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FREQUENCY ANALYSIS OF T-S FUZZY CONTROL SYSTEMS

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Abstract. *This paper investigates the problem of analysis of a Takagi-Sugeno (T-S) fuzzy control system in the frequency domain. First, a linear plant with a T-S fuzzy controller is considered, and the complex dynamic behaviors of the system generated by the nonlinearity in the controller are analyzed. The global stability, instability and existence of limit cycles of the system are predicted with the aid of the describing function method. Then, the method is extended to the robust control of an uncertain system, and the condition is given under which the T-S fuzzy controller can guarantee the global stability of the closed-loop system. All the results are obtained straightforwardly by the frequency domain graphic methods and simulations are finally provided to show the effectiveness of the proposed analysis methods.*
Keywords: T-S fuzzy controllers, Describing function method, Frequency response methods, Global stability, Limit cycles

1. **Introduction.** In recent years, fuzzy logic control has become an alternative control method relative to the traditional control theory, and its successful applications can be found in many fields ranging from control engineering [7], pattern recognition [6] and signal processing to decision making [16], etc. The advantages of fuzzy logic control compared with the traditional control methods lie in its robustness and effectiveness in dealing with both linear and nonlinear plants and those with incomplete knowledge of models by virtue of the human intelligence involved. A fuzzy control system is composed of a set of linguistic rules where skilled human operators' experiences are involved, and the precise control action is realized by the representation and evaluation of those rules through fuzzy mathematical tools. Among many categories of fuzzy logic control methods, the T-S fuzzy control has experienced a rapid and full development in the time domain because it is model-based and thus capable to overcome the shortage of the conventional fuzzy control method that the general theory of stability analysis and synthesis is out of reach [11, 15, 18, 21]. This advantage has attracted remarkable attention from researchers, and a great number of results have been reported [4, 8, 9, 14, 19, 20, 24, 25, 26, 27].

In the fuzzy control system, besides the nonlinearities in generation of fuzzy rules, the fuzzification and defuzzification parts accompanied with the finite length of the universe

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of discourse of the input and output variables can introduce nonlinearities. The dynamic behavior of the system becomes complicated as a result and it is necessary that we consider both local stability and global stability problems. The local stability analysis for fuzzy control systems can be found in much literature and various methods have been utilized. For instance, the describing function method is used for Mamdani-type fuzzy control systems in [1, 2]; the circle criterion and the multivariable circle criterion are applied for Mamdani-type fuzzy control systems in [23] and for the MIMO case in [13]; the Popov criterion has also been utilized for Mamdani-type fuzzy control systems in [3]. Moreover, these years have witnessed the exploration of some conventional nonlinear control theories and the popular application of Lyapunov methods based on linear matrix inequality tools [22, 28]. The global stability problem of fuzzy control systems is more complicated and the describing function method is often applied. The transient response of Mamdani-type fuzzy control systems is analyzed and the global stability analysis is performed in [1, 2] and the existence of limit cycles is predicted in [12].

The methods mentioned above with respect to frequency domain are mostly concerned with the model-free fuzzy control systems. As the frequency domain method provides direct graphic insight of the control system, which makes the controller synthesis more straightforward, in recent years, researchers are switching the efforts to performing analysis and synthesis of T-S fuzzy control systems with frequency domain methods. For example, in [3] a conicity criterion is employed to analyze the input-output stability of the reactive navigation under fuzzy perception and fuzzy control; in [5] the circle criterion is applied to obtain the sufficient conditions guaranteeing the stability of the simplest T-S fuzzy control system with a linear plant; in [10] the describing function method is utilized for MIMO T-S fuzzy models to predict the existence of multiple equilibria and limit cycles, where the control synthesis is not involved.

Among those methods, the describing function method can help give a better understanding of the complex dynamic behaviors of the control systems [5]. Yet the existing results using the describing function method are mostly related to the stability analysis problem; the stabilization problem has not been addressed, which motivates this research. In addition, many advanced control methods in time-domain are actually not applicable in practice, and the frequency methods are often preferred by engineers which can effectively depict the dynamic behavior of control systems. Moreover, it is proved that the nonlinear fuzzy controller can guarantee a better performance of the system than the linear controller. Besides, stability analysis and synthesis of the T-S fuzzy control system are more simplified than those traditional ones. So it is of practical values to investigate the frequency characteristic of the T-S fuzzy control system via the describing function method, and the results contribute to many practical problems such as the robot control, cement kiln control.

In this paper, we investigate the problem of analysis of the T-S fuzzy control system in the frequency domain. First, a linear plant with a T-S fuzzy controller is considered, and the complex dynamic behaviors of the system generated by the nonlinearity in the controller are analyzed. The global stability, instability and existence of limit cycles of the system are predicted with the aid of the describing function method. Then, the method is extended to the robust control of an uncertain system, and the condition is given such that the T-S fuzzy controller guarantees the global stability of the closed-loop system. All the results are obtained straightforwardly by the frequency domain graphic methods and simulations are finally provided to show the effectiveness of the proposed analysis methods.

The organization of the paper is as follows. Section 2 formulates the system under investigation, which is a plant with a T-S fuzzy controller. The T-S fuzzy controller is analyzed and the method is extended to an uncertain plant case in Section 3. Numerical examples are given to illustrate the effectiveness of the proposed approaches in Section 4 and the paper is concluded in Section 5.

Notations: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, the notation $|\cdot|$ refers to the amplitude of the transfer function. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. **Problem Formulation.** In this section, the problem will be formulated mathematically. A simple T-S fuzzy controller is utilized to control a linear plant. For the convenience of the analysis of the closed-loop system in the frequency domain, the control loop is arranged in Figure 1.

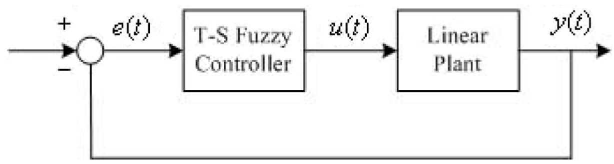

FIGURE 1. The control loop

2.1. **Plant.** Consider a linear system with the following realized state space model:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^{n\times n}$ is the state vector, and matrices $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^n$ and $C \in \mathbb{R}^{1\times n}$ are known.

2.2. **T-S fuzzy controllers.** The T-S fuzzy system approximates a nonlinear system by a "blending" of some linear functions, which express the local linear properties of the nonlinear system by fuzzy implications. Specifically, the T-S fuzzy system is of the following form:

Rule i: IF $\theta_1(t)$ is M_{i1} and $\theta_2(t)$ is M_{i2} and $\cdots$ and $\theta_n(t)$ is M_{in}, THEN

$$\begin{aligned} \hat{y}_i(t) &= a_i\theta(t), \\ i &= 1, \cdots, r, \end{aligned} \tag{2}$$

where r is the number of IF-THEN rules; $\theta(t) = \begin{bmatrix} \theta_1(t), & \theta_2(t), & \ldots, & \theta_n(t) \end{bmatrix}$ is the premise variable vector and M_{ij} is the fuzzy set. The linear function $\hat{y}(t) = a_i\theta(t)$ is the consequence of the ith IF-THEN rule, where $a_i \in \mathbb{R}^{1\times n}$. The possibility that the ith rule will fire is given by the product of all the membership functions associated with the ith rule:

$$h_i(\theta(t)) = \prod_{j=1}^{n} M_{ij}(\theta_j(t)), \tag{3}$$

where $M_{ij}(\theta_j(t))$ represents the grade of membership of $\theta_j(t)$ in M_{ij}. Then, it can be seen that for all t we have

$$\begin{aligned} h_i(\theta(t)) &\geq 0, \qquad i = 1, 2, \cdots, r, \\ \sum_{i=1}^{r} h_i(\theta(t)) &= 1. \end{aligned} \tag{4}$$

By using the center-of-gravity method for defuzzification, the whole T-S fuzzy system can be represented as follows:

$$\hat{y}(t) = \sum_{i=1}^{r} h_i(\theta(t)) a_i \theta(t). \tag{5}$$

The approximation sense of the T-S fuzzy system in (5) is twofold: one is the approximator of the n-dimensional nonlinear dynamic system with enough fuzzy rules, and the other is a universal approximator of any nonlinear state feedback controller. Here, the latter sense is represented by the simplest one of the T-S fuzzy controllers which consists of the following two rules:

Controller Rule i: IF $e(t)$ is M_i, THEN

$$u_i(t) = K_i e(t), \quad i = 1, 2, \tag{6}$$

where $e(t) \in \mathbb{R}^1$ is the input of the local controller and $u_i(t) \in \mathbb{R}^1$ is the output of the local controller; K_i is the gain of the feedback controller. Thus, the controller can be represented by the following input-output form:

$$u(t) = \sum_{i=1}^{r} h_i(e(t)) K_i e(t). \tag{7}$$

The triangular membership functions are utilized for $h_i(e(t))$, as shown in Figure 2.

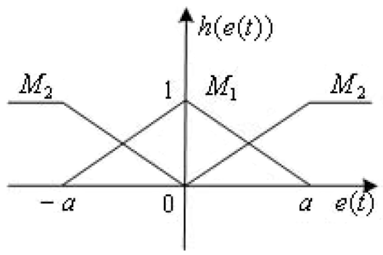

FIGURE 2. The fuzzy membership function

3. **Analysis and Synthesis of T-S Fuzzy Controllers.** In this section, the describing function method, which is well known as a successful technique for solving the nonlinear control problems, will be reviewed and applied to the analysis of the T-S fuzzy control system in the frequency domain.

3.1. **Describing function method.** The describing function method fits the control loop composed of a nonlinear element and a linear element as in Figure 1. The amplitude of the higher order harmonics is usually smaller than that of the first harmonic in the response signal of the nonlinear element; moreover, most linear plants have the characteristic of low-pass filtering. Thus only the first harmonic can survive the linear part while the higher order ones are exhausted. Therefore, if the input $e(t)$ is set to be a sinusoidal signal $A\sin(\omega t)$, we can assume that only the first harmonic of $x(t)$ is back to $e(t)$ in the loop. So the nonlinear part can be replaced by a describing function which changes the amplitude and phase of the input sinusoidal signal. The definition of the describing function is as follows [12]:

$$N(A, \omega) = \frac{B}{A} e^{j\phi}$$

with $j = \sqrt{-1}$, where $N(A, \omega)$ is the describing function; B and A are the amplitudes of the first harmonic response and the input signal, respectively. By replacing the nonlinear part with the describing function, the solutions of the equation

$$1 + N(A, \omega) G(j\omega) = 0 \tag{8}$$

will correspond to limit cycles of the system, where $G(s)$ is the transfer function of the linear plant, that is, if $A = A_1$ and $\omega = \omega_1$ satisfy Equation (8), a limit cycle will exist with the amplitude A_1 and period $2\pi/\omega_1$. The solutions of this equation can be found by sketching the curves $G(j\omega)$ for $0 < \omega < \infty$ and $-1/N(A, \omega_i)$ for $0 < A < \infty$ and different values of ω_i. If the curve of $G(j\omega)$ for $\omega = \omega_i$ intersects the curve corresponding to $-1/N(A, \omega_i)$ for a certain A this intersection is the reflection of the existence of a limit cycle. The stability of this limit cycle can be analyzed by the Loeb criterion.

3.2. **Application to T-S fuzzy controllers.** By considering the membership function in Figure 2, the output of the T-S fuzzy controller can be obtained by computing (7):

$$u(t) = \begin{cases} K_2 e(t) & e(t) < -a \\ \frac{K_1 - K_2}{a} e(t)^2 + K_1 e(t) & -a \le e(t) < 0 \\ \frac{K_2 - K_1}{a} e(t)^2 + K_1 e(t) & 0 \le e(t) < a \\ K_2 e(t) & e \ge a \end{cases}.$$

Given the input signal $e(t) = A\sin(\omega t)$, the formulation of the output control signal $u(t)$ between $[0,\ \pi]$ is

$$u(t) = \begin{cases} \frac{K_2 - K_1}{a} (A\sin(\omega t))^2 + K_1 A \sin(\omega t) & 0 \le \omega t < \alpha \\ \frac{K_1 - K_2}{a} e(t)^2 + K_1 e(t) & \alpha < \omega t < \pi - \alpha \\ K_2 e(t) & \alpha \le \omega t \le \pi - \alpha \end{cases},$$

where $\alpha = \sin^{-1} \frac{a}{A}$, and the formulation between $[\pi,\ 2\pi]$ can be obtained similarly.

Therefore, the first harmonic of the output control signal is

$$u(t) = A_1 \cos\omega t + B_1 \sin\omega t,$$

where

$$A_1 = \frac{1}{\pi} \int_0^{2\pi} u(t) \cos\omega t d(\omega t), \quad B_1 = \frac{1}{\pi} \int_0^{2\pi} u(t) \sin\omega t d(\omega t).$$

By simple computation, we can obtain the describing function of the nonlinear part

$$N(A) = \frac{4}{\pi} \left[\frac{K_2 - K_1}{a} A \left(\frac{2}{3} + \frac{1}{3} \left(1 - \left(\frac{a}{A}\right)^2\right)^{\frac{3}{2}} - \sqrt{1 - \left(\frac{a}{A}\right)^2} \right) \right.$$
$$\left. + K_1 \left(\frac{1}{2} \sin^{-1} \frac{a}{A} - \frac{a}{2A} \sqrt{1 - \left(\frac{a}{A}\right)^2} \right) + K_2 \left(\frac{\pi}{4} - \frac{1}{2} \sin^{-1} \frac{a}{A} + \frac{a}{2A} \sqrt{1 - \left(\frac{a}{A}\right)^2} \right) \right].$$

When $A \to \infty$, $-1/N(A) = -\frac{1}{K_2}$ and when $A \to a$, $-1/N(A) = -\frac{1}{K_1 + \frac{8}{3\pi}(K_2 - K_1)}$, which shows that the curve of $-1/N(A)$ is a line beginning at the point $\left(-\frac{1}{K_1 + \frac{8}{3\pi}(K_2 - K_1)}, 0\right)$ and terminating at $\left(-\frac{1}{K_2}, 0\right)$ on the complex plane.

When the controller is designed such that the line $-1/N(A)$ is on the left side of the Nyquist curve of the linear plant, the closed-loop system will be stable; when the line $-1/N(A)$ intersects the Nyquist plot of $G(s)$ with amplitude A and frequency ω, there will be a limit cycle with the same amplitude and frequency, and when the line $-1/N(A)$ is on the right side of the Nyquist plot without any intersection, the closed-loop system will be instable.

3.3. **Plants with uncertainties.** In this subsection, we will extend the result to the robust control problem of an uncertain system. The condition will be obtained such that the closed-loop system is globally stable and the field where limit cycles may appear will be estimated.

Suppose there are some uncertainties in linear plant (1) and the model has the following form:

$$\begin{aligned} \dot{x}(t) &= (A_0 + \Delta A)\, x(t) + (B_0 + \Delta B)\, u\,(t)\,, \\ y &= Cx\,(t)\,, \end{aligned}$$

where ΔA and ΔB are unknown matrices with appropriate dimensions. The transfer function of the uncertain system can be represented as

$$G\,(s) = G_0\,(s) + \Delta G\,(s)\,,$$

where $G_0\,(s)$, $G\,(s)$ and $\Delta G\,(s)$ are the transfer functions of the nominal system, uncertain system and the unknown dynamics, respectively. If $\Delta G\,(s)$ is bounded and there exists a rational function $r\,(jw)$ such that

$$|\Delta G\,(jw)| \le |r\,(jw)|\,, \quad \forall \omega \in \mathbb{R},$$

then the Nyquist plot of the uncertain system will distribute in the field between the envelope curve of the circles with radius $|r\,(jw)|$ and centers on the Nyquist curve of $G_0\,(jw)$, where the bound function $r\,(jw)$ can be found through the Bode plot method. Therefore, when the controller is designed such that the line of $-1/N\,(A)$ intersects the field swept by the circles, that is, the field between the envelop curves of the circles, limit cycles may appear; when the controller is designed such that the line $-1/N\,(A)$ is on the left side of the envelop curves without any intersection the closed-loop system will be stable, and when the line $-1/N\,(A)$ is on the right side of the envelop curves, the closed-loop system will be instable.

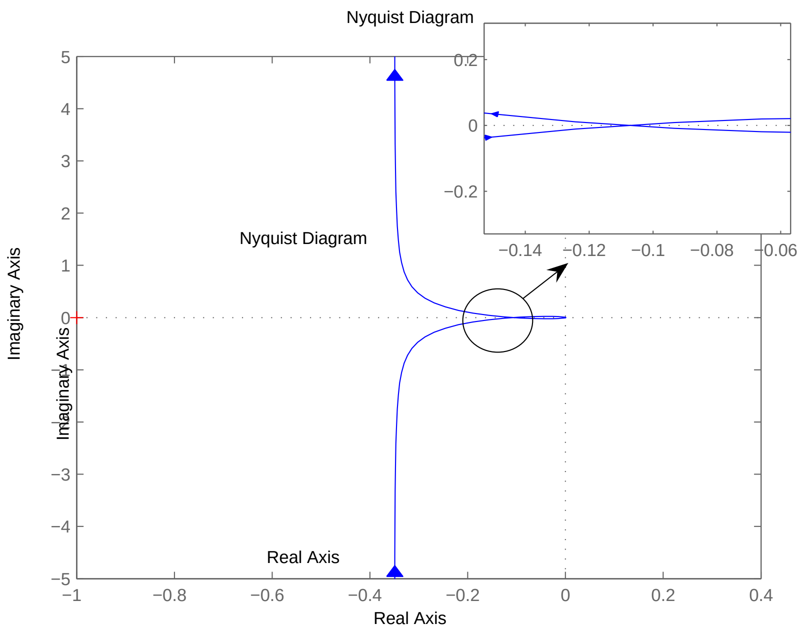

FIGURE 3. The Nyquist plot of $G\,(s)$

$r(jw)$ can be obtained in Bode plot: first calculate the difference between the frequency responses of $G(jw)$ and $G_0(jw)$, i.e., $G(jw)-G_0(jw)$ and represent it in Bode plot. Then find a function $r(jw)$ such that $|r(jw)|$ covers $|G(jw)-G_0(jw)|$.

It is noticed that for the uncertain system, the limit cycles with accurate amplitude and frequency cannot be obtained here. However, we give a field where the limit cycles may appear, and the condition guaranteeing the global stability of the fuzzy system.

4. **Simulations.** In this section, simulation examples will be illustrated to show the effectiveness of the proposed analysis methods. A linear plant will be considered and different behaviors of the closed-loop system with different designed T-S fuzzy controllers will be shown.

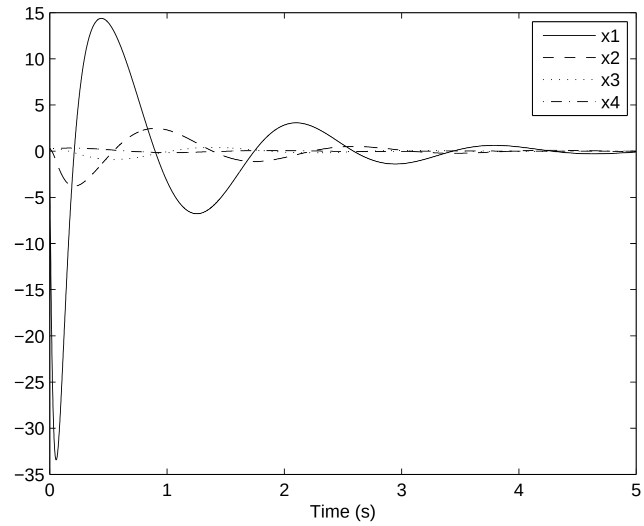

FIGURE 4. State response of the system

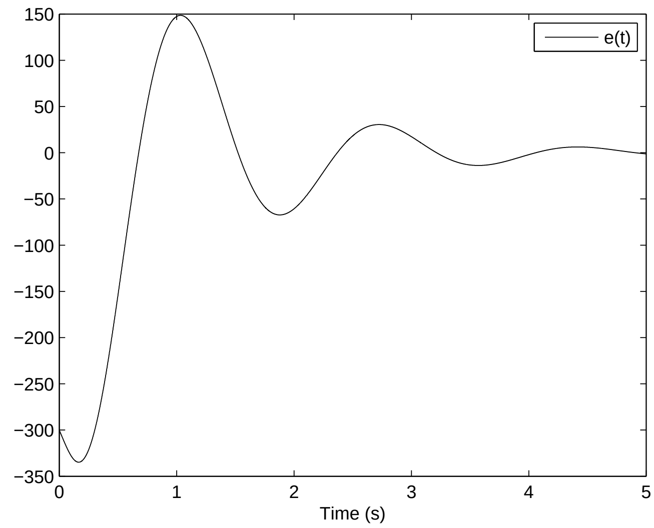

FIGURE 5. The error of the system

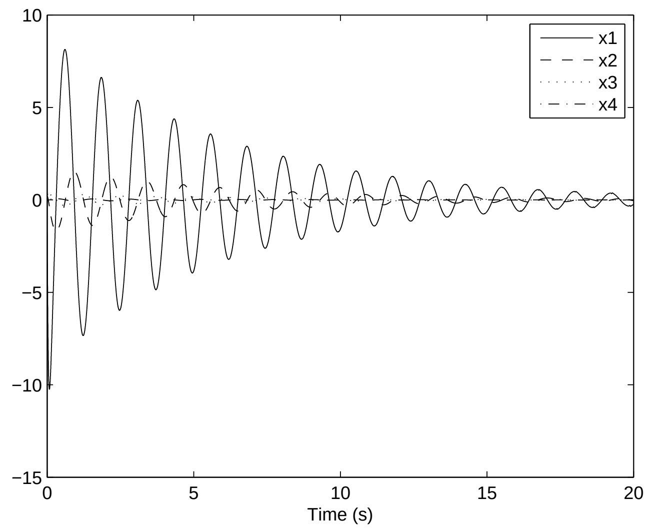

FIGURE 6. State response of the system

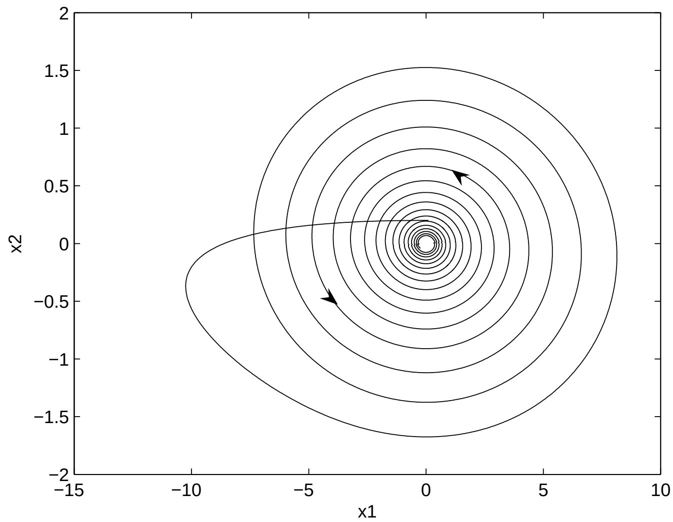

FIGURE 7. The trajectory of the system

Consider the following system

$$G(s) = \frac{1}{s(0.2s+1)(0.1s+1)(0.05s+1)}.$$

The Nyquist plot of the system is shown in Figure 3, where the plot intersects the real axis of the complex plane at point $(-0.107, 0)$, that is, the limit cycle will appear with amplitude $A = 0.107$ and frequency $\omega = 5.42$ rad/s. When the control gains in T-S fuzzy controller (6) are designed as $K_1 = 2$ and $K_2 = 4$, the line of $-1/N(A)$ with start point $(-0.2704, 0)$ and end point $(-0.2500, 0)$ does not intersect the Nyquist plot of $G(s)$. The closed-loop system is stable. The state response is shown in Figure 4 and the error is shown in Figure 5, where the initial state is $(0.1, 0.2, 0.3, 0.3)$.

When the control gains are chosen as $K_1 = 35$ and $K_2 = 8$, the line of $-1/N(A)$ begins at $(-0.1250, 0)$ and terminates at $(-0.0828, 0)$ intersecting the Nyquist plot of $G(s)$ at

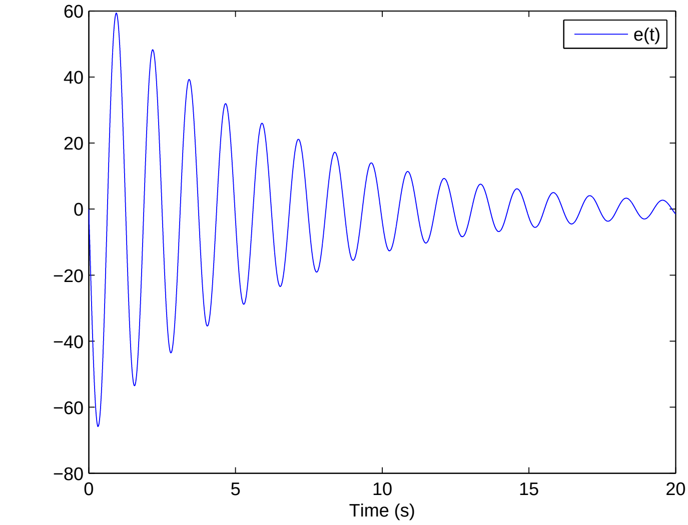

FIGURE 8. The error of the system

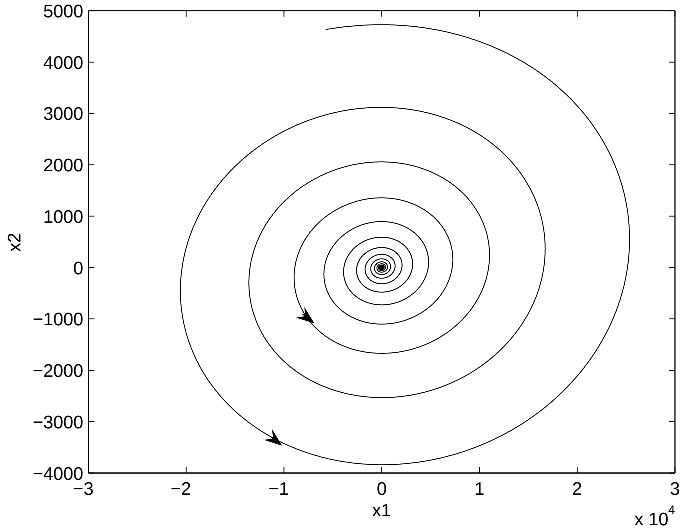

FIGURE 9. The trajectory of the system

$(-0.107, 0)$. According to the Loeb criterion, there is a stable limit cycle with amplitude $A = -0.107$ and $\omega = 5.42$ rad/s. Figure 6 shows the state response of the system, where the states are oscillating with decreasing amplitudes; Figure 7 depicts the limit cycle and Figure 8 gives the error response of the system, where the initial state (0.1, 0.2, 0.3, 0.00003) is in the limit circle.

When the control gains are chosen as $K_1 = 8$ and $K_2 = 12$, the line of $-1/N(A)$ begins at $(-0.0878, 0)$ and terminates at $(-0.0833, 0)$ on the right side of the Nyquist plot of $G(s)$ without any intersection. The closed-loop system is instable, as shown in Figure 9 and Figure 10.

5. **Concluding Remarks.** This paper has investigated the problem of analysis of a T-S fuzzy control system. Different dynamic behaviors of a system composed of a plant with a T-S fuzzy controller have been predicted based on the describing function method. The

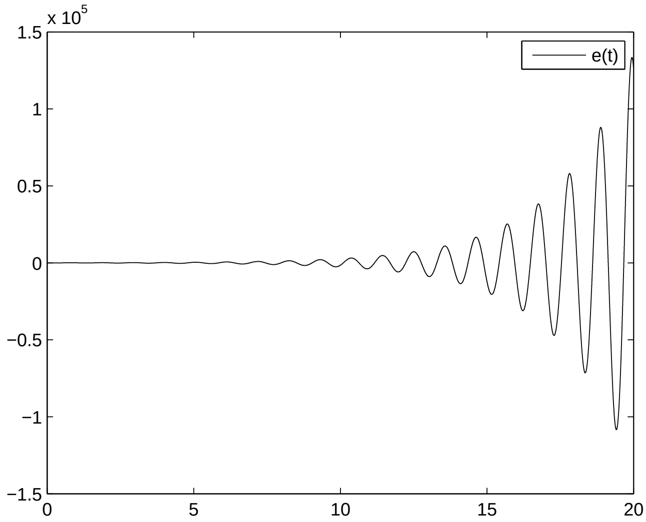

FIGURE 10. The error of the system

method has been extended to robust control of the uncertain system. The results are all obtained straightforwardly by frequency domain graphic methods. Finally, the simulations have been provided to show the effectiveness of the proposed analysis methods. The future work should include the investigation of the frequency analysis of the fuzzy systems with more rules and time delay [17].

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