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BANK SPREAD BEHAVIOR AND DEFAULT RISK IN RESPONSE TO CAPITAL REGULATION IN MERTON, BLACK AND BLACK-MERTON STRUCTURAL FRAMEWORKS

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Abstract. *This paper examines the relationship between capital regulation and default risk prediction with the bank interest margin determination under the standard Merton-type and Black-type structural models. The former can be identified as a narrow banking framework while the latter can be identified as a synergy banking framework. In addition, we also introduce a Black-Merton-type structural model in a non-exclusive, narrow-synergy framework. We compare the three structural models for their default prediction capabilities under capital regulation. We find a consistent result from these three models: higher capital requirements lead to lower default risks in the bank's equity return. The ranking of the significance effect on default risk is sorted in the following order: Merton-type, Black-Merton-type and Black-type one. This analysis provides important strategic and policy implications for bank managers and regulators.*
Keywords: Bank spread behavior, Default risk, Capital regulation, Call option, Cap option

1. **Introduction.** Motivated by the ongoing literature concerning the impact of capital regulation on bank profits and risks, and the fact that the research thus far has primarily focused on either narrowing banking or synergy banking, we assess the extent to which risk-based capital requirements such as the Basel III system, affect bank profits and default risks under a non-exclusive narrow-synergy banking proposal. The theoretical banking literature is sharply divided on the effects of capital requirements on bank behavior and, hence, on the risks faced by individual institutions and the banking system as a whole. Some recent academic works indicates that capital requirements unambiguously contribute to various possible measures of bank stability [3-5]. In contrast, other works conclude that, if anything, capital requirements make banks riskier institutions than they would have been in the absence of such requirements [6-8].

Our paper makes several important contributions to the literature due to the following extensions in methodology and scope. First, on the methodological side, we propose a framework of bank equity valuation based on a call-cap option model instead of the commonly-known call or cap approach. The standard contingent claim approach to corporate security valuation views bank equity as a call option on the assets of the banking firm with the strike price of the book value of the banks' liabilities [1]. An alternative contingent claims approach views bank equity as a cap option on the book-value equity return of the banking firm with the strike price of the Libor rate [2]. The former "Merton-type" approach is motivated based on a narrow banking proposal, which effectively calls for the breaking-up of the bank into separate lending (the underlying assets in the call) and deposit-taking (the strike price). The latter "Black-type" approach is motivated based on a synergy banking proposal, which effectively calls for the integration of the lending and deposit-taking activities (the underlying equity in the cap) and the opportunity cost in terms of the Libor rate (the strike price). A substantial number of analyses have been devoted to understanding the circumstances behind the narrow-banking or the synergy-banking operations management. Much has been learned from this work but not addressed in an alternative aspect: a bank may carry out both functions under a non-exclusive, narrow-synergy banking structure since the valuation of bank equity and risk are in general under the constraints of balance sheets. Banking firm theory, however, implies that constraints of balance sheets in banks' operation management must be satisfied by Merton-type as well as Black-type prices. In this case, the equity of a bank creates the need for model equity as a cap option on the market value rather than the book value of the banks' equity, while the strike price of the cap is referred to as the Libor rate. Such approach is known as "Black-Merton-type" valuation.

Regarding scope, to the best of our knowledge, we are the first researchers to analyze bank interest margin determination under capital regulation in the Black-Merton-type valuation. The broader contingent claims approach has been found with a natural application of commercial bank behavior in response to capital regulation. Commercial banks are institutions that engage in two distinct types of activities, namely the balance sheet, lending and deposit-taking, respectively. Lending involves acquiring costly information on opaque borrowers and credit extension based on such information. Deposit-taking involves issuing claims that are riskless and demandable; in other words, claims can be recognized for a fixed value at any time [9,10]. The bank interest margin or, the spread between the loan rate and the deposit rate is one of the principal elements of bank net cash flows and earnings [11,12]. The bank interest margin conveys vital information on the efficiency of financial operation management related to narrowing banking and/or synergy banking [13]. It is interesting to study and compare contingent claims under the Merton-type, Black-type and Black-Merton type frameworks in order to understand the effect of capital regulation on bank spread behavior in return to the retail banking [14]. The purpose of this paper is to address this interest.

In light of previous works, the purpose of this paper is to use the various structural approaches to examine how bank interest margin is determined when the banks' alternative objectives admit the Merton-type, the Black-type and the Black-Merton-type equity representations under capital regulation. Bank claims are evaluated with alternative options under three frameworks, thus providing more suitable, alternative views of bank equity and debts while further exhibiting a strong link to the alternatives inducing probability of default. In this paper, we compare structural models, including Merton-type, Black-type and Black-Merton-type based on their default prediction capability with bank interest determination under capital regulation. It is consistently found that higher capital requirements lead to lower loan volume at an increased margin and further lower default risks of the banks' equity return in Merton-type, Black-type and Black-Merton-type of structural frameworks. Our results are considerably supported by the empirical evidence of Kashyap et al. [3], Slovik and Cournede [4], and Cosimano and Hakura [5]. A further contribution of the paper shows that the ranking of the significant degree of capital regulation effects on the default risk includes the Merton-type, Black-Merton-type and Black-type. This ranking explains a real synergy or narrow-synergy banking is a forced switch narrowing banking that could lead to large inefficiencies. The effect of capital regulation on default prediction which ignores the narrow banking functions in the synergy banking lead to overestimation. In general, we conclude that capital requirements contribute to various possible measures of bank stability.

One immediate application of this paper is to evaluate the plethora of bank equity valuation arrangements under the capital regulation proposed as alternatives for future lending activities which are related to the issues of efficiency and effectiveness. One tends to be sympathetic to a narrow banking proposal, which effectively calls for the breaking up of banks into separate lending and deposit-taking operations that would resemble finance companies and mutual funds, respectively. Under this view, it may be appropriate to define bank equity as the Merton-type value of the call option effectively purchased by the shareholders of the bank. One tends to be sympathetic to a synergy banking proposal, which effectively calls for the integration of the two activities that would resemble credit or liquidity providers. Under this view, it may be appropriate to define bank equity as the Black-type value of the cap option effectively purchased by the shareholders of the bank. Finally, one tends to be sympathetic to a narrow-synergy banking proposal, which effectively calls for the breaking up as well as the integration of the two activities that would resemble conglomerate banks under the Gramm-Leach-Bliley Act. Under this view, it may be appropriate to define bank equity as the Black-Merton-type value of the cap-call option effectively purchased by the shareholders of the bank. Across the types of financial institution, we could observe bank equity valuation arrangements to avoid substantial inefficiencies. Our argument is applicable to bank analysts supervising agencies, and policy makers.

The rest of the paper is organized as follows. Section 2 briefly reviews the related literature. Section 3 outlines the theoretical foundations of the three alternative contingent claims approaches to bank equity valuation and further to default risks. Section 4 derives equilibrium solutions and comparative static results. Section 5 presents numerical analyses to explain and compare the possible comparative static results. We draw our conclusions in the last section.

2. **Background.** There is an approach in pricing credit risk, the structural approach pioneered by Merton [1] which regards default and recovery endogenously as a result of a bad operation of the firm. The principal advantage of this approach is that equity prices carry useful credit information which can be used to price credit derivatives. Vassalou and Xing [15] calculate default risk using Merton's [1] model and find the size and the book-to-market of a firm exhibiting a strong link to the default probability. This paper attempts to fill the void in the literature by applying the received theory of Merton [1] and then Vassalou and Xing [15] to the case of a regulated bank.

In Merton's [1] model, the equity of a firm is viewed as a call option on the firm's assets. It is assumed that asset market is perfectly competitive so that quantity-setting is the relevant behavioral model in the market. This assumption is not applicable to a loan market since such a market is virtually always highly concentrated whereas a bank sets a loan rate [11,16]. This paper, first, allows the inclusion of more realistic market along with the more appropriate behavioral modes of loan rate-setting in the contingent claim approach to bank equity valuation under the capital regulation.

The strike price of the call option on the bank's assets is the book value of its liabilities. This approach can be motivated based on a narrow banking proposal, which effectively calls for the breaking-up of the bank into separate lending and deposit-taking operation management, according to Kobayakawa and Nakamura [9] and Bossone [17]. Alternatively, Kashyap et al. [10] suggested that there is a synergy effect between the liquidity needs in the lending and deposit-taking activities.[1] In the case of a real synergy, a forced switch to narrow banking could lead to large inefficiencies. Under this view, Tsai and Hung [18] indicated that a contingent claim approach to corporate security valuation views bank equity as a cap option [2] on the underlying equity value of the bank, with its expected investment opportunity cost of the strike price of the Libor rate. This paper secondly argues that there may be some significant synergies between the lending and deposit-taking while further examining the optimal loan rate (and thus the optimal bank interest margin) under the capital regulation in the cap option framework.

Both the Merton-type and the Black-type contingent claim approaches in abovementioned showed a natural application in the bank interest margin determination under the capital regulation. These models imply that both approaches are used to value bank equity specifically. Banking firm theory, however, implies that both approaches are subject to a balance sheet constraint of the bank. In view of this, this paper develops a Black-Merton-type model of bank behavior that integrates the narrow banking with the synergy banking under capital regulation.

The three types of works have recognized that bank equity is viewed as a path-independent option that its payoff depends on the underlying asset value or equity value only at maturity.[2] This fundamental concept of the path-independent structural approach allows the application of Vassalou and Xing [15] assessing the effect of default risk on equity returns under capital regulation. We select these particular three models that cover distinctly different assumptions so we can study how and why certain models can predict default concerning different applications. Chen et al. [19] argued that the structural models do not price corporate securities well; however, they are quite effective in predicting defaults. This paper, in fourth, aims to compare those three types of structural models for their default prediction capability through focusing on the bank interest margin determination under the capital regulation.

3. **Three Alternative Objectives.**

The basics of the modeling approach used in this paper or, the industrial economics approach to banking, can be found in Freixas and Rochet [20]. We applied the methodology in the literature and followed its lead with respect to market structure, i.e., the consideration of a monopolistic bank.[3] This enables us to include market power without having to deal with strategic interaction in a banking oligopoly. Our analysis includes capital regulation and banking lending, accounting for their interaction and joint impact on default prediction.

Consider a bank that makes decisions on one-year period with 360 days and $t \in [0, 360]$. At $t = 1$, the bank has the following balance sheet:

$$L + B = D + K \tag{1}$$

where $L > 0$ is the volume of loans, $B > 0$ is amount of liquid assets, $D > 0$ is the quantity of deposits, and $K > 0$ is the stock of equity capital.

[1] This paper focuses on an alternative of synergy banking proposal. Recent related literature includes, i.e., structural pricing model based on dynamic investment strategy [21], loan pricing model based on recovery rate distribution [22], and real options analysis based on fuzzy random variables [23].

[2] We remain silent on the case of path dependence. See Episcopos [24] for bank capital regulation and Tsai et al. [25] for bank interest margin determination in a path-dependent, barrier option framework.

[3] i.e., Wahl and Broll [26], Wong [11] and Pausch and Welzel [16].

The bank enjoys market power in the loan market [16]. L can be interpreted as the total number of homogenous loans. The decision on the loans is made via the setting of loan rate R_L at $t = 1$. The bank faces a loan demand function $L(R_L)$ with $\partial L/\partial R_L < 0$ and $\partial^2 L/\partial R_L^2 < 0$. The demand for the loans is assumed to be a concave function. Loans are risky in that they are subject to non-performance. In addition to loans, the bank can also hold an amount B of liquid assets, i.e., bonds, on its balance sheet between dates 1 and 360. These assets earn the security-market interest rate of R. The bank's deposits are insured by a government-funded deposit insurance scheme. The supply of deposits is perfectly elastic at the fixed deposit rate, $R_D > 0$ [11]. K needs to satisfy the following capital adequacy requirements by regulation, $K \geq qD$, whereas the required capital-to-deposits ratio q is assumed to be an increasing function of L specified as $\partial q/\partial L = q' > 0$. q' explicitly captures the operational risk as shown in the Basel Committee on Banking Supervision [8]. When the capital is binding, Equation (1) can be further expressed as $L + B = K(1/q + 1)$.[4]

3.1. **Merton-type objective.** We apply Merton [1] and define the equity of a bank viewed as a call option on the bank's assets. The strike price of the call option is the book value of the bank's liabilities. Based on Equation (1), the capital structure of the bank includes both the equity and debt. The market value of the bank's underlying assets follows a geometric Brownian motion of the form:

$$dV = \mu V dt + \sigma V dW \tag{2}$$

where

$$V = (1 + R_L)L$$

and V is the value of the bank's loan repayments at $t = 360$, with an instantaneous drift μ, and an instantaneous volatility σ. A standard Wiener process is known as W.

Z is denoted as the book value of the net debt at $t = 1$, which has maturity equal to $t = 360$. The net debt is the difference value between the payments to depositors and the repayments from the liquid assets held by the bank. As noted earlier, Z plays the role of the strike price of the call option, since the market value of the bank's equity can be thought as a call option on V with $t = 1$ to expiration equal to $t = 360$. The market value of equity, S, will be given by the Merton [1] formula for call options:

$$S = VN(d_1) - Ze^{-\delta}N(d_2) \tag{3}$$

where

$$Z = \frac{(1 + R_D)K}{q} - (1 + R)\left[K\left(\frac{1}{q} + 1\right) - L\right]$$

$$d_1 = \frac{1}{\sigma}\left(\ln\frac{V}{Z} + \delta + \frac{\sigma^2}{2}\right), \quad d_2 = d_1 - \sigma$$

where $\delta = R - R_D$ is the risk-free spread and $N(\cdot)$ is the cumulative density function of the standard normal distribution. Under this Merton-type [1] contingent claim approach to corporate security valuation, it is inevitable to be sympathetic to a narrow banking proposal, which effectively calls for the breaking-up of S into $VN(d_1)$ and $Ze^{-\delta}N(d_2)$ in Equation (3).

The structural approach in pricing credit risk pioneered by Merton [1] regards default endogenously as a result of a bad operation of the bank. Our approach in calculating default risk measures using Equation (3) is very similar to the one used by Vassalou and Xing [15]. The default risk in the bank's equity return is the default probability that V is

[4]This is the case as long as R is sufficiently higher than R_D.

less than Z. We defined the distance to default d_3 in order to capture the bad operation of the bank as follows:

$$d_3 = \frac{1}{\sigma}\left(\ln\frac{V}{Z} + \mu - \frac{\sigma^2}{2}\right) \tag{4}$$

Default occurs when the ratio of V to Z is less than 1, or when its log is negative. The d_3 tells us how many standard deviations the log of this ratio needs to deviate from its mean before the occurrence of default. Note that although S in Equation (3) does not depend on μ, d_3 on the other hand does. This is because d_3 depends on the future value of V which is given in d_1 and d_2. In this case, the probability of default will be given by:

$$P = N(-d_3) = 1 - N(d_3) \tag{5}$$

3.2. **Black-type objective.** A cap is designed to provide insurance against the interest rate on a floating rate rising from a certain level known as the opportunity cost of the cap, usually denoted as the Libor rate [2]. We apply Black [2] and define the equity of a bank viewed as a cap option on the bank's equity returns, subject to Equation (1). Specifically, the cap option gives to the bank's equity holders a series of European call options or caplets on the Libor rate, whereas each caplet has the same strike price as the others, but a different expiration date. The expiration dates for the caplets are on the same cycle as the frequency of the underlying Libor rate. Specifically, let $S_b(1, 360)$ denote the value at $t = 1$ with a discount bond maturing at $t = 360$, $F(1, \tau, 360)$, whereas $1 < \tau < 360$, denoting the value at $t = 1$ for the Libor forward rate applicable to the period from $t = \tau$ to $t = 360$. Let $a = 360 - \tau$ be the actual number of days during the period from $t = \tau$ to $t = 360$, and let G be the strike price. Applying the Black [2] model to this forward rate will result in the following closed-form expression for the $t = 1$ value of a caplet with expiration date, $t = 360$:

$$S_b(1, \tau, 360) = S_b(1, 360)\frac{360 - \tau}{360}[F(1, \tau, 360)N(b_1) - GN(b_2)] \tag{6}$$

where

$$\begin{aligned}
&F(1, \tau, 360) = \frac{360}{360 - \tau}\left(\frac{S_b(1, \tau)}{S_b(1, 360)} - 1\right)\\
&S_b(1, 360) = (V - Z)e^{-\rho}, \quad \rho = R_L + R - R_D\\
&S_b(1, \tau) = (V - Z)e^{-\tau\rho/360}\\
&b_1 = \frac{1}{\sqrt{\sigma^2\tau}}\left(\ln\frac{F(1, \tau, 360)}{G} + \frac{\sqrt{\sigma^2\tau}}{2}\right), \quad b_2 = b_1 - \sqrt{\sigma^2\tau}
\end{aligned}$$

and where σ is the volatility of changes in the logarithm of the forward rate. Under this Black-type [2] contingent claim approach to corporate security valuation, it is inevitable to be sympathetic to a synergy banking proposal, which effectively calls for the integration of V and Z denoted as $S_b(1, 360)$ and $S_b(1, \tau)$ in Equation (6).

We further apply Vassalou and Xing [15] to define the default risk in the bank's equity return by using information as shown in Equation (6). Again, equity price carries useful credit information. The default probability is governed only by the underlying ratio of $F(1, \tau, 360)$ and its strike price of G, since the term $S_b(1, 360)$ is expressed as a book value. Specifically, the default probability occurs when $F(1, \tau, 360)$ is less than G. Under the circumstances, we define the probability of the distance to default b_3 to capture the probability of the bad operation of the bank as follows:

$$P_b = N(-b_3) = 1 - N(b_3) \tag{7}$$

where

$$b_3 = \frac{1}{\sqrt{\sigma^2\tau}}\left(\ln\frac{F(1,\ \tau,\ 360)}{G} - \frac{\sqrt{\sigma^2\tau}}{2}\right)$$

3.3. **Black-Merton-type objective.** The Merton-type [1] and the Black-type [2] framework are generally used as separate contingent claim approaches for corporate security valuation. Banking firm theory, however, implies that the constraint of balance sheet must satisfy Merton-type and Black-type prices. The constraint of balance sheet in Equation (1) captures the bank's operations management in lending, since the total assets on the left-hand side are financed by deposits and equity capital on the right-hand side. In view of this, the lending function of the bank creates an alternative need for model equity as a Black-Merton-type (a cap-call-type) option. Specifically, the Black-type [2] equity valuation is based on the book value of $S_b(1,\ 360)$ multiplied by the market value of the caplet of $(a/360)\ \max\,[0,\ F(1,\ \tau,\ 360) - G]$. We replace this book-value term by S as defined in Equation (3) and model equity as a Black-Merton-type form:

$$S_m(1,\ \tau,\ 360) = S\frac{360-\tau}{360}[F(1,\ \tau,\ 360)N(b_1) - GN(b_2)] \tag{8}$$

The form of the Black-Merton-type contingent claims approach for corporate security valuation is multiplicative, whereas mentioned earlier, the term $S = VN(d_1) - Ze^{-\delta}N(d_2)$ implies a monetary transmission mechanism of narrow banking and the term $[F(1,\ \tau,\ 360)N(b_1) - GN(b_2)]$ implies that of synergy banking. Using the information from Equation (8) above, we define an additive default probability as follows:

$$P_m = P + P_b \tag{9}$$

It is interesting to compare default risk assessments based on the contingent claims in the Merton-type, Black-type and Black-Merton-type frameworks. First, the conceptual transition from the Merton-type to the Black-Merton-type shows that the default probability is increased additionally by P_b. This makes intuitive sense because, in the Black-Merton-type framework, the bank has an additional option to consider the synergy banking caused by the balance-sheet operation management. The Merton-type default probability becomes a special case if this consideration is ignored. Second, in a conceptual transition from the Black-type to the Black-Merton-type, it can be shown that the default probability is increased additionally by P. This is because, in the Black-Merton-type framework, the bank has an additional option to consider the narrow banking operations management. The Black-type default probability becomes a special case if this consideration is ignored.

4. Solutions and Comparative Static Results.

4.1. **Merton-type equilibrium and result.** Partially differentiating Equation (3) with respect to R_L, the first-order condition is given by:

$$\frac{\partial S}{\partial R_L} = \frac{\partial V}{\partial R_L}N(d_1) + V\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial R_L} - \frac{\partial Z}{\partial R_L}e^{-\delta}N(d_2) - Ze^{-\delta}\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial R_L} = 0 \tag{10}$$

where

$$\frac{\partial V}{\partial R_L} = L + (1+R_L)\frac{\partial L}{\partial R_L} < 0, \quad \frac{\partial Z}{\partial R_L} = \left[\frac{(R-R_D)Kq'}{q^2} + (1+R)\right]\frac{\partial L}{\partial R_L} < 0$$

$$V\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial R_L} = Ze^{-\delta}\frac{\partial N(d_2)}{\partial d_2}\frac{\partial d_2}{\partial R_L}$$

A sufficient condition for an optimum of Equation (10) is shown in $\partial^2 S/\partial R_L^2 < 0$. The sign of the term $\partial V/\partial R_L$ is negative since the bank operates on the elastic portion of its loan demand curve. The sign of the term $\partial Z/\partial R_L$ is negative since the risk-free spread is positive. The optimal loan rate is set as the equity return maximization in Equation (3) where both risk-adjusted marginal values are equal. We can further substitute the optimal loan rate to obtain the default risk in Equation (5) by staying on the maximization optimization. The optimal bank interest margin is given by the difference between the optimal loan rate and the fixed deposit rate. Since the deposit rate is not a choice variable, the examination of the impact of capital regulation on the optimal bank interest margin is tantamount to examining that on the optimal loan rate.

Consider next the impact on the optimal bank interest margin from changes in the capital-to-deposits ratio. The implicit differentiation of Equation (10) shown in the form of Merton-type valuation with respect to q, yields as follows:

$$\left.\frac{\partial R_L}{\partial q}\right|_{Merton} = -\frac{\partial^2 S}{\partial R_L \partial q} \Big/ \frac{\partial^2 S}{\partial R_L^2} \tag{11}$$

where

$$\frac{\partial^2 S}{\partial R_L \partial q} = \frac{\partial^2 V}{\partial R_L \partial q} N(d_1) - \frac{\partial^2 Z}{\partial R_L \partial q} e^{-\delta} N(d_2) + \frac{\partial V}{\partial R_L}\left(1 - \frac{VN(d_1)}{Ze^{-\delta}N(d_2)}\right)\frac{\partial N(d_1)}{\partial d_1}\frac{\partial d_1}{\partial q}$$

$$\frac{\partial^2 V}{\partial R_L \partial q} = 0, \quad \frac{\partial^2 Z}{\partial R_L \partial q} = -\frac{2(R - R_D)Kq'}{q^3}\frac{\partial L}{\partial R_L}$$

$$\frac{\partial d_2}{\partial q} = \frac{\partial d_1}{\partial q} = -\frac{1}{\sigma Z}\frac{\partial Z}{\partial q}, \quad \frac{\partial Z}{\partial q} = \frac{(R - R_D)K}{q^2}$$

The first two terms on the right-hand side of $\partial^2 S/\partial R_L \partial q$ can be interpreted as the mean profit effect on $\partial S/\partial R_L$ form a change in q, while the third term can be interpreted as the variance or risk effect.

We further consider the impact on the default risk in the bank's equity return changes in terms of the capital-to-deposits ratio. The differentiation of Equation (5) evaluated at the optimal loan rate with respect to q yields:

$$\frac{dP}{dq} = \frac{\partial P}{\partial q} + \left.\frac{\partial P}{\partial R_L}\frac{\partial R_L}{\partial q}\right|_{Merton} \tag{12}$$

where

$$\frac{\partial P}{\partial q} = -\frac{\partial N(d_3)}{\partial d_3}\frac{\partial d_3}{\partial q}, \quad \frac{\partial P}{\partial R_L} = -\frac{\partial N(d_3)}{\partial d_3}\frac{\partial d_3}{\partial R_L}, \quad \frac{\partial d_3}{\partial R_L} = \frac{1}{\sigma R_L}\left(\frac{R_L}{V}\frac{\partial V}{\partial R_L} - \frac{R_L}{Z}\frac{\partial Z}{\partial R_L}\right)$$

In Equation (12), the first term on the right-hand side is identified as the direct effect while the second term is identified as the indirect effect. The direct effect captures the changes in the default risk due to the increase in the capital-to-deposits ratio, holding the optimal loan rate constant. The indirect effect arises because an increase in q changes the default risk by $L(R_L)$ in every possible state.

4.2. **Black-type equilibrium and result.** For the partial differentiation of Equation (6) with respect to R_L, the first-order condition is given as:

$$\begin{aligned}\frac{\partial S_d(1,\ \tau,\ 360)}{\partial R_L} &= \frac{\partial S_b(1,\ 360)}{\partial R_L}\frac{360-\tau}{360}[F(1,\ \tau,\ 360)N(b_1) - GN(b_2)] \\ &\quad + S_b(1,\ 360)\frac{360-\tau}{360}\left[\frac{\partial F(1,\ \tau,\ 360)}{\partial R_L}N(b_1)\right. \\ &\quad \left. + F(1,\ \tau,\ 360)\frac{\partial N(b_1)}{\partial b_1}\frac{\partial b_1}{\partial R_L} - G\frac{\partial N(b_2)}{\partial b_2}\frac{\partial b_2}{\partial R_L}\right] = 0\end{aligned} \tag{13}$$

where

$$\frac{\partial S_b(1,\ 360)}{\partial R_L} = \left[\left(\frac{\partial V}{\partial R_L} - \frac{\partial Z}{\partial R_L}\right) - (V-Z)\right]e^{-\rho}$$

$$\frac{\partial F(1,\ \tau,\ 360)}{\partial R_L} = \frac{360}{360-\tau}\frac{1}{S_b(1,\ 360)}\left(\frac{\partial S_b(1,\ \tau)}{\partial R_L} - \frac{S_b(1,\ \tau)}{S_b(1,\ 360)}\frac{\partial S_b(1,\ 360)}{\partial R_L}\right)$$

$$\frac{\partial S_b(1,\ \tau)}{\partial R_L} = \left[\left(\frac{\partial V}{\partial R_L} - \frac{\partial Z}{\partial R_L}\right) - (V-Z)\left(\frac{\tau}{360}\right)\right]e^{-\tau\rho/360}$$

$$F(1,\ \tau,\ 360)\frac{\partial N(b_1)}{\partial b_1}\frac{\partial b_1}{\partial R_L} = G\frac{\partial N(b_2)}{\partial b_2}\frac{\partial b_2}{\partial R_L}$$

A sufficient condition for an optimum of Equation (13) is $\partial^2 S_b(1,\ \tau,\ 360)/\partial R_L^2 < 0$. The first term on the right-hand side of Equation (13) can be interpreted as the marginal book-value equity associated with the caplet, while the second term as the marginal market-value caplet associated with the book-value equity. Equation (13) defines the optimal loan rate when both the marginal values are equal. We can further substitute the optimal loan rate to obtain the default risk in Equation (7) by staying on the optimization.

The implicit differentiation of the Black-type Equation (3) with respect to q yields:

$$\left.\frac{\partial R_L}{\partial q}\right|_{Black} = -\frac{\partial^2 S_b(1,\ \tau,\ 360)}{\partial R_L \partial q}\Bigg/\frac{\partial^2 S_b(1,\ \tau,\ 360)}{\partial R_L^2} \tag{14}$$

where

$$\begin{aligned}\frac{\partial^2 S_b(1,\ \tau,\ 360)}{\partial R_L \partial q} &= \frac{360-\tau}{360}\left\{\frac{\partial^2 S_b(1,\ 360)}{\partial R_L \partial q}[F(1,\ \tau,\ 360)N(b_1) - GN(b_2)]\right. \\ &\quad + \frac{\partial S_b(1,\ 360)}{\partial R_L}\frac{\partial F(1,\ \tau,\ 360)}{\partial q}N(b_1) + \frac{\partial S_b(1,\ 360)}{\partial q}\frac{\partial F(1,\ \tau,\ 360)}{\partial R_L}N(b_1) \\ &\quad \left. + S_b(1,\ 360)\left[\frac{\partial^2 F(1,\ \tau,\ 360)}{\partial R_L \partial q}N(b_1) + \frac{\partial F(1,\ \tau,\ 360)}{\partial R_L}\frac{\partial N(b_1)}{\partial b_1}\frac{\partial b_1}{\partial q}\right]\right\}\end{aligned}$$

$$\frac{\partial^2 S_b(1,\ 360)}{\partial R_L \partial q} = \left(-\frac{\partial^2 Z}{\partial R_L \partial q} + \frac{\partial Z}{\partial q}\right)e^{-\rho}$$

$$\frac{\partial F(1,\ \tau,\ 360)}{\partial q} = \frac{360}{360-\tau}\frac{1}{S_b(1,\ 360)}\left(\frac{\partial S_b(1,\ \tau)}{\partial q} - \frac{S_b(1,\ \tau)}{S_b(1,\ 360)}\frac{\partial S_b(1,\ 360)}{\partial q}\right)$$

$$\frac{\partial S_b(1,\ \tau)}{\partial q} = -\frac{\partial Z}{\partial q}e^{-\tau\rho/360} < 0$$

$$\frac{\partial S_b(1,\ 360)}{\partial q} = -\frac{\partial Z}{\partial q}e^{-\rho} < 0$$

The sign of Equation (14) is governed by its numerator due to the validity of the second-order condition of $\partial^2 S_b(1,\ \tau,\ 360)/\partial R_L^2 < 0$. The first term on the right-hand side of the numerator explains the effect on the marginal book-value equity from a change in q, the second term captures the effect on the caplet, the third term demonstrates the effect on the marginal caplet, and the last term indicates the effect on the book-value equity.

The differentiation of Equation (7) evaluated at the optimal loan rate with respect to q yields:

$$\frac{dP_b}{dq} = \frac{\partial P_b}{\partial q} + \frac{\partial P_b}{\partial R_L}\frac{\partial R_L}{\partial q}\bigg|_{Black} \tag{15}$$

where

$$\frac{\partial P_b}{\partial q} = \frac{1}{\sqrt{\sigma^2\tau}}\frac{1}{F(1,\ \tau,\ 360)}\frac{\partial F(1,\ \tau,\ 360)}{\partial q}$$
$$\frac{\partial P_b}{\partial R_L} = \frac{1}{\sqrt{\sigma^2\tau}}\frac{1}{F(1,\ \tau,\ 360)}\frac{\partial F(1,\ \tau,\ 360)}{\partial R_L}$$

In Equation (15), the first term on the right-hand side is identified as the direct effect, while the second term is identified as the indirect effect through optimal loan rate adjustments.

4.3. **Black-Merton-type equilibrium and result.** For the partial differentiation of Equation (8) with respect to R_L, the first-order condition is given as:

$$\frac{\partial S_m(1,\ \tau,\ 360)}{\partial R_L} = \frac{360-\tau}{360}\left\{\frac{\partial S}{\partial R_L}[F(1,\ \tau,\ 360)N(b_1) - GN(b_2)] + S\frac{\partial F(1,\ \tau,\ 360)}{\partial R_L}\right\} = 0 \tag{16}$$

A sufficient condition for an optimum of Equation (16) is $\partial^2 S_m(1,\ \tau,\ 360)/\partial R_L^2 < 0$. The first term on the right-hand side of Equation (16) can be identified as the marginal market-value equity associated with the caplet, while the second term as the marginal market-value capital with the market-value equity. Equation (16) defines the optimal loan rate for the Black-Merton-equity maximizations whereas both the marginal values are equal. We can further substitute the optimal loan rate to obtain the default risk of Equation (9) by staying on the optimization.

The implicit differentiation of Equation (16) with respect to q yields:

$$\frac{\partial R_L}{\partial q}\bigg|_{Black\text{-}Merton} = -\frac{\partial^2 S_m(1,\ \tau,\ 360)}{\partial R_L \partial q}\bigg/\frac{\partial^2 S_m(1,\ \tau,\ 360)}{\partial R_L^2} \tag{17}$$

where

$$\frac{\partial^2 S_m(1,\ \tau,\ 360)}{\partial R_L \partial q} = \frac{360-\tau}{360}\left\{\frac{\partial^2 S}{\partial R_L \partial q}[F(1,\ \tau,\ 360)N(b_1) - GN(b_2)]\right.$$
$$\left. + \frac{\partial S}{\partial R_L}\frac{\partial F(1,\ \tau,\ 360)}{\partial q}N(b_1) + \frac{\partial S}{\partial q}\frac{\partial F(1,\ \tau,\ 360)}{\partial R_L} + S\frac{\partial^2 F(1,\ \tau,\ 360)}{\partial R_L \partial q}\right\}$$

The sign of Equation (17) is determined by its numerator due to the second-order condition of Equation (16) while the denominator of Equation (17) should be negative in sign. The interpretation of the numerator follows a similar argument as in the case of Equation (14) but with basis on the market value of the bank's equity rather than the book value of the bank's equity.

The differentiation of Equation (9) evaluated at the optimal loan rate with respect to q yields:

$$\frac{dP_m}{dq} = \left(\frac{\partial P}{\partial q} + \frac{\partial P_b}{\partial q}\right) + \left(\frac{\partial P}{\partial R_L} + \frac{\partial P_b}{\partial R_L}\right)\frac{\partial R_L}{\partial q}\bigg|_{Black\text{-}Merton} \tag{18}$$

The first term on the right-hand side of Equation (18) can be identified as the direct effect, while the second term can be identified as the indirect effect. The direct effect includes the impacts on the default risk from changes in the capital-to-deposits ratio based on both the Merton-type and Black-type approaches. The indirect effect includes the impact on the loan rate from changes in the capital-to-deposits ratio based on the Black-Merton-type approach as well as the impacts on the default from changes in the loan rate based on the Merton-type and Black-type approaches.

5. **Numerical Examples.** In the bank risk management under capital regulation, computing changes in the option value to small changes in constituent variables is essential for risks hedging. To work toward that end, we compute partial derivatives of the value functions of Merton-type, Black-type and Black-Merton-type options. The numerical examples are non-exhaustive and they provide intuition regarding the problems at hand. The added complexity of the structural options, in general, does not always lead to clear-cut results, but we can certainly speak of tendencies for reasonable parameter levels corresponding roughly to a bank with rather risky assets.

Unless otherwise indicated, the parameter values and assume to be $R = 3.00\%$, $R_D = 2.50\%$, $D = 200$, $\sigma = 0.20$ and $\mu = 0.10$. Let $(R_L\%, L)$ change from $(4.50, 200)$ to $(6.00, 179)$ due to the conditions of $\partial L/\partial R_L < 0$ and $\partial^2 L/\partial R_L^2 < 0$ in the model, and let q increase from 8.0% to 13.0%. Note that (i) the condition of $R_L > R_D$ indicates the positive interest margin as a proxy for the efficiency of financial intermediation [13], (ii) the constant value of $R_D = 2.50\%$ is not a choice variable of the bank [11], (iii) the condition of $R_L > R = 3.00\%$ implies asset substitution in the bank's earning-asset portfolio [10], (iv) the condition of $R > R_D$ indicates a possible binding case of the capital requirement constraint [27], and (v) the specification of capital adequacy requirement is consistent with the approach of the Basel [8]. The numerical parameters used above can be given with an intuitive interpretation roughly approaching to a real state of a hypothetical bank.

5.1. **Merton-type case.** In this subsection, we compute the Merton-type value of the bank's equity based on Equation (3). Using the information on Equation (10), we further calculate $\partial^2 S/\partial R_L \partial q$ and $\partial^2 S/\partial R_L^2$ to obtain the comparative static results of $\partial R_L/\partial q$ in Equation (11), which will be used to compute the indirect effect of Equation (12). The findings are summarized in Table 1.

In Table 1, we have the result of $S > 0$, $\partial^2 S/\partial R_L \partial q > 0$, $\partial^2 S/\partial R_L^2 < 0$ and $\partial R_L/\partial q > 0$. Note that $\partial^2 S/\partial R_L^2 < 0$ confirms the second-order condition. It is interesting that, as the capital-to-deposits ratio increases, the loan rate (the bank interest margin) is increased. Intuitively, as the bank is forced to increase its capital relative to its deposit level, it must now provide a return to a larger equity base. One way that the bank may attempt to augment its total returns, is to shift its investments to the liquid assets and away from its loan portfolio. If loan demand is relatively rate-elastic, a less loan portfolio is possible at an increased loan rate. This result is consistent with the empirical findings of Cosimano and Hakura [5] and Pausch and Welzel [16] that capital regulation as such makes the bank more prudent and less prone to risk-taking. In addition, this result is supported by Kobayakawa and Nakamura [9] that a desirable narrow bank is one that carries out both deposit-taking and lending activities, though restrictively, and allowed to invest in safe assets.

In Table 2, we have the negative direct effect on the second panel, the negative indirect effect on the third panel, and the total negative effect on the last panel. The direct effect captures the change in the default risk due to an increase in q, holding the optimal loan rate constant. It is unambiguously negative because an increase in the capital makes the bank less prone to risk-taking and thus resulting in less default risk in the bank's equity returns, ceteris paribus. The indirect effect arises because an increase in q changes the

TABLE 1. Values of S and $\partial R_L/\partial q$ in the Merton-type case*

$q\%$	$(R_L\%, L)$						
	(4.50, 200)	(4.75, 199)	(5.00, 197)	(5.25, 194)	(5.50, 190)	(5.75, 185)	(6.00, 179)
	S						
8.0	28.7486	29.0323	29.2267	29.3278	29.3316	29.2344	29.0329
8.5	29.4406	29.7295	29.9301	30.0382	30.0500	29.9619	29.7706
9.0	30.1427	30.4368	30.6436	30.7588	30.7786	30.6997	30.5188
9.5	30.8547	31.1541	31.3670	31.4894	31.5174	31.4478	31.2775
10.0	31.5767	31.8813	32.1004	32.2299	32.2663	32.2061	32.0466
10.5	32.3084	32.6182	32.8435	32.9804	33.0251	32.9745	32.8258
11.0	33.0499	33.3649	33.5964	33.7405	33.7936	33.7527	33.6151
11.5	33.8009	34.1211	34.3588	34.5102	34.5718	34.5407	34.4142
12.0	34.5615	34.8867	35.1306	35.2894	35.3596	35.3382	35.2230
12.5	35.3314	35.6617	35.9118	36.0779	36.1566	36.1452	36.0413
13.0	36.1106	36.4458	36.7021	36.8756	36.9629	36.9614	36.8689
	$\partial^2 S/\partial R_L \partial q$						
8.0∼8.5	0.0053	0.0061	0.0070	0.0080	0.0090	0.0102	
8.5∼9.0	0.0053	0.0062	0.0071	0.0081	0.0092	0.0104	
9.0∼9.5	0.0052	0.0062	0.0072	0.0082	0.0093	0.0106	
9.5∼10.0	0.0052	0.0062	0.0072	0.0083	0.0095	0.0108	
10.0∼10.5	0.0052	0.0062	0.0073	0.0084	0.0096	0.0109	
10.5∼11.0	0.0052	0.0062	0.0073	0.0084	0.0097	0.0110	
11.0∼11.5	0.0051	0.0062	0.0073	0.0085	0.0098	0.0111	
11.5∼12.0	0.0051	0.0062	0.0073	0.0085	0.0098	0.0112	
12.0∼12.5	0.0050	0.0062	0.0074	0.0086	0.0099	0.0113	
12.5∼13.0	0.0050	0.0062	0.0074	0.0086	0.0099	0.0114	
	$\partial^2 S/\partial R_L^2$						
8.0	–	−0.0892	−0.0933	−0.0973	−0.1010	−0.1043	–
8.5	–	−0.0884	−0.0924	−0.0963	−0.0999	−0.1032	–
9.0	–	−0.0875	−0.0915	−0.0953	−0.0988	−0.1019	–
9.5	–	−0.0865	−0.0905	−0.0943	−0.0977	−0.1007	–
10.0	–	−0.0856	−0.0895	−0.0932	−0.0965	−0.0994	–
10.5	–	−0.0845	−0.0885	−0.0921	−0.0953	−0.0980	–
11.0	–	−0.0835	−0.0874	−0.0910	−0.0941	−0.0967	–
11.5	–	−0.0824	−0.0863	−0.0898	−0.0928	−0.0953	–
12.0	–	−0.0813	−0.0851	−0.0886	−0.0915	−0.0939	–
12.5	–	−0.0802	−0.0840	−0.0874	−0.0902	−0.0924	–
13.0	–	−0.0790	−0.0828	−0.0861	−0.0889	−0.0910	–
	$\partial R_L/\partial q = -(\partial^2 S/\partial R_L \partial q)/(\partial^2 S/\partial R_L^2)$						
8.0∼8.5	–	0.0694	0.0760	0.0828	0.0903	0.0989	–
8.5∼9.0	–	0.0704	0.0775	0.0849	0.0929	0.1021	–
9.0∼9.5	–	0.0714	0.0790	0.0869	0.0954	0.1052	–
9.5∼10.0	–	0.0725	0.0805	0.0889	0.0979	0.1082	–
10.0∼10.5	–	0.0734	0.0820	0.0908	0.1004	0.1113	–
10.5∼11.0	–	0.0744	0.0834	0.0927	0.1028	0.1142	–
11.0∼11.5	–	0.0753	0.0848	0.0946	0.1051	0.1170	–
11.5∼12.0	–	0.0763	0.0862	0.0964	0.1074	0.1198	–
12.0∼12.5	–	0.0771	0.0875	0.0981	0.1096	0.1224	–
12.5∼13.0	–	0.0780	0.0888	0.0998	0.1117	0.1250	–

*Parameter values, unless stated otherwise, $R = 3.00\%$, $R_D = 2.50\%$, $D = 200$, $K = qD$, $\sigma = 0.20$, $\mu = 0.10$, and $L + B = D + K$.

TABLE 2. Impact on P from changes in q in the Merton-type case*

q%	$(R_L\%, L)$						
	(4.50, 200)	(4.75, 199)	(5.00, 197)	(5.25, 194)	(5.50, 190)	(5.75, 185)	(6.00, 179)
	P%						
8.0	17.9925	17.6204	17.1926	16.7080	16.1642	15.5569	14.8799
8.5	17.2829	16.9164	16.4921	16.0089	15.4645	14.8548	14.1736
9.0	16.5877	16.2271	15.8066	15.3254	14.7813	14.1701	13.4862
9.5	15.9071	15.5525	15.1363	14.6578	14.1147	13.5032	12.8179
10.0	15.2414	14.8932	14.4816	14.0063	13.4650	12.8543	12.1689
10.5	14.5909	14.2492	13.8427	13.3711	12.8325	12.2237	11.5396
11.0	13.9558	13.6208	13.2198	12.7525	12.2174	11.6113	10.9299
11.5	13.3364	13.0083	12.6130	12.1506	11.6197	11.0175	10.3400
12.0	12.7328	12.4117	12.0226	11.5657	11.0397	10.4423	9.7700
12.5	12.1451	11.8313	11.4487	10.9977	10.4774	9.8858	9.2198
13.0	11.5736	11.2671	10.8914	10.4468	9.9329	9.3480	8.6896
	$\partial P/\partial q$(%)						
8.0∼8.5	−0.7096	−0.7040	−0.7005	−0.6991	−0.6997	−0.7021	−0.7063
8.5∼9.0	−0.6952	−0.6894	−0.6855	−0.6835	−0.6833	−0.6846	−0.6874
9.0∼9.5	−0.6806	−0.6745	−0.6702	−0.6676	−0.6666	−0.6669	−0.6683
9.5∼10.0	−0.6657	−0.6594	−0.6547	−0.6515	−0.6497	−0.6489	−0.6489
10.0∼10.5	−0.6505	−0.6440	−0.6389	−0.6352	−0.6325	−0.6307	−0.6294
10.5∼11.0	−0.6351	−0.6284	−0.6229	−0.6186	−0.6152	−0.6123	−0.6097
11.0∼11.5	−0.6194	−0.6125	−0.6067	−0.6019	−0.5977	−0.5938	−0.5899
11.5∼12.0	−0.6036	−0.5966	−0.5904	−0.5850	−0.5800	−0.5752	−0.5700
12.0∼12.5	−0.5877	−0.5804	−0.5739	−0.5680	−0.5623	−0.5565	−0.5501
12.5∼13.0	−0.5715	−0.5642	−0.5573	−0.5509	−0.5445	−0.5378	−0.5302
	$(\partial P/\partial R_L)(\partial R_L/\partial q)$(%)						
8.0∼8.5	–	−0.0294	−0.0367	−0.0451	−0.0551	−0.0674	–
8.5∼9.0	–	−0.0296	−0.0373	−0.0462	−0.0568	−0.0698	–
9.0∼9.5	–	−0.0297	−0.0378	−0.0472	−0.0584	−0.0721	–
9.5∼10.0	–	−0.0298	−0.0383	−0.0481	−0.0598	−0.0742	–
10.0∼10.5	–	−0.0299	−0.0387	−0.0489	−0.0611	−0.0761	–
10.5∼11.0	–	−0.0298	−0.0390	−0.0496	−0.0623	−0.0778	–
11.0∼11.5	–	−0.0298	−0.0392	−0.0502	−0.0633	−0.0793	–
11.5∼12.0	–	−0.0297	−0.0394	−0.0507	−0.0641	−0.0805	–
12.0∼12.5	–	−0.0295	−0.0395	−0.0511	−0.0648	−0.0815	–
12.5∼13.0	–	−0.0293	−0.0395	−0.0513	−0.0653	−0.0823	–
	dP/dq						
8.0∼8.5	–	−0.0073	−0.0074	−0.0074	−0.0076	−0.0077	–
8.5∼9.0	–	−0.0072	−0.0072	−0.0073	−0.0074	−0.0076	–
9.0∼9.5	–	−0.0070	−0.0071	−0.0071	−0.0073	−0.0074	–
9.5∼10.0	–	−0.0068	−0.0069	−0.0070	−0.0071	−0.0072	–
10.0∼10.5	–	−0.0067	−0.0067	−0.0068	−0.0069	−0.0071	–
10.5∼11.0	–	−0.0065	−0.0066	−0.0066	−0.0067	−0.0069	–
11.0∼11.5	–	−0.0064	−0.0064	−0.0065	−0.0066	−0.0067	–
11.5∼12.0	–	−0.0062	−0.0062	−0.0063	−0.0064	−0.0065	–
12.0∼12.5	–	−0.0060	−0.0061	−0.0061	−0.0062	−0.0063	–
12.5∼13.0	–	−0.0059	−0.0059	−0.0060	−0.0060	−0.0061	–

*Parameter values, unless stated otherwise, $R = 3.00\%$, $R_D = 2.50\%$, $D = 200$, $K = qD$, $\sigma = 0.20$, $\mu = 0.10$, and $L + B = D + K$.

default risk by $L(R_L)$ in every possible state. As observed from Table 2, this indirect effect is negative in sign. The indirect effect reinforces the direct effect to give an overall negative response of P to an increase in q. We thus have the following result: an increase in the capital-to-deposits ratio decreases the default risk in the bank's equity returns. This result is consistent with the findings of Wang [6] and VanHoose [28] that capital regulation as such makes the bank less prone to risk-taking, thereby resulting in decreasing the default risk in the bank's equity returns and contributing to the stability of the banking system. This result may be resulted from a desirable narrow banking proposal as mentioned by Kobayakawa and Nakamura [9].

5.2. **Black-type case.** In this case, we compute the Black-type value of the bank's equity based on Equation (6). Using the information on Equation (13), we further compute $\partial^2 S_b(1,\tau,360)/\partial R_L \partial q$ and $\partial^2 S_b(1,\tau,360)/\partial R_L^2$ to obtain the result of $\partial R_L/\partial q$ in Equation (14) which will be used to compute the indirect effect in Equation (15). The findings are summarized in Table 3.

TABLE 3. Values of $S_b(1,\tau,360)$ and $\partial R_L/\partial q$ in the Black-type case*

$q\%$	$(R_L\%, L)$						
	(4.50, 200)	(4.75, 199)	(5.00, 197)	(5.25, 194)	(5.50, 190)	(5.75, 185)	(6.00, 179)
	$S_b(1,\tau,360)$						
8.0	0.4792	0.5214	0.5645	0.6083	0.6522	0.6960	0.7391
8.5	0.5033	0.5470	0.5917	0.6370	0.6825	0.7278	0.7724
9.0	0.5274	0.5727	0.6188	0.6656	0.7127	0.7595	0.8058
9.5	0.5515	0.5983	0.6460	0.6943	0.7429	0.7913	0.8391
10.0	0.5756	0.6239	0.6731	0.7230	0.7731	0.8231	0.8724
10.5	0.5997	0.6495	0.7003	0.7517	0.8033	0.8548	0.9057
11.0	0.6239	0.6751	0.7274	0.7804	0.8336	0.8866	0.9390
11.5	0.6480	0.7008	0.7546	0.8090	0.8638	0.9184	0.9723
12.0	0.6721	0.7264	0.7817	0.8377	0.8940	0.9501	1.0057
12.5	0.6962	0.7520	0.8089	0.8664	0.9242	0.9819	1.0390
13.0	0.7203	0.7776	0.8360	0.8951	0.9544	1.0137	1.0723
	$\partial^2 S_b(1,\tau,360)/\partial R_L \partial q$						
		1.5181	1.5263	1.5336	1.5401	1.5458	1.5509
	$\partial^2 S_b(1,\tau,360)/\partial R_L^2$						
8.0	–	0.0009	0.0006	0.0002	–0.0002	–0.0007	–
8.5	–	0.0009	0.0006	0.0002	–0.0002	–0.0006	–
9.0	–	0.0010	0.0006	0.0002	–0.0002	–0.0006	–
9.5	–	0.0010	0.0006	0.0003	–0.0002	–0.0006	–
10.0	–	0.0010	0.0006	0.0003	–0.0002	–0.0006	–
10.5	–	0.0010	0.0006	0.0003	–0.0002	–0.0006	–
11.0	–	0.0010	0.0007	0.0003	–0.0002	–0.0006	–
11.5	–	0.0010	0.0007	0.0003	–0.0001	–0.0006	–
12.0	–	0.0010	0.0007	0.0003	–0.0001	–0.0006	–
12.5	–	0.0010	0.0007	0.0003	–0.0001	–0.0006	–
13.0	–	0.0010	0.0007	0.0003	–0.0001	–0.0006	–
	$\partial R_L/\partial q = -(\partial^2 S_b(1,\tau,360)/\partial R_L \partial q)/(\partial^2 S_b(1,\tau,360)/\partial R_L^2)$						
8.0∼8.5				–	8.4894	2.3907	–
8.5∼9.0				–	8.7652	2.4094	–
9.0∼9.5				–	9.0595	2.4284	–
9.5∼10.0				–	9.3743	2.4478	–
10.0∼10.5				–	9.7117	2.4675	–
10.5∼11.0				–	10.0743	2.4874	–
11.0∼11.5				–	10.4651	2.5077	–
11.5∼12.0				–	10.8873	2.5284	–
12.0∼12.5				–	11.3451	2.5494	–
12.5∼13.0				–	11.8431	2.5707	–

*Parameter values, unless stated otherwise, $R = 3.00\%$, $R_D = 2.50\%$, $D = 200$, $K = qD$, $\sigma = 0.20$, $\tau = 90$, $G = 4.25\%$, and $L + B = D + K$.

In Table 3, we observe the positive value of equity of the bank on the first panel. On the second panel, we have the result of $\partial^2 S_b(1,\tau,360)/\partial R_L \partial q > 0$, which is not invariant but very insignificant based on the parameter values assumed in the numerical exercise. On the third panel, the term $\partial^2 S_b(1,\tau,360)/\partial R_L^2$ indicates a sufficient condition for an optimum of Equation (13). According to the numerical case, the valid range for lending is no less than $(R_L\%, L) = (5.25, 194)$, since the sign of the sufficient condition should be negative. Thus, the result of $\partial R_L/\partial q$ on the last panel is presented and is limited to the range from $(R_L\%, L) = (5.25, 194)$ to $(6.00, 176)$. We have the result of $\partial R_L/\partial q > 0$: higher capital requirements will lead to higher loan rates. The interpretation of this result follows a similar argument as in the case of a change in q, under a narrow banking proposal. Capital regulation may lead to superior performance under a synergy banking proposal and based on an efficient liquidity provision argument in the spirit of Kashyap and Stein [29] and Kashyap et al. [10].

TABLE 4. Impact on P_b from changes in q in the Black-type case*

$q\%$	$(R_L\%, L)$						
	(4.50, 200)	(4.75, 199)	(5.00, 197)	(5.25, 194)	(5.50, 190)	(5.75, 185)	(6.00, 179)
	$P_b\%$						
8.0				62.9027	62.0331	61.1924	60.3789
8.5				62.9027	62.0331	61.1924	60.3789
9.0				62.9027	62.0331	61.1924	60.3789
9.5				62.9027	62.0331	61.1924	60.3789
10.0				62.9027	62.0331	61.1924	60.3789
10.5				62.9027	62.0331	61.1924	60.3789
11.0				62.9027	62.0331	61.1924	60.3789
11.5				62.9027	62.0331	61.1924	60.3789
12.0				62.9027	62.0331	61.1924	60.3789
12.5				62.9027	62.0331	61.1924	60.3789
13.0				62.9027	62.0331	61.1924	60.3789
	$\partial P_b/\partial q$						
8.0∼8.5				0.0000	0.0000	0.0000	0.0000
8.5∼9.0				0.0000	–0.0000	0.0000	0.0000
9.0∼9.5				0.0000	0.0000	0.0000	0.0000
9.5∼10.0				0.0000	0.0000	0.0000	0.0000
10.0∼10.5				0.0000	–0.0000	0.0000	0.0000
10.5∼11.0				–0.0000	0.0000	0.0000	–0.0000
11.0∼11.5				0.0000	0.0000	0.0000	0.0000
11.5∼12.0				0.0000	0.0000	0.0000	0.0000
12.0∼12.5				0.0000	0.0000	0.0000	0.0000
12.5∼13.0				0.0000	0.0000	0.0000	0.0000
	$(\partial P_b/\partial R_L)(\partial R_L/\partial q) \approx dP_b/dq$						
8.0∼8.5				–	–0.0714	–0.0194	–
8.5∼9.0				–	–0.0737	–0.0196	–
9.0∼9.5				–	–0.0762	–0.0198	–
9.5∼10.0				–	–0.0788	–0.0199	–
10.0∼10.5				–	–0.0816	–0.0201	–
10.5∼11.0				–	–0.0847	–0.0202	–
11.0∼11.5				–	–0.0880	–0.0204	–
11.5∼12.0				–	–0.0915	–0.0206	–
12.0∼12.5				–	–0.0954	–0.0207	–
12.5∼13.0				–	–0.0996	–0.0209	–

*Parameter values, unless stated otherwise, $R = 3.00\%$, $R_D = 2.50\%$, $D = 200$, $K = qD$, $\sigma = 0.20$, $\tau = 90$, $G = 4.25\%$, and $L + B = D + K$.

We use the computed results in Table 4 to explain Equation (15). On the first panel, we have $P_b > 0$ limited to the range from $(R_L\%, L) = (5.25, 194)$ to (6.00, 179) due to the validness of the second-order condition in Equation (14). Again, the value of P_b is not constant but insignificantly various at different levels of q. Thus, the direct effect presented on the second panel, $\partial P_b/\partial q$, is either positively or negatively approaching to zero. The total effect is dominated by the indirect effect, $(\partial P_b/\partial R_L)(\partial R_L/\partial q) \approx dP_d/dq$, and is negative in sign presented on the last panel. Note that the term $\partial R_L/\partial q$ is obtained from Table 3. As in Table 2, we conclude that higher capital requirements will lead to lower default risk in the bank's equity returns. Capital regulation as such may produce superior performance and greater safety for the bank, which is consistent with the findings of Wang [6] and VanHoose [28]. Our result indicates the efficient provision of liquidity under a synergy banking proposal by Kashyap et al. [10], which enables us to better understand the impact of capital requirements on lending strategies and default risks in the Black-type bank equity valuation.

5.3. **Black-Merton-type case.** In this case, we compute the Black-Merton-type value of the bank's equity based on Equation (8). Using the information on Equation (16), we further calculate the terms of $\partial^2 S_m(1, \tau, 360)/\partial R_L \partial q$ and $\partial^2 S_m(1, \tau, 360)/\partial R_L^2$ to obtain the result of $\partial R_L/\partial q$ in Equation (17). This result is used to compute the indirect effect in Equation (18). The findings are summarized in Table 5.

In this case, we have the results of $S_m(1, \tau, 360) > 0$, $\partial^2 S_m/\partial R_L \partial q > 0$, $\partial^2 S_m/\partial R_L^2 < 0$, and $\partial R_L/\partial q > 0$. Again, the term of $\partial^2 S_m/\partial R_L^2 < 0$ confirms the second-order condition of Equation (16). As in Table 1, we show that capital adequacy regulation increases the interest on loans (and thus the bank interest margin) and lowers loan volume. The interpretation of this result follows a similar argument as in the case of a change in q under a synergy banking proposal. Capital regulation may lead to superior performance for the bank, which is consistent with the findings of Cosimano and Hakura [5] and Pausch and Welzel [16]. In particular, the superior performance from an efficient liquidity provision is associated with narrow banking operation in the Black-Merton-type bank equity valuation.

In Table 6, we observe the following results: (i) P_m is consistently positive in sign; (ii) $\partial P/\partial q$ is negative in sign obtained from the second panel of Table 2; (iii) the sign of the term $\partial P_b/\partial q$ is indeterminable but also very insignificant; (iv) the term in the fourth panel can be explained as the negative indirect effect where the term $\partial R_L/\partial q$ is obtained from the last panel of Table 5; (iv) the result presented in the last panel denoted by Equation (18) is the negative total effect on the default risk due to an increase in the capital-to-deposits ratio. Again, we have the following result: higher capital requirements lead to lower default risk in the bank's equity returns. Capital requirements as such may produce superior performance and greater safety for the bank, which is largely supported by Wang [6] and VanHoose [28].

5.4. **Comparison of predicting defaults under capital regulation.** In the three cases (Tables 2, 4 and 6) reported, we have consistent results of the negative impact on the default risk in the bank's equity returns due to an increase in the capital requirements. Furthermore, the Merton-type estimates of the negative effects are less significant than the Black-type estimates and the Black-Merton-type estimates. Nonetheless, the Black-type estimates of the negative effects are more significant than the Black-Merton-type estimates. The reasons that explain the differences are as follows. The market value of the bank's equity in the Merton-type valuation is the call on its underlying assets with $t = 1$ to expiration to $t = 360$. In this call valuation, the actual number of days is 360. The market

Table 5. Values of $S_m(1,\tau,360)$ and $\partial R_L/\partial q$ in the Black-Merton-type case*

$q\%$	$(R_L\%, L)$						
	(4.50, 200)	(4.75, 199)	(5.00, 197)	(5.25, 194)	(5.50, 190)	(5.75, 185)	(6.00, 179)
	$S_m(1,\tau,360)$						
8.0	0.7072	0.7611	0.8139	0.8650	0.9138	0.9598	1.0022
8.5	0.7242	0.7794	0.8334	0.8859	0.9362	0.9836	1.0277
9.0	0.7415	0.7979	0.8533	0.9072	0.9589	1.0079	1.0535
9.5	0.7590	0.8167	0.8735	0.9287	0.9819	1.0324	1.0797
10.0	0.7768	0.8358	0.8939	0.9506	1.0052	1.0573	1.1062
10.5	0.7948	0.8551	0.9146	0.9727	1.0289	1.0826	1.1331
11.0	0.8130	0.8747	0.9355	0.9951	1.0528	1.1081	1.1604
11.5	0.8315	0.8945	0.9568	1.0178	1.0771	1.1340	1.1879
12.0	0.8502	0.9146	0.9783	1.0408	1.1016	1.1602	1.2159
12.5	0.8692	0.9349	1.0000	1.0640	1.1264	1.1866	1.2441
13.0	0.8883	0.9554	1.0220	1.0876	1.1516	1.2134	1.2727
	$\partial^2 S_m(1,\tau,360)/\partial R_L \partial q$						
8.0∼8.5	0.0013	0.0013	0.0014	0.0014	0.0015	0.0016	
8.5∼9.0	0.0013	0.0013	0.0014	0.0014	0.0015	0.0016	
9.0∼9.5	0.0013	0.0013	0.0014	0.0015	0.0015	0.0016	
9.5∼10.0	0.0013	0.0014	0.0014	0.0015	0.0016	0.0017	
10.0∼10.5	0.0013	0.0014	0.0014	0.0015	0.0016	0.0017	
10.5∼11.0	0.0013	0.0014	0.0015	0.0015	0.0016	0.0017	
11.0∼11.5	0.0013	0.0014	0.0015	0.0015	0.0016	0.0017	
11.5∼12.0	0.0014	0.0014	0.0015	0.0016	0.0016	0.0017	
12.0∼12.5	0.0014	0.0014	0.0015	0.0016	0.0017	0.0018	
12.5∼13.0	0.0014	0.0015	0.0015	0.0016	0.0017	0.0018	
	$\partial^2 S_m(1,\tau,360)/\partial R_L^2$						
8.0	–	–0.0011	–0.0017	–0.0023	–0.0029	–0.0035	–
8.5	–	–0.0010	–0.0016	–0.0022	–0.0028	–0.0035	–
9.0	–	–0.0010	–0.0015	–0.0021	–0.0027	–0.0034	–
9.5	–	–0.0009	–0.0015	–0.0021	–0.0027	–0.0033	–
10.0	–	–0.0009	–0.0014	–0.0020	–0.0026	–0.0032	–
10.5	–	–0.0008	–0.0014	–0.0019	–0.0025	–0.0031	–
11.0	–	–0.0008	–0.0013	–0.0019	–0.0024	–0.0030	–
11.5	–	–0.0007	–0.0012	–0.0018	–0.0024	–0.0029	–
12.0	–	–0.0006	–0.0012	–0.0017	–0.0023	–0.0028	–
12.5	–	–0.0006	–0.0011	–0.0016	–0.0022	–0.0027	–
13.0	–	–0.0005	–0.0010	–0.0016	–0.0021	–0.0026	–
	$\partial R_L/\partial q = -(\partial^2 S_m(1,\tau,360)/\partial R_L \partial q)/(\partial^2 S_m(1,\tau,360)/\partial R_L^2)$						
8.0∼8.5	–	1.2631	0.8511	0.6496	0.5322	0.4575	–
8.5∼9.0	–	1.3496	0.8955	0.6789	0.5545	0.4760	–
9.0∼9.5	–	1.4477	0.9441	0.7104	0.5782	0.4956	–
9.5∼10.0	–	1.5599	0.9975	0.7444	0.6034	0.5162	–
10.0∼10.5	–	1.6894	1.0563	0.7810	0.6302	0.5380	–
10.5∼11.0	–	1.8404	1.1214	0.8207	0.6589	0.5611	–
11.0∼11.5	–	2.0189	1.1940	0.8637	0.6895	0.5856	–
11.5∼12.0	–	2.2329	1.2753	0.9106	0.7224	0.6116	–
12.0∼12.5	–	2.4943	1.3671	0.9618	0.7577	0.6391	–
12.5∼13.0	–	2.8205	1.4713	1.0180	0.7958	0.6684	–

*Parameter values, unless stated otherwise, $R = 3.00\%$, $R_D = 2.50\%$, $D = 200$, $K = qD$, $\sigma = 0.20$, $\mu = 0.10$, $\tau = 90$, $G = 4.25\%$, and $L + B = D + K$.

value of the bank's equity in the Black-type valuation as well as that in the Black-Merton-type valuation is the cap on its underlying assets with $t = 90$ ($\tau = 90$) to expiration to $t = 360$. In both types, the actual number of days is $360 - \tau$. The default risk in the Merton-type estimates is expected to be lower than that in the Black-type estimates and

TABLE 6. Impact on P_m from changes in q in the Black-Merton-type case*

$q\%$	$(R_L\%, L)$						
	(4.50, 200)	(4.75, 199)	(5.00, 197)	(5.25, 194)	(5.50, 190)	(5.75, 185)	(6.00, 179)
	$P_m\%$						
8.0	83.6978	82.3571	80.9958	79.6108	78.1973	76.7493	75.2588
8.5	82.9882	81.6531	80.2953	78.9117	77.4976	76.0472	74.5525
9.0	82.2929	80.9637	79.6098	78.2282	76.8144	75.3625	73.8651
9.5	81.6123	80.2892	78.9396	77.5605	76.1478	74.6956	73.1968
10.0	80.9467	79.6298	78.2849	76.9090	75.4981	74.0467	72.5479
10.5	80.2962	78.9858	77.6459	76.2738	74.8656	73.4161	71.9185
11.0	79.6611	78.3575	77.0230	75.6552	74.2505	72.8037	71.3088
11.5	79.0416	77.7449	76.4163	75.0534	73.6528	72.2099	70.7189
12.0	78.4380	77.1484	75.8259	74.4684	73.0728	71.6347	70.1489
12.5	77.8503	76.5679	75.2519	73.9004	72.5105	71.0782	69.5988
13.0	77.2788	76.0038	74.6946	73.3495	71.9660	70.5404	69.0686
	$\partial P/\partial q(\%)$						
8.0∼8.5	–0.7096	–0.7040	–0.7005	–0.6991	–0.6997	–0.7021	–0.7063
8.5∼9.0	–0.6952	–0.6894	–0.6855	–0.6835	–0.6833	–0.6846	–0.6874
9.0∼9.5	–0.6806	–0.6745	–0.6702	–0.6676	–0.6666	–0.6669	–0.6683
9.5∼10.0	–0.6657	–0.6594	–0.6547	–0.6515	–0.6497	–0.6489	–0.6489
10.0∼10.5	–0.6505	–0.6440	–0.6389	–0.6352	–0.6325	–0.6307	–0.6294
10.5∼11.0	–0.6351	–0.6284	–0.6229	–0.6186	–0.6152	–0.6123	–0.6097
11.0∼11.5	–0.6194	–0.6125	–0.6067	–0.6019	–0.5977	–0.5938	–0.5899
11.5∼12.0	–0.6036	–0.5966	–0.5904	–0.5850	–0.5800	–0.5752	–0.5700
12.0∼12.5	–0.5877	–0.5804	–0.5739	–0.5680	–0.5623	–0.5565	–0.5501
12.5∼13.0	–0.5715	–0.5642	–0.5573	–0.5509	–0.5445	–0.5378	–0.5302
	$\partial P_b/\partial q$						
8.0∼8.5	0.0000	–0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
8.5∼9.0	–0.0000	0.0000	0.0000	0.0000	–0.0000	0.0000	0.0000
9.0∼9.5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
9.5∼10.0	0.0000	0.0000	–0.0000	0.0000	0.0000	0.0000	0.0000
10.0∼10.5	–0.0000	0.0000	0.0000	0.0000	–0.0000	0.0000	0.0000
10.5∼11.0	0.0000	0.0000	0.0000	–0.0000	0.0000	0.0000	–0.0000
11.0∼11.5	0.0000	0.0000	–0.0000	0.0000	0.0000	0.0000	0.0000
11.5∼12.0	0.0000	–0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
12.0∼12.5	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
12.5∼13.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	$(\partial P/\partial R_L + \partial P_b/\partial R_L)(\partial R_L/\partial q)$						
8.0∼8.5	–	–0.0171	–0.0118	–0.0092	–0.0077	–0.0068	–
8.5∼9.0	–	–0.0183	–0.0124	–0.0096	–0.0081	–0.0071	–
9.0∼9.5	–	–0.0195	–0.0130	–0.0100	–0.0084	–0.0074	–
9.5∼10.0	–	–0.0210	–0.0137	–0.0105	–0.0088	–0.0077	–
10.0∼10.5	–	–0.0226	–0.0145	–0.0110	–0.0091	–0.0081	–
10.5∼11.0	–	–0.0246	–0.0153	–0.0115	–0.0095	–0.0084	–
11.0∼11.5	–	–0.0268	–0.0163	–0.0121	–0.0099	–0.0087	–
11.5∼12.0	–	–0.0295	–0.0173	–0.0127	–0.0104	–0.0091	–
12.0∼12.5	–	–0.0328	–0.0185	–0.0134	–0.0109	–0.0095	–
12.5∼13.0	–	–0.0369	–0.0198	–0.0141	–0.0113	–0.0098	–

*Parameter values, unless stated otherwise, $R = 3.00\%$, $R_D = 2.50\%$, $D = 200$, $K = qD$, $\sigma = 0.20$, $\mu = 0.10$, $\tau = 90$, $G = 4.25\%$, and $L + B = D + K$.

Table 6. Impact on P_m from changes in q in the Black-Merton-type case (continued)*

$q\%$	$(R_L\%, L)$						
	(4.50, 200)	(4.75, 199)	(5.00, 197)	(5.25, 194)	(5.50, 190)	(5.75, 185)	(6.00, 179)
	$dP_m/dq = (\partial P/\partial q + \partial P_b/\partial q) + (\partial P/\partial R_L + \partial P_b/\partial R_L)(\partial R_L/\partial q)$						
8.0∼8.5	–	–0.0242	–0.0188	–0.0162	–0.0147	–0.0139	–
8.5∼9.0	–	–0.0251	–0.0192	–0.0164	–0.0149	–0.0140	–
9.0∼9.5	–	–0.0262	–0.0197	–0.0167	–0.0151	–0.0141	–
9.5∼10.0	–	–0.0275	–0.0202	–0.0170	–0.0152	–0.0142	–
10.0∼10.5	–	–0.0290	–0.0208	–0.0173	–0.0154	–0.0144	–
10.5∼11.0	–	–0.0308	–0.0215	–0.0177	–0.0157	–0.0145	–
11.0∼11.5	–	–0.0329	–0.0223	–0.0181	–0.0159	–0.0146	–
11.5∼12.0	–	–0.0354	–0.0232	–0.0185	–0.0161	–0.0148	–
12.0∼12.5	–	–0.0386	–0.0242	–0.0190	–0.0164	–0.0150	–
12.5∼13.0	–	–0.0425	–0.0253	–0.0195	–0.0167	–0.0151	–

*Parameter values, unless stated otherwise, $R = 3.00\%$, $R_D = 2.50\%$, $D = 200$, $K = qD$, $\sigma = 0.20$, $\mu = 0.10$, $\tau = 90$, $G = 4.25\%$, and $L + B = D + K$.

that in the Black-Merton-type estimates. These less significant effects of capital regulation are expected. In addition, if one uses the Black-type valuation approach with the book value of $S_b(1, 360)$ multiplied by the market value of the caplet, the risk-adjusted function is the $S_b(1, 360)$ which is ignored. In this case, the probability of default is expected to increase if the book value of $S_b(1, 360)$ is replaced by the market value of S in the Black-Merton-type valuation approach because the risk-adjusted function in S is explicitly treated in the Black-Merton-type approach. Both the comparative results indicated the well-functioned capital regulation designed to force the bank's capital position reflects its asset portfolio risk.

One immediate application of this comparison is to evaluate the plethora of lending arrangements at various structural models as alternatives for defaults prediction. We argue that a main expected benefit of narrowing banking in accordance with the Merton-type equity valuation is the reduced cost of capital regulation, which is consistent with the findings of Chamley and Kotlifoff [30], and Philips and Roselli [31]. Nevertheless, the narrow banking proposal has received some criticism. In particular, we argue that the efficiency gains afforded by the integration of financial services within a bank have been brought forward within the synergy banking of the Black-type equity valuation, consistent with the findings of Bossone [17] and Kashyap et al. [10]. Furthermore, we argue that narrowing banking implies efficiency losses in predicting defaults requiring a trade-off made between stability and efficiency under the capital regulation due to the synergies between lending and deposit-taking. If there is real, non-exclusive narrow-synergy banking, a forced switch to narrow banking could lead to large efficiency losses and a forced switch to synergy banking could lead to overestimation of efficiency gains in predicting defaults under capital regulation.

Our results indicate an implication. In 1999, US Congress passed the Gramm-Leach-Bliley Act (GLBA) that allowed bank holding companies to convert to financial holding companies and conduct securities and insurance activities without limit in subsidiaries separated from their commercial banks [32]. In related works, Lin et al. [33] use a down-and-out call option to study the valuation effects of the GLBA. They argue that the GLBA has created incentives and choices that expose commercial banks to increased risk and uncertainty. Unlike Lin et al. [33], we may use a Black-Merton-type option to study the valuation of a real, non-exclusive narrow-synergy banking proposal permitted by the GLBA, and conclude that capital requirements may lead to superior performance and greater safety for the bank. These results document that the choice of an appropriate goal in modeling the bank's optimization problem remains a controversial issue and highlights the importance that banks tend to attach to the government's incentives in the design of capital regulation.

6. **Conclusions.** In this paper, we compute and compare the powers of various structural models in defaults prediction. In particular, we measure the power of each model by its capability to separate the underlying asset and the strike price. The measure used across all models is the distance to default. We apply the Vassalou and Xing's [15] methodology that provides an estimate on the distance to default with the bank interest margin determination under capital regulation, which allows us to compare different models. We find consistent results in the Merton-type, Black-type and Black-Merton-type: higher capital requirements lower loan volume and increase the bank's interest margin while leading to lower default risk in the bank's equity return. Furthermore, the ranking of the significance impact on default risk from increasing capital requirements is Merton-type, Black-Merton-type and Black-type.

The three structural models, Merton-type, Black-type and Black-Merton-type for bank equity valuation are based on path-independent approach because its payoff depends on the underlying asset value only at maturity. One issue that has not been addressed is the structural model for bank equity valuation based on the path-dependent, barrier option model which payoff depends on the particular path followed up to maturity. Banking firms with high asset variability, high operating leverage or low capitalization, are likely to exhibit a higher probability of hitting the barrier before the expiration date than banking firms without such characteristics [34]. Such concerns are beyond the scope of this paper and are thus not addressed here. The aforementioned issues may provide an ample opportunity for future capital regulation research.

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