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SEISMIC DESIGN OF BUILDINGS BY THE DYNAMIC METHOD USED THREE DIFFERENT MODELS

Arnulfo Luévanos Rojas

Facultad de Ingenieria, Ciencias y Arquitectura
Universidad Juarez del Estado de Durango
Av. Universidad S/N, Fracc. Filadelfia, CP 35010, Gómez Palacio, Durango, México
arnulfol_2007@hotmail.com

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Abstract. *This paper proposes a new model (model 1) for seismic analysis of buildings. In this model are taken into account four lumped masses at each level and are applied, one at each end and the two others intermediate per level, the three degrees of freedom in the joints are considered. Also, a comparison is made with model 2 that considers two equal lumped masses per level and applied in the free joints of the structure and with the model 3 (classical model), which takes into account a lumped mass at each level, i.e., a mass per floor of all the building. About the results obtained, the model 1 is much more economical in relation to the model 3 and the model 2 is not very secure, because it ignores certain modal shapes that the model 1, if takes into account. Therefore, the usual practice considering one and two lumped masses per level will not be a recommendable solution and it is proposed to consider four lumped masses at each level and is also more related to real conditions.*

Keywords: Lumped masses, Modal analysis, Spectral analysis, Eigenvalues and eigenvectors, Modal participation factor, Spectral acceleration

1. **Introduction.** Three types of methods can be used for the seismic analysis of building structures: simplified, static and dynamic methods. The simplified method is applicable to regular structures with not greater height 13 m and simultaneously fulfilling all requirements indicated by the building regulations. The static method is applicable to buildings whose height is less than or equal to 30 m for regular structures and irregular structures standing less than 20 m high; these limits increase to 40 m and 30 m, respectively, for structures sited on rocky terrain. The dynamic method consists of the same basic steps as that for the static method, with the reservation that applicable lateral forces in floor's mass centre are determined from a structure's dynamic response. Modal spectral analysis and step-by-step analysis or calculation of responses having specific acceleration registries can be used for the dynamic method [1-3].

In dynamic method have been made comparisons considering and neglecting shear deformations, resulting more economical the first case and also adhering more to the real conditions [4].

The models used are: model 3 (classical model) which takes into account a lumped mass per level and one degree of freedom is considered at each floor (horizontal displacement per each level) [4], and the model 2 considers two lumped masses at each level are applied in the structure's free nodes and the three degrees of freedom are taken into account at free joints (horizontal displacement, vertical displacement and rotation per free node) [5,6].

This paper proposes a new model which takes into account four equal lumped masses per level and are applied, one at each end and the two others intermediate at each floor, the three degrees of freedom at the free joints are considered. Also a comparison is realized with the model 2 considering two lumped masses per level applied at the free joints of the structure taking into account three degrees of freedom at the free joints (horizontal displacement, vertical displacement and rotation at each free node), and with the model 3 (classical model) that considers a lumped mass at each level considering a degree of freedom at each floor (horizontal displacement per level). The three models take into account the shear deformations.

2. Methodology.

2.1. Equations of motion in a structural dynamic system.

Overall equations of motion in a structural dynamic system, without including border conditions, can be written in matrix form as follows [7-9]:

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \ddot{U}_1 \\ \ddot{U}_2 \end{bmatrix} + \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} \tag{1}$$

where U_1 is a vector of $n \times 1$ generalized absolute displacements (unknowns) corresponding to the degrees of freedom non-restricted "n", U_2 is a vector of $m \times 1$ generalized absolute displacements (nulls or known) corresponding to the degrees of freedom in supports "m", M_{ij}, C_{ij}, K_{ij} are mass matrices, damping and stiffness are associated with degrees of freedom "n" and/or "m" respectively, P_1 is a vector of $n \times 1$ representing dynamic's associate solicitations to degrees of freedom "n", and P_2 is a vector of $m \times 1$ representing associate reactions (unknowns) to degrees of freedom of supports "m".

For the case of seismic excitations, "$P_1 = 0$" and the values of "$\ddot{U}_2$, $\dot{U}_2$, U_2" these are considered known. Therefore, the first expression of Equation (1) of the system is:

$$M_{11}\ddot{U}_1 + C_{11}\dot{U}_1 + K_{11}U_1 = -M_{12}\ddot{U}_2 - C_{12}\dot{U}_2 - K_{12}U_2 \tag{2}$$

The total displacement "U_1" can be expressed as the sum of the relative displacement "U_1^r" and the pseudostatic displacement "U_1^s" that would be a static displacement of the support according as seen in Figure 1, this is

$$U_1 = U_1^r + U_1^s \tag{3}$$

where

$$U_1 = U_1^r + U_1^s; \quad \dot{U}_1 = \dot{U}_1^r + \dot{U}_1^s; \quad \ddot{U}_1 = \ddot{U}_1^r + \ddot{U}_1^s \tag{4}$$

$$U_2 = U_2^s; \quad U_2^r = 0; \quad \dot{U}_2 = \dot{U}_2^s; \quad \ddot{U}_2 = \ddot{U}_2^s \tag{5}$$

Equation (4) is substituted into Equation (2), which gives:

$$M_{11}(\ddot{U}_1^r + \ddot{U}_1^s) + C_{11}(\dot{U}_1^r + \dot{U}_1^s) + K_{11}(U_1^r + U_1^s) = -M_{12}\ddot{U}_2 - C_{12}\dot{U}_2 - K_{12}U_2 \tag{6}$$

The pseudostatic displacement will be evaluated by the static equilibrium condition, which is obtained from Equation (6), that is

$$K_{11}U_1^s = -K_{12}U_2 \tag{7}$$

$$U_1^s = \hat{r}U_2 \tag{8}$$

where "$\hat{r}$" the pseudostatic influence matrix can be expressed as:

$$\hat{r} = -K_{11}^{-1}K_{12} \tag{9}$$

Equation (8) is substituted into Equation (4) gives:

$$U_1 = U_1^r + \hat{r}U_2 \tag{10}$$

Displacement dynamic components will be expressed as:

$$U_1^r = U_1 - \hat{r}U_2 \tag{11}$$

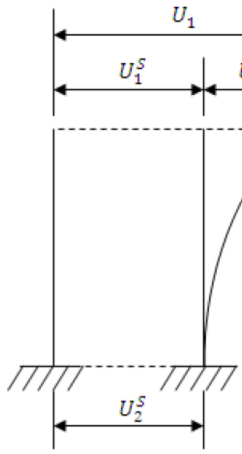


FIGURE 1. Total displacements

Equation (10) is substituted into Equation (2); the equations of motion in terms of the displacement dynamic components are obtained:

$$M_{11}(\ddot{U}_1^r + \hat{r}\ddot{U}_2) + C_{11}(\dot{U}_1^r + \hat{r}\dot{U}_2) + K_{11}(U_1^r + \hat{r}U_2) = -M_{12}\ddot{U}_2 - C_{12}\dot{U}_2 - K_{12}U_2 \quad (12)$$

or

$$M_{11}\ddot{U}_1^r + C_{11}\dot{U}_1^r + K_{11}U_1^r = -(M_{11}\hat{r} + M_{12})\ddot{U}_2 - (C_{11}\hat{r} + C_{12})\dot{U}_2 - (K_{11}\hat{r} + K_{12})U_2 \quad (13)$$

Then Equation (9) is substituted into the last member of Equation (13); the equations of motion in terms of the displacement dynamic components may be rewritten:

$$M_{11}\ddot{U}_1^r + C_{11}\dot{U}_1^r + K_{11}U_1^r = -(M_{11}\hat{r} + M_{12})\ddot{U}_2 - (C_{11}\hat{r} + C_{12})\dot{U}_2 \quad (14)$$

It is important to indicate that when a lumped mass formulation is used as is normal, implies that the term "$M_{12}\ddot{U}_2$" is null.

By other part, the damping in the excitation is demonstrated that the second term of the right side in Equation (14) is very small in comparison with first, by general, it is not considered. In addition, when a spectral analysis is realized, the damping effect of excitation in the response spectrum comes implicit.

Then, Equation (14) may be written:

$$M_{11}\ddot{U}_1^r + C_{11}\dot{U}_1^r + K_{11}U_1^r = -M_{11}\hat{r}\ddot{U}_2 \quad (15)$$

Equation (9) is substituted into Equation (15) gives:

$$M_{11}\ddot{U}_1^r + C_{11}\dot{U}_1^r + K_{11}U_1^r = M_{11}K_{11}^{-1}K_{12}\ddot{U}_2 \quad (16)$$

Then, if the normal coordinate orthogonality property are used to simplify the equations of motion Multi-Degree-of-Freedom system. These equations are given in general form by Equation (1). For system of undamped free vibration becomes [8,9]:

$$M_{11}\ddot{U}_1^r + K_{11}U_1^r = 0 \quad (17)$$

where M_{11} is mass matrix corresponding to the degrees of freedom non-restricted "n", K_{11} is stiffness matrix corresponding to the degrees of freedom non-restricted "n", U_1^r is a relative displacements vector, and $\ddot{U}_1^r$ is relative accelerations vector.

Its solution is defined as:

$$U_1^r = \bar{\varnothing}e^{i\omega t} \quad (18)$$

where ω is vibration natural frequency; $\bar{\varnothing}$ is modal vector (mode-shaped vector) associated to "ω", $i = \sqrt{-1}$, $t =$ time.

The values of "ω" and "$\bar{\varnothing}$" are determined by resolving eigenproblems as:

$$(K_{11} - \omega^2 M_{11})\bar{\varnothing} = 0 \tag{19}$$

With this, the equations of motion in the system are defined by Equation (16), and can be diagonalized to transform to a modal normal coordinates system "$Y_n(t)$" is defined as:

$$U_1^r = \sum_{n=1}^{N} \bar{\varnothing}_n Y_n = \Phi \bar{Y} \tag{20}$$

where Φ is modal matrix (mode-shape matrix); $\bar{Y}$ is normal coordinates vector.

Equation (20) is Substituted into Equation (16) and is pre-multiplied by the transpose of the modal vector corresponding to mode "n" and applying the orthogonality conditions are obtained the equations of motion undocked [8-10]. Being the equation corresponding to mode "n", it is presented as:

$$\bar{\varnothing}_n^t M_{11} \bar{\varnothing}_n \ddot{Y}_n + \bar{\varnothing}_n^t C_{11} \bar{\varnothing}_n \dot{Y}_n + \bar{\varnothing}_n^t K_{11} \bar{\varnothing}_n Y_n = \bar{\varnothing}_n^t M_{11} K_{11}^{-1} K_{12} \ddot{U}_2 \tag{21}$$

being

$$M_n = \bar{\varnothing}_n^t M_{11} \bar{\varnothing}_n \tag{22}$$

$$\eta_n = \bar{\varnothing}_n^t C_{11} \bar{\varnothing}_n / (2 M_n \omega_n) \tag{23}$$

$$\omega^2 M_n = \bar{\varnothing}_n^t K_{11} \bar{\varnothing}_n \tag{24}$$

where η_n is percent of damping to the mode "n"; $\bar{\varnothing}_n^t$ is the transpose matrix of the modal vector corresponding to mode "n" [8,9].

Now, the system of equations of motion in Equation (16) is transformed in modal normal coordinate to obtain the system of "n" degrees of freedom, the equations of motion uncoupled in each mode "n". Equation (21) may be written as:

$$M_n \ddot{Y}_n + 2 M_n \eta_n \omega_n \dot{Y}_n + M_n \omega_n^2 Y_n = \bar{\varnothing}_n^t M_{11} K_{11}^{-1} K_{12} \ddot{U}_2 \tag{25}$$

where "ω_n" and "$\bar{\varnothing}_n$" are eigenvalues and eigenvectors corresponding to mode "n".

Equation (25) is simplified

$$\ddot{Y}_n + 2 \eta_n \omega_n \dot{Y}_n + \omega_n^2 Y_n = \bar{\varnothing}_n^t M_{11} K_{11}^{-1} K_{12} \ddot{U}_2 / M_n \tag{26}$$

or

$$\ddot{Y}_n + 2 \eta_n \omega_n \dot{Y}_n + \omega_n^2 Y_n = \Gamma_n \ddot{U}_j(t) \tag{27}$$

where "Γ_n" is modal participation factor and is defined as [8,11]:

$$\Gamma_n = \frac{\bar{\varnothing}_n^t M_{11} \hat{r}}{\bar{\varnothing}_n^t M_{11} \bar{\varnothing}_n} \tag{28}$$

The solution of Equation (27) can be obtained considering the first integral of Duhamel as follows [8-12]:

$$Y_n = \left(\frac{\Gamma_n}{\omega_n}\right) \int_0^t \ddot{U}_2(\tau) [e^{-\eta_n \omega_n (t-\tau)}] \sin \omega_n (t - \tau) d\tau \tag{29}$$

It is denoted:

$$S_{vn}(t) = \int_0^t \ddot{U}_2(\tau) [e^{-\eta_n \omega_n (t-\tau)}] \sin \omega_n (t - \tau) d\tau \tag{30}$$

Thus, in general

$$Y_n = \left(\frac{\Gamma_n}{\omega_n}\right) S_{vn} \tag{31}$$

where S_{vn} is spectral velocity.

Now, according to the procedure, the seismic response spectrum will be sufficient to determine solely the response maximum values, and not all the complete history. For

Equation (29) and Equation (30) are observed that the maximum responses are defined considering the maximum value of the response function [8-12].

In terms of the spectral acceleration "S_{an}" is obtained for mode "n" to from the response spectrum corresponding to the excitation of the support as follows:

$$S_{vn} = \frac{S_{an}}{\omega_n} \tag{32}$$

Equation (32) is substitutied into Equation (31) is obtained the maximum modal response due to excitation of the support "$(Y_n)_{\max}$" is presented [8-12]:

$$(Y_n)_{\max} = \frac{\Gamma_n S_{an}}{\omega_n^2} \tag{33}$$

The vectors corresponding to the components of the maximum relative displacement vector for each mode "$\{U_{1n}^r\}_{\max}$" are defined as:

$$\{U_{1n}^r\}_{\max} = \{\bar{\varnothing}_n\}(Y_n)_{\max} \tag{34}$$

The maximum value of relative displacements vector in the structural dynamic system "$\{U_1^r\}_{\max}$" is obtained as:

$$\{U_1^r\}_{\max} = \left\{ \sum_{j=1}^{n} (U_{1j}^r)_{\max}^2 \right\}^{1/2} \tag{35}$$

or

$$\{U_1^r\}_{\max} = \{(U_{11}^r)_{\max}^2 + (U_{12}^r)_{\max}^2 + \ldots + (U_{1n}^r)_{\max}^2\}^{1/2} \tag{36}$$

The value of the equivalent elements mechanical acting in the free joints "P" may be expressed as [9,11-14]:

$$P = K_{11}\{U_1^r\}_{\max} \tag{37}$$

Finally, the mechanical elements acting on members "F_i" are defined as [9,11-14]:

$$F_i = K_i U_{ij} \tag{38}$$

where K_i is stiffness matrix of member "i", in the overall or general system, U_{ij} is displacement vector of member "ij", in overall system.

3. **Application.** An example of the dynamic method for seismic design is presented, using three different models and taking into account the shear deformations for an offices building built with structural steel profiles. The analysis is developed only in the transverse direction, i.e., in the sense of 10 m. The offices building in plan and elevation are shown in Figure 2, and the spectrum of horizontal response is observed in Figure 3, this is the motion of soil, where the building is supporting. In Table 1, the properties of the steel profiles are presented. The loads to be considered in the analysis per level are:

Weight of level 1 = 700 kg/m^2,
Weight of level 2 = 600 kg/m^2,
Weight of level 3 = 500 kg/m^2,
Weight of level 4 = 300 kg/m^2,
Elasticity modulus = 2040734 kg/cm^2,
Poisson's ratio = 0.32

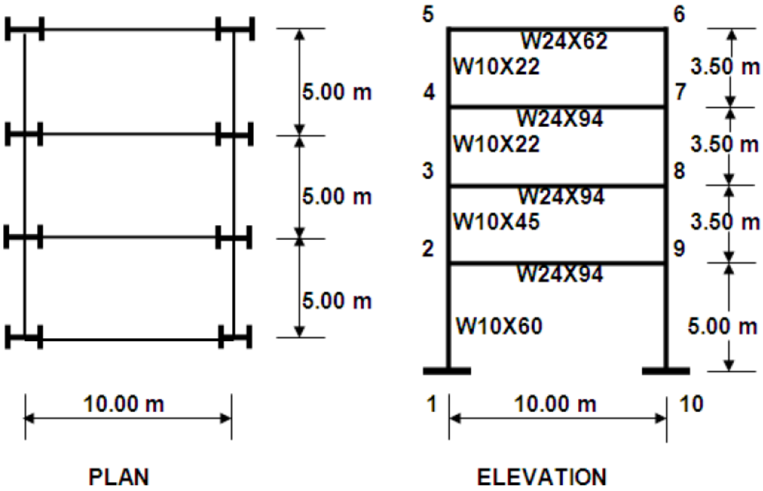


FIGURE 2. Plan and elevation of office building built with structural steel profiles

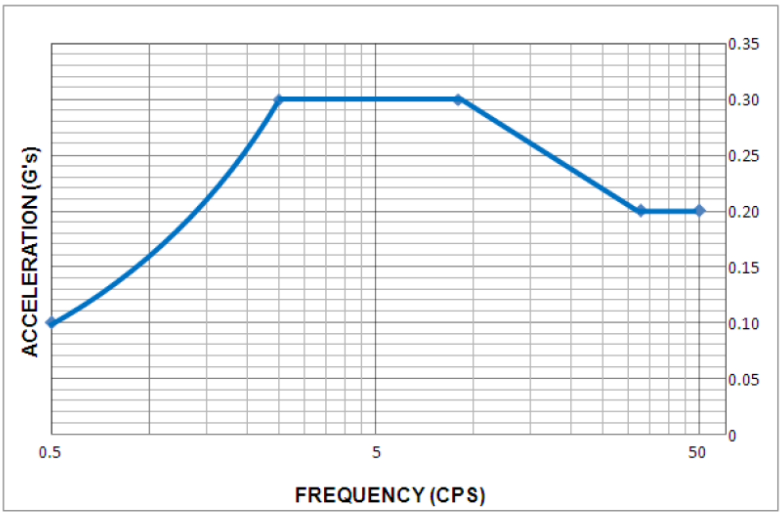


FIGURE 3. The spectrum of horizontal response

TABLE 1. Properties of the steel profiles

Profiles	Total area (cm^2)	Shear area (cm^2)	Moment of inertia (cm^4)
W10X60	113.55	27.69	14193
W10X45	85.81	22.81	10323
W10X22	41.87	15.75	4912
W24X94	178.71	80.77	112382
W24X62	117.42	65.86	64516

3.1. **Model 1 (proposed model).** This model considers beams and the columns for the analysis taking into account four lumped masses per level applied one at each end and the others two intermediate, and three degrees of freedom at each joint. The building is analyzed solely in the transverse direction. The mathematical model is presented in Figure 4.

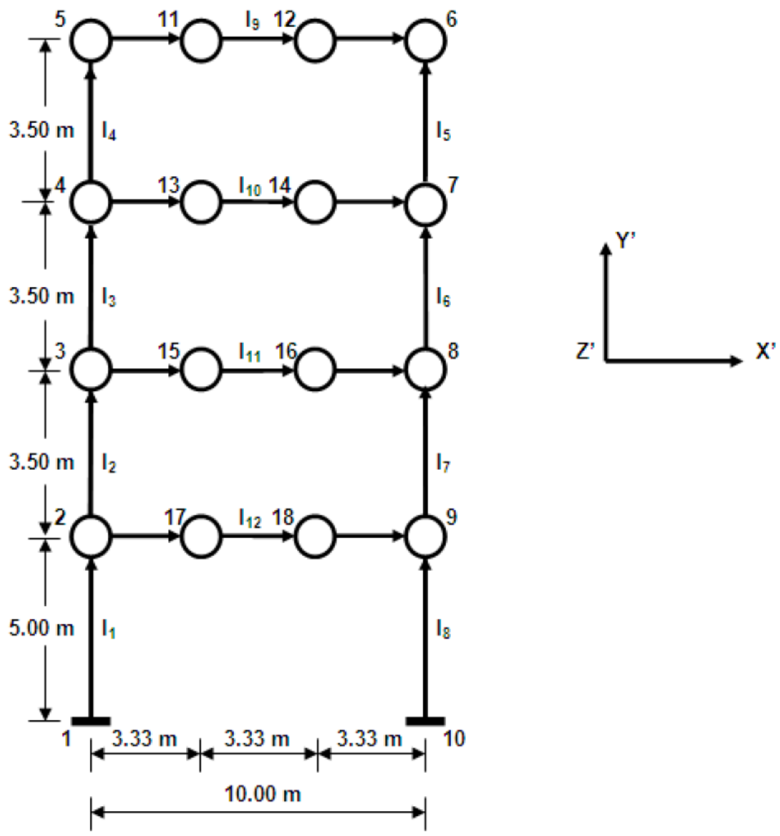


FIGURE 4. Vectorial model considers four lumped masses per level of the building

Matrix of lumped mass, "M_i" of a joint is:

$$M_i = \begin{bmatrix} M_{ix} & 0 & 0 \\ 0 & M_{iy} & 0 \\ 0 & 0 & M_{iz} \end{bmatrix}$$

where i goes 2 to 9, M_{ix} is the mass in direction X' of the joint i, M_{iy} is the mass in direction Y' of the joint i, and M_{iz} is the rotational mass, i.e., around Z' axis of the joint i.

For our problem is $M_2 = M_9$, $M_3 = M_8$, $M_4 = M_7$ and $M_5 = M_6$.

Mass matrix in the overall system, "M_{11}" is defined as:

$$M_{11} = \begin{bmatrix} M_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & M_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & M_4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & M_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & M_7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & M_8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_9 \end{bmatrix}$$

Lumped masses on the building are obtained:

$$M_2 = M_9 = \frac{(700\text{kg/m}^2)(10\text{m}/6)(15\text{m})}{980.665\text{cm/sec}^2} = 17.85\text{kg-sec}^2/\text{cm}$$

$$M_3 = M_8 = \frac{(600\text{kg/m}^2)(10\text{m}/6)(15\text{m})}{980.665\text{cm/sec}^2} = 15.30\text{kg-sec}^2/\text{cm}$$

$$M_4 = M_7 = \frac{(500\text{kg/m}^2)(10\text{m}/6)(15\text{m})}{980.665\text{cm/sec}^2} = 12.75\text{kg-sec}^2/\text{cm}$$
$$M_5 = M_6 = \frac{(300\text{kg/m}^2)(10\text{m}/6)(15\text{m})}{980.665\text{cm/sec}^2} = 7.65\text{kg-sec}^2/\text{cm}$$
$$M_{11} = M_{12} = \frac{(300\text{kg/m}^2)(10\text{m}/3)(15\text{m})}{980.665\text{cm/sec}^2} = 15.30\text{kg-sec}^2/\text{cm}$$
$$M_{13} = M_{14} = \frac{(500\text{kg/m}^2)(10\text{m}/3)(15\text{m})}{980.665\text{cm/sec}^2} = 25.49\text{kg-sec}^2/\text{cm}$$
$$M_{15} = M_{16} = \frac{(600\text{kg/m}^2)(10\text{m}/3)(15\text{m})}{980.665\text{cm/sec}^2} = 30.59\text{kg-sec}^2/\text{cm}$$
$$M_{17} = M_{18} = \frac{(700\text{kg/m}^2)(10\text{m}/3)(15\text{m})}{980.665\text{cm/sec}^2} = 35.69\text{kg-sec}^2/\text{cm}$$

Table 2 shows the masses acting on degrees of freedom of the building.

TABLE 2. The masses on joints

Joint	Mass in X (kg-sec^2/cm)	Mass in Y (kg-sec^2/cm)	Rotational mass Mass by distance^2 (kg-cm-sec^2)
2 and 9	35.69 + 17.85 = 53.54	35.69 + 17.85 = 53.54	35.69 X $(1000/3)^2$ = 3965563.05
3 and 8	30.59 + 15.30 = 45.89	30.59 + 15.30 = 45.89	30.59 X $(1000/3)^2$= 3399054.04
4 and 7	25.49 + 12.75 = 38.24	25.49 + 12.75 = 38.24	25.49 X $(1000/3)^2$ = 2832545.04
5 and 6	15.30 + 7.65 = 22.95	15.30 + 7.65 = 22.95	15.30 X $(1000/3)^2$ = 1699527.02

The mass matrix "M_{11}" is presented in [17].

Stiffness matrix of a structural member, for this case "K_j" is the stiffness of the four columns or four beams of the rigid frame:

$$K_j = \begin{bmatrix} K_{11}^{(j)} & K_{12}^{(j)} \\ K_{21}^{(j)} & K_{22}^{(j)} \end{bmatrix}$$

where j goes 1 to 12.

Stiffness matrix of a column in the global system is [9,15-17]:

$$K_j = \begin{bmatrix} \frac{12EI}{L^3(1+\alpha)} & 0 & -\left[\frac{6EI}{L^2(1+\alpha)}\right] & -\frac{12EI}{L^3(1+\alpha)} & 0 & -\left[\frac{6EI}{L^2(1+\alpha)}\right] \\ 0 & \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 \\ -\left[\frac{6EI}{L^2(1+\alpha)}\right] & 0 & \left[\frac{4+\alpha}{1+\alpha}\right]\frac{EI}{L} & \frac{6EI}{L^2(1+\alpha)} & 0 & \left[\frac{2-\alpha}{1+\alpha}\right]\frac{EI}{L} \\ -\frac{12EI}{L^3(1+\alpha)} & 0 & \frac{6EI}{L^2(1+\alpha)} & \frac{12EI}{L^3(1+\alpha)} & 0 & \frac{6EI}{L^2(1+\alpha)} \\ 0 & -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 \\ -\left[\frac{6EI}{L^2(1+\alpha)}\right] & 0 & \left[\frac{2-\alpha}{1+\alpha}\right]\frac{EI}{L} & \frac{6EI}{L^2(1+\alpha)} & 0 & \left[\frac{4+\alpha}{1+\alpha}\right]\frac{EI}{L} \end{bmatrix}$$

Stiffness matrix of a beam in the global system may be expressed as [9,15-17]:

$$K_j = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3(1+\alpha)} & \frac{6EI}{L^2(1+\alpha)} & 0 & -\frac{12EI}{L^3(1+\alpha)} & \frac{6EI}{L^2(1+\alpha)} \\ 0 & \frac{6EI}{L^2(1+\alpha)} & \left[\frac{4+\alpha}{1+\alpha}\right]\frac{EI}{L} & 0 & -\frac{6EI}{L^2(1+\alpha)} & \left[\frac{2-\alpha}{1+\alpha}\right]\frac{EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3(1+\alpha)} & -\frac{6EI}{L^2(1+\alpha)} & 0 & \frac{12EI}{L^3(1+\alpha)} & -\frac{6EI}{L^2(1+\alpha)} \\ 0 & \frac{6EI}{L^2(1+\alpha)} & \left[\frac{2-\alpha}{1+\alpha}\right]\frac{EI}{L} & 0 & -\frac{6EI}{L^2(1+\alpha)} & \left[\frac{4+\alpha}{1+\alpha}\right]\frac{EI}{L} \end{bmatrix}$$

Values "α" and "G" are obtained:

$$\alpha = \frac{12EI}{GA_cL^2}; \quad G = \frac{E}{2(1+v)}$$

where E is elasticity modulus, I is inertia moment, L is member length, A is total area, α is form factor, G is shear modulus, A_c is shear area, and v is Poisson's ratio.

3.2. **Model 2.** This model considers beams and the columns for the analysis taking into account two lumped masses per level and three degrees of freedom at each joint. The building is analyzed solely in the transverse direction [6]. The mathematical model is presented in Figure 5.

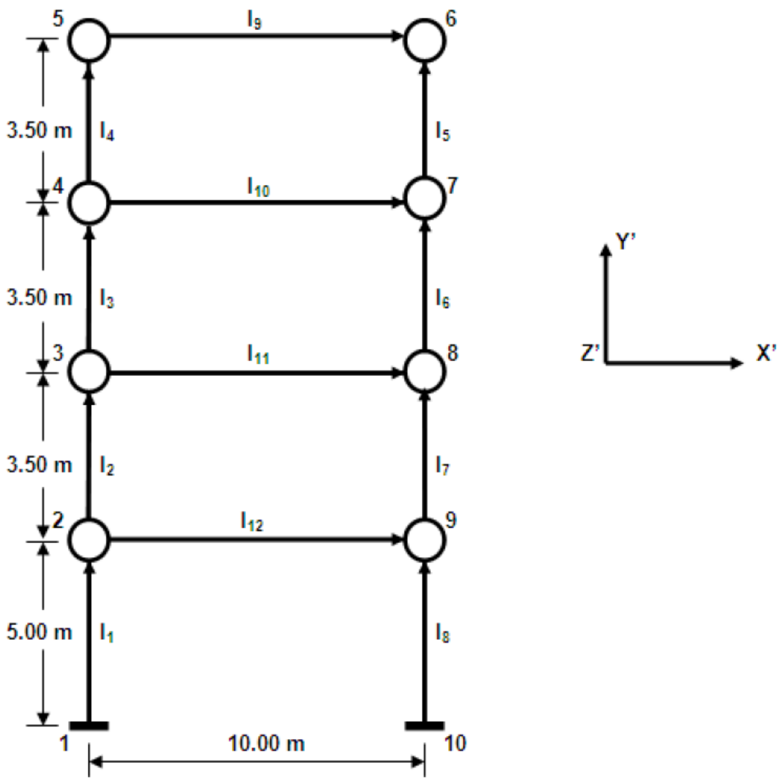


FIGURE 5. Vectorial model considers two lumped masses per level of the building

In model 2, the matrix of lumped mass in a joint (M_i), the mass matrix in overall system (M_{11}), the stiffness matrix of a structural member (K_j), the stiffness matrix of a column in overall system and the stiffness matrix of a beam in overall system, these are the same of the model 1, the only difference is that in model 2 there is not rotational mass (M_z) and in model 1 if exists by the application of intermediate masses in beams.

3.3. **Model 3 (classical model).** This model considers that the beams are rigid in comparison to the columns, and therefore the beams do not influence at the dynamic analysis of the building. It also considers one degree of freedom per level, i.e., horizontal displacement [4]. The mathematical model is presented in Figure 6.

Mass matrix in the overall system, "M_{11}" is defined as:

$$M_{11} = \begin{bmatrix} M_2 & 0 & 0 & 0 \\ 0 & M_3 & 0 & 0 \\ 0 & 0 & M_4 & 0 \\ 0 & 0 & 0 & M_5 \end{bmatrix}$$

Stiffness matrix of a structural member, for this case "K_j" is the stiffness of the eight columns for each level:

$$K_j = \begin{bmatrix} K_{11}^{(j)} & K_{12}^{(j)} \\ K_{21}^{(j)} & K_{22}^{(j)} \end{bmatrix} = \begin{bmatrix} \frac{12EI}{L^3(1+\alpha)} & -\frac{12EI}{L^3(1+\alpha)} \\ -\frac{12EI}{L^3(1+\alpha)} & \frac{12EI}{L^3(1+\alpha)} \end{bmatrix}$$

Stiffness matrix of all the building in overall system for the model 1 and 2 is:

$$K_{11} = \begin{bmatrix} K_{22}^{(1)} + K_{11}^{(2)} + K_{11}^{(12)} & K_{12}^{(2)} & 0 & 0 & 0 & 0 & 0 & K_{12}^{(12)} \\ K_{21}^{(2)} & K_{22}^{(2)} + K_{11}^{(3)} + K_{11}^{(11)} & K_{12}^{(13)} & 0 & 0 & 0 & K_{12}^{(11)} & 0 \\ 0 & K_{21}^{(3)} & K_{22}^{(3)} + K_{11}^{(4)} + K_{11}^{(10)} & K_{12}^{(4)} & 0 & K_{12}^{(10)} & 0 & 0 \\ 0 & 0 & K_{21}^{(4)} & K_{22}^{(4)} + K_{11}^{(9)} & K_{12}^{(9)} & 0 & 0 & 0 \\ 0 & 0 & 0 & K_{21}^{(9)} & K_{22}^{(9)} + K_{22}^{(5)} & K_{21}^{(5)} & 0 & 0 \\ 0 & 0 & K_{21}^{(10)} & 0 & K_{12}^{(5)} & K_{22}^{(6)} + K_{11}^{(5)} + K_{22}^{(10)} & K_{21}^{(6)} & 0 \\ 0 & K_{21}^{(11)} & 0 & 0 & 0 & K_{12}^{(6)} & K_{22}^{(7)} + K_{11}^{(6)} + K_{22}^{(11)} & K_{21}^{(7)} \\ K_{21}^{(12)} & 0 & 0 & 0 & 0 & 0 & K_{12}^{(7)} & K_{22}^{(9)} + K_{11}^{(7)} + K_{22}^{(12)} \end{bmatrix}$$

Stiffness matrix of all the building in overall system is:

$$K_{11} = \begin{bmatrix} K_{22}^{(1)} + K_{11}^{(2)} & K_{12}^{(2)} & 0 & 0 \\ K_{21}^{(2)} & K_{22}^{(2)} + K_{11}^{(3)} & K_{12}^{(3)} & 0 \\ 0 & K_{21}^{(3)} & K_{22}^{(3)} + K_{11}^{(4)} & K_{12}^{(4)} \\ 0 & 0 & K_{21}^{(4)} & K_{22}^{(4)} \end{bmatrix}$$

The mass matrix and stiffness in overall system of the three models for the entire building appear in [17].

3.4. **Procedure of seismic analysis for the three models.** The mass matrices and stiffness for each member are evaluated, followed by change of local system to overall system. The mass matrices and stiffness in each member's overall system are then coupled and the system's general matrix is obtained. This general matrix was organized to separate the degrees of freedom in the structure (M_{11} and K_{11}) and degrees of freedom in the supports (M_{22} and K_{22}). A similar transformation is applied through exchange of rows and columns matrix (permutation matrix).

Taking into account the condition of free vibration given by Equation (17), being "U_1^r", a vector of relative displacements (24 × 1 for model 1 and 2, and of 4 × 1 for model 3) corresponding to degrees of freedom in the building's structural system. Subsequently, the eigenvalues and eigenvectors are obtained by solving the determinant resulting of Equation (19).

MATLAB software was used for solving the determinant; we obtain the polynomial and the roots. The results are presented in Table 3; twenty-four modes for the model 1 (M1), sixteen modes for the model 2 (M2) and four modes for model 3 (M3).

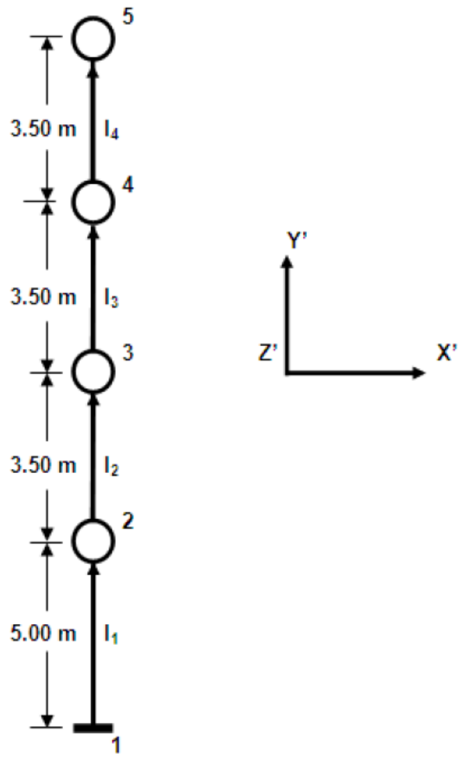
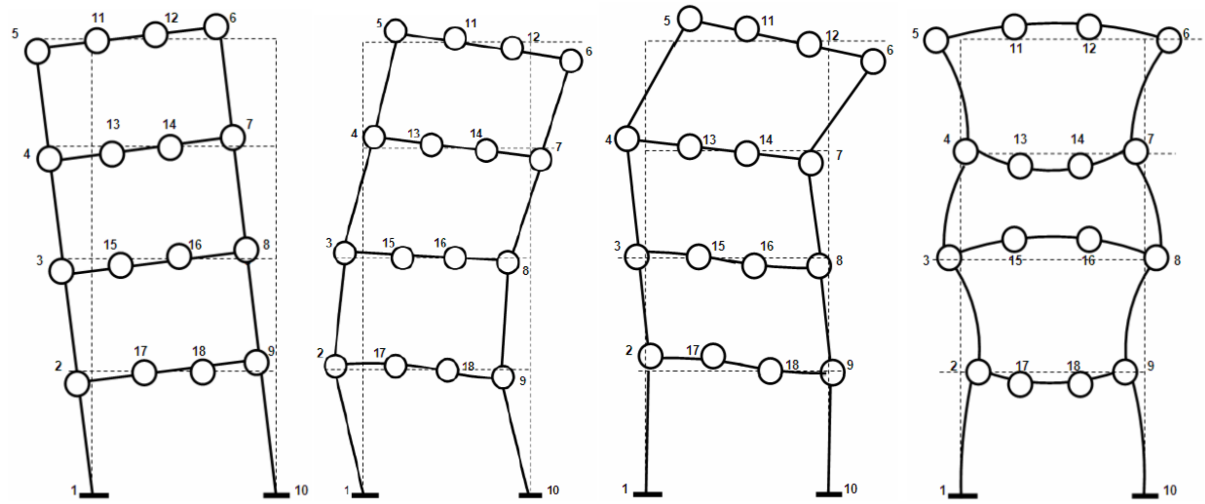


Figure 6. Vectorial model considers a lumped mass per level of the building



(a) (b) (c) (d)

Figure 7. Vibration modes: (a) first, (b) second, (c) third, (d) fourth

Spectral accelerations for the first four modes of the model 1 and 2, and the four modes of the model 3 are shown in Table 4.

The values of the first four modes of model 1 are presented in Table 5.

The first four modes of the model 1, as well as the building's configuration are shown in Figure 7.

The values of the first four modes of model 2 are presented in Table 6.

The first four modes of the model 2, as well as the building's configuration are shown in Figure 8.

Table 3. Eigenvalues

Mode	Circular frequency (rad/sec)			Frequency (Hz)			Period (sec)			ω_n^2 (rad/sec)^2		
	M1	M2	M3	M1	M2	M3	M1	M2	M3	M1	M2	M3
1	5.870	5.885	6.632	0.934	0.937	1.056	1.071	1.067	0.947	34.5	34.6	44.0
2	15.065	15.235	16.448	2.398	2.425	2.618	0.417	0.412	0.382	227.0	232.1	270.5
3	25.663	26.591	28.391	4.084	4.232	4.519	0.245	0.236	0.221	658.6	707.1	806.1
4	27.936	33.068	33.939	4.446	5.263	5.402	0.225	0.190	0.185	780.4	1093.5	1151.8
5	28.806	76.039		4.585	12.102		0.218	0.083		829.8	5782.0	
6	31.368	76.747		4.992	12.215		0.200	0.082		984.0	5890.1	
7	31.985	177.861		5.091	28.307		0.196	0.035		1023.0	31634.5	
8	32.652	178.149		5.197	28.353		0.192	0.035		1066.1	31737.0	
9	41.728	235.663		6.641	37.507		0.151	0.027		1741.2	55537.0	
10	45.092	253.323		7.177	40.318		0.139	0.025		2033.3	64172.6	
11	45.422	277.085		7.229	44.099		0.138	0.023		2063.1	76776.2	
12	46.746	280.879		7.440	44.703		0.134	0.022		2185.2	78892.8	
13	76.039	280.985		12.102	44.720		0.083	0.022		5782.0	78952.4	
14	80.702	289.667		12.844	46.102		0.078	0.022		6512.9	83906.8	
15	177.861	339.121		28.307	53.973		0.035	0.019		31634.5	115003.4	
16	179.221	339.220		28.524	53.988		0.035	0.019		32120.3	115069.9	
17	235.833			37.534			0.027			55617.1		
18	253.591			40.360			0.025			64308.4		
19	277.229			44.122			0.023			76856.2		
20	280.879			44.703			0.022			78892.8		
21	281.822			44.853			0.022			79423.8		
22	289.917			46.142			0.022			84051.9		
23	339.121			53.973			0.018			115003.4		
24	339.786			54.079			0.018			115454.2		

Table 4. Spectral acceleration

Mode	Frequency ω_n (Hz)			Acceleration S_{an} (cm/sec^2)		
	M1	M2	M3	M1	M2	M3
1	0.934	0.937	1.056	0.153g = 150.0417	0.153g = 150.0417	0.167g = 163.7711
2	2.398	2.425	2.618	0.292g = 286.3542	0.294g = 288.3155	0.300g = 294.1995
3	4.084	4.232	4.519	0.300g = 294.1995	0.300g = 294.1995	0.300g = 294.1995
4	4.446	5.263	5.402	0.300g = 294.1995	0.300g = 294.1995	0.300g = 294.1995

The 4 modes of the model 3, as well as the building's configuration are shown in Figure 9.

Equation (28) is used to obtain the modal participation factor "Γ_n". The maximum normal coordinates "$(Y_n)_{\max}$" of the system for each mode are located by Equation (33) and the first four values for the model 1 and 2, and the four values for the model 3, appear in Table 7.

The vectors corresponding to the components of the maximum relative displacements vector for each mode "$\{U_{1n}^r\}_{\max}$" are given by Equation (34) and finally, the maximum value of the relative displacements vector for the structural system of the building "$\{U_1^r\}_{\max}$" is obtained by Equation (35). These values for the three models appear in Table 8.

Once the deformations are obtained, Equation (37) is used to find the values of the forces in "X", the forces in "Y" and moments; these are applied at the free joints. Such

Table 5. Vibration modes of model 1

Joint	Mode											
	1			2			3			4		
	X	Y	θ	X	Y	θ	X	Y	θ	X	Y	θ
1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	−0.432	−0.003	0.028	−0.714	0.005	0.026	0.551	0.001	0.026	0.009	0.000	−0.700
3	−0.655	−0.005	0.018	−0.607	0.011	−0.038	−0.259	0.004	0.090	−0.002	0.000	0.889
4	−0.895	−0.006	0.011	0.336	0.018	−0.054	−0.797	0.019	−0.056	0.001	0.000	−0.784
5	−1.000	−0.006	0.006	1.000	0.021	−0.039	1.000	0.028	−0.140	−0.002	0.000	1.000
6	−1.000	0.006	0.006	1.000	−0.021	−0.039	1.000	−0.028	−0.140	0.002	0.000	−1.000
7	−0.895	0.006	0.011	0.336	−0.018	−0.054	−0.797	−0.019	−0.056	−0.001	0.000	0.784
8	−0.655	0.005	0.018	−0.607	−0.011	−0.038	−0.259	−0.004	0.090	0.002	0.000	−0.889
9	−0.432	0.003	0.028	−0.714	−0.005	0.026	0.551	−0.001	0.026	−0.009	0.000	0.700
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

X is displacement in direction X
Y is displacement in direction Y
θ is rotation in joint

Table 6. Vibration modes of model 2

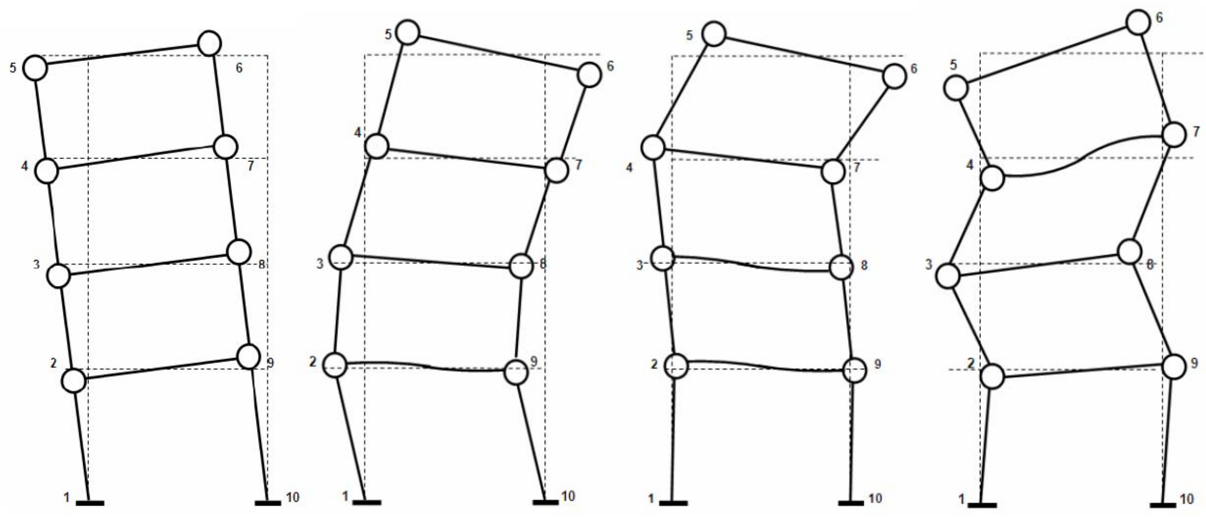
Joint	Mode											
	1			2			3			4		
	X	Y	θ	X	Y	θ	X	Y	θ	X	Y	θ
1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	−0.433	−0.003	0.027	−0.717	0.004	0.022	0.517	0.001	0.018	0.708	−0.008	0.062
3	−0.655	−0.005	0.018	−0.606	0.010	−0.034	−0.255	0.003	0.058	−1.000	−0.012	0.048
4	−0.895	−0.006	0.010	0.342	0.016	−0.048	−0.794	0.013	−0.038	0.696	−0.014	−0.018
5	−1.000	−0.006	0.006	1.000	0.018	−0.035	1.000	0.019	−0.094	−0.436	−0.019	0.063
6	−1.000	0.006	0.006	1.000	−0.018	−0.035	1.000	−0.019	−0.094	−0.436	0.019	0.063
7	−0.895	0.006	0.010	0.342	−0.016	−0.048	−0.794	−0.013	−0.038	0.696	0.014	−0.018
8	−0.655	0.005	0.018	−0.606	−0.010	−0.034	−0.255	−0.003	0.058	−1.000	0.012	0.048
9	−0.433	0.003	0.027	−0.717	−0.004	0.022	0.517	−0.001	0.018	0.708	0.008	0.062
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Table 7. The modal participation factor "Γ_n" and the maximum normal coordinates "$(Y_n)_{\max}$" of the system for each mode

Mode	Modal participation factor Γ_n			Maximum normal coordinates of the system for each mode $(Y_n)_{\max}$		
	M1	M2	M3	M1	M2	M3
1	−1.3174	−1.3245	+1.3132	−5.7294	−5.7436	+4.8878
2	−0.4096	−0.4162	+0.3819	−0.5167	−0.5170	+0.4154
3	+0.1329	+0.1293	−0.0971	+0.0594	+0.0538	−0.0354
4	0.0000	+0.0864	−0.0698	0.0000	+0.0232	−0.0178

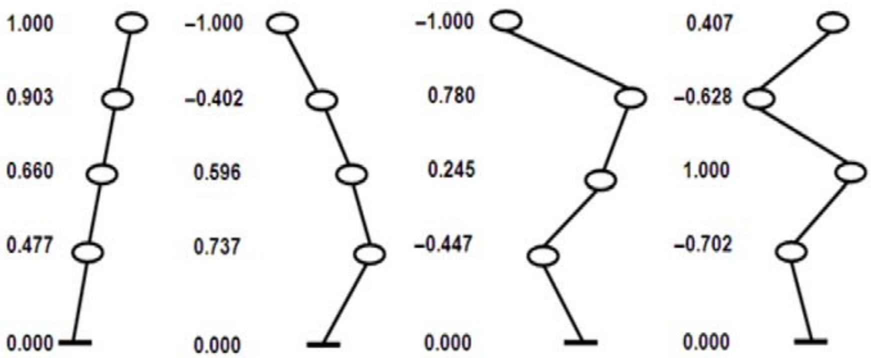
effects are equivalent to what would have occurred due to a movement in the soil where the building is located. Then, mechanical elements at the joints on the members of the whole building are determined by Equation (38) and subsequently these are obtained for each of the building's rigid frames. The axial forces, the shear forces and the moments for a central frame are presented in Figures 10-12 respectively.

4. **Results and Discussions.** The values of the frequencies of the vibration modes of the building for the three models appear in Table 2. It is observed that values for model



(a) (b) (c) (d)

FIGURE 8. Vibration modes: (a) first, (b) second, (c) third, (d) fourth



(a) (b) (c) (d)

FIGURE 9. Vibration modes: (a) first, (b) second, (c) third, (d) fourth

1 are lower with respect to the other two models in terms of frequency and logically the periods are inverses. The first four modes of model 1 and 2 are presented in this table, but the work was developed with modes twenty-four of the model 1 and sixteen modes of the model 2 resulting from the dynamic analysis.

Table 3 shows spectral acceleration. These values are obtained from the frequency of each of the structure's vibration modes and these results are found by means of the horizontal response spectrum of the soil where the building is constructed, this spectrum is presented in Figure 3.

The participation factors and the maximum normal coordinates of the system for each mode are observed in Table 6, all the values in the model 3 are lower in absolute value, with respect to the others two models, with exception in mode 4, where the model 1 is lower, since this value is zero.

Table 7 gives the structural system's relative deformations, also all the values are lower in model 3, with respect to the others two models. Now the model 1 and 2 are compared, the model 1 is lower in terms of the displacements in direction X and in direction Y is equal, in terms of rotations these are the same with the exception in joints 3 and 8, where the model 1 is greater than the model 2.

TABLE 8. Vector of deformations

Relative deformations	Joint	Concept	Unit	Amount		
				M1	M2	M3
$U_1^r 1$	2	Displacement X	cm	2.51	2.52	2.35
$U_1^r 2$		Displacement Y	cm	0.02	0.02	0.00
$U_1^r 3$		Rotation	rad	0.16	0.16	0.00
$U_1^r 4$	3	Displacement X	cm	3.78	3.78	3.23
$U_1^r 5$		Displacement Y	cm	0.03	0.03	0.00
$U_1^r 6$		Rotation	rad	0.11	0.10	0.00
$U_1^r 7$	4	Displacement X	cm	5.14	5.16	4.41
$U_1^r 8$		Displacement Y	cm	0.04	0.04	0.00
$U_1^r 9$		Rotation	rad	0.07	0.07	0.00
$U_1^r 10$	5	Displacement X	cm	5.77	5.78	4.49
$U_1^r 11$		Displacement Y	cm	0.04	0.04	0.00
$U_1^r 12$		Rotation	rad	0.04	0.04	0.00
$U_1^r 13$	6	Displacement X	cm	5.77	5.78	4.49
$U_1^r 14$		Displacement Y	cm	0.04	0.04	0.00
$U_1^r 15$		Rotation	rad	0.04	0.04	0.00
$U_1^r 16$	7	Displacement X	cm	5.14	5.16	4.41
$U_1^r 17$		Displacement Y	cm	0.04	0.04	0.00
$U_1^r 18$		Rotation	rad	0.07	0.07	0.00
$U_1^r 19$	8	Displacement X	cm	3.78	3.78	3.23
$U_1^r 20$		Displacement Y	cm	0.03	0.03	0.00
$U_1^r 21$		Rotation	rad	0.11	0.10	0.00
$U_1^r 22$	9	Displacement X	cm	2.51	2.52	2.35
$U_1^r 23$		Displacement Y	cm	0.02	0.02	0.00
$U_1^r 24$		Rotation	rad	0.16	0.16	0.00

Figure 10 shows the axial forces of the structure, all the values in model 3 are greater in absolute value with respect to the others two models. Now the model 1 and 2 are compared, the model 1 is greater, there is an increase in model 1, up to of 7.5% in members 4 and 5 with respect to model 2, this only occurs in the columns, and the axial force is not presented in beams in model 1 and 2, and in model 3, if values exist, in this case are exceeded designs.

The shear forces are presented in Figure 11, also all the values in model 3 are greater with respect to the others two models. Now the model 1 and 2 are compared, the model 1 is greater in all beams, there is an increase in model 1, up to of 7.7% with respect to model 2 in member 9, and in columns practically are equals.

The moments acting on the members of the structure are shown in Figure 12, also all the values in model 3 are greater with respect to the others two models, with the exception the member 9 in where the model 1 is greater. Now the model 1 and 2 are compared, the model 1 is greater in all beams, there is an increase in model 3, up to of 7.7% with respect to model 2 in member 9, and in the columns practically are equals.

5. **Conclusions.** Through the application of three different models it is possible to conclude the following: in terms of model 3 taking into account four degrees of freedom per floor, i.e., the horizontal displacement at each level, and the models 1 and 2 taking into account twenty-four degrees of freedom, i.e., the horizontal displacement, vertical displacement and rotation in each joints are not restricted. Now with respect to the model

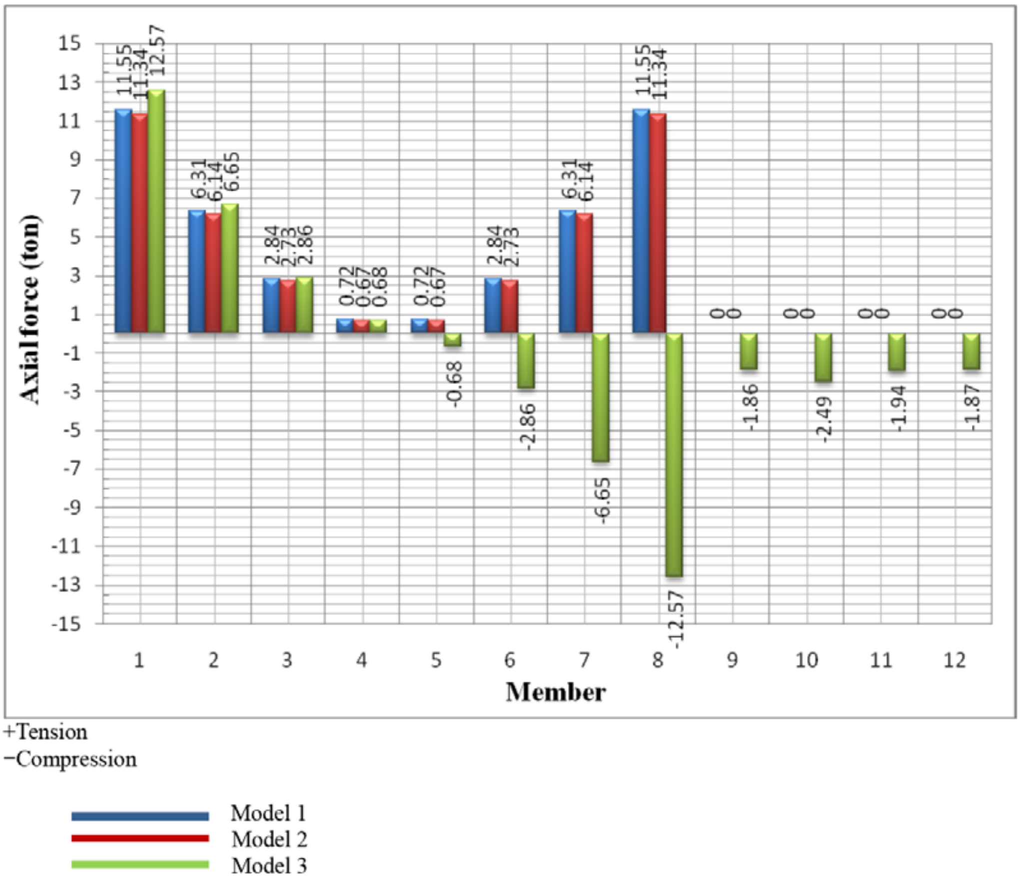


Figure 10. Axial force

3 considering four vibration modes, the model 2 takes into account sixteen modes and model 1 considers twenty-four modes. According to the above, we are observed that in model 3 are not taken into account several degrees of freedom, which are reflected in the response of the system and not of the conservative side. On the other hand, when the frequencies analysis is realized, these demonstrate that considering the models 2 and 3, beforehand implies not to consider certain modal shapes (symmetrical modes and/or anti-symmetrical) of the structure, which in the case of soil excitations are present in certain situations and these must be considered, since in some cases correspond to relatively low frequencies.

Finally, it is observed that the differences, between the model 1 (proposed model) and the model 3 in the last are greater in all members, with the exception the member 9 in which the model 1 is greater. Now the models 1 and 2 are compared, the model 1 is greater in all beams, with respect to model 2, and in the columns practically are equals, as it is presented in the tables and figures of the problem considered. Therefore, the general practice to consider one and two masses lumped at each level will not be a recommendable solution, since the model 1 is much more economical in relation to the model 3 and model 2 is not very secure because it ignores modal shapes several, that model 1 if we take into account, as the slogan of civil engineering, that is safe and economic. Then, the model 1 (proposed model) in this paper is the more appropriate for the seismic analysis of structural systems in buildings.

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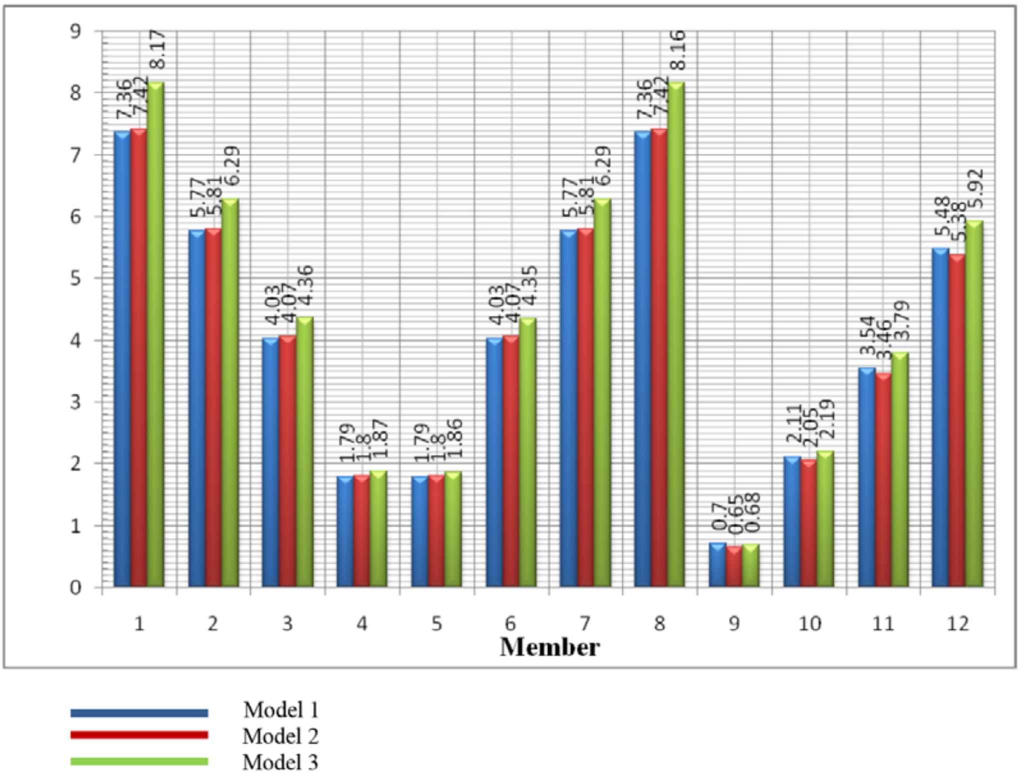

FIGURE 11. Shear force

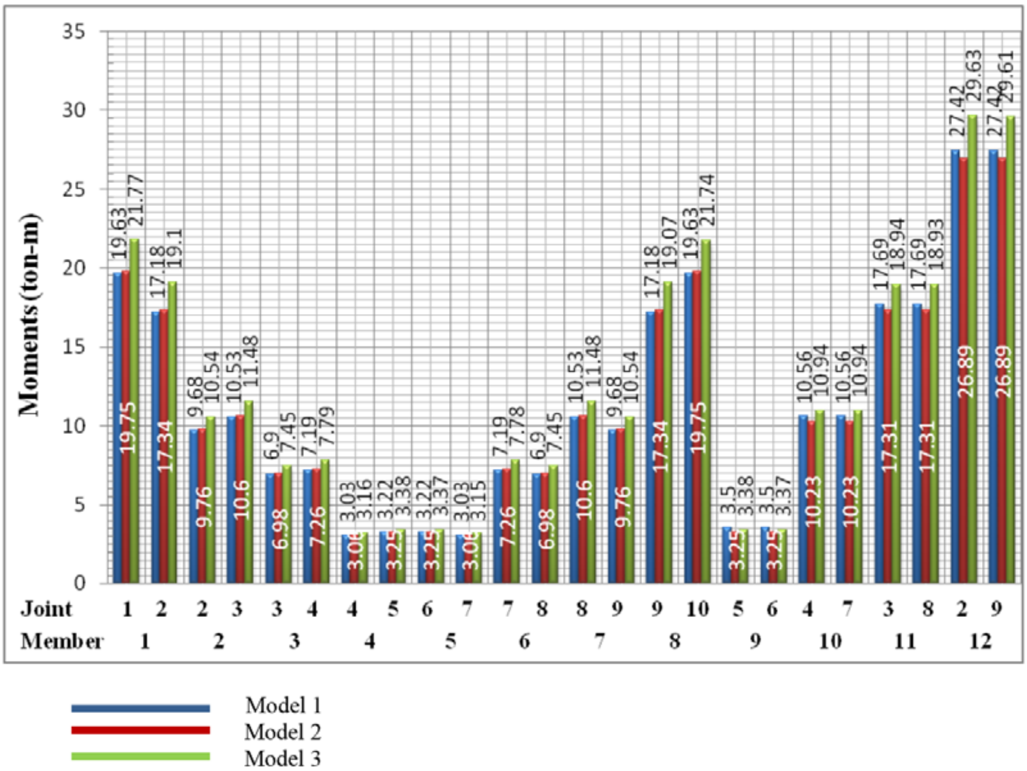

FIGURE 12. Moments

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