{M_y h}{M_x} \tag{45}$$

Step 5: The mechanical elements (P, M_x, M_y) acting on the footing are factored [24].

Step 6: The maximum moment acting on the footing is obtained from Equations (14) and (22), said critical section is located in the junction of the column with the footing as shown in Figure 3.

Step 7: The effective cant "d" for the maximum moment is found by means of the following expression [24]:

$$d = \sqrt{\frac{M_u}{\varnothing_f b_w \rho f_y \left[1 - \frac{0.59 \rho f_y}{f'_c}\right]}}, \tag{46}$$

where M_u is the factored maximum moment at section acting on the footing, $\varnothing_f$ is the strength reduction factor by flexure and its value is 0.90, b_w is width of analysis in structural member, ρ is ratio of "A_s" to "$b_w d$", f_y is the specified yield strength of reinforcement of steel, f'_c is the specified compressive strength of concrete at 28 days.

Step 8: Shear force by flexure (unidirectional shear force), which resists the concrete "V_{cf}", it is given [24]:

$$\varnothing_v V_{cf} = 0.53 \varnothing_v \sqrt{f'_c} b_w d \tag{47}$$

Shear force by flexure acting on the footing (V_f) is compared with shear force by flexure resisting by concrete (V_{cf}) and must comply with the following expression [23]:

$$V_f \leq \varnothing_v V_{cf} \tag{48}$$

where $\varnothing_v$ is the strength reduction factor by shear and its value is 0.85.

Step 9: Shear force by penetration (shear force bidirectional), which resists the concrete "V_{cp}" is given [24]:

$$\varnothing_v V_{cp} = 0.53 \varnothing_v \left(1 + \frac{2}{\beta_c}\right) \sqrt{f'_c} b_0 d \tag{49a}$$

where β_c is the ratio of long side to short side of the column and b_0 is the perimeter of the critical section.

$$\varnothing_v V_{cp} = 0.27 \varnothing_v \left(\frac{\alpha_s d}{b_0} + 2\right) \sqrt{f'_c} b_0 d \tag{49b}$$

where α_s is 40 for interior columns, 30 for edge columns, and 20 for corner columns.

$$\varnothing_v V_{cp} = \varnothing_v \sqrt{f'_c} b_0 d \tag{49c}$$

where $\varnothing_v V_{cp}$ must be the smallest value of Equations (49a), (49b) and (49c).

Shear force by penetration acting on the footing (V_p) is compared with shear force by penetration resisting by concrete (V_{cp}) and must comply with the following expression [24]:

$$V_p \leq \varnothing_v V_{cp} \tag{50}$$

Step 10: The main reinforcement steel (parallel reinforcement steel to the direction of the long side of the footing) "A_{sp}" is calculated with the following expression [24]:

$$A_{sp} = wb_w d - \sqrt{(wb_w d)^2 - \frac{2M_u wb_w}{\varnothing_f f_y}} \tag{51}$$

where w is $0.85f'_c/f_y$.

The minimum steel "$A_{s\,\min}$" by rule is [24]:

$$A_{s\,\min} = \rho_{\min} b_w d \tag{52}$$

where $\rho_{\min}$ is the minimum percentage of reinforcement steel which is obtained [24]:

$$\rho_{\min} = \frac{14}{f_y} \tag{53}$$

The parallel reinforcement steel in the short direction a portion of the total reinforcement steel, "$\gamma_s A_s$", is distributed uniformly on a band (centered with respect to the axis of the column or pedestal) whose width is equal to the length of the short side of the footing. The rest of reinforcement steel in the short direction required, "$(1 - \gamma_s)A_s$", should be uniformly distributed in the areas which are outside the central band of the footing. "γ_s" is obtained from [24]:

$$\gamma_s = \frac{2}{\beta + 1} \tag{54}$$

where β is the ratio of long side to short side of the footing.

Later the spacing of the bars "s" is obtained:

$$s = \frac{b_w a_s}{A_s} \tag{55}$$

where a_s is the rod area used.

Step 11: The development length for deformed bars "l_d" is expressed [24]:

$$l_d = \frac{f_y \psi_t \psi_e}{6.6\sqrt{f'_c}} d_b \tag{56}$$

where l_d is the minimum length that should have a deformed bar to prevent slippage, ψ_t is the traditional factor of location of the reinforcing steel which reflects the adverse effects of the position of the bars of the upper part of the section with respect to the height of fresh concrete located beneath them, ψ_e is a coating factor which reflects the effects of the epoxy coating, and d_b is the diameter of the bars.

The development length for deformed bars "l_d" is compared with the available length of the footing "l_a" and must comply with the following expression [24]:

$$l_d \leq l_a \tag{57}$$

3. **Application.** The design of an isolated footing of rectangular form that supports a square column is presented in Figure 7, with the basic information following:

$c_1 = 40$ cm
$c_2 = 40$ cm
$H = 1.5$ m
$P_D = 70$ ton
$P_L = 50$ ton
$M_{Dx} = 14$ ton-m
$M_{Lx} = 10$ ton-m
$M_{Dy} = 12$ ton-m

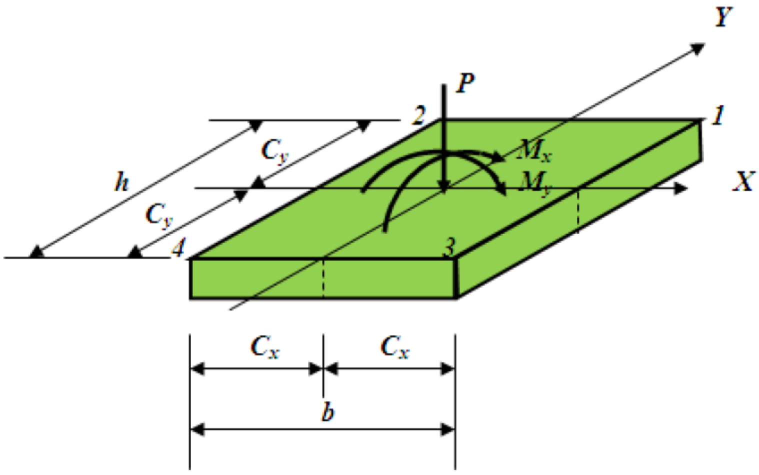

FIGURE 7. Isolated footing of rectangular form

$M_{Ly} = 8$ ton-m
$f'_c = 210$ kg/cm^2
$f_y = 4200$ kg/cm^2
$q_a = 22$ ton/m^2
$\gamma_{ppz} = 2400$ kg/m^3
$\gamma_{pps} = 1500$ kg/m^3
where H is the depth of the footing, P_D is the dead load, P_L is the live load, M_{Dx} is the moment of around dead load of axis "X-X", M_{Lx} is the moment of around live load of axis "X-X", M_{Dy} is the moment of around dead load of axis "Y-Y", and M_{Ly} is the moment of around live load of axis "Y-Y".

3.1. **Traditional model.**

Step 1: The loads and moments acting on soil:

$$P = P_D + P_L = 70 + 50 = 120 \text{ ton}$$

$$M_x = M_{Dx} + M_{Lx} = 14 + 10 = 24 \text{ ton-m}$$

$$M_y = M_{Dy} + M_{Ly} = 12 + 8 = 20 \text{ ton-m}$$

Step 2: The available load capacity of the soil:

The thickness "t" of the footing is proposed, and the first proposal is the minimum thickness of 25 cm marking regulations, subsequently the thickness is revised to meet the following conditions: moment, shear force by flexure and shear force by penetration. If such conditions are not satisfied a greater thickness is proposed until it fulfills the three conditions mentioned.

The thickness of the footing that fulfills the three conditions listed above is 65 cm.

$$\sigma_{\max} = q_a - \gamma_{ppz} - \gamma_{pps} = 22 - 2.4(0.65) - 1.5(1.5 - 0.65) = 19.165 \text{ ton/m}^2$$

Step 3: The value of "h" is:

First condition:

$$h = \frac{12M_x}{P} = \frac{12(24)}{120} = 2.4 \text{ m}$$

Second condition:

$$\sigma_{\max} M_y h^3 - PM_x h - 12M_x^2 = 0$$

$$(19.165)(20)h^3 - (120)(24)h - 12(24)^2 = 0$$

$$383.3h^3 - 2880h - 6912 = 0$$

$$h = 3.549 \text{ m}$$

Then, the greater value of "h" considered to meet the two mentioned conditions is 3.549 cm.

Step 4: The value of "b" is:

$$b = \frac{M_y h}{M_x} = \frac{(20)(3.549)}{24} = 2.958 \text{ m}$$

Therefore, the dimension of the footing is:

$$h = 3.55 \text{ m}; \quad b = 3.00 \text{ m}$$

Step 5: The mechanical elements (P, M_x, M_y) acting on the footing is factored:

$$P_u = 1.2P_D + 1.6P_L = 1.2(70) + 1.6(50) = 164 \text{ ton}$$

$$M_{ux} = 1.2M_{Dx} + 1.6M_{Lx} = 1.2(14) + 1.6(10) = 32.8 \text{ ton-m}$$

$$M_{uy} = 1.2M_{Dy} + 1.6M_{Ly} = 1.2(12) + 1.6(8) = 27.2 \text{ ton-m}$$

Step 6: The maximum moment acting on the footing is:
The maximum pressure is obtained:

$$\sigma_{u\max} = \frac{P_u}{bh} + \frac{6M_{ux}}{bh^2} + \frac{6M_{uy}}{hb^2} = \frac{164}{(3.00)(3.55)} + \frac{6(32.8)}{(3.00)(3.55)^2} + \frac{6(27.2)}{(3.55)(3.00)^2} = 25.71 \text{ ton/m}^2$$

The maximum moment acting on the footing according to Figure 3 is presented:

$$M_{a'\text{-}a'} = \frac{\sigma_{u\max} b(h - c_1)^2}{8} = \frac{(25.71)(3.00)(3.55 - 0.40)^2}{8} = 95.67 \text{ ton-m}$$

Step 7: The effective cant for the maximum moment is found:
where $M_{a'\text{-}a'} = M_u$

$$d = \sqrt{\frac{M_u}{\varnothing_f b_w \rho f_y \left[1 - \frac{0.59\rho f_y}{f'_c}\right]}} = \sqrt{\frac{9567000}{0.90(300)(0.016)(4200)\left[1 - \frac{0.59(0.016)(4200)}{210}\right]}}$$

$$d = 25.50 \text{ cm}$$

Then, we are proposed the final dimensions of footing after performing different proposals:

$$d = 57 \text{ cm}; \quad r_1 = 8 \text{ cm}; \quad t = 65 \text{ cm}$$

where r_1 is the coating.

Step 8: Shear force by flexure (unidirectional shear force) is:

$$\varnothing_v V_{cf} = 0.53\varnothing_v \sqrt{f'_c} b_w d = 0.53(0.85)\sqrt{210}(300)(57) = 111635.05 \text{ kg}$$

$$V_f = \sigma_{u\max} b\left(\frac{h - c_1}{2} - d\right) = (25.71)(3.00)\left(\frac{3.55 - 0.40}{2} - 0.57\right) = 71.52 \text{ ton}$$

$$V_f \leq \varnothing_v V_{cf}, \textit{cumple}$$

Step 9: Shear force by penetration (bidirectional shear force) is:

$$\begin{aligned}\varnothing_v V_{cp} &= 0.53\varnothing_v \left(1 + \frac{2}{\beta_c}\right)\sqrt{f'_c} b_0 d \\ &= 0.53(0.85)\left(1 + \frac{2}{1}\right)\sqrt{210}[4(40 + 57)](57) = 433143.98 \text{ kg}\end{aligned}$$

$$\begin{aligned}\varnothing_v V_{cp} &= 0.27\varnothing_v\left(\frac{\alpha_s d}{b_0}+2\right)\sqrt{f_c'}b_0 d\\ &= 0.27(0.85)\left[\frac{(40)(57)}{4(40+57)}+2\right]\sqrt{210}[4(40+57)](57) = 579322.70 \text{ kg}\\ \varnothing_v V_{cp} &= \varnothing_v\sqrt{f_c'}b_0 d = (0.85)\sqrt{210}[4(40+57)](57) = 272417.59 \text{ kg}\\ V_p &= \sigma_{u\,\max}[bh-(c_1+d)(c_2+d)]\\ &= (25.71)[(3.00)(3.55)-(0.40+0.57)(0.40+0.57)] = 249.62 \text{ ton}\end{aligned}$$

$$V_p \le \varnothing_v V_{cp}, cumple$$

Step 10: The reinforcement steel is:

* The parallel reinforcement steel to the direction of the long side of the footing is:

$$w = \frac{0.85 f_c'}{f_y} = \frac{0.85(210)}{4200} = 0.0425$$

$$\begin{aligned}A_{sp} &= wb_w d - \sqrt{(wb_w d)^2 - \frac{2M_u w b_w}{\varnothing_f f_y}}\\ &= 0.0425(300)(57) - \sqrt{[0.0425(300)(57)]^2 - \frac{2(9567000)(0.0425)(300)}{0.90(4200)}}\\ &= 45.85 \text{ cm}^2\end{aligned}$$

$$\rho_{\min} = \frac{14}{f_y} = \frac{14}{4200} = 0.00333; \quad A_{s\,\min} = \rho_{\min} b_w d = 0.00333(300)(57) = 56.43 \text{ cm}^2$$

Therefore, minimum steel is proposed "$A_{s\,\min}$".
Rod "3/4" diameter is used:

$$s = \frac{b_w a_s}{A_s} = \frac{300(2.85)}{56.43} = 15.15 \text{ cm} \approx 15 \text{ cm}$$

* The parallel reinforcement steel to the direction of the short side of the footing is:

The around maximum moment of axis b'-b' acting on the footing according to Figure 3 is presented:

$$M_{b'\text{-}b'} = \frac{\sigma_{u\,\max} h(b-c_2)^2}{8} = \frac{(25.71)(3.55)(3.00-0.40)^2}{8} = 77.12 \text{ ton-m}$$

where $M_{b'\text{-}b'} = M_u$

$$\begin{aligned}A_s &= wb_w d - \sqrt{(wb_w d)^2 - \frac{2M_u w b_w}{\varnothing_f f_y}}\\ &= 0.0425(355)(57) - \sqrt{[0.0425(355)(57)]^2 - \frac{2(7712000)(0.0425)(355)}{0.90(4200)}} = 36.57 \text{ cm}^2\end{aligned}$$

$$A_{s\,\min} = \rho_{\min} b_w d = 0.00333(355)(57) = 67.38 \text{ cm}^2$$

therefore, minimum steel is proposed "$A_{s\,\min}$".

The reinforcing steel in the central band is:
where $\beta = 355/300 = 1.183$

$$\gamma_s = \frac{2}{\beta+1} = \frac{2}{1.183+1} = 0.916; \quad \gamma_s A_s = 0.916(67.38) = 61.72 \text{ cm}^2$$

Rod "3/4" diameter is used:

$$s = \frac{b_w a_s}{\gamma_s A_s} = \frac{300(2.85)}{61.72} = 13.85 \text{ cm} \approx 13 \text{ cm}$$

The reinforcing steel in the lateral bands is:

$$(1 - \gamma_s)A_s = (1 - 0.916)(67.38) = 5.66 \text{ cm}^2$$

Rod "3/4" diameter is used:

$$s = \frac{b_w a_s}{(1 - \gamma_s)A_s} = \frac{(355 - 300)(2.85)}{5.66} = 27.69 \text{ cm} \approx 27 \text{ cm}$$

Step 11: The minimum development length for deformed bars is:
where $\psi_t = 1$ and $\psi_e = 1$.

$$l_d = \frac{f_y \psi_t \psi_e}{6.6\sqrt{f'_c}} d_b = \frac{(4200)(1)(1)}{6.6\sqrt{210}}(2.85) = 125.15 \text{ cm}$$

The available length of the rod in the direction of short side of the footing is

$$(300 - 40)/2 = 130 \text{ cm}.$$

The available length of the rod in the direction of long side of the footing is

$$(355 - 40)/2 = 157 \text{ cm}.$$

The minimum development length is less than the available length. Therefore, not requires hook.

3.2. **Proposed model.**

Steps 1 to 5: Those are the same as the traditional model.

Step 6: The maximum moment acting on the footing is:

The maximum moment acting on the footing through Equation (14) according to Figure 3 is presented:

$$\begin{aligned} M_{a'\text{-}a'} &= \left[\frac{P(h - c_1)}{2h} + \frac{3M_x(h^2 - c_1^2)}{2h^3}\right]\left[\frac{Ph^2(h^2 - c_1^2) + 4M_x(h^3 - c_1^3)}{4Ph^2(h - c_1) + 12M_x(h^2 - c_1^2)} - \frac{c_1}{2}\right] \\ &= \left[\frac{164(3.55 - 0.40)}{2(3.55)} + \frac{3(32.8)[(3.55)^2 - (0.40)^2]}{2(3.55)^3}\right] \\ &\quad \left[\frac{164(3.55)^2[(3.55)^2 - (0.40)^2] + 4(32.8)[(3.55)^3 - (0.40)^3]}{4(164)(3.55)^2(3.55 - 0.40) + 12(32.8)[(3.55)^2 - (0.40)^2]} - \frac{0.40}{2}\right] \\ &= 70.94 \text{ ton-m} \end{aligned}$$

Step 7: The effective cant for the maximum moment is found:
where $M_{a'\text{-}a'} = M_u$

$$d = \sqrt{\frac{M_u}{\varnothing_f b_w \rho f_y \left[1 - \frac{0.59\rho f_y}{f'_c}\right]}} = \sqrt{\frac{7094000}{0.90(300)(0.016)(4200)\left[1 - \frac{0.59(0.016)(4200)}{210}\right]}}$$

$$d = 21.95 \text{ cm}$$

Then, we are proposed the final dimensions of footing after performing different proposals:

$$d = 42 \text{ cm}; \quad r_1 = 8 \text{ cm}; \quad t = 50 \text{ cm}$$

Step 8: Shear force by flexure (shear force unidirectional) is:

$$\varnothing_v V_{cf} = 0.53\varnothing_v \sqrt{f'_c} b_w d = 0.53(0.85)\sqrt{210}(300)(42) = 82257.40 \text{ kg}$$

$$V_f = \frac{P(h - c_1 - 2d)}{2h} + \frac{3M_x(h^2 - c_1^2 - 4c_1 d - 4d^2)}{2h^3}$$
$$= \frac{164[3.55 - 0.40 - 2(0.42)]}{2(3.55)} + \frac{3(32.8)[(3.55)^2 - (0.40)^2 - 4(0.40)(0.42) - 4(0.42)^2]}{2(3.55)^2}$$
$$= 63.53 \text{ ton}$$

$$V_f \leq \varnothing_v V_{cf}, cumple$$

Step 9: Shear force by penetration (shear force bidirectional) is:

$$\varnothing_v V_{cp} = 0.53\varnothing_v \left(1 + \frac{2}{\beta_c}\right)\sqrt{f_c'} b_0 d$$
$$= 0.53(0.85)\left(1 + \frac{2}{1}\right)\sqrt{210}[4(40 + 42)](42) = 269804.28 \text{ kg}$$

$$\varnothing_v V_{cp} = 0.27\varnothing_v \left(\frac{\alpha_s d}{b_0} + 2\right)\sqrt{f_c'} b_0 d$$
$$= 0.27(0.85)\left[\frac{(40)(42)}{4(40 + 42)} + 2\right]\sqrt{210}[4(40 + 42)](42) = 326298.04 \text{ kg}$$

$$\varnothing_v V_{cp} = \varnothing_v \sqrt{f_c'} b_0 d = (0.85)\sqrt{210}[4(40 + 42)](42) = 169688.23 \text{ kg}$$

$$V_p = P - \frac{P(c_1 + d)(c_2 + d)}{bh} = 164 - \frac{164(0.4 + 0.42)(0.4 + 0.42)}{(3.00)(3.55)} = 153.65 \text{ ton}$$

$$V_p \leq \varnothing_v V_{cp}, cumple$$

Step 10: The reinforcement steel is:
* The parallel reinforcement steel to the direction of the long side of the footing is:

$$w = \frac{0.85 f_c'}{f_y} = \frac{0.85(210)}{4200} = 0.0425$$

$$A_{sp} = w b_w d - \sqrt{(w b_w d)^2 - \frac{2 M_u w b_w}{\varnothing_f f_y}}$$
$$= 0.0425(300)(42) - \sqrt{[0.0425(300)(42)]^2 - \frac{2(7094000)(0.0425)(300)}{0.90(4200)}}$$
$$= 46.72 \text{ cm}^2$$

$$\rho_{\min} = \frac{14}{f_y} = \frac{14}{4200} = 0.00333; \quad A_{s\min} = \rho_{\min} b_w d = 0.00333(300)(42) = 41.58 \text{ cm}^2$$

Thus, main reinforcement steel is proposed "A_{sp}".
Rod "3/4" diameter is used:

$$s = \frac{b_w a_s}{A_s} = \frac{300(2.85)}{46.72} = 18.30 \text{ cm} \approx 18 \text{ cm}$$

* The parallel reinforcement steel to the direction of the short side of the footing is:

The around maximum moment of axis b'-b' acting on the footing according to Figure 3 is presented:

$$
\begin{aligned}
M_{b'\text{-}b'} &= \left[\frac{P(b-c_2)}{2b} + \frac{3M_y(b^2-c_2^2)}{2b^3}\right]\left[\frac{Pb^2(b^2-c_2^2)+4M_y(b^3-c_2^3)}{4Pb^2(b-c_2)+12M_y(b^2-c_2^2)} - \frac{c_2}{2}\right] \\
&= \left[\frac{164(3.00-0.40)}{2(3.00)} + \frac{3(27.2)[(3.00)^2-(0.40)^2]}{2(3.00)^3}\right] \\
&\quad \left[\frac{164(3.00)^2[(3.00)^2-(0.40)^2]+4(27.2)[(3.00)^3-(0.40)^3]}{4(164)(3.00)^2(3.00-0.40)+12(27.2)[(3.00)^2-(0.40)^2]} - \frac{0.40}{2}\right] \\
&= 57.09 \text{ ton-m}
\end{aligned}
$$

where $M_{b'-b'} = M_u$

$$
\begin{aligned}
A_s &= wb_wd - \sqrt{(wb_wd)^2 - \frac{2M_uwb_w}{\varnothing_f f_y}} \\
&= 0.0425(355)(42) - \sqrt{[0.0425(355)(42)]^2 - \frac{2(5709000)(0.0425)(355)}{0.90(4200)}} = 37.04 \text{ cm}^2
\end{aligned}
$$

$$A_{s\,\min} = \rho_{\min} b_w d = 0.00333(355)(42) = 49.65 \text{ cm}^2$$

therefore, minimum steel is proposed "$A_{s\,\min}$".

The reinforcing steel in the central band is:

$$\gamma_s = \frac{2}{\beta+1} = \frac{2}{1.183+1} = 0.916; \quad \gamma_s A_s = 0.916(49.65) = 45.48 \text{ cm}^2$$

where $\beta = 355/300 = 1.183$.

Rod "3/4" diameter is used:

$$s = \frac{b_w a_s}{\gamma_s A_s} = \frac{300(2.85)}{45.48} = 18.80 \text{ cm} \approx 18 \text{ cm}$$

The reinforcing steel in the lateral bands is:

$$(1-\gamma_s)A_s = (1-0.916)(49.65) = 4.17 \text{ cm}^2$$

Rod "3/4" diameter is used:

$$s = \frac{b_w a_s}{(1-\gamma_s)A_s} = \frac{(355-300)(2.85)}{4.17} = 37.59 \text{ cm} \approx 37 \text{ cm}$$

Step 11: This is the same as the traditional model.

4. **Results and Discussion.**

Table 1 shows the differences between the two models and Figure 8 presents the concrete dimensions and reinforcement steel of the two footings.

In all cases the proposed model is less with respect to the traditional model.

Effects that govern the design for isolated footings are: moments, shear force by flexure and shear force by penetration.

a) The maximum moments acting on the footing in the two directions are increased in a 35% the traditional model with respect to proposed model.

b) The shear force by flexure acting on the footing has an increase of 9% in traditional model with respect to the proposed model.

c) According to shear force by penetration acting on the footing, in this concept is presented the greater increase that is of 62% in traditional model with respect to the proposed model.

Table 1. Comparison of results

Concept	Traditional model TM	Proposed model PM	TM/PM
Maximum moment acting $M_{a'\text{-}a'}$ (ton-m)	95.67	70.94	1.35
Maximum moment acting $M_{b'\text{-}b'}$ (ton-m)	77.12	57.09	1.35
More economic effective cant d (cm)	57	42	1.36
Coating r_1 (cm)	8	8	1.00
Total thickness t (cm)	65	50	1.30
Volume of concrete (m^3)	6.92	5.32	1.30
Shear force by flexure acting V_f (ton)	71.52	65.53	1.09
Shear force by penetration acting V_p (ton)	249.62	153.65	1.62
Parallel reinforcement steel in direction of the long side of the footing A_s (cm^2)	56.43	46.72	1.21
Parallel reinforcement steel in direction of the short side of the footing A_s (cm^2)	67.38	49.65	1.36

Materials used for the construction of an isolated footing are: concrete and reinforcement steel.

a) In terms of concrete it has a saving of 30% in the proposed model with respect to the traditional model.

b) For reinforcement steel in the parallel direction to the long side of the footing it has a saving of 21% in the proposed model with respect to the traditional model and in the parallel direction to the short side of the footing it has a saving of 36 % in the proposed model with respect to the traditional model.

5. Conclusions.

The results of the problem considered, through the application of two different models, are possible to conclude as the following.

- According to the maximum moments acting on the isolated footing, it is observed that it is greater in traditional model with respect to the proposed model. This is a logical situation, because in traditional model, the design pressure is the same in all the contact area of the footing on soil, being this the maximum pressure that is presented in said structural member, but the pressure in the proposed model is reduced, which has a linear variation along all its contact area that goes from a maximum pressure up the minimum pressure, which is as it presents the real pressure, consequently the effective cant is less; therefore, the thickness of the footing is more slender.

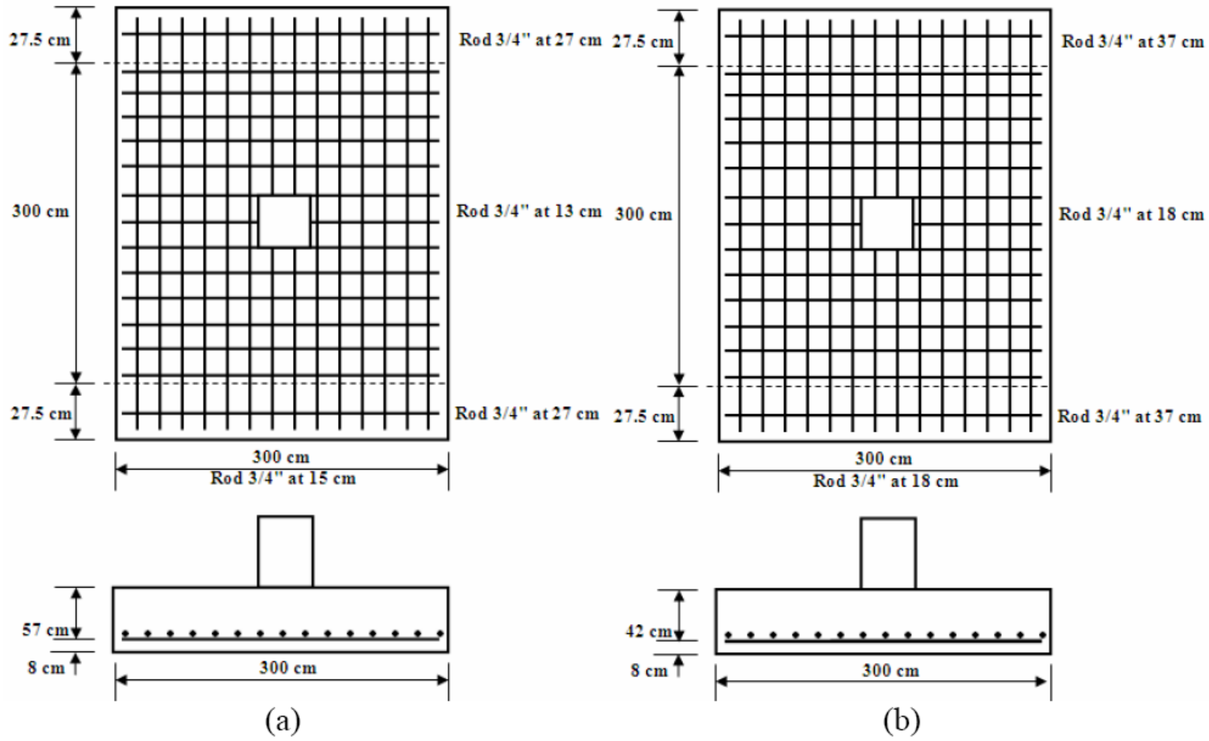

FIGURE 8. Isolated footing in plan and elevation: (a) traditional model, (b) proposed model

- In terms of the dimensions in the isolated footing it is shown that long side "h" and the short side "b" are equal in the two models, but the thickness of the footing "t" is different, being less than the proposed model with respect to the traditional model.
- With respect to parallel reinforcement steel in direction of the long side of the footing, the traditional model is greater with respect to the proposed model, because the design in the traditional model is governed by the minimum steel, and because it presents a much greater thickness and in the proposed model is governed by the design is the maximum moment acting on the isolated footing.
- We examine the parallel reinforcement steel in direction of the short side of the footing, the traditional model is greater with respect to the proposed model, in both models the design is governed by minimum steel.

This means that it can have great savings in terms of materials used (reinforcing steel and concrete) for the fabrication of footings isolated under conditions mentioned above. Since the principle in civil engineering, in terms of structural conditions is safe and economical, and the latter is not met in traditional model.

Therefore, the practice of using the traditional model is not a recommended solution, because are very exceeded the materials in some cases, with regard to the design of these structural members.

Then, we propose using the model developed in this paper for the structural design of isolated footings subject to axial load and moment in two directions (bidirectional flexure), also, it can be applied to the other cases: 1) The footings subject to concentric axial load; 2) The footings subject to axial load and moment in one direction (unidirectional flexure). Moreover, the proposed model is the most appropriate, since it is more economic and also is adjusted to real conditions.

The mathematical model developed in this paper applies only to rigid soils that meet expression of the bidirectional flexion, i.e., the variation of pressure is linear. The suggestions for future research, when presented by other types of soil; for example, in cohesive soils and granular soils, the pressures diagram is not linear and should be treated differently.

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