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IMPULSIVE CONTROL AND SYNCHRONIZATION ANALYSIS OF COMPLEX DYNAMICAL NETWORKS WITH NON-DELAYED AND DELAYED COUPLING

Jianeng Tang and Chengti Huang

College of Engineering
Huaqiao University
No. 269, Chenghua North Rd., Fengze Dist., Quanzhou 362021, P. R. China
tangjianeng@sina.com; qzhct@163.com

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Abstract. *In the paper, the synchronization stability of a class of complex dynamical networks with non-delayed and delayed coupling is investigated. By establishing impulsive delay differential inequalities, sufficient conditions on the asymptotical stability of complex dynamical networks are derived. Some impulsive controllers are explicitly designed not only to achieve asymptotically stable for the complex networks based on the analysis, but also to ensure the convergence rate of complex networks. In the end, some numerical examples are also given to illustrate the effectiveness of the proposed method.*
Keywords: Impulsive control, Synchronization, Complex dynamical networks, Non-delayed and delayed coupling, Differential inequalities

1. **Introduction.** Complex networks have been paid much attention to since they are shown to widely exist in various fields of real world [1,2]. A complex network is a large set of interconnected nodes, in which a node is a fundamental unit with specific contents. Network topology structure provides a powerful metaphor for describing sophisticated collaborative dynamics of many practical systems in essence with the dynamical units regarded as nodes and the interplay between them expressed by the links of nodes. Examples of complex networks include World Wide Web (WWW), the brain, food webs, social networks, telecommunication networks, etc. [3-8].

Stabilization problem of complex networks with time delay is receiving much attention. The existence of a delay in complex networks may induce more complex dynamical behaviors such as instability, oscillations, and chaos. Wu et al. [9] investigated the problem of master-slave synchronization for neural networks with discrete and distributed delays, and proposed an improved method to capture the characteristic of sampled-data systems. Some important control methods have been developed for stabilizing dynamic systems without delay or with delay, which include linear feedback control [10], adaptive control [11], fuzzy control [12], variable structure control [13], asynchronously switched control [14], hybrid feedback control [15], etc. Impulsive control is also an important control method. As is known, impulsive control is characterized by the abrupt changes in the system dynamics at certain instants. And impulsive control has an advantage in reducing the amount of information transmission and improving the security and robustness against disturbances especially in telecommunication networks and power grid, orbital transfer of satellite [16-22]. In some cases, the scheme of impulsive control cannot be substituted by continuous control. By employing the stability theory of impulsive delayed differential equations, a unified approach was proposed for impulsive lag-synchronization of a class of chaotic systems with time delay [23]; sufficient conditions for impulsive synchronization with a bound on the synchronization error were derived [24]. In addition, impulsive control has been introduced into complex networks to achieve the synchronous dynamics [25-27]. Based on impulsive control theory on delayed dynamical systems, some simple yet generic criteria for robust impulsive synchronization were established [28]. By establishing the extended Halanay differential inequality on impulsive delayed dynamical systems, some simple yet generic sufficient conditions for global exponential synchronization of the impulsive controlled delayed dynamical networks were derived analytically [29]. However, there are few results for dynamical networks with non-delayed and delayed coupling, moreover with time delays in the dynamical nodes. Thence, synchronization stability analysis for complex dynamical networks still remains unsolved and challenging. In light of the above analysis, it is our purpose to propose new less conservative delay-dependent conditions for continuous-time networks.

In this paper, we investigate synchronization of a class of complex dynamical networks. However, most available literature in impulsive synchronization takes no account of time delay for simplicity. As a matter of fact, time delays commonly exist in the world. Some of them are trivial so that they can be ignored whilst some of them cannot be ignored, such as in long-distance communication and traffic congestion. By establishing impulsive delay differential inequalities, sufficient conditions on the asymptotical stability of complex dynamical networks are derived. Based on the analysis, the impulsive controller is designed.

The rest of the paper is organized as follows. In Section 2, complex networks model is presented, and some relevant definitions and lemmas are provided. Section 3 deals with the stability analysis. In Section 4, the controller design procedures are given in detail and theoretical results are verified by several numerical examples to illustrate their effectiveness. Conclusions are drawn in Section 5.

Notation: The notation used throughout the paper is fairly standard. Let N^+ be the set of natural number; R^n denotes the n-dimensional Euclidean space. For a matrix A, the largest eigenvalue and the smallest one are denoted by $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$, respectively; its induced matrix norm and matrix measure are

$$\|A\| = \sqrt{\lambda_{\max}(A^T A)}, \quad \mu(A) = \frac{1}{2}\lambda_{\max}(A^T + A)$$

2. **Complex Dynamical Networks Model.** In what follows, we consider a general complex dynamical network consisting of N identical linearly and diffusively coupled nodes, with each node being an n-dimensional dynamical system and introduce a complex delayed dynamical network model. This dynamical network is described by

$$\dot{x}_i = f(x_i(t), x_i(t-\tau(t))) + c_1 \sum_{j=1}^{N} C_{ij} G_1 x_j(t) + c_2 \sum_{j=1}^{N} C_{ij} G_2 x_j(t-\tau(t)), \quad i = 1, 2, \ldots, N \quad (1)$$

and the initial condition function

$$x_i(\theta) = \varphi(\theta), \quad \theta \in [-\tau_m, 0]$$

where $x_i = (x_{i1}, x_{i2}, \ldots, x_{in})^T \in R^n$ is a state vector representing the state variables of node i, $f(\cdot) \in R^n$ is a continuously differentiable vector function, $G_l = (G_{lij})_{n\times n} \in R^{n\times n}$ is a constant inner-coupling matrix between node i and node j ($i \neq j$) for all $1 \leq i, j \leq N$, the constant $c_i > 0$ ($i = 1, 2$) is the coupling strength, $\tau(t)$ is bounded time-varying delay with $0 < \tau(t) \leq \tau_m$, $C(t) = (C_{ij})_{N\times N}$ is the coupling configuration matrix representing topological structure of the network at time t, in which C_{ij} is defined as follows: if there is a connection from node i to node j ($i \neq j$), then $C_{ij} = C_{ji} = 1$, otherwise $C_{ij} = C_{ji} = 0$, τ ($\tau > 0$) is the time delay and the diagonal elements of matrix $C(t)$ are defined by

$$C_{ii}(t) = -\sum_{j=1, j\neq i}^{N} C_{ij}(t), \quad i = 1, 2, \ldots, N \tag{2}$$

Network (1) is a general complex network model in which there exist both non-delayed coupling and delayed coupling. It means that each node communicates with other nodes the information at time t as well as at time $t - \tau(t)$. In fact, this phenomenon exists commonly in our real world. For example, in the stock market, decision-making of single trader is influenced by that of others at time t as well as at time $t - \tau(t)$.

There have been various definitions of synchronization in the literature [4,30-32]. Hereafter, the delayed dynamical network (1) is said to achieve (asymptotical) synchronization if

$$x_1(t) = x_2(t) = \ldots x_N(t) = s(t), \quad as\ t \to \infty \tag{3}$$

where $s(t) \in R^n$ is a solution of an isolate node, namely, $\dot{s(t)} = f(s(t), s(t - \tau(t)))$.

To obtain the main results, we will need the following lemmas.

Lemma 2.1. *Consider the delayed dynamical network (1). Let*

$$0 = \lambda_1 \leq \lambda_1 \leq \ldots \leq \lambda_N \tag{4}$$

be the eigenvalues of the outer-coupling matrix C. If the following $N-1$ of n-dimensional delayed differential equations are asymptotically stable about their zero solutions:

$$\dot{w} = (J_1 + c_1\lambda_i G_1)w(t) + (J_2 + c_2\lambda_i G_2)w(t - \tau(t)), \quad i = 2, \ldots, N \tag{5}$$

where $J_1(t)$ is the Jacobian of $f(x_i(t), x_i(t - \tau(t)))$ at $s(t)$ and $J_2(t)$ is the Jacobian of $f(x_i(t), x_i(t - \tau(t)))$ at $s(t - \tau(t))$, then the synchronized states (3) are asymptotically stable for the dynamical network (1) [32].

Form Lemma 2.1, we know that the synchronized states (3) are asymptotically stable for the dynamical network (1) when Equation (5) is asymptotically stable about its zero solutions. For simplicity, we let $J_1(t)$ and $J_2(t)$ be constant matrices. To asymptotically stabilize their zero solutions of Equation (5), we design an impulsive control law as follows:

$$U_i(k, w(t_k)) = Dw(t_k), \quad i = 1, 2, \ldots, N, \quad k \in N^+$$

Then Equation (5) under impulsive control can be rewritten as follows:

$$\begin{cases} \dot{w} = (J_1 + c_1\lambda_i G_1)w(t) + (J_2 + c_2\lambda_i G_2)w(t - \tau(t)), \quad t \in (t_k, t_{k+1}], \quad i = 2, \ldots, N \\ \Delta w = w(t_k^+) - w(t_k) = Dw(t_k), \quad k \in N^+ \\ w(\theta) = \phi(\theta), \quad \theta \in [-\tau_m, 0] \end{cases} \tag{6}$$

3. **Stability Criteria.** In this section, we shall analyze the global asymptotic stability of the impulsive control system (6) by employing impulsive delay differential inequalities. We first present a basic result [33] and the basic result will play an important role in the following qualitative analysis of impulsive control systems with time delay.

Lemma 3.1. *Let $P \in R^{n\times n}$ be a symmetric positive definite matrix and $P = Q^TQ$. For any x, $x \in R^n$, $A \in R^{n\times n}$ then (1) $x^TA^TPAx \leq \|QAQ^{-1}\|^2 x^TPx$, (2) $x^T(A^TP + A)x \leq 2\mu(QAQ^{-1})x^TPx$, (3) $|x^TP\overline{x}| \leq \sqrt{x}^T Px\sqrt{(\overline{x})}^T P\overline{x}$.*

Then we shall establish some stability criteria for system (6). The following theorem gives some conditions for asymptotic stability of the trivial solution of system (6).

Theorem 3.1. *If there exists* $0 < \rho = \sup_{k \in N^+}(t_k - t_{k+1})$ *and a non-singular matrix* $Q \in R^{n\times n}$ *such that*

$$\|I + QDQ^{-1}\| \le \gamma, \ \ 0 < \gamma \le 1 \tag{7}$$

and

$$2\ln(\gamma) + 2\rho\mu(QJQ^{-1}) + \rho(\gamma)^2|c_2\lambda_i|(\|QGQ^{-1}\|)^2 < 0, \ \ i = 2, \dots, N \tag{8}$$

where I *is an n-dimension identity matrix,* $J = J_1 + c_1\lambda_i G_1$ *and* $G = J_2/(c_2\lambda_i) + G_2$*, then zero solution of system (6) is asymptotically stable in the following sense*

$$\|w(t)\| \le Me^{-\lambda/2t}\sup_{-\tau_m \le \theta \le 0}(\|\phi(\theta)\|), \ \ t \ge 0 \tag{9}$$

where $M = 1/\alpha\sqrt{\lambda_{\max}(Q^TQ)/\lambda_{\min}(Q^TQ)}$*,* $\lambda > 0$ *is the solution of*

$$\lambda - a + be^{\lambda\tau_m} = 0 \tag{10}$$

in which $a = -2\ln(\gamma)/\rho + 2\mu(QJQ^{-1}) - |c_2\gamma_i|$*,* $b = \gamma^{-2}|c_2\gamma_i|(\|QGQ^{-1}\|)^2$*. That is, the synchronized states (3) are asymptotically stable for the dynamical network (1).*

Proof: Denote $g_1(\lambda) = \lambda - a + be^{\lambda\tau_m}$. From (8), we obtain $a > 0$, $b > 0$, $-a + b < 0$, and so $g_1(0) < 0$, $g_1(+\infty) > 0$, $\dot{g}_1(\lambda) > 0$. Using the continuity and the monotonicity of $g_1(\lambda)$, Equation (10) has a unique solution $\lambda > 0$.

Define the Lyapunov functional $V(t) = w^T(t)Pw(t)$ and $P = Q^TQ$. According to Lemma 3.1, the derivative of V along the trajectories of system (6) is given by

$$\begin{aligned}
\dot{V} &= w^T(J^TP + PJ)w + 2w^T(c_2\lambda_i PG)w(t-\tau) \\
&\le 2\mu(QJQ^{-1})w^TPw + 2|c_2\lambda_i|\sqrt{(}w^TPw)\sqrt{(}w^T(t-\tau(t))PGw(t-\tau(t))) \\
&\le 2\mu(QJQ^{-1})w^TPw + |c_2\lambda_i|w^TPw + |c_2\lambda_i|(\|QGQ^{-1}\|)^2w^T(t-\tau(t))Pw(t-\tau(t)) \\
&\le (2\mu(QJQ^{-1}) + |c_2\lambda_i|)V(t) + |c_2\lambda_i|(\|QGQ^{-1}\|)^2V(t-\tau(t)), \ \ t \in (t_{k-1}, t_k]
\end{aligned}$$

where $J = J_1 + c_1\lambda_i G_1$ and $G = J_2/(c_2\lambda_i) + G_2$, and

$$\begin{aligned}
V(t_k^+) &= ((I+D)w(t_k))^TP(I+D)w(t_k) = w^T(t_k)(I+D)^TP(I+D)w(t_k) \\
&\le (\|I+D\|)^2w^T(t_k)Pw(t_k)\lambda_{\max}(Q^TQ)/\lambda_{\min}(Q^TQ) \le \gamma^2V(t_k)
\end{aligned}$$

For any $\sigma > 0$, let $x(t)$ be a unique solution of the following impulsive delay system

$$\begin{cases}
\dot{v} = (2\mu(QJQ^{-1}) + |c_2\lambda_i|)v(t) + |c_2\lambda_i|(\|QGQ^{-1}\|)^2v(t-\tau(t)), \ \ t \in (t_{k-1}, t_k] \\
v(t_k^+) = \gamma^2v(t_k), \ \ k \in N^+ \\
v(\theta) = \lambda_{\max}(P)\|\phi(\theta)\|^2, \ \ \theta \in [-\tau_m, 0]
\end{cases} \tag{11}$$

It may be clear that $V(\theta) < v(\theta)$, for $\theta \in [-\tau_m, 0]$. Based on [27,33], it leads to $v(t) \ge V(t) \ge 0$, $t \ge 0$. By the formula for the variation of parameters [34], the zero solution of system (11) is

$$v(t) = W(t,0)v(0) + \int_0^t W(t,s)|c_2\lambda_i|\|QGQ^{-1}\|^2v(s-\tau(s)+\sigma))ds, \ \ t \ge 0$$

where $W(t,s)$ is the Cauchy matrix and it is estimated by

$$W(t,s) = e^{(2\mu(QJQ^{-1})+|c_2\lambda_i|)(t-s)}\prod_{s<t_k<t}\gamma^2 \le e^{\left(-a-\frac{2\ln\gamma}{\rho}\right)(t-s)}\gamma^{2\frac{t-s}{\rho}-1} = \gamma^{-2}e^{-a(t-s)}, \ \ t \ge s \ge 0$$

Accordingly, we have

$$\begin{aligned}
v(t) \le& \gamma^{-2}e^{-at}\lambda_{\max}(P)\|\phi(0)\|^2 + \int_0^t \gamma^{-2}e^{-a(t-s)}|c_2\lambda_i|\|QGQ^{-1}\|^2v(s-\tau(s)+\sigma))ds \\
\le& \varsigma e^{-at} + \int_0^t e^{-a(t-s)}bv(s-\tau(s)+\sigma\gamma^{-2}))ds, \ \ t \ge 0
\end{aligned} \tag{12}$$

where $\varsigma = \gamma^{-2}\lambda_{\max}(P)\text{sup}_{-\tau_m \le s \le 0}\|\phi(s)\|^2$.

In the following, we will verify that

$$v(t) \le \varsigma e^{\lambda t} + \sigma\gamma^{-2}(a-b)^{-1}, \quad t \ge 0 \tag{13}$$

Because $\sigma \ge 0$, $-a+b<0$, then we obtain $\sigma\gamma^{-2}(a-b)^{-1} > 0$. At first, it is assumed that there exists a $t^* > 0$ such that

$$v(t^*) \ge \varsigma e^{\lambda t^*} + \sigma\sigma\gamma^{-2}(a-b)^{-1} \tag{14}$$

and

$$v(t) \le \varsigma e^{\lambda t} + \sigma\gamma^{-2}(a-b)^{-1}, \quad t \le t^* \tag{15}$$

From (10), (12), (15), we can get

$$\begin{aligned}
v(t^*) &\le \varsigma e^{-at^*} + \int_0^{t^*} e^{-a(t^*-s)} bv(s-\tau(s)+\sigma\gamma^{-2}))ds \\
&< e^{-at^*}\left(\varsigma + \sigma\gamma^{-2}(a-b)^{-1} + \int_0^{t^*} e^{as}(\varsigma b e^{-\lambda(s-\tau(s))} + \sigma b\gamma^{-2}(a-b)^{-1} + \sigma\gamma^{-2})ds\right) \\
&\le e^{-at^*}\left(\varsigma + \sigma\gamma^{-2}(a-b)^{-1} + \varsigma b e^{\lambda\tau_m}\int_0^{t^*} e^{(a-\lambda)s}ds + \sigma b\gamma^{-2}(a-b)^{-1}\int_0^{t^*} e^{as}ds\right) \\
&= \varsigma e^{\lambda t^*} + \sigma\sigma\gamma^{-2}(a-b)^{-1}
\end{aligned}$$

This contradicts (14), and so the estimate (13) holds. Letting $\sigma \to 0$, then $V(t) \le v(t) \le \varsigma e^{\lambda t}$, $t \ge 0$. It implies the conclusion and this completes the proof.

Similarly, we can obtain the following conclusion for the case. Here we omit its proof to avoid the repetition.

Theorem 3.2. *For the case $\gamma > 1$, we can replace the condition $0 < \rho = \sup_{k\in N^+}(t_k - t_{k+1})$ with $0 < \rho = inf_{k\in N^+}(t_k - t_{k+1})$, then the conclusion the above theorem yet holds when the following inequalities replace inequalities (7) and (8).*

$$\|I + QDQ^{-1}\| \le \gamma, \quad \gamma > 1$$

and

$$2\ln(\gamma) + 2\rho\mu(QJQ^{-1}) + \rho(\gamma)^2|c_2\lambda_i|(\|QGQ^{-1}\|)^2 < 0, \quad i = 2, \dots, N$$

with $a = -2\ln(\gamma)/\rho + 2\mu(QJQ^{-1}) - |c_2\gamma_i|$, $b = \gamma^2|c_2\gamma_i|(\|QGQ^{-1}\|)^2$.

4. **Impulsive Controlled with Numerical Simulations.**

The above synchronization conditions can be applied to networks with different topologies and different sizes. In order to illustrate the main results of the above theoretical analysis clearly, we consider a lower-dimensional network model with 5 nodes. In the section, two design procedures of impulsive controller are provided based on Theorem 3.1 and Theorem 3.2. After that, the corresponding numerical simulations are given.

Firstly, based on the analysis of Theorem 3.1, we give the following design procedure:

(1) Select the parameters c_1, c_2, J_1, J_2, Q, G_1, G_2.

(2) Calculate J, λ_i, τ_m, G, P, $\mu(QJQ^{-1})$. Choose a matrix D such that $\|I+QDQ^{-1}\| \le \gamma$, $0 < \gamma \le 1$.

(3) For a given λ_0, determine a set of impulsive control instants t_k, $k \in N^+$ as below.

1) Choose a $\lambda \ge \lambda_0 \ge 0$ such that

$$\vartheta \triangleq \lambda + 2\mu(QJQ^{-1}) + (\gamma)^{-2}|c_2\lambda_i|(\|QGQ^{-1}\|)^2(1+e^{\lambda\tau_m}) > 0, \quad i = 2, \dots, N$$

2) Take $\rho = \sup_{k\in N^+}(t_k - t_{k+1}) = -2\ln\gamma/\vartheta$.

In order to illustrate the design procedure, we consider a lower-dimensional network model with five nodes, in which each node is a simple three-dimensional linear system described [32].

$$\begin{cases} \dot{x_1} = -x_1 - x_1(t-\tau(t)) \\ \dot{x_2} = -2x_2 + x_1(t-\tau(t)) - x_2(t-\tau(t)) \\ \dot{x_3} = -3x_3 - x_3(t-\tau(t)) \end{cases} \tag{16}$$

which is asymptotically stable at the equilibrium point $s(t) = 0$, Jacobin matrices are $J_1 = diag{-1, -2, -3}$ and $J_2 = [-1, 0, 0; 1, -1, 0; 0, 0, -1]$. For simplicity, we shall choose matrix Q to be the 3×3 identity matrix and we suppose that the inner-coupling matrix is $G_1 = G_2 = Q$ and $c_1 = c_2 = 1$, and the outer-coupling matrix G is

$$\begin{pmatrix} -2 & 1 & 0 & 0 & 1 \\ 1 & -3 & 1 & 1 & 0 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 1 & 1 & -3 & 1 \\ 1 & 0 & 0 & 1 & -2 \end{pmatrix}$$

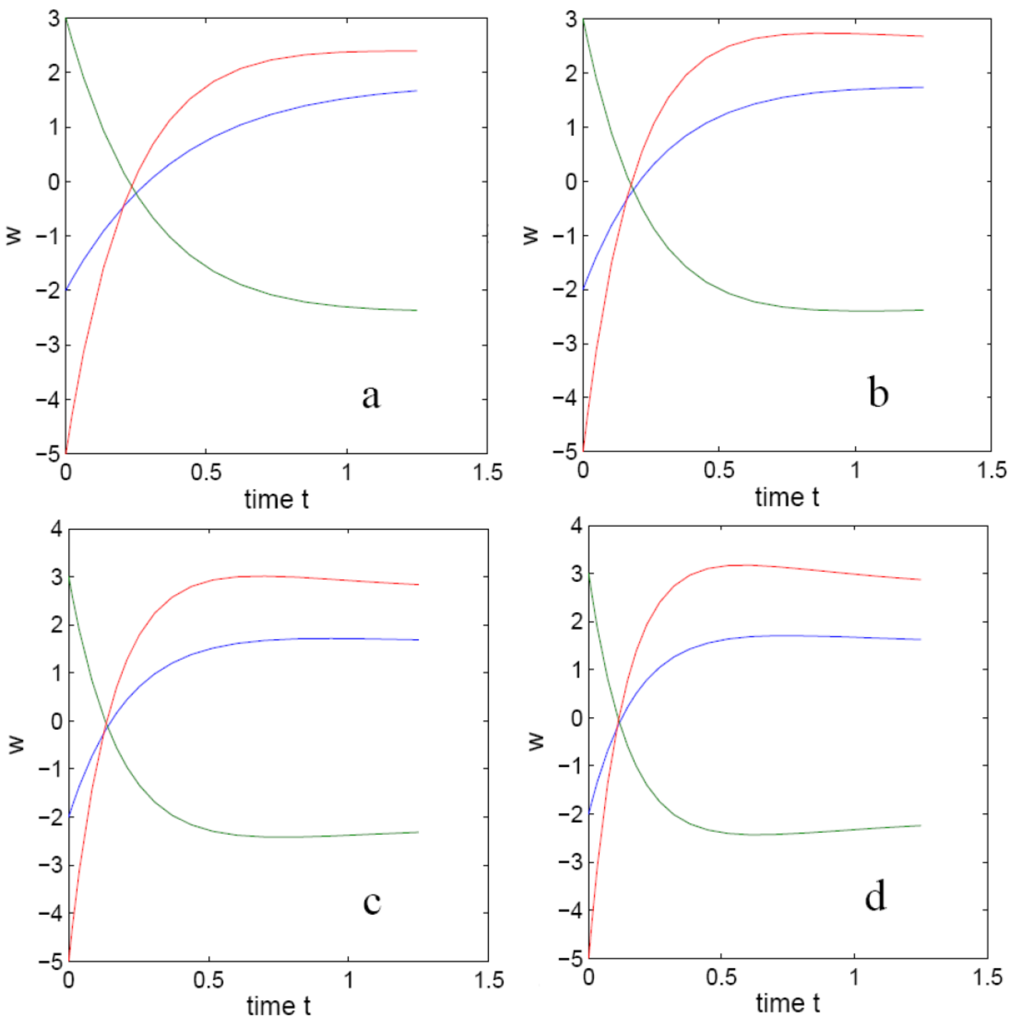

FIGURE 1. Time response for system (5) without impulsive control. (a) $\lambda_2 = -1.382$, (b) $\lambda_3 = -2.382$, (c) $\lambda_4 = -3.618$, (d) $\lambda_5 = -4.618$.

Obviously, C is an irreducible symmetric matrix. The eigenvalues of C are $\lambda_i = 0, -1.382, -2.382, -3.618, -4.618$. Here we choose $D = diag{-0.6, -0.6, -0.6}$. Let $\tau(t) = 0.2|\sin(t)|$, which is bounded by $\tau_m = 0.2$. Then we can get $\gamma = \|I + D\| = 0.4 < 1$. When $\lambda = \lambda_0 = 3$ and $\lambda_i = -1.382, -2.382, -3.618, -4.618$, according to the last step of the above design procedure, then we can obtain $\rho = 0.0561, 0.0421, 0.0317, 0.0264$, respectively.

We implement the equidistant impulsive control and denote $\Delta T = t_k - t_{k-1} = 0.025$, $k \in N_+$. Figure 1 shows time response for system (5). As shown in Figure 1, we can know that 4 of 3-dimensional delayed differential equations are not asymptotically stable about their zero solutions. However, the following 4 of 3-dimensional delayed differential equations are asymptotically stable about their zero solutions via impulsive control as shown in Figure 2. According to Lemma 2.1, it is obvious that the synchronized states (3) are asymptotically stable for the dynamical network (1).

Then we give another design procedure based on the analysis of Theorem 3.2:

(1) Select the parameters c_1, c_2, J_1, J_2, Q, G_1, G_2.

(2) Calculate J, λ_i, τ_m, G, P, $\mu(QJQ^{-1})$. Choose a matrix D such that $\|I+QDQ^{-1}\| \leq \gamma$, $\gamma \geq 1$.

(3) For a given λ_0, determine a set of impulsive control instants t_k, $k \in N^+$ as below.

1) Choose a $\lambda \geq \lambda_0 \geq 0$ such that

$$\vartheta \triangleq \lambda + 2\mu(QJQ^{-1}) + (\gamma)^{-2}|c_2\lambda_i|(\|QGQ^{-1}\|)^2(1 + e^{\lambda\tau_m}) > 0, \quad i = 2, \dots, N$$

2) Take $\rho = inf_{k\in N^+}(t_k - t_{k+1}) = 2\ln\gamma/\vartheta$.

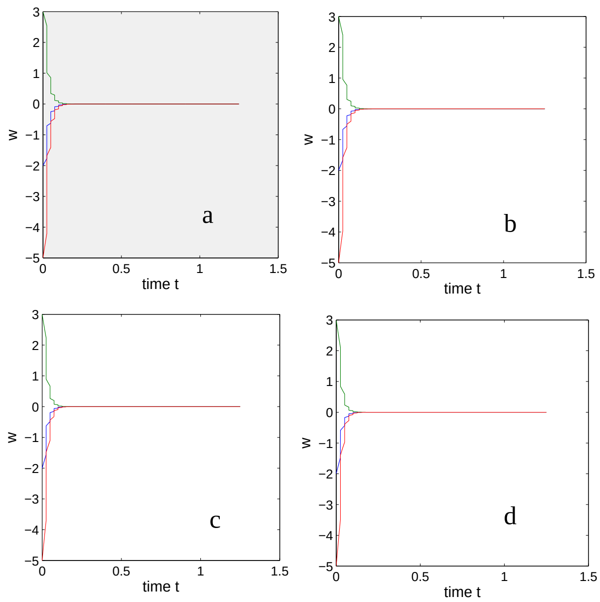

FIGURE 2. Impulsive time response for system (5) with $\lambda_0 = 3$ and $\rho = 2.5\times10^{-2}$, where $\rho = t_k - t_{k-1}$ and the initial values: $w_1(t) = -2$, $w_2(t) = 3$, $w_3(t) = -5$, $t \in [-0.02, 0]$, (a) $\lambda_2 = -1.382$, (b) $\lambda_3 = -2.382$, (c) $\lambda_4 = -3.618$, (d) $\lambda_5 = -4.618$.

In order to illustrate the main results of the above theoretical analysis, we choose $D = diag0.4, 0.4, 0.4$ here and other parameters are the same as the previous example. Then we can get $\gamma = \|I + D\| = 1.4 > 1$. When $\lambda = \lambda_0 = 3$ and $\lambda_i = 0, -1.382, -2.382, -3.618, -4.618$, according to the last step of the above design procedure, then we can obtain $\rho = 0.2006, 0.1517, 0.1412, 0.0947$, respectively. We implement the equidistant impulsive control and denote $\Delta T = t_k - t_{k-1} = 0.3$, $k \in N_+$. As shown in Figure 3, the following 4 of 3-dimensional delayed differential equations are asymptotically stable about their zero solutions via impulsive control. Therefore, it is obvious that the synchronized states (3) are asymptotically stable for the dynamical network (1).

5. **Conclusions.** In this paper, we investigate synchronization of a class of complex dynamical networks. Based on impulsive delay differential inequalities, we give sufficient conditions on the asymptotical stability of complex dynamical networks with non-delayed and delayed coupling. Then two procedures for designing impulsive controller have been outlined. Finally, some numerical examples are also given to illustrate that the conditions are practical and the designed methods are effective.

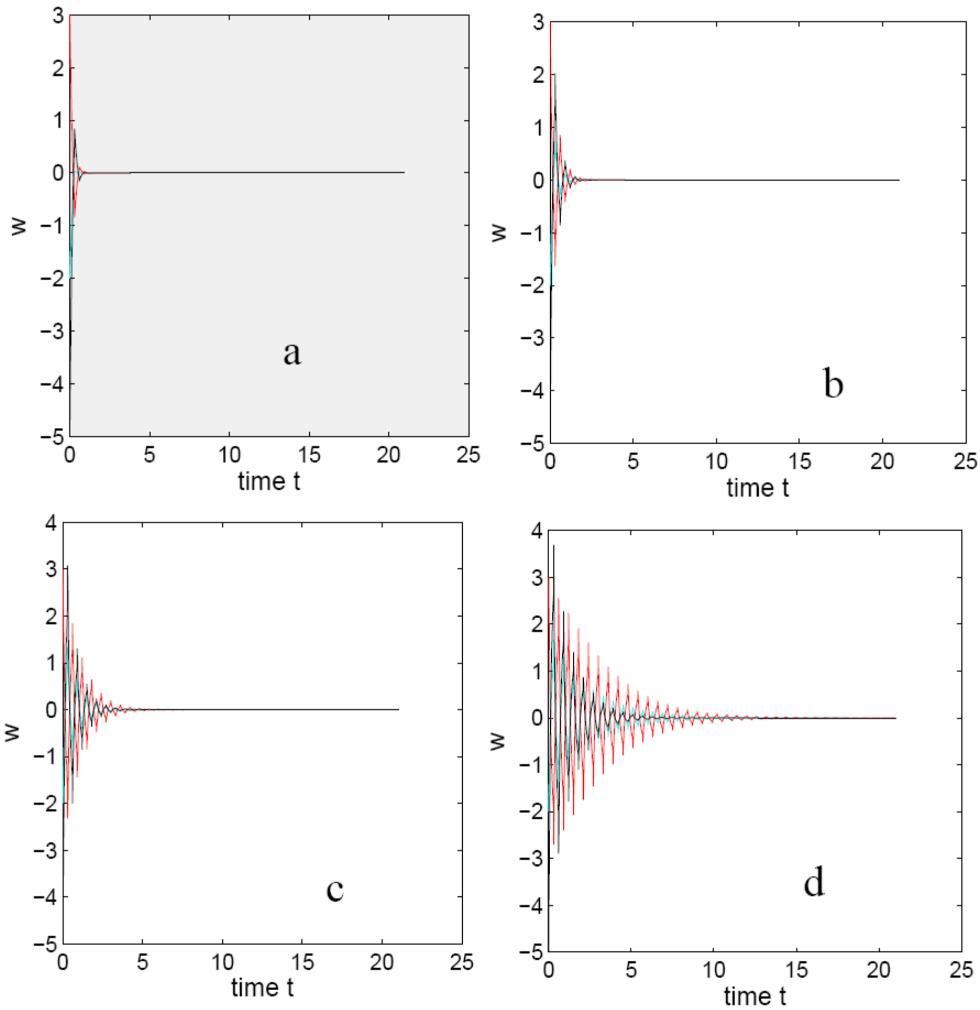

FIGURE 3. Impulsive time response for system (5) with $\lambda_0 = 3$ and $\rho = 0.3$, where $\rho = t_k - t_{k-1}$ and the initial values: $w_1(t) = -2$, $w_2(t) = 3$, $w_3(t) = -5$, $t \in [-0.02, 0]$, (a) $\lambda_2 = -1.382$, (b) $\lambda_3 = -2.382$, (c) $\lambda_4 = -3.618$, (d) $\lambda_5 = -4.618$.

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