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ROBUST MPC CONTROLLER DESIGN FOR SWITCHED SYSTEMS USING MULTI-PARAMETER DEPENDENT LYAPUNOV FUNCTION

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Abstract. *The paper addresses the problem of designing a robust output/state model predictive control for linear polytopic switched systems. We propose a new method for calculation of control algorithm parameters for predictive robust control of a linear switched polytopic system. Lyapunov function approach guarantees the multi-parameter-dependent quadratic stability (MPDQS) and guaranteed cost for a closed-loop system. In the proposed control scheme the required on line computation load is significantly less than that in MPC references, which opens possibility to use this control design scheme not only for plants with slow dynamics but also for faster ones. Sufficient robust stability conditions are given in the form of BMI and respective heuristic LMI iterative algorithm. The examples show the effectiveness of the proposed output feedback design method.*
Keywords: Model predictive control, Switched systems, Robust control, Lyapunov function, Polytopic system

1. **Introduction.** Model predictive control (MPC) is an attractive control methodology widely used in the academic and industry field. The popularity of MPC is mostly due to its ability to directly deal with constraints leading to a safe operation of the plant under all circumstances. In addition, performance criteria can be also embedded into MPC problem hence improving the economics or quality of the operation. Based on the plant model, constraints, measurements and cost function at every instant time, MPC predicts the output plant variables up to time $t + N_y$ – prediction horizon and requires the on-line solution of an optimization problem to compute optimal control inputs over a fixed number of future time instants $t+N_u$ – control horizon. MPC is usually implemented in the Receding Horizon fashion. In this setup only the first input variable $u(t)$ is implemented on the real plant. At the next sampling time, the optimization problem is reformulated and solved with new measurement (Camacho and Bordons, 2004 [3]; Maciejovski, 2002 [15]; Rossiter, 2003 [25]). Therefore, the presence of the plant model is a necessary condition for the development of the predictive control. The success of MPC depends on the precision of the plant model. In practice, modelling real plants inherently includes uncertainties that have to be considered in control design; that control design procedure has to guarantee stability, performance and robustness properties of closed-loop systems in the whole uncertainty domain. The survey of present state of MPC design can be consulted in excellent papers of (Mayne et al., 2000 [16]; Zafiriou and Marchal, 1991 [31]; Polak and Yang, 1993 [21]; Zheng and Morari, 1993 [32]; Casavola et al., 2004 [4]; Kuwata et al., 2007 [9]; Orukpe et al., 2007 [18]; Huang et al., 2011 [7]; Xia et al., 2008 [30]). There exist various approaches to robustness issue in MPC, we concentrate on those, based on LMI-BMI formulation.

In the seminal work of (Kothare et al., 1996 [8]), the polytopic model or structured feedback uncertainty model has been proposed for a design of robust MPC algorithm. The main idea of (Kothare et al., 1996 [8]) is the use of infinite horizon ($N_y \to \infty$) and the respective design of control laws which guarantee the stability and robustness properties with simple state feedback. In (Ding et al., 2008 [5]), output feedback robust MPC for systems with both polytopic and bounded uncertainty with input/state constraints is presented. Off-line, a sequence of output feedback laws based on the state estimators is calculated, by solving LMI optimization problem. On-line, at each sampling time, an appropriate output feedback law from this sequence is chosen. In (Veselý et al., 2010 [28]) the new concept of robust MPC control design procedure for polytopic systems and input constraints for finite prediction horizon is proposed. In this paper the authors assume that the model prediction is known and plant model belongs to the class of polytopic systems. In (Nguyen et al., 2013 [22]), the original robust MPC control design procedure with input constraints is proposed. A plant model and prediction model are uncertain and design procedure is based on parameter-dependent Lyapunov function and guaranteed cost. One step ahead prediction robust MPC controller design is proposed in (Veselý and Rosinová, 2009 [26]).

The topic of hybrid systems has attracted considerable attention from the industrial and research community in recent decades. Wherever continuous and discrete dynamics interact, hybrid system arises; that hybrid system theory studies the behaviour of dynamical systems, which involve continuous models described by differential or difference equations and discrete model such as finite state machines or Petri net that describes the software and logical behaviour. There are several approaches to model hybrid systems, (Lunze and Lagarrigue, 2009 [13]). (Branicky et al., 1998 [1]) model a large class of hybrid systems as they consider a discrete event system and continuous dynamics modelled by differential or difference equation. Such models are used to formulate a general stability analysis and controller synthesis framework for hybrid systems. Results for modelling and stability analysis of hybrid systems have been presented in (Lygeros et al., 2005 [14]; Goebel and Teel, 2006 [6]; Lunze and Lagarrigue, 2009 [13]; Wang et al., 2012 [33]). The survey of present state of hybrid systems can be consulted in the excellent paper and book of (Lygeros et al., 2005 [14]) and (Lunze and Lagarrigue, 2009 [13]).

The research of MPC control of hybrid systems focuses on efficient ways to solve the finite horizon constrained optimization problem and on techniques to a priori guarantee stability and robustness properties of controlled system. The techniques developed for standard MPC such as stabilization condition and others, (Mayne et al., 2000 [16]) do not work for switched systems.

In this paper, we consider the class of hybrid system known as switched systems, (Liberzon, 2003 [11]). We pursue the idea of stability analysis of switched systems, (Lunze and Lagarrigue, 2009 [13], Lazar, 2006 [10]) and combine these results with those obtained in (Veselý et al., 2010 [28]) for a design of robust MPC controller using parameter-dependent Lyapunov function and guaranteed cost approach. In this way. the new robust MPC design procedure for hybrid system is obtained. The main contribution of present paper is that all time demanding computations of robust output feedback MPC control of hybrid system are realized off-line. The developed control design scheme employs multi-parameter-dependent quadratic stability (MPDQS) to guarantee the robustness and performance over the whole uncertainty domain.

The paper is organized as follows. Problem formulation and preliminaries on a prediction output/state model as a polytopic system as well as stability conditions for switched systems are given in Section 2. In Section 3, robust output feedback predictive controller design using bilinear matrix inequality and heuristic LMI approach is presented.

Two examples illustrate the effectiveness of the proposed method in Section 4. Finally, conclusions are given.

Hereafter, the following notational conventions will be adopted: given a symmetric matrix $P = P^T \in R^{n\times n}$, the inequality $P > 0$ ($P \geq 0$) denotes matrix positive definiteness (semi-definiteness). Given two symmetric matrices P, Q, the inequality $P > Q$ indicates that $P - Q > 0$. The notation $x(t+k)$ will be used to define at time t k-steps ahead prediction of a system variable x from time t onwards under specified initial state and input scenario. I denotes the identity matrix of corresponding dimensions. $q \in I_q$ indicates the arbitrary switching algorithm and $q+1$ is the first next mode to mode q for switching system.

2. **Problem Statement and Preliminaries.** Consider the following linear discrete-time switched uncertain system

$$x(t+1) = A_q(\alpha)x(t) + B_q(\alpha)u(t) \tag{1}$$
$$y(t) = C_q x(t), \quad q \in I_q = \{1, 2, \dots, N\}$$

where $x(t) \in R^n$, $u(t) \in R^m$, $y(t) \in R^l$ are state, control and output variables of the system, respectively; $A_q(\alpha)$, $B_q(\alpha)$ belong to the convex set S_q

$$S_q := \{A_q(\alpha) \in R^{n\times n}, B_q(\alpha) \in R^{n\times m}\} \tag{2}$$
$$\left\{A_q(\alpha), B_q(\alpha) = \sum_{j=1}^{K}(A_{qj}, B_{qj})\alpha_j, \alpha_j \geq 0\right\}$$
$$j = 1, 2, \dots, K, \quad \sum_{j=1}^{K} \alpha_j = 1$$

N-number of switched modes of uncertain system and I_q a finite set of indices; $q \in I_q$ indicates the arbitrary switching algorithm for switched system. We assume polytopic uncertainty in each mode, where K is a number of vertices of uncertainty box. We assume that for each mode number of vertices does not change. A_{qj}, B_{qj} and C_q are known matrices of corresponding dimensions with constant entries.

2.1. **Prediction model for a switched system.** Simultaneously with (1) we consider the nominal model of system (1) for each mode $q \in I_q$ in the form

$$x(t+1) = A_{qo}x(t) + B_{qo}u(t) \tag{3}$$
$$y(t) = C_q x(t), \quad q = 1, 2, \dots, N$$

where A_{qo}, B_{qo} are any constant matrices from the convex bounded domain S_q (2). The nominal model (3) will be used for calculation of $y(t+k)$, $k = 1, 2, \dots, N_y$ on output prediction horizon up to N_y, while (1) is considered as a real plant description providing plant output. Therefore, in the robust controller design we assume that for time t, output $y(t)$ for each mode of switched system is obtained from the uncertain real plant model (1), and the predicted outputs for time $t+1, \dots, t+N_y$ are obtained from model prediction, which is constructed by nominal model (3). Thus, the uncertain model for robust control calculations with predicted states and outputs for the time instants $t+k$, $k = 1, 2, \dots, N$ is given by

- $k = 2$
  $x(t+2) = A_{qo}x(t+1) + B_{qo}u(t+1) = A_{qo}A_q(\alpha)x(t) + A_{qo}B_q(\alpha)u(t) + B_{qo}u(t+1)$
- $k = 3$
  $x(t+3) = A_{qo}^2 A_q(\alpha)x(t) + A_{qo}^2 B_q(\alpha)u(t) + A_{qo}B_{qo}u(t+1) + B_{qo}u(t+2)$

- for k

$$x(t+k) = A_{qo}^{k-1}A_q(\alpha)x(t) + A_{qo}^{k-1}B_q(\alpha)u(t) + \sum_{i=0}^{k-2} A_{qo}^{k-i-2}B_{qo}u(t+1+i) \tag{4}$$

and the corresponding output is

$$y(t+k-1) = C_q x(t+k-1) \tag{5}$$

Uncertain model of plant (1) with predicted states (4), (5) for $k = 1, 2, \dots, N_y - 1$ can be written in a compact form as

$$\begin{aligned} z(t+1) &= A_{qf}(\alpha)z(t) + B_{qf}(\alpha)v(t) \\ y_f(t) &= C_{qf}z(t) \end{aligned} \tag{6}$$

where

$$\begin{aligned} z(t)^T &= [x(t)^T \dots x(t+N_y-1)^T], \\ v(t)^T &= [u(t)^T \dots u(t+N_u-1)^T], \\ y_f(t)^T &= [y(t)^T \dots y(t+N_y-1)^T] \end{aligned} \tag{7}$$

N_u is control horizon. For the next developments we assume that $N_y = N_u$, then matrices in (6) are

$$B_{qf}(\alpha) = \begin{bmatrix} B_q(\alpha) & 0 & \dots & 0 \\ A_{qo}B_q(\alpha) & B_{qo} & \dots & 0 \\ \dots & \dots & \dots & 0 \\ A_{qo}^{N_y-1}B_q(\alpha) & A_{qo}^{N_y-2}B_{qo} & \dots & B_{qo} \end{bmatrix} \tag{8}$$

$$A_{qf}(\alpha) = \begin{bmatrix} A_q(\alpha) & 0 & \dots & 0 \\ A_{qo}A_q(\alpha) & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ A_{qo}^{N_y-1}A_q(\alpha) & 0 & \dots & 0 \end{bmatrix} \tag{9}$$

$$C_{qf} = blockdiag\{C_q\} \in R^{N_y(l\times n)}$$

Matrix dimensions are $A_{qf}(\alpha) \in R^{nN_y\times nN_y}$, $B_{qf}(\alpha) \in R^{nN_y\times mN_y}$ and $C_f \in R^{lN_y\times nN_y}$.

It is well known, that the fixed order dynamic output feedback control design problem is a special case of the static output feedback problem (SOF), since the closed-loop system for the fixed order case has exactly the same structure as the SOF case with appropriately augmented system matrices. To assess the performance quality a quadratic cost function known from LQ theory is often used. However, in practice the response rate or overshoot are often limited. Therefore we include into the LQR cost function the additional term for state variable to open the possibility to damp the oscillations and limit the response rate. Consider the cost function, associated with the system (6), in the form

$$J = \sum_{t=0}^{\infty} J(t) \tag{10}$$

where

$$J(t) = z(t)^T Q z(t) + v(t)^T R v(t) + z(t+1)^T S z(t+1)$$

$Q = blockdiag\{Q_i\}$, $S = blockdiag\{S_i\}$, $i = 0, 1, \dots, N_y - 1$ and $R = blockdiag\{R_i\}$ are positive definite (semidefinite) and positive definite constant matrices of corresponding dimensions respectively.

The problem studied in this paper can be formulated as follows. Design the robust model predictive controller for switched uncertain system (1) and given prediction horizons N_y, N_u using output feedback in the form

$$v(t) = F_q y_f(t) = F_q C_{qf} z(t) \tag{11}$$

where F_q is the static output feedback gain matrix for $q = 1, 2, \ldots, N$ modes of uncertain plant which guarantee the closed-loop system (12) stability, robustness and guaranteed cost. In this paper, the output feedback gain matrix F_q is calculated off line and there is no need to update it every sample time. Equations (10) and (11) imply that for output feedback model predictive control design for switched systems we will use the infinite optimization horizon and finite output/input prediction horizon. For closed-loop system one obtain

$$z(t+1) = (A_{qf}(\alpha) + B_{qf}(\alpha)F_q C_{qf})z(t) = A_{qc}(\alpha)z(t) \tag{12}$$

Note that due to receding horizon strategy, only the first m rows of matrix F_q are used for real plant control. The other part of matrix F_q serves for predicted output variables calculation. Model prediction for output can be obtained from (4) and (5) substituting $A_q(\alpha) = A_{qo}$, $B_q(\alpha) = B_{qo}$. If control horizon $N_u < N_y$ then corresponding entries of matrix F_q are equal to zero.

2.2. **Robust stability of switched system.** To guarantee closed-loop stability of uncertain system over the whole uncertainty domain S_q, for $q = 1, 2, \ldots, N$, the concept of quadratic stability (QS) is frequently used. That is, one Lyapunov function is considered for the whole uncertainty domain S_q. Experience and stability analysis have shown that quadratic stability is rather conservative in many cases, therefore robust stability with parameter dependent Lyapunov function $P(\alpha)$ has been introduced in (Peaucelle et al., 2000 [19]). Using the concept of parameter dependent Lyapunov stability, it is possible to formulate the following definition and lemma for stability of switched systems.

Definition 2.1. *(Lunze and Lagarrigue, 2009 [13]) We say that a switched linear discrete-time system (1) is multi-parameter-dependent quadratically stable (MPDQS) if there exist three positive constants a_0, a_1, a_2 and a Lyapunov function*

$$V(x(t), q) = x(t)^T P_q(\alpha)x(t) \tag{13}$$

such that

$$a_1||x(t)||^2 \le V(x(t), q) \le a_2||x(t)||^2 \tag{14}$$

where $P_q(\alpha) = \sum_{j=1}^{K} P_{q,j}\alpha_j$ and whose difference along the system solutions is negative decreasing, that is for every $x(0) \in R^n$, every $q \in I_q$ and every t we have

$$\begin{aligned} \Delta V(x(t), q) =& V(x(t+1, q+1)) - V(x(t, q)) \\ \le& -a_0||x(t)||^2 \end{aligned} \tag{15}$$

Lemma 2.1. *(Lunze and Lagarrigue, 2009 [13]) The following statements are equivalent*

- *There exists a quadratic Lyapunov function*

  $$V(x(t), q) = x(t)^T P_q(\alpha)x(t)$$

  strictly decreasing along the system trajectories for all $q \in I_q$.
- *There exist N symmetric matrices $P_i(\alpha) = P_i(\alpha)^T$, $i = 1, 2, \ldots, N$ satisfying the LMI's*

  $$\begin{bmatrix} P_i(\alpha) & A_{ic}(\alpha)^T P_j(\alpha) \\ P_j(\alpha)A_{ic}(\alpha) & P_j(\alpha) \end{bmatrix} > 0 \tag{16}$$
- *There exist N symmetric positive definite matrices $S_i(\alpha)$ and N matrices G_i, $i = 1, 2, \ldots, N$, satisfying the LMI's*

  $$\begin{bmatrix} G_i + G_i^T - S_i(\alpha) & G_i^T A_{ic}(\alpha) \\ A_{ic}(\alpha)G_i & S_j(\alpha) \end{bmatrix} > 0 \tag{17}$$

  $$(i, j) \in I_q \times I_q$$

The notion of guaranteed cost is defined in a standard way.

Definition 2.2. *Consider the system (6). If there exists a control law $v(t)^*$ and a positive scalar J^* such that the closed-loop system (12) is stable and the closed-loop cost function (10) value J satisfies $J \leq J^*$ then J^* is said to be the guaranteed cost and $v(t)^*$ is said to be the guaranteed cost control law for the system (6).*

Lemma 2.2. *(Rosinová et al., 2003 [24]) Consider the closed-loop system (12) with control algorithm (11). Control algorithm (11) is the guaranteed cost control law if and only if there exists a positive definite matrix $P_q(\alpha)$, $q = 1, 2, \dots, N$ and matrix F_q such that the following condition holds*

$$B_e = \Delta V(x(t), q) + J(t) \leq 0, \quad q = 1, 2, \dots, N \tag{18}$$

Considering the above robust stability results, the problem of robust MPC design for switched systems can be summarized as:

$$\min_{F_q}\{z(t)^T Q z(t) + v(t)^T R v(t) + z(t+1)^T S z(t+1)\} \tag{19}$$

Subject to:

- $z(t+1) = A_{qc}(\alpha) z(t)$
- robust stability condition (16)
- guaranteed cost condition (18)
- input, output, state or other constraints conditions.

For input constraints see (Veselý et al., 2010 [28]). The above described procedure (instead of constraints) will be formulated in the Bilinear Matrix Inequality (BMI) form in Theorem 3.1 and in the Linear Matrix Inequality (LMI) form in the two step heuristic switched controller design approach.

3. **Robust Model Predictive Controller Design for Switched Systems.** In this section we present a new procedure to design a static output feedback robust predictive controller for switched systems which ensures the guaranteed cost using PDQS. Our main result is summarized in the following theorem.

Theorem 3.1. *Consider the uncertain switched system (6) with control algorithm (11) and cost function (10). The closed-loop system (12) is robustly MPDQS in convex set S_q, $q \in I_q$ defined by (2), if there exist positive definite matrix $P_q(\alpha)$, matrices N_{1q}, N_{2q}, and gain matrix F_q for $q = 1, 2, \dots, N$ such that the following BMI is satisfied*

$$W_q(\alpha) = \{w_{ijq}(\alpha)\}_{2\times 2} < 0 \tag{20}$$

where

$$\begin{aligned} w_{11q} &= P_{q+1}(\alpha) + N_{1q} + N_{1q}^T + S \\ w_{12q} &= -N_{1q} A_{qc}(\alpha) + N_{2q}^T \\ w_{22q} &= Q - P_q(\alpha) - N_{2q} A_{qc}(\alpha) - A_{qc}(\alpha)^T N_{2q}^T + C_{qf}^T F_q^T R F_q C_{qf} \end{aligned}$$

for all $q \in I_q$. In our denotation mode $q+1$ is the first next mode to mode q for arbitrary switching mode ($q \in I_q$).

Proof: Since the matrix $[A_{qc}(\alpha); I]$ has a full column rank, (20) implies that

$$[A_{qc}(\alpha)^T \quad I] W_q(\alpha) [A_{qc}(\alpha)^T \quad I]^T < 0$$

from which one obtains

$$A_{qc}(\alpha)^T P_{q+1}(\alpha) A_{qc}(\alpha) - P_q(\alpha) + Q + C'_{qf} F'_q R F_q C_{fq} + A_{qc}(\alpha)' S A_{qc}(\alpha) < 0 \tag{21}$$

which is equivalent to (18). It completes the proof of robust sufficient stability condition given by Theorem 3.1.

Because of linearity with respect to α_i, $i = 1, 2, \ldots, K$, the inequality (20) for jth vertex and qth mode can be written as follows.

Corollary 3.1. *The closed-loop system (14) is robust MPDQS in a convex set S_q, $q \in I_q$ if there exist positive definite matrix $P_q(\alpha)$, matrices N_{1q}, N_{2q}, and gain matrix F_q for $q = 1, 2, \ldots, N$ such that*

$$\begin{bmatrix} P_{q+1,j} + N_{1q} + N_{1q}^T + S & * \\ -A_{qcj}^T N_{1q}^T + N_{2q} & \vartheta_{qj} \end{bmatrix} < 0 \tag{22}$$

where

$$\vartheta_{qj} = Q - P_{qj} - N_{2q}A_{qcj} - A_{qcj}^T N_{2q}^T + C_{qf}^T F_q^T R F_q C_{qf}$$

$j = 1, 2, \ldots, K$, and for arbitrary switching $(q + 1, q) \in I_q \times I_q$).

We have used the multi Lyapunov function approach for robust controller design for each mode of hybrid system where for guaranteeing the robust stability and performance the parameter-dependent Lyapunov function has been used. Finally, we have multi-parameter-dependent Lyapunov function and multi-parameter-dependent quadratic stability (MPDQS). We can conclude that if the BMI's (20) are feasible with respect to $\varrho * I > P_{q,j} = P_{q,j}^T > 0$, N_{1q}, N_{2q} and static output feedback gain matrix F_q (it need to be stressed that F_q is constant output feedback gain matrix and there is no need to be updated at every sample time) then the closed-loop system (12) with control algorithm (11) is multi-parameter-dependent quadratically stable with guaranteed cost (10) in the convex set (2). Note that due to control horizon strategy only the first m rows of matrix F_q are used for real uncertain switched plant control, the other part of matrix F_q serves for prediction of output variables future values.

3.1. **Two step heuristic robust switched controller design approach.** One of the most important issues raised in numerical solution of optimization problem is feasibility or convergence in dependence on the problem size. Since the controller design requires numerical solution of BMI, and the size of the optimization problem significantly increases with increased number of step prediction $N_y(N_u)$, this limits the choice of $N_y(N_u)$. This feature restricts the effectiveness of proposed design procedures especially for large-scale systems. Solution of this issue can be then realized by reformulating the optimization problem into the form of LMI instead of BMI. A frequent approach to cope with nonlinear terms in (20) is to employ linearization. In our case, the nonlinear term on the main diagonal $(-N_{2q}A_{qc}(\alpha) - A_{qc}(\alpha)^T N_{2q}^T)$ can be linearized by its upper bound. It is important to note that the off-diagonal nonlinear term $(N_{1q}A_{qc}(\alpha))$ is much problematic. The problem is that the off-diagonal element should be bounded as a whole. For our case, the elimination approach proposed in Veselý et al. 2011 [29] can be used. In this section an alternative new two step heuristic robust switched controller design LMI approach is presented. In the first step, the classical elimination lemma is applied to (20) simultaneously for individual vertices with the same feedback gain matrix. In the second step, the overall system robust stability is checked through sufficient condition (20).

Heuristic approach

Let there exist MPDLM (13), static output feedback gain matrix F_q and matrix N_{q2} such that for the j-th vertex and mode $q \in I$, the following conditions hold:

$$A_{qcj}^T(P_{q+1j} + S)A_{qcj} - P_{qj} + Q + C_{fq}^T F_q^T R F_q C_{fq} < 0 \tag{23}$$

$$Q - P_{qj} - N_{q2}A_{qcj} - A_{qcj}^T N_{q2}^T + C_{fq}^T F_q^T R F_q C_{fq} < 0 \tag{24}$$

$$j = 1, 2, \dots, K; \quad (q, q+1) \in I \times I$$

From (23) and (24) after small manipulation and using Schur complement formula one obtains

$$\begin{bmatrix} P_{qj} - Q & A_{qcj}^T & A_{qcj}^T & C_{fq}^T F_q^T \\ A_{qcj} & S^{-1} & 0 & 0 \\ A_{qcj} & 0 & P_{q+1j}^{-1} & 0 \\ F_q C_{fq} & 0 & 0 & R^{-1} \end{bmatrix} > 0 \tag{25}$$

and

$$\begin{bmatrix} \phi_{qj} & G_{qj}^T \\ G_{qj} & -R \end{bmatrix} < 0 \tag{26}$$

where

$$\phi_{qj} = Q - P_{qj} - N_{q2} A_{qfj} - A_{qfj}^T N_{q2}^T - N_{q2} B_{qfj} R^{-1} B_{qfj}^T N_{q2}^T$$

and $G_{qj} = B_{qfj}^T N_{q2}^T - R F_q C_{fqj}$.

Note that the off-diagonal terms in (25) and (26) are in LMI's form, (linear with respect to unknown matrices) the only two nonlinear terms are in diagonal part, they are P_{q+1j}^{-1} and $-N_{q2} B_{qfj} R^{-1} B_{qfj}^T N_{q2}^T$. Using classical linearization approach, the above two BMI terms can be reduced to two LMI ones. On the base of heuristic approaches (25) and (26) the robust switched controller design procedure for a design of static output feedback controller is summarized as follows:

- For a given performance matrices Q, R, if the solutions of (25) and (26) are feasible with respect to matrices $F_q, P_{qj}, N_{q2}, q = 1, 2, \dots, N; j = 1, 2, \dots, K$ then the stability and performance is guaranteed for each mode and vertex of uncertainty box.
- For known gain matrix F_q checks the robust stability and performance using (20) which reduces to LMI conditions. If the solution is feasible, the designed robust controller guarantees the robust stability and performance in the uncertainty box defined by (2). If (20) is not feasible return to first step. For first step calculation increases the eigenvalues of matrix Q and obtains the new matrices F_q, $P_{q,j}$, N_{q2}. Robust stability and performance is checked in second step with original value of matrix Q.
- If there is no solution with proposed algorithm the two step heuristic algorithm fails.

4. **Examples.**

Example 4.1. *This example illustrates the methodology of the control design procedure proposed above (20), namely its ability to cope with robust stability of predictive control for hybrid system. Stability is assessed using spectral radius of closed-loop system matrix. This example was generated by computer. The basic data are: order of system $n = 3$, number of modes $N = 2$, number of inputs and outputs $m = 1$, number of uncertainty $p = 1$, eigenvalues of extended hybrid systems (extended system has been obtained using standard approach, see Veselý and Rosinová, 2013 [27]). The open loop system (including I term) eigenvalues are:*
mode $q = 1$, $eigHS1$:
$firstpolsystem = \{1.000; -0.6685; 0.1213; -0.0726\}$,
$secondpolsystem = \{1.000; -0.6649; 0.1132; -0.0605\}$;
mode $q = 2$, $eigHS2$:
$firstpolsystem = \{1.000; 0.6057; -0.6751; 0.0639\}$,
$secondpolsystem = \{1.000; 0.6265; -0.6581; 0.0492\}$.
Prediction and control horizons $N_y = 3$; $N_u = 3$. *The problem is to design PI robust predictive controller for the above hybrid system with performance matrices* $Q = qI$, $q =$

0.0001; $R = rI$; $r = 1$; $S = sI$; $s = 0$. *The obtained gain matrices which guarantee the closed-loop stability, robustness properties of predictive control of hybrid system for* $q = 1, 2$; F_q, *are*
Mode $q = 1$

$$F_1 = \begin{bmatrix} 1.2137 & 0.5588 & -0.0036 & -0.1363 & 0.3130 & 0.9458 \\ 0.1255 & 0.0807 & 0.0636 & -0.0496 & 0.1638 & 0.3993 \\ 0.1880 & 0.0535 & -0.0122 & -0.0254 & 0.0404 & 0.1506 \end{bmatrix}$$

Maximal abs(eigenvalue) of the closed-loop polytopic system is $MaxEig = 0.5618$. *Maximal eigenvalue of Lyapunov matrices for evaluation of the guaranteed cost is* $\max \lambda(P_i) = 457.8764$.
Mode $q = 2$

$$F_2 = \begin{bmatrix} 0.4336 & 0.5028 & -0.0965 & 0.1049 & -0.1791 & -0.6000 \\ 0.0092 & -0.4784 & 0.1124 & -0.0989 & 0.1778 & 0.7549 \\ 0.2983 & 0.1513 & -0.0168 & 0.0414 & -0.0902 & -0.2132 \end{bmatrix}$$

Maximal abs(eigenvalue) of the closed-loop polytopic system is $MaxEig = 0.7086$. *Maximal eigenvalue of Lyapunov matrices for evaluation of the guaranteed cost is* $\max \lambda(P_i) = 522.2856$. *Inequality (22) is feasible, the proposed predictive controller for hybrid system guarantees the robustness properties of the closed-loop system and its performance. Note that from the obtained gain matrices, the proportional and integral part of PI controller can be obtained in the following way (Veselý and Rosinová, 2013 [27]). The gain matrix* $F_q = [K_p + K_i \quad K_i]$ *or, last half part of gain matrix* F_q, $q = 1, 2$ *is equal to integral part of controller,* K_i *and first half part is equal to a sum of proportional* K_p *and integral* K_i *gain matrices. Note that due to control horizon strategy only the first* m *rows of matrix* F_q, $q = 1, 2$ *are used for real uncertain switched plant control that is for the first mode the control algorithm is given as* $(m = 1)$*:*

$$\begin{aligned} u(t) = &((1.2137 + .1363) - 0.1363/s)y(t) + ((.5588 - 0.3130) + 0.3130/s)y(t + 1) \\ &+ ((-0.0036 - 0.9458) + 0.9458/s)y(t + 2) \end{aligned}$$

and for the second mode

$$\begin{aligned} u(t) = &((0.4336 - 0.1049) - 0.1049/s)y(t) + ((.5028 + 0.1791) - 0.1791/s)y(t + 1) \\ &+ ((-0.0965 + 0.6000) - 0.6000/s)y(t + 2). \end{aligned}$$

From the above control algorithm it is clear that predictive control algorithm proposed in (Kothare et al., 1996 [8]) is a special case of algorithm proposed in this paper.

Example 4.2. *This example illustrates the methodology of the control design procedure proposed as heuristic approach to design robust controller which ensures MPDQS, guaranteed cost for hybrid system controlled by predictive algorithm. The discrete model of system (2) is generated by computer and extended for controller I part.*
Mode $q = 1$, *first vertex*

$$A_{q1} = \begin{bmatrix} -0.7186 & -0.0613 & -0.3136 & 0.0430 & -0.0597 & 0 \\ 0.1620 & 0.0167 & -0.2309 & -0.5262 & -0.0756 & 0 \\ -0.2881 & -0.1218 & 0.1070 & -0.1291 & -0.2105 & 0 \\ 0.2673 & -0.0575 & -0.1647 & -0.1866 & -0.0155 & 0 \\ -0.6844 & -0.4282 & 0.4393 & 0.0301 & -0.4115 & 0 \\ 0 & 1.0000 & 1.0000 & 1.0000 & 0 & 1.0000 \end{bmatrix};$$

$$B_{q1} = \begin{bmatrix} 0.1906 \\ -0.4126 \\ -0.2042 \\ -0.0475 \\ 0.0730 \\ 0 \end{bmatrix}; \quad C_q^T = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

second vertex

$$A_{q2} = \begin{bmatrix} -0.7157 & -0.0661 & -0.3276 & 0.0367 & -0.0659 & 0 \\ 0.1709 & 0.0163 & -0.2330 & -0.5341 & -0.0832 & 0 \\ -0.2857 & -0.1199 & 0.1016 & -0.1294 & -0.2071 & 0 \\ 0.2702 & -0.0580 & -0.1689 & -0.1846 & -0.0212 & 0 \\ -0.6953 & -0.4330 & 0.4358 & 0.0287 & -0.4104 & 0 \\ 0 & 1.0000 & 1.0000 & 1.0000 & 0 & 1.0000 \end{bmatrix};$$

$$B_{q2}^T = \begin{bmatrix} 0.1928 & -0.4160 & -0.2100 & -0.0440 & 0.0824 & 0 \end{bmatrix};$$

Mode $q = 2$, first vertex

$$A_{q1} = \begin{bmatrix} -0.4369 & 0.1016 & -0.4134 & -0.1973 & -0.1591 & 0 \\ 0.1907 & -0.1266 & -0.2434 & -0.4720 & -0.2845 & 0 \\ -0.1585 & -0.2925 & -0.1955 & -0.2475 & -0.0671 & 0 \\ 0.4310 & 0.0793 & -0.0017 & -0.1270 & 0.0971 & 0 \\ -0.4664 & -0.4959 & 0.2207 & -0.3991 & -0.2188 & 0 \\ 0.4000 & 1.0000 & 1.0000 & 1.0000 & 0.4000 & 1.0000 \end{bmatrix};$$

$$B_{q1} = \begin{bmatrix} 0.0937 \\ -0.7478 \\ -0.1154 \\ -0.1972 \\ -0.0063 \\ 0 \end{bmatrix}; \quad C_q^T = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

second vertex

$$A_{q2} = \begin{bmatrix} -0.4412 & 0.0969 & -0.4209 & -0.2022 & -0.1564 & 0 \\ 0.2002 & -0.1249 & -0.2441 & -0.4713 & -0.2918 & 0 \\ -0.1569 & -0.2892 & -0.1940 & -0.2486 & -0.0694 & 0 \\ 0.4369 & 0.0819 & -0.0054 & -0.1204 & 0.0878 & 0 \\ -0.4758 & -0.5023 & 0.2084 & -0.4005 & -0.2234 & 0 \\ 0.4000 & 1.0000 & 1.0000 & 1.0000 & 0.4000 & 1.0000 \end{bmatrix};$$

$$B_{q2}^T = \begin{bmatrix} 0.0978 & -0.7515 & -0.1224 & -0.1910 & -0.0015 & 0 \end{bmatrix}$$

For prediction and control horizons $N_y = 4$; $N_u = 4$ the size of resulting matrices are $A_{qf} \in R^{24\times 24}$, $B_{qf} \in R^{24\times 4}$ and $C_{qf} \in R^{8\times 24}$. The aim is to design PI robust predictive controller for the above switched system with performance specified by matrices $Q = qI$, $q = 0.001$; $R = rI$; $r = 1$; $S = sI$; $s = 0$ and $0 < P_{qj} < \varrho = 200$. After the first step (6 iteration procedure) and second step (no iteration) of heuristic algorithm the obtained gain matrices which guarantee the MPDQS of closed-loop predictive control of switched system (11) for mode $q = 1, 2$; F_q, we have obtained approximately the same results (27).

$$F_q = \begin{bmatrix} 0.3365 & 0.4815 & 0.3008 & 0.1453 & 0.1551 & -0.0979 & -0.2619 & 0.0900 \\ 0.5520 & 0.7268 & 0.0681 & -0.0059 & -0.0225 & 0.1848 & 0.0422 & 0.0775 \\ 0.0307 & -0.2350 & -0.0128 & 0.0903 & 0.0503 & -0.1999 & -0.0653 & -0.0697 \\ 0.2462 & 0.4764 & 0.0172 & -0.1505 & -0.0171 & 0.2547 & 0.0512 & 0.1561 \end{bmatrix} \tag{27}$$

Maximal abs(eigenvalues) of closed-loop polytopic system for the first and the second modes are: $MaxEigs = 0.9122, \quad 0.7190$ *respectively. Maximal eigenvalues of Lyapunov matrices for the first and the second modes for evaluation of the guaranteed cost are* $\max \lambda(P_i) = 178.1927, \quad 186.4939$ *respectively. For calculation of parameters P and I of controller gain matrices see Example 4.1. Note that due to control horizon strategy only the first row of matrix F_q, $q = 1, 2$ is used for real uncertain predicted switched plant control that is for the first and second mode the control algorithm is given as*

$$\begin{aligned} u(t) = &((0.3365 - 0.1551) + 0.1551/s)y(t) + ((0.4815 + 0.0979) - 0.0979/s)y(t+1) \\ &+ ((0.3008 + 0.2619) - 0.2619/s)y(t+2) + ((0.1453 - 0.09) + 0.09/s)y(t+3). \end{aligned}$$

5. **Conclusions.** The paper addresses the problem of designing the output/state feedback robust model predictive controller for hybrid systems with N_y and N_u output and control prediction horizons. The main contribution of the presented results is twofold: The obtained robust control algorithm guarantees the closed-loop system multi-parameter-dependent quadratic stability and guaranteed cost in the whole uncertainty domain. The required on-line computation load is significantly less than in robust MPC and hybrid control literature (according to the best knowledge of authors), which opens possibility to use this control design scheme not only for plants with slow dynamics but also for faster ones. All calculation (gain matrices) have been realized off-line. The design procedure leads to using BMI approach for small size systems and heuristic LMI iterative one for large scale systems. The proposed heuristic approach has been proved in practical examples as extremely successful. In each tested example the robust stability test was successful and approved the results obtained by the first step of the proposed heuristic procedure. Finally, two examples illustrate the effectiveness of the proposed method.

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