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MODIFIED VIRTUAL REFERENCE FEEDBACK TUNING AND ITS APPLICATION TO ULTRASONIC MOTORS

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Abstract. *This paper proposes a virtual reference feedback tuning (VRFT) method that can compensate for a dead zone. To this end, first, by focusing on the control error, we modify the performance index such that it is minimized in VRFT. Then, we present analysis results pertaining to the optimality conditions and their related filters in the modified VRFT method. Next, a VRFT method with dead-zone compensation is proposed by combining the modified VRFT method and a dead-zone compensation technique. The proposed VRFT method with dead-zone compensation is applied to an experimental ultrasonic motor control system, and its effectiveness is verified.*
Keywords: Controller parameter tuning, Virtual reference feedback tuning (VRFT), Dead-zone, Ultrasonic motor

1. **Introduction.** Ultrasonic motors (USMs) have excellent features such as compact size, low weight, no driving sound, high torque even at low speeds, and high holding torque. In addition, theoretically, USMs do not generate electromagnetic noise and are not influenced by electromagnetic fields. Given their electromagnetic compatibility, USMs show promise for use in the development of nursing aids and surgery robots that can work safely near patients with pacemakers and can work reliably under magnetic resonance imaging devices. However, USMs cannot be modeled easily from physical analysis because of their friction force. Furthermore, USMs have a dead zone that is sensitive to temperature and load changes, thus leading to the deterioration of control performance. Therefore, a simple and effective controller tuning method is desirable.

Many types of USM control methods have been investigated thus far. However, as stated above, model-based control methods are not practical because it is difficult to model USMs. Moreover, although a fuzzy-based control method [1] and a particle swarm optimization (PSO)-based control method [2] have been proposed as model-free control methods for USMs, these methods require iterative experiments or periodic reference signals, thus resulting in an extended controller tuning duration.

Direct controller parameter tuning methods have received considerable attention as alternative model-free controller tuning schemes in the past decade. Iterative feedback tuning (IFT) [3], which was the first such proposed method, requires iterative experiments. In contrast, virtual reference feedback tuning (VRFT) [4] and fictitious reference iterative tuning (FRIT) [5, 6] are based on input and output data for noniterative experiments, which means that these methods have greater practicality than IFT. Furthermore, VRFT and FRIT have been analyzed theoretically from the viewpoint of optimality [4, 7]. The results of these analyses point out that VRFT and FRIT are suitable for controlling USMs; however, because these methods were developed for linear systems, they do not often perform well with systems having a dead zone such as USMs. To cope with the dead-zone nonlinearity, an FRIT scheme with dead-zone compensation was proposed and its effectiveness was verified [8]. VRFT is known to provide control performance comparable to that of FRIT. However, VRFT with dead-zone compensation has not been fully investigated, and it is unclear whether VRFT based on the same technique as in [8] would afford good control performance.

In this paper, we propose a VRFT scheme that compensates for dead zones. To this end, we first modify the performance index to be minimized in VRFT by focusing on the control error. The standard VRFT method focuses on control input, whereas the standard FRIT method focuses on control output. Therefore, the abovementioned modified VRFT framework is different from the existing direct controller parameter tuning methods. We present the optimality conditions and their related filters for the modified VRFT method. Next, we propose a VRFT method with dead-zone compensation by combining the modified VRFT method and the dead-zone compensation technique proposed in [8]. The proposed VRFT method with dead-zone compensation is then applied to an experimental USM control system, and its effectiveness is verified.

2. **Modified VRFT.** We consider a closed-loop system configuration, shown in Figure 1, where the plant described by $G(z)$ is assumed to be a linear single-input and single-output discrete-time system and the controller described by $C(z,\boldsymbol{\theta})$ is assumed to be parameterized as follows:

$$C(z,\boldsymbol{\theta}) = \frac{\boldsymbol{\beta}^a(z)^T\boldsymbol{\theta}^a}{\boldsymbol{\beta}^b(z)^T\boldsymbol{\theta}^b} = \frac{\sum_{i=1}^{n_a}\theta_i^a\beta_i^a(z)}{\sum_{i=1}^{n_b}\theta_i^b\beta_i^b(z)}, \tag{1}$$

where $\boldsymbol{\beta} = [\boldsymbol{\beta}^a(z)^T, \boldsymbol{\beta}^b(z)^T]^T$ is a known (n_a+n_b)-dimensional vector of linear discrete time rational transfer functions and $\boldsymbol{\theta} = [(\boldsymbol{\theta}^a)^T, (\boldsymbol{\theta}^b)^T]^T \in \mathbb{R}^{n_a+n_b}$ is a tunable controller parameter vector. In the figure, $u(k)$, $y(k)$, $r(k)$ and $e(k)$ denote the control input, control output, reference signal, and control error, respectively.

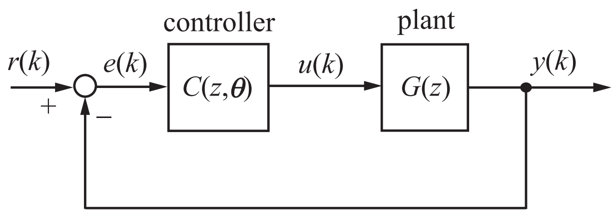

FIGURE 1. Standard feedback control system

Although there are many expressions of control objectives, one natural objective is to find a parameter by minimizing the following performance index:

$$J^N(\boldsymbol{\theta}) = \frac{1}{N}\sum_{k=1}^{N}(T(z,\boldsymbol{\theta})r(k) - M(z)r(k))^2,$$

where $T(z,\boldsymbol{\theta}) = G(z)C(z,\boldsymbol{\theta})/(1+G(z)C(z,\boldsymbol{\theta}))$ is the closed-loop transfer function and $M(z)$ is a reference model. VRFT is a parameter tuning method for approximately achieving this goal. In the standard VRFT method, we first calculate a virtual reference signal $\bar{r}(k)$ such that $y_0(k) = M(z)\bar{r}(k)$ and then find an optimal parameter by minimizing the performance index as follows:

$$J_{\mathrm{V}}^N(\boldsymbol{\theta}) = \frac{1}{N}\sum_{k=1}^{N}(u_0(k) - C(z,\boldsymbol{\theta})(\bar{r}(k) - y_0(k)))^2,$$

where $u_0(k)$, $y_0(k)$, $k = 1, \ldots, N$ are the (initial) input and output data from a one-shot experiment.

Instead of J_{V}^N, we now consider the following modified performance index:

$$J_{\mathrm{MV}}^N(\boldsymbol{\theta}) = \frac{1}{N}\sum_{k=1}^{N}(e_0(\boldsymbol{\theta}, k) - \bar{e}(k))^2,$$

where $e_0(\boldsymbol{\theta}, k) = C^{-1}(z, \boldsymbol{\theta})u_0(k)$ and $\bar{e}(k) = \bar{r}(k) - y_0(k)$. In VRFT, the performance index J_{V}^N is used for evaluating the difference between the ideal and actual control inputs. Similarly, the performance index used in FRIT [5] yields the difference between the ideal and actual control outputs. In contrast, J_{MV}^N focuses on the difference between the ideal and actual control errors. Therefore, the minimization of J_{MV}^N affords a novel controller-tuning framework from among several direct controller-tuning methods. We refer to this J_{MV}^N-based controller parameter tuning as "modified VRFT".

3. **Analysis of Optimality for Modified VRFT.** In [4, 7], the optimality of VRFT and FRIT is investigated and the related filter conditions are presented. In this section, we present the results related to the optimality and filter conditions of the modified VRFT by applying the techniques used in [4, 7].

For analysis, we introduce an ideal controller $C_d(z)$ that satisfies the following condition:

$$M(z) = \frac{G(z)C_d(z)}{1 + G(z)C_d(z)}.$$

Furthermore, we use a filter $F(z)$ for the input and output data and consider the following performance index:

$$J_{\mathrm{MVF}}^N(\boldsymbol{\theta}) = \frac{1}{N}\sum_{k=1}^{N}(e_0^F(\boldsymbol{\theta}, k) - \bar{e}^F(k))^2,$$

where $e_0^F(\boldsymbol{\theta}, k) = C^{-1}(z, \boldsymbol{\theta})F(z)u_0(k)$ and $\bar{e}^F(k) = \bar{r}(k) - F(z)y_0(k)$. By denoting a minimal point of $J^N(\boldsymbol{\theta})$ as $\boldsymbol{\theta}^* = [(\boldsymbol{\theta}^{a*})^T, (\boldsymbol{\theta}^{b*})^T]^T$, we define $\Delta C^a(z)$ and $\Delta C^b(z)$ such that

$$C_d(z) = \frac{\boldsymbol{\beta}^a(z)^T\boldsymbol{\theta}^{a*} + \Delta C^a(z)}{\boldsymbol{\beta}^b(z)^T\boldsymbol{\theta}^{b*} + \Delta C^b(z)}.$$

Additionally, we introduce the following extended family of controllers

$$C^+(z, \boldsymbol{\theta}^+) = \frac{\boldsymbol{\beta}^{a+}(z)^T\boldsymbol{\theta}^{a+}}{\boldsymbol{\beta}^{b+}(z)^T\boldsymbol{\theta}^{b+}},$$

where $\boldsymbol{\beta}^{a+}(z) = [\boldsymbol{\beta}^a(z)^T, \Delta C^a(z)]^T$, $\boldsymbol{\beta}^{b+}(z) = [\boldsymbol{\beta}^b(z)^T, \Delta C^b(z)]^T$, $\boldsymbol{\theta}^+ = [(\boldsymbol{\theta}^{a+})^T, (\boldsymbol{\theta}^{b+})^T]^T$, $\boldsymbol{\theta}^{a+} = [(\boldsymbol{\theta}^a)^T, \theta^a_{n_a+1}]^T$ and $\boldsymbol{\theta}^{b+} = [(\boldsymbol{\theta}^b)^T, \theta^b_{n_b+1}]^T$. Using the extended controller $C^+(z, \boldsymbol{\theta}^+)$, we consider the extended performance index

$$J^{N+}(\boldsymbol{\theta}^+) = \frac{1}{N}\sum_{k=1}^{N}\left(\frac{G(z)C^+(z, \boldsymbol{\theta}^+)}{1 + G(z)C^+(z, \boldsymbol{\theta}^+)}r(k) - M(z)r(k)\right)^2.$$

Note that $\boldsymbol{\theta}^{+*} = [(\boldsymbol{\theta}^{a+*})^T, (\boldsymbol{\theta}^{b+*})^T]^T = [[(\boldsymbol{\theta}^{a*})^T, 1], [(\boldsymbol{\theta}^{b*})^T, 1]]^T$ is a global minimizer of $J^{N+}(\boldsymbol{\theta}^+)$, and $C_d(z) = C^+(z, \boldsymbol{\theta}^{+*})$ holds.

As in [4, 7], we assume that the measured signals can be considered as realizations of stationary and ergodic stochastic processes when $N \to \infty$. Then, we can obtain the frequency-domain representations of $J_{\mathrm{MVF}}^N(\boldsymbol{\theta})$ and $J^{N+}(\boldsymbol{\theta}^+)$ as follows:

$$J_{\mathrm{MVF}}(\boldsymbol{\theta}) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\left(1 - \frac{C_d(e^{j\omega})}{C(e^{j\omega}, \boldsymbol{\theta})}\right)\frac{1 - M(e^{j\omega})}{M(e^{j\omega})}F(e^{j\omega})\right|^2\Phi_{y_0}d\omega,$$

$$J^+(\boldsymbol{\theta}^+) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\frac{G(e^{j\omega})C^+(e^{j\omega},\boldsymbol{\theta}^+)}{1+G(e^{j\omega})C^+(e^{j\omega},\boldsymbol{\theta}^+)} - M(e^{j\omega})\right|^2 \Phi_r d\omega,$$

where Φ_{y_0} and Φ_r are the spectral densities of $y_0(k)$ and $r(k)$, respectively.

The second-order Taylor expansion of $J^+(\boldsymbol{\theta}^+)$ around its global minimizer $\boldsymbol{\theta}^{+*}$ can be represented in two ways:

$$\hat{J}^+(\boldsymbol{\theta}^+) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\left(1-\frac{C^+(e^{j\omega},\boldsymbol{\theta}^+)}{C_d(e^{j\omega})}\right)M(e^{j\omega})(1-M(e^{j\omega}))\frac{\boldsymbol{\beta}^{b+}(e^{j\omega})^T\boldsymbol{\theta}^{b+}}{\boldsymbol{\beta}^{b+}(e^{j\omega})^T\boldsymbol{\theta}^{b+*}}\right|^2 \Phi_r d\omega,$$

and

$$\hat{J}^+(\boldsymbol{\theta}^+) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\left(\frac{C_d(e^{j\omega})}{C^+(e^{j\omega},\boldsymbol{\theta}^+)}-1\right)M(e^{j\omega})(1-M(e^{j\omega}))\frac{\boldsymbol{\beta}^{a+}(e^{j\omega})^T\boldsymbol{\theta}^{a+}}{\boldsymbol{\beta}^{a+}(e^{j\omega})^T\boldsymbol{\theta}^{a+*}}\right|^2 \Phi_r d\omega.$$

Comparing $\hat{J}^+$ and J_{MVF}, we obtain the following theorem.

Theorem 3.1. *Suppose that the controller is parameterized in (1). Then, the relationship*

$$\arg\min_{\boldsymbol{\theta}} J_{\mathrm{MVF}}(\boldsymbol{\theta}) = \arg\min_{\boldsymbol{\theta}} \hat{J}^+([[(\boldsymbol{\theta}^a)^T, 0], [(\boldsymbol{\theta}^b)^T, 0]]^T)$$

holds if $F(z)$ satisfies one of the following two conditions:

$$|F(e^{j\omega})|^2 = \left|M^2(e^{j\omega})\frac{\boldsymbol{\beta}^{a+}(e^{j\omega})^T\boldsymbol{\beta}^{a+}}{\boldsymbol{\beta}^{a+}(e^{j\omega})^T\boldsymbol{\beta}^{a+*}}\right|^2 \frac{\Phi_r}{\Phi_{y_0}},$$

$$|F(e^{j\omega})|^2 = \left|(1-M(e^{j\omega}))M(e^{j\omega})C(e^{j\omega},\boldsymbol{\theta})\frac{\boldsymbol{\beta}^{b+}(e^{j\omega})^T\boldsymbol{\beta}^{b+}}{\boldsymbol{\beta}^{b+}(e^{j\omega})^T\boldsymbol{\beta}^{b+*}}\right|^2 \frac{\Phi_r}{\Phi_{u_0}}.$$

From the above theorem, we obtain the following theorems for two special cases wherein the controller or its inverse is linearly parameterized. We first consider the case when the controller is expressed as $C(z,\boldsymbol{\theta}) = \boldsymbol{\beta}^a(z)^T\boldsymbol{\theta}^a$. In the same manner as that discussed above, we introduce the extended performance index $J^{a+}(\boldsymbol{\theta}^{a+})$ and define its second-order approximation $\hat{J}^{a+}(\boldsymbol{\theta}^{a+})$. Then, we obtain the following theorem.

Theorem 3.2. *Suppose that the controller is linearly parameterized as given by $C(z,\boldsymbol{\theta}) = \boldsymbol{\beta}^a(z)^T\boldsymbol{\theta}^a$. Then, the relationship*

$$\arg\min_{\boldsymbol{\theta}^a} J_{\mathrm{MVF}}(\boldsymbol{\theta}^a) = \arg\min_{\boldsymbol{\theta}^a} \hat{J}^{a+}([(\boldsymbol{\theta}^a)^T, 0]^T)$$

holds if $F(z)$ satisfies the following condition:

$$|F(e^{j\omega})|^2 = \left|1-M(e^{j\omega})C(e^{j\omega},\boldsymbol{\theta})\right|^2 \frac{\Phi_r}{\Phi_{u_0}}.$$

When the controller is expressed as $C(z,\boldsymbol{\theta}) = 1/(\boldsymbol{\beta}^b(z)^T\boldsymbol{\theta}^b)$, we obtain the following theorem. The notations here are the same as those in the above mentioned case.

Theorem 3.3. *Suppose that the inverse of the controller is linearly parameterized as given by $C(z,\boldsymbol{\theta}) = 1/(\boldsymbol{\beta}^b(z)^T\boldsymbol{\theta}^b)$. Then, the relationship*

$$\arg\min_{\boldsymbol{\theta}^b} J_{\mathrm{MVF}}(\boldsymbol{\theta}^b) = \arg\min_{\boldsymbol{\theta}^b} \hat{J}^{b+}([(\boldsymbol{\theta}^b)^T, 0]^T)$$

holds if $F(z)$ satisfies the following condition:

$$|F(e^{j\omega})|^2 = \left|M(e^{j\omega})\right|^4 \frac{\Phi_r}{\Phi_{y_0}}.$$

The above results show that the modified VRFT scheme with appropriate filters yields the minimizer for the restricted second-order approximation of the original performance index. This suggests that we can obtain a reasonable controller parameter tuning result by using the modified VRFT as in the standard VRFT and FRIT [4, 7].

4. **Modified VRFT with Dead-Zone Compensation.** Herein, we propose a controller tuning method by combining the modified VRFT method with a dead-zone compensation technique such that systems with a dead zone, such as ultrasonic motors, can be controlled more precisely.

We consider a closed-loop system configuration, as shown in Figure 2, where the plant is assumed to consist of a dead-zone property and a linear system $G(z)$ connected in series, and the overall controller consists of a proportional-integral-derivative (PID) controller $C(z, \boldsymbol{\theta})$ and a dead-zone compensator. The transfer function of the PID controller can be expressed as follows:

$$C(z, \boldsymbol{\theta}) = \frac{K_P(1 - z^{-1}) + K_I + K_D(1 - z^{-1})^2}{1 - z^{-1}},$$

where K_P, K_I and K_D are the proportional, integral, and derivative gains, respectively, and $\boldsymbol{\theta} = [K_P, K_I, K_D]^T$ contains the PID gains to be tuned. Although in this setup, we consider a PID controller as the typical controller, we can use parameterized controllers with other structures, as described in the previous sections.

The basic system configuration is explained as follows [2, 8]. If the inverse of a dead-zone property is used as the dead-zone compensator, the dead-zone property in the plant is neutralized by the dead-zone compensator. In such an ideal case, only the linear system G must be controlled appropriately using the controller C, thereby resulting in a simple but effective control method.

To realize the above mentioned situation, we consider the following dead-zone function D_a representing a dead-zone property:

$$D_a(u) = \begin{cases} u + a & u < -a \\ 0 & -a \leq u \leq a \\ u - a & u > a \end{cases}$$

where a is a dead-zone parameter that shows that $[-a, a]$ $(a > 0)$ is a dead-zone interval. For the dead zone D_a, we define the following function.

$$\hat{D}_a(\hat{u}) = \begin{cases} \hat{u} - a & \hat{u} < 0 \\ 0 & \hat{u} = 0 \\ \hat{u} + a & \hat{u} > 0 \end{cases}$$

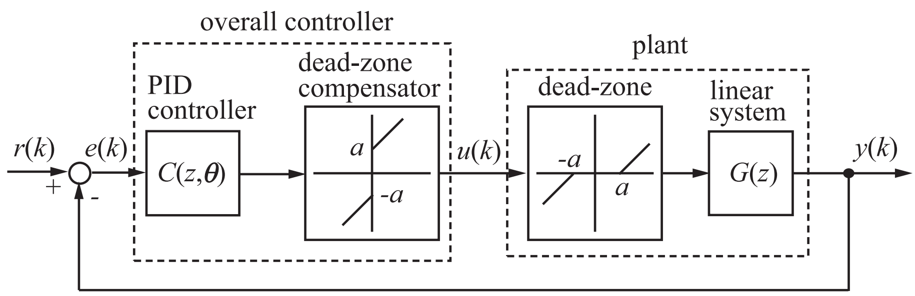

Figure 2. System configuration with dead-zone compensation

The function $\hat{D}_a$ is a right inverse function of D_a because $\hat{u} = D_a(\hat{D}_a(\hat{u}))$ holds for any $\hat{u} \in \Re$. Therefore, using $\hat{D}_a$, we compensate for the dead zone in the plant.

In [8], the above mentioned dead-zone compensation technique is combined with FRIT and its effectiveness is demonstrated. When the same technique is applied to the standard VRFT, the resultant performance index is as follows:

$$J_{\mathrm{VD}}^N(\boldsymbol{\theta}) = \frac{1}{N}\sum_{k=1}^{N}(u_0(k) - \hat{D}_a(C(z,\boldsymbol{\theta})(\bar{r}(k) - y_0(k))))^2.$$

However, the minimization of this function is slightly difficult because the output signal of $\hat{D}_a$ cannot take the values in $[-a, a]$. Therefore, even with dead-zone compensation, the standard VRFT technique cannot yield good tuning results.

To overcome this difficulty, we apply the dead-zone compensation technique to the modified VRFT method. In this case, the performance index is represented by

$$J_{\mathrm{MVD}}^N(\boldsymbol{x}) = \frac{1}{N}\sum_{k=1}^{N}(\hat{e}(\boldsymbol{x},k) - \bar{e}(k))^2, \tag{2}$$

where $\hat{e}(\boldsymbol{x},k) = C^{-1}(z,\boldsymbol{\theta})D_a(u_0(k))$, $\bar{e}(k) = \bar{r}(k) - y_0(k)$ and $\boldsymbol{x} = [\boldsymbol{\theta}^T, a]^T$.

We summarize the modified VRFT procedure with dead-zone compensation as follows.

Modified VRFT procedure with dead-zone compensation

Step 1: Set initial PID parameter $\boldsymbol{\theta}_0$, reference signal $r(k)$, for $k = 1, \ldots, N$, and reference model $M(z)$.

Step 2: From a closed-loop experiment that is performed without dead-zone compensation, obtain input data $u_0(k)$ and output data $y_0(k)$.

Step 3: Calculate control errors $\hat{e}(\boldsymbol{x},k)$ and $\bar{e}(k)$. Next, find the optimal (or suboptimal) parameter, $\boldsymbol{x}^*$, that minimizes the performance index (2) to obtain optimal (or suboptimal) PID parameter $\boldsymbol{\theta}^*$ and dead-zone parameter a^*.

Because the minimization problem in Step 3 is usually nonconvex, the use of stochastic multi-point search techniques is practical and effective for solving it. In this study, we use the covariance matrix adaptation evolution strategy (CMA-ES) algorithm [9] to solve this problem.

5. **Experimental Results.** In this section, we verify the effectiveness of the proposed modified VRFT method with dead-zone compensation (M-VRFT-D) by applying it to an experimental USM control system (Canon UA60), as shown in Figure 3. The USM, magnetic brake, and encoder are connected to the same shaft. The USM is driven using a phase difference scheme, and its rotation angle (i.e., the control output) is obtained using

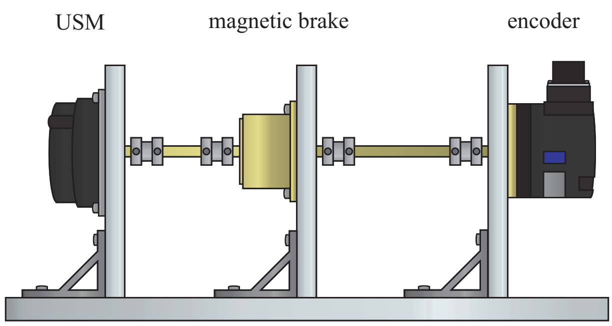

Figure 3. Experimental USM control system

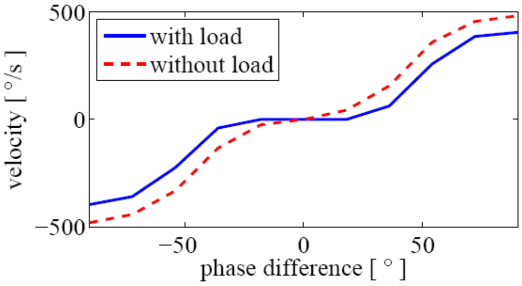

FIGURE 4. Dead-zone characteristics of USM

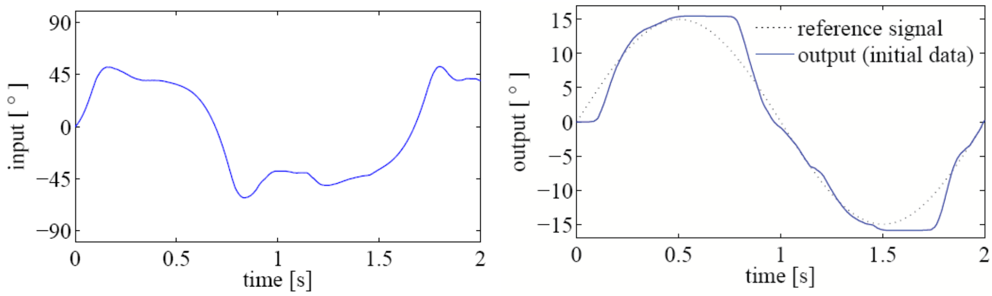

FIGURE 5. Initial control input and output

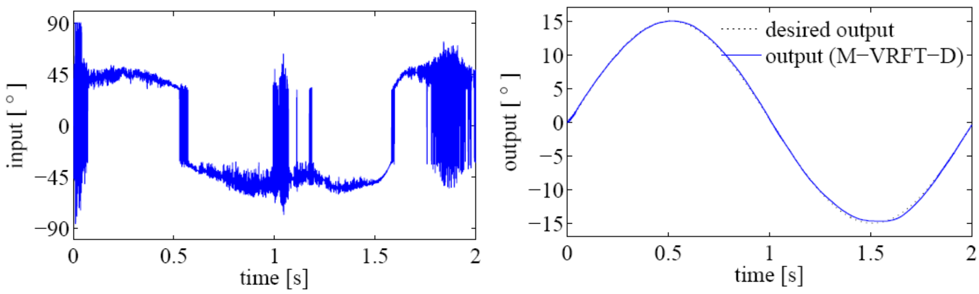

FIGURE 6. Control input and output using modified VRFT with dead-zone compensation

the encoder. The phase difference (i.e., the control input) can be adjusted between $-90°$ and $90°$ in steps of $1.406°$. A 0.1-N·m load can be generated using the electromagnetic brake. Figure 4 shows the USM velocity both with and without the load. This figure shows that the dead zone for the input changes depending on the presence of the load. In fact, the dead zone is sensitive to the load magnitude. Therefore, we should be able to compensate for the dead zone easily and achieve good control performance, particularly when it is either difficult or impossible to estimate the load magnitude.

5.1. **Typical case and detailed discussion.** We adopt a PID controller and tune the PID gains and dead-zone parameter according to the method presented in the previous section. We set initial PID gains as $\boldsymbol{\theta}_0 = [K_{\mathrm{P}}, K_{\mathrm{I}}, K_{\mathrm{D}}]^T = [4, 0.1, 1]^T$ and a reference model as $M(z) = (0.04z^2 + 0.003722z + 0.01086)/(z^2 - 1.637z + 0.6703)$ which is obtained by discretizing $(0.001s + 1)^2/(0.005s + 1)^2$ with a sampling time of 1 ms. The reference signal is a sinusoid with a magnitude of $15°$ and a period of 2 s. We set $N = 2000$ to evaluate the control performance within 2 s and impose the load. The resulting initial control input and output data are shown in Figure 5.

From a pre-experiment calculations, the search region is given by $\{\boldsymbol{x}|\boldsymbol{x}_{\mathrm{lb}} \leq \boldsymbol{x} \leq \boldsymbol{x}_{\mathrm{ub}}\}$ with $\boldsymbol{x}_{\mathrm{lb}} = [0, 0, 0, 0]^T$ and $\boldsymbol{x}_{\mathrm{ub}} = [400, 4, 2000, 90]^T$. In the CMA-ES algorithm, the

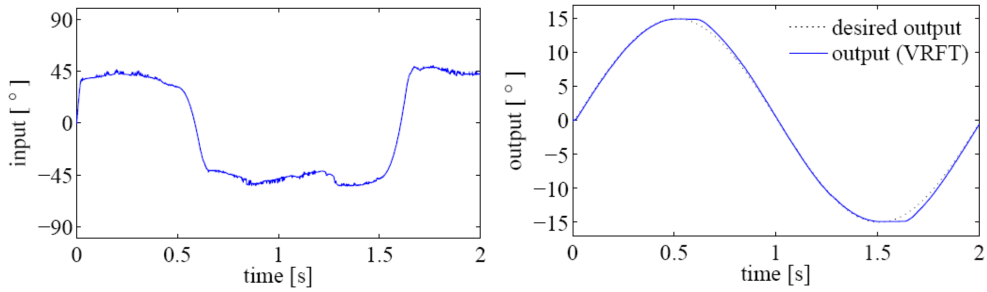

FIGURE 7. Control input and output using standard VRFT

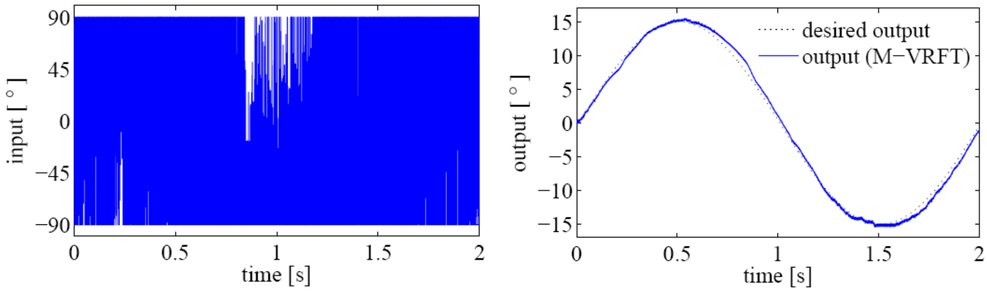

FIGURE 8. Control input and output using modified VRFT

maximum number of generations was set to 500 as the stopping criterion. The algorithm, which was programmed in MATLAB, was run on a computer with a 2.4 GHz Core 2 Duo CPU and 1024 MB RAM. The computation time was 41.8374 s. As a result, we obtained the PID gains $\boldsymbol{\theta}^* = [33.5975, 0.0274, 175.7096]^T$ and dead-zone parameter $a^* = 30.9896$. The control input and output are shown in Figure 6.

For comparison, we executed the standard and modified VRFT methods without dead-zone compensation. Under the standard VRFT method without dead-zone compensation, we obtained PID gains of $\boldsymbol{\theta}^* = [57.6561, 0.1452, 0]^T$. Under the modified VRFT without dead-zone compensation, we obtained PID gains of $\boldsymbol{\theta}^* = [103.9016, 0.1760, 2000]^T$. The control inputs and outputs are shown in Figures 7 and 8, respectively. In these figures, VRFT and M-VRFT denote the standard VRFT and the modified VRFT methods, respectively. These figures show that the modified VRFT method with dead-zone compensation affords the best control performance, especially at low speeds.

5.2. **Comparison of control performance under various conditions.** To show the effectiveness of the proposed VRFT method for various cases, we carried out experiments by changing some of the conditions used in the previous subsection. The control performance is evaluated using the performance index $J = \sum_{k=1}^{N}(y(k) - M(z)r(k))^2$. For the initial experiment, we use $J_0^1 = \sum_{k=1}^{N}(y_0(k) - r(k))^2$ and $J_0^2 = \sum_{k=1}^{N}(y_0(k) - M(z)r(k))^2$. We refer to the case in the previous subsection as "Case 1". In all other cases, the conditions that we changed from Case 1 are as follows.

Case 2: Coefficient of the denominator in the reference model is changed to 0.0025.
Case 3: Coefficient of the denominator in the reference model is changed to 0.01.
Case 4: Amplitude of the reference signal is changed to 10°.
Case 5: Amplitude of the reference signal is changed to 20°.
Case 6: Initial PID gains are changed to $\boldsymbol{\theta}_0 = [2, 0.1, 1]^T$.
Case 7: Initial PID gains are changed to $\boldsymbol{\theta}_0 = [6, 0.1, 1]^T$.

In Table 1, we list the performance index values for Cases 1-7. Although it might be slightly difficult to see from Figures 6-8 that the modified VRFT method with dead-zone

Table 1. Control performance

	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7
Initial exp. (J_0^1)	8151.0	8151.0	8151.0	7374.4	10591.0	1386.7	9780.0
Initial exp. (J_0^2)	7252.1	7743.5	6656.5	6553.9	9429.6	1326.0	8481.0
VRFT	326.2	**22.5**	5786.7	118.3	633.8	650.4	674.0
M-VRFT	780.3	181.6	389.1	338.7	1006.9	132.0	325.0
M-VRFT-D	**82.2**	124.2	**142.7**	**49.6**	**139.0**	**30.2**	**62.4**

compensation is superior to the other methods, the same is clear in Table 1, with Case 2 being the only exception.

6. **Conclusion.** In this paper, we proposed a modified VRFT method and presented the related optimality conditions. In addition, we proposed a VRFT method with dead-zone compensation by combining the modified VRFT method and a dead-zone compensation technique. Finally, we applied it to an experimental USM control system and verified its effectiveness. The main contributions of this paper are summarized as follows.

- A novel framework for direct controller parameter tuning was developed as the modified VRFT, and its effectiveness was verified through optimality analysis.
- It was shown that the modified VRFT method can be used effectively by combining it with dead-zone compensation.

Because many motors and actuators, which are not limited to USMs, have intrinsic dead-zone properties, the proposed method is widely applicable and effective for controlling such devices. Moreover, the dead-zone compensation technique used in this paper could be extended to other nonlinearities such as hysteresis and saturation, which we will pursue in future works.

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