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FUZZY ADAPTIVE ACTUATOR FAILURE COMPENSATION DYNAMIC SURFACE CONTROL OF MULTI-INPUT AND MULTI-OUTPUT NONLINEAR SYSTEMS

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Abstract. *This paper develops an adaptive fuzzy control method for accommodating actuator faults in a class of uncertain multi-input and multi-output (MIMO) nonlinear systems with immeasurable states. The considered faults are modeled as both loss of effectiveness and lock-in-place (stuck at unknown place). With the help of fuzzy logic systems to approximate the unknown nonlinear functions, a fuzzy adaptive observer is developed for estimating the unmeasured states. Combining the dynamic surface control (DSC) approach with the backstepping design technique, a novel adaptive fuzzy faults-tolerant control (FTC) approach is constructed. It is proved that the proposed control approach can guarantee that all the signals of the resulting closed-loop system are semi-globally uniformly ultimately bounded (SUUB) and the tracking errors and observer errors converge to a small neighborhood of the origin. Simulation results are provided to show the effectiveness of the proposed approach.*

Keywords: Uncertain MIMO nonlinear systems, Fuzzy tolerant-control, Fuzzy state observer, Dynamic surface control, Stability analysis

1. **Introduction.** In the past decades, many approximation-based adaptive backstepping control approaches have been developed to deal with uncertain nonlinear systems in strict-feedback form via fuzzy logic systems [1] or neural networks (NNs) [2], for example, see [3-11] and the references therein. Adaptive fuzzy and NN backstepping control approaches in [3-5] are for single-input and single-output (SISO) nonlinear systems, and in [6,7] are for multiple-input and multiple-output (MIMO) nonlinear systems, while those in [8-11] are for SISO/MIMO nonlinear systems with immeasurable states. Adaptive fuzzy and NN backstepping control approaches can provide a systematic methodology of solving control problems for a larger class of unknown nonlinear systems. The main features of the above adaptive control approaches are as follows: (i) they can be used to deal with those nonlinear systems without satisfying the matching condition, and (ii) they do not require the unknown nonlinear functions are linearly parameterized. Therefore, the approximator-based adaptive fuzzy and NN backstepping control becomes one of the most popular design approaches to a large class of uncertain nonlinear systems.

Although the adaptive backstepping control has achieved a great progress, the aforementioned control approaches assume that all the components of the considered nonlinear systems are in good operating conditions. As we know, some faults, such as actuators and sensors usually exist in many real processes, which often degrade the control performances and even result in the instability of the control system or even catastrophic accidents [12]. It is thus important to develop an FTC scheme against actuator or sensor failures.

To handle the problem of nonlinear system with actuator or sensor faults, many FTC approaches have been developed, see for examples [13-19] and the references therein. [13,14] presented adaptive FTC methods for linear systems with both loss of effectiveness and lock-in-place actuator faults. [15,16] developed adaptive FTC methods for a class of SISO nonlinear systems and MIMO nonlinear systems with the same actuator faults as in [13,14], while [17-19] developed observer-based fuzzy FTC approaches for some nonlinear systems with additive profile faults. However, the above mentioned FTC schemes all require that the considered nonlinear systems satisfy the matching conditions or that the nonlinear functions in the controlled systems are known. To remove the above limitations, authors in [20,21] investigated a class of unknown SISO nonlinear systems in strict-feedback form with both loss of effectiveness and lock-in-place actuator faults, in which fuzzy logic systems are employed to approximate the unknown functions, and two adaptive fuzzy backstepping FTC schemes were developed by using the backstepping technique. The proposed control schemes guarantee not only the stability, but also the robust performance of the failed system. On the basis of the results of [20,21], authors in [22] proposed an adaptive fuzzy backstepping FTC scheme for unknown MIMO nonlinear systems in strict-feedback form, and the stability of the control system was proved. Two main limitations in [20-22] are as follows: i) The states of the systems are required to be available for measurement. Thus they cannot be applied to solve the problem of those nonlinear systems with the immeasurable states. ii) These control methods in [20-22] has the problem of "explosion of complexity". In fact, the "explosion of complexity" is caused by repeated differentiations of certain nonlinear functions, such as virtual controls [23,24]. To overcome the problem of "explosion of complexity", an adaptive neural backstepping control approach was first proposed by [23] for a class of SISO uncertain nonlinear systems based on the so-called dynamic surface control (DSC) technique. The proposed controller eliminates this problem by introducing a filter at each step of the neural backstepping approach. Therefore, the proposed controller becomes much simpler than the existing neural backstepping controllers. After [23], an adaptive fuzzy backstepping DSC robust fault-tolerant control scheme has been developed. However, there are two limitations in [24]: one is that the proposed control approach is based on an assumption that the states of the controlled system can be measured directly, and the other is that the considered plant is the type of SISO systems.

It is worth pointing out that the problems of actuator or sensor faults and unmeasured states widely exist in the complex nonlinear practice systems, for induction motor systems, hypersonic vehicle systems, chemical process systems, and power systems. Therefore, the failure compensation control design for nonlinear systems with unmeasured states and actuator or sensor faults is an important issue, which motivated us for this study.

Motivated by the aforementioned observations, in this paper, an adaptive fuzzy fault-tolerant control method is developed for a class of unknown MIMO nonlinear systems with the actuator faults of both the loss of effectiveness and lock-in-place, and without assuming that the states are available for measurements. With the help of fuzzy logic systems to approximate the unknown nonlinear functions, a fuzzy adaptive observer is developed to estimate the unmeasured states. Using the backstepping design technique and combining the backstepping DSC technique, a novel adaptive fuzzy fault-tolerant scheme is constructed. It is shown that the proposed control approach can guarantee that all the signals of the resulting closed-loop system are bounded, and the tracking error converges to a small neighborhood of zero even in the presence of the actuator faults and immeasurable states. The main advantages of the proposed FTC scheme are as follows: (i) by designing a state observer, the proposed FTC method removes the restrictive assumption in [20-22,24] that all the states of the system be measured directly;

and (ii) the proposed FTC method can overcome the problem of "explosion of complexity" inherent in [20-22,24]. It is proved that the proposed FTC method can guarantee that all the signals of the resulting closed-loop system are bounded, and the tracking error converges to a small neighborhood of zero.

2. Problem Formulations.

2.1. Nonlinear system descriptions.

Consider the following MIMO nonlinear system in strict-feedback form [6,7,11]:

$$\begin{cases} \dot{x}_{i,1} = x_{i,2} + f_{i,1}(\bar{x}_{i,1}) \\ \dot{x}_{i,2} = x_{i,3} + f_{i,2}(\bar{x}_{i,2}) \\ \quad\vdots \\ \dot{x}_{i,n_i-1} = x_{i,n_i} + f_{i,n_i-1}(\bar{x}_{i,n_i-1}) \\ \dot{x}_{i,n_i} = \bar{\omega}_{i,n_i}^T \bar{u}_i + f_{i,n_i}(X, \bar{u}_1, \bar{u}_2, \cdots, \bar{u}_{i-1}) \\ y_i = x_{i,1}, \quad 1 \le i \le m \end{cases} \tag{1}$$

where $\bar{x}_{i,j} = [x_{i,1}, \cdots, x_{i,j}]^T \in R^j$, $j = 1, 2, \cdots, n_i$ is the state vector for the first j differential equations of the ith subsystem, $X = [\bar{x}_{1,n_1}^T, \bar{x}_{2,n_2}^T, \cdots, \bar{x}_{m,n_m}^T]^T$, n_i is the order of the ith subsystem. $\bar{u}_i = [u_{i,1}, u_{i,2}, \cdots, u_{i,l_i}]^T \in R^{l_i}$ is the control input vector of ith subsystem, where l_i is the number of its elements, and $y_i \in R$ is the output of the ith subsystem. $f_{i,j}(\cdot)$ are unknown smooth nonlinear functions. $\bar{\omega}_{i,n_i} = [\omega_{i,n_i,1}, \omega_{i,n_i,2}, \cdots, \omega_{i,n_i,l_i}]^T$ are constant vectors. This paper assumes that the only variables $y_i = x_{i,1}$ $(i = 1, 2, \cdots, m)$ are available for measurement.

The actuator faults considered in this paper are both lock-in-place and loss of effectiveness, which are defined by [13-16,20,21] as follows:

Lock-in-place model:

$$u_{i,j}^F(t) = \bar{u}_{i,j}(t), \quad t \ge t_{i,j}, \quad j \in \{j_1, j_2, \cdots, j_p\} \tag{2}$$

where $\bar{u}_{i,j}(t)$ stands for the place, which expresses the jth actuator of the ith subsystem is stuck; $t_{i,j}$ is the time instant at which the lock-in-place fault occurs.

Loss of effectiveness model:

$$u_{i,k}^F(t) = \phi_{i,k}(t) \upsilon_{i,k}(t), \quad t \ge t_{i,k}, \quad k \in \overline{\{j_1, j_2, \cdots, j_p\}} \tag{3}$$

where $\upsilon_{i,k}(t)$ is the kth applied control input of the ith subsystem, and $t_{i,k}$ is the time instant at which the loss of effectiveness fault takes place. $\phi_{i,k}(t) \in [\underline{\phi}_{i,k}, 1]$ is an effective factor of the corresponding actuator $u_{i,k}^F$, and $0 < \underline{\phi}_{i,k} \le 1$ is the lower bound of $\phi_{i,k}(t)$. In particular, $\underline{\phi}_{i,k} = 1$ means that the actuator $u_{i,k}(t)$ is normal.

Taking (2) and (3) into account, the control input $u_{i,j}(t)$ can be written as

$$u_{i,j}(t) = (1 - \sigma_{i,j}) \phi_{i,j}(t) \upsilon_{i,j}(t) + \sigma_{i,j} \bar{u}_{i,j}(t), \quad i = 1, 2, \cdots, m, \quad j = 1, 2, \cdots, l_i \tag{4}$$

where $\sigma_{i,j}$ is the lock factor defined as follows:

$$\sigma_{i,j} = \begin{cases} 1 & \text{if the } j\text{th actuator in the } i\text{th subsystem is stucked} \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

Our control objective is to design a stable output feedback control scheme for the plant (1) with the actuator faults (2) and (3) such that all the signals in the closed-loop system are bounded and the output $y_i(t)$ can track the given reference signals $y_{i,d}(t)$ as closely as possible. In order to accomplish the control task, the following assumption is demanded for our design.

Assumption 1 [13-16]: System (1) is so constructed that, for the ith subsystem, when any up to the $(l_i - 1)$ actuators stuck at some unknown places, the other(s) may lose effectiveness as (3), the closed-loop system can still be driven to achieve the desired control objective.

3. **Fuzzy State Observer Design.** Write (1) in the state space form:

$$\begin{cases} \dot{\bar{x}}_{i,n_i} = A_i\bar{x}_{i,n_i} + K_iy_i + \sum_{j=1}^{n_i-1} B_{i,j}f_{i,j}(\bar{x}_{i,j}) + B_{i,n_i}(\bar{\omega}_{i,n_i}^T\bar{u}_i + f_{i,n_i}(X,\bar{u}_1,\cdots,\bar{u}_{i-1})) \\ y_i = C_i^T\bar{x}_{i,n_i} \end{cases} \tag{6}$$

where $A_i = \begin{bmatrix} -k_{i,1} & & \\ \vdots & I & \\ -k_{i,n_i} & 0 \ \ldots & 0 \end{bmatrix}_{n_i\times n_i}$, $K_i = \begin{bmatrix} k_{i,1} \\ \vdots \\ k_{i,n_i} \end{bmatrix}$, $B_{i,j}^T = [\underbrace{0\cdots 1}_{j}\cdots 0]_{1\times n_i}$, $C_i^T = [1\cdots 0\cdots 0]_{1\times n_i}$. Choose the vector K_i such that A_i is a Hurwitz matrix. Thus, given a $Q_i > 0$, there exists a $P_i > 0$ satisfying

$$A_i^TP_i + P_iA_i = -2Q_i \tag{7}$$

We can assume that the nonlinear terms in (1) can be approximated by the following fuzzy logic systems

$$\hat{f}_{i,j}(\hat{\bar{x}}_{i,j}|\theta_{i,j}) = \theta_{i,j}^T\varphi_{i,j}(\hat{\bar{x}}_{i,j}), \quad j = 1,\cdots,n_i-1 \tag{8}$$

$$\hat{f}_{i,n_i}(\hat{X}|\theta_{i,n_i}) = \theta_{i,n_i}^T\varphi_{i,n_i}(\hat{X}) \tag{9}$$

where $\hat{\bar{x}}_{i,j}$ and $\hat{X}$ are the estimates of $\bar{x}_{i,j}$ and X, respectively.

Define the optimal parameter vectors $\theta_{i,j}^*$ and θ_{i,n_i}^* as

$$\theta_{i,j}^* = \arg\min_{\theta_{i,j}\in\Omega_{i,j}}\left[\sup_{\left(\hat{\bar{x}}_{i,j},\hat{\bar{x}}_{i,j}\right)\in U_{i,j}}\left|\hat{f}_{i,j}\left(\hat{\bar{x}}_{i,j}|\theta_{i,j}\right) - f_{i,j}(\bar{x}_{i,j})\right|\right], \quad j = 1,\cdots,n_i-1$$

$$\theta_{i,n_i}^* = \arg\min_{\theta_{i,n_i}\in\Omega_{i,n_i}}\left[\sup_{(X,\hat{X})\in U}\left|\hat{f}_{i,n_i}(\hat{X}|\theta_{i,n_i}) - f_{i,n_i}(X)\right|\right]$$

where $\Omega_{i,j}$, Ω_{i,n_i}, $U_{i,j}$, U are compact regions for $\theta_{i,j}$, θ_{i,n_i}, $\bar{x}_{i,j}$, $\hat{\bar{x}}_{i,j}$, X, $\hat{X}$, respectively.

Also the fuzzy minimum approximation errors $\varepsilon_{i,j}$ are defined as

$$\varepsilon_{i,j} = f_{i,j}(\bar{x}_{i,j}) - \hat{f}_{i,j}(\hat{\bar{x}}_{i,j}|\theta_{i,j}^*), \quad j = 1,\cdots,n_i-1 \tag{10}$$

$$\varepsilon_{i,n_i} = f_{i,n_i}(X) - \hat{f}_{i,n_i}(\hat{X}|\theta_{i,n_i}^*) \tag{11}$$

In this paper, we assume that the fuzzy minimum approximation errors $\varepsilon_{i,j}$, $j = 1,\cdots,n_i$, satisfying $|\varepsilon_{i,j}| \le \varepsilon_{i,j}^*$, where $\varepsilon_{i,j}^*$ are known constants.

A fuzzy adaptive observer is designed for (11) as

$$\begin{cases} \dot{\hat{\bar{x}}}_{i,n_i} = A_i\hat{\bar{x}}_{i,n_i} + K_iy_i + \sum_{j=1}^{n_i-1} B_{i,j}\hat{f}_{i,j}(\hat{\bar{x}}_{i,j}|\theta_{i,j}) + B_{i,n_i}(\bar{\omega}_{i,n_i}^T\bar{u}_i + \hat{f}_{i,n_i}(\hat{X}|\theta_{i,n_i})) \\ y_i = C_i^T\hat{\bar{x}}_{i,n_i}, \quad i = 1,2,\cdots,m \end{cases} \tag{12}$$

Let $e_i = \bar{x}_{i,n_i} - \hat{\bar{x}}_{i,n_i}$ be observer errorthen from (6) and (12), we have the observer error equation

$$\dot{e}_i = A_ie_i + \varepsilon_i + \sum_{j=1}^{n_i-1} B_{i,j}\tilde{\theta}_{i,j}^T\varphi_{i,j}(\hat{\bar{x}}_{i,j}) + B_{i,n_i}\tilde{\theta}_{i,n_i}^T\varphi_{i,n_i}(\hat{X}) \tag{13}$$

where $\varepsilon_i = [\varepsilon_{i,1},\cdots,\varepsilon_{i,n_i}]^T$ and $\tilde{\theta}_{i,j} = \theta_{i,j}^* - \theta_{i,j}$, $j = 1,\cdots,n_i$.

Consider the Lyapunov function candidate $V_{i,0} = \frac{1}{2}e_i^T P_i e_i$. The time derivative of $V_{i,0}$ along with (13) is

$$\dot{V}_{i,0} = -e_i^T Q_i e_i + e_i^T P_i \varepsilon_i + e_i^T P_i \sum_{j=1}^{n_i-1} B_{i,j}\tilde{\theta}_{i,j}^T \varphi_{i,j}(\hat{\bar{x}}_{i,j}) + e_i^T P_i B_{i,n_i}\tilde{\theta}_{i,n_i}^T \varphi_{i,n_i}(\hat{X}) \tag{14}$$

By using Young's inequality $2ab \le a^2 + b^2$, we have

$$e_i^T P_i \varepsilon_i \le \frac{1}{2}\|e_i\|^2 + \frac{1}{2}\|P_i\|^2\|\varepsilon_i^*\|^2 \tag{15}$$

where $\varepsilon_i^* = [\varepsilon_{i,1}^*, \cdots, \varepsilon_{i,n_i}^*]^T$. For any a positive constant τ_i, by using Young's inequality, the fact $\varphi_{i,j}^T(\hat{\bar{x}}_{i,j})\varphi_{i,j}(\hat{\bar{x}}_{i,j}) \le 1$ and $\varphi_{i,n_i}^T(\hat{X})\varphi_{i,n_i}(\hat{X}) \le 1$, we have

$$\begin{aligned} & e_i^T P_i \sum_{j=1}^{n_i-1} B_{i,j}\tilde{\theta}_{i,j}^T \varphi_{i,j}(\hat{\bar{x}}_{i,j}) + e_i^T P_i B_{i,n_i}\tilde{\theta}_{i,n_i}^T \varphi_{i,n_i}(\hat{X}) \\ \le & 2\tau_i \lambda_{\max}^2(P_i)\|e_i\|^2 + \frac{1}{\tau_i}\sum_{j=1}^{n_i}\tilde{\theta}_{i,j}^T\tilde{\theta}_{i,j} \end{aligned} \tag{16}$$

where $\lambda_{\max}(P_i)$ is the largest eigenvalue of matrix P_i. Substituting (15) and (16) into (14) gives

$$\dot{V}_{i,0} \le -q_{i,0}\|e_i\|^2 + \frac{1}{2}\|P_i\|^2\|\varepsilon_i^*\|^2 + \frac{1}{\tau_i}\sum_{j=1}^{n_i}\tilde{\theta}_{i,j}^T\tilde{\theta}_{i,j} \tag{17}$$

where $q_{i,0} = \lambda_{\min}(Q_i) - (\frac{1}{2} + 2\tau_i\lambda_{\max}^2(P))$.

4. **Faults-Tolerant Control Design and Stability Analysis.** In this section, an adaptive fuzzy controller and parameters adaptive laws are to be developed in the framework of the backstepping design and dynamic surface control technique, so that all the signals in the closed-loop system are SUUB and the tracking errors $S_{i,1} = y_i - y_{i,d}$ are as small as desired.

To develop a solution to the control problem, we first use the following proportional-actuation scheme adopted by [13-16,20,21]:

$$\upsilon_{i,j} = b_{i,j}(\hat{\bar{x}}_{i,n_i})u_i$$

where $0 < \underline{b}_{i,j} \le b_{i,j}(\hat{\bar{x}}_{i,n_i}) \le \bar{b}_{i,j}$, $j = 1, 2, \cdots, l_i$; $\underline{b}_{i,j}$ and $\bar{b}_{i,j}$ are the lower and upper bounds of $b_{i,j}(\hat{\bar{x}}_{i,n_i})$, respectively. u_i is the adaptive fuzzy controller to be designed in the last step by the backstepping design.

The n_i-step adaptive fuzzy output feedback backstepping design is based on the following changes of coordinates.

$$S_{i,j} = \hat{x}_{i,j} - z_{i,j} \tag{18}$$

$$\chi_{i,j} = z_{i,j} - \alpha_{i,j-1}, \quad i = 1, \cdots, m, \quad j = 2, \cdots, n_i \tag{19}$$

where $S_{i,j}$ is called an error surface, $z_{i,j}$ is a state variable, which is obtained through a first-order filter on intermediate control function, $\alpha_{i,j-1}$ and $\chi_{i,j}$ is called the output error of the first-order filter.

Step 1: Expressing $x_{i,2}$ in terms of its estimate as $x_{i,2} = \hat{x}_{i,2} + e_{i,2}$, we obtain

$$\dot{S}_{i,1} = \hat{x}_{i,2} + \varepsilon_{i,1} + \theta_{i,1}^T\varphi_{i,1}(\hat{\bar{x}}_{i,1}) + \tilde{\theta}_{i,1}^T\varphi_{i,1}(\hat{\bar{x}}_{i,1}) + e_{i,2} + \varepsilon_{i,1} - \dot{y}_{i,d} \tag{20}$$

Consider the following Lyapunov function candidate

$$V_{i,1} = V_{i,0} + \frac{1}{2}S_{i,1}^2 + \frac{1}{2\gamma_{i,1}}\tilde{\theta}_{i,1}^T\tilde{\theta}_{i,1} \tag{21}$$

where $\gamma_{i,1} > 0$ is a design constant.

The time derivative of $V_{i,1}$ along with the solutions of (17) and (20) is

$$\begin{aligned}\dot{V}_{i,1} =& \dot{V}_{i,0} + S_{i,1}(\hat{x}_{i,2} + \varepsilon_{i,1} + \theta_{i,1}^T \varphi_{i,1}(\hat{\bar{x}}_{i,1}) + \tilde{\theta}_{i,1}^T \varphi_{i,1}(\hat{\bar{x}}_{i,1}) \\ &+ e_{i,2} + \varepsilon_{i,1} - \dot{y}_{i,d}) - \frac{1}{\gamma_{i,1}} \tilde{\theta}_{i,1}^T \dot{\theta}_{i,1} \\ \leq& - q_{i,0} \|e_i\|^2 + \frac{1}{2} \|P_i\|^2 \|\varepsilon_i^*\|^2 + \frac{1}{\tau_i} \sum_{j=1}^{n_i} \tilde{\theta}_{i,j}^T \tilde{\theta}_{i,j} \\ &+ S_{i,1}(\hat{x}_{i,2} + \theta_{i,1}^T \varphi_{i,1}(\hat{\bar{x}}_{i,1}) - \dot{y}_{i,d}) + S_{i,1} e_{i,2} + S_{i,1} \varepsilon_{i,1} \\ &+ \frac{1}{\gamma_{i,1}} \tilde{\theta}_{i,1}^T (\gamma_{i,1} S_{i,1} \varphi_{i,1}(\hat{\bar{x}}_{i,1}) - \dot{\theta}_{i,1})\end{aligned} \tag{22}$$

By using Young's inequality, we have

$$S_{i,1} e_{i,2} \leq \frac{1}{2} \|e_i\|^2 + \frac{1}{2} S_{i,1}^2 \tag{23}$$

$$S_{i,1} \varepsilon_{i,1} \leq \frac{1}{2} S_{i,1}^2 + \frac{1}{2} \varepsilon_{i,1}^{*2} \tag{24}$$

From (18) and (19), we obtain $\hat{x}_{i,2} = \chi_{i,2} + S_{i,2} + \alpha_{i,1}$. Take $\hat{x}_{i,2}$ as a virtual control and substitute (23) and (24) into (22) yields

$$\begin{aligned}\dot{V}_{i,1} \leq& - q_{i,1} \|e_i\|^2 + \frac{1}{2} \|P_i\|^2 \|\varepsilon_i^*\|^2 + \frac{1}{\tau_i} \sum_{j=1}^{n_i} \tilde{\theta}_{i,j}^T \tilde{\theta}_{i,j} \\ &+ S_{i,1}(\chi_{i,2} + S_{i,2} + \alpha_{i,1} + S_{i,1} + \theta_{i,1}^T \varphi_{i,1}(\hat{\bar{x}}_{i,1}) - \dot{y}_{i,d}) \\ &+ \frac{1}{2} \varepsilon_{i,1}^{*2} + \frac{1}{\gamma_{i,1}} \tilde{\theta}_{i,1}^T (\gamma_{i,1} S_{i,1} \varphi_{i,1}(\hat{\bar{x}}_{i,1}) - \dot{\theta}_{i,1})\end{aligned} \tag{25}$$

where $q_{i,1} = q_{i,0} + 1/2$.

Design the first intermediate control function $\alpha_{i,1}$ and the adaptation function $\theta_{i,1}$ as

$$\alpha_{i,1} = -c_{i,1} S_{i,1} - S_{i,1} - \theta_{i,1}^T \varphi_{i,1}(\hat{\bar{x}}_{i,1}) + \dot{y}_{i,d} \tag{26}$$

$$\dot{\theta}_{i,1} = \gamma_{i,1} \varphi_{i,1}(\hat{\bar{x}}_{i,1}) S_{i,1} - \sigma_i \theta_{i,1} \tag{27}$$

where $c_{i,1} > 0$ and $\sigma_i > 0$ are design parameters, $\theta_{i,1}(0) = 0$.

Substituting (26) and (27) into (25) results in

$$\begin{aligned}\dot{V}_{i,1} \leq& - q_{i,1} \|e_i\|^2 + \frac{1}{2} \|P_i\|^2 \|\varepsilon_i^*\|^2 + \frac{1}{\tau_i} \sum_{j=1}^{n_i} \tilde{\theta}_{i,j}^T \tilde{\theta}_{i,j} \\ &+ S_{i,1} \chi_{i,2} + S_{i,1} S_{i,2} - c_{i,1} S_{i,1}^2 + \frac{1}{2} \varepsilon_{i,1}^{*2} + \frac{\sigma_i}{\gamma_{i,1}} \tilde{\theta}_{i,1}^T \theta_{i,1}\end{aligned} \tag{28}$$

To avoid repeatedly differentiating $\alpha_{i,1}$ in the traditional backstepping design, which leads to the so called "explosion of complexity", in the sequel, we can introduce a new state variable $z_{i,2}$ and let $\alpha_{i,1}$ pass a first-order filter with constant $\eta_{i,2}$ to obtain $z_{i,2}$, i.e.,

$$\eta_{i,2} \dot{z}_{i,2} + z_{i,2} = \alpha_{i,1}, \quad z_{i,2}(0) = \alpha_{i,1}(0) \tag{29}$$

By the definition of $\chi_{i,2} = z_{i,2} - \alpha_{i,1}$, it yields $\dot{z}_{i,2} = -\frac{\chi_{i,2}}{\eta_{i,2}}$ and

$$\begin{aligned}\dot{\chi}_{i,2} &= \dot{z}_{i,2} - \dot{\alpha}_{i,1} \\ &= - \frac{\chi_{i,2}}{\eta_{i,2}} + B_{i,2}(S_{i,1}, S_{i,2}, \chi_{i,2}, \theta_{i,1}, y_{i,d}, \dot{y}_{i,d}, \ddot{y}_{i,d})\end{aligned} \tag{30}$$

where $B_{i,2}(\cdot)$ is a continuous function of variables $S_{i,1}, S_{i,2}, \chi_{i,2}, W_{i,1}, y_{i,d}, \dot{y}_{i,d}$ and $\ddot{y}_{i,d}$ with the following expression,

$$B_{i,2}(\cdot) = c_{i,1}\dot{S}_{i,1} + \dot{S}_{i,1} + \dot{\theta}_{i,1}^T\varphi_{i,1}(\hat{\bar{x}}_{i,1}) + \frac{\theta_{i,1}^T\partial\varphi_{i,1}(\hat{\bar{x}}_{i,1})}{\partial\hat{\bar{x}}_{i,1}}\dot{\hat{\bar{x}}}_{i,1} - \ddot{y}_{i,d}$$

Step j ($j = 2, \cdots, n_i - 1$): From (12), we obtain

$$\dot{\hat{x}}_{i,j} = -k_{i,j}\hat{x}_{i,1} + \hat{x}_{i,j+1} + k_{i,j}x_{i,1} + \theta_{i,j}^T\varphi_{i,j}(\hat{\bar{x}}_{i,j}) \tag{31}$$

The time derivative of $S_{i,j} = \hat{x}_{i,j} - z_{i,j}$ is

$$\dot{S}_{i,j} = -k_{i,j}\hat{x}_{i,1} + \hat{x}_{i,j+1} + k_{i,j}x_{i,1} + \theta_{i,j}^T\varphi_{i,j}(\hat{\bar{x}}_{i,j}) - \dot{z}_{i,j} \tag{32}$$

Introduce a new state variable $z_{i,j+1}$ and let $\alpha_{i,j}$ pass through a first-order filter with the constant $\eta_{i,j+1}$ to obtain $z_{i,j+1}$

$$\eta_{i,j+1}\dot{z}_{i,j+1} + z_{i,j+1} = \alpha_{i,j}, \quad z_{i,j+1}(0) = \alpha_{i,j}(0) \tag{33}$$

Let $\chi_{i,j+1} = z_{i,j+1} - \alpha_{i,j}$, it yields $\dot{z}_{i,j+1} = -\frac{\chi_{i,j+1}}{\eta_{i,j+1}}$ and

$$\begin{aligned}\dot{\chi}_{i,j+1} &= \dot{z}_{i,j+1} - \dot{\alpha}_{i,j}\\ &= -\frac{\chi_{i,j+1}}{\eta_{i,j+1}} + B_{i,j+1}(S_{i,1}, \cdots, S_{i,j+1}, \chi_{i,2}, \cdots, \chi_{i,j}, \theta_{i,1}, \cdots, \theta_{i,j}, y_{i,d}, \dot{y}_{i,d}, \ddot{y}_{i,d})\end{aligned} \tag{34}$$

where

$$B_{i,j+1}(\cdot) = -k_{i,j}\dot{\hat{x}}_{i,1} + \dot{S}_{i,j-1} + k_{i,j}\dot{x}_{i,1} + \dot{\theta}_{i,j}^T\varphi_{i,j}(\hat{\bar{x}}_{i,j}) + \frac{\theta_{i,j}^T\partial\varphi_{i,j}}{\partial\hat{\bar{x}}_{i,j}}\dot{\hat{\bar{x}}}_{i,j} + \frac{\dot{\chi}_{i,j}}{\eta_{i,j}} + \tau_i\dot{S}_{i,j} + c_{i,j}\dot{S}_{i,j}$$

Substituting $\chi_{i,j} = z_{i,j+1} - \alpha_{i,j}$ into (32) results in

$$\dot{S}_{i,j} = \theta_{i,j}^T\varphi_{i,j}(\hat{\bar{x}}_{i,j}) + S_{i,j+1} + \chi_{i,j+1} + \alpha_{i,j} - k_{i,j}\hat{x}_{i,1} + k_{i,j}x_{i,1} - \dot{z}_{i,j} \tag{35}$$

Consider the Lyapunov function candidate

$$V_{i,j} = V_{i,j-1} + \frac{1}{2}S_{i,j}^2 + \frac{1}{2}\chi_{i,j}^2 + \frac{\tilde{\theta}_{i,j}^T\tilde{\theta}_{i,j}}{2\gamma_{i,j}} \tag{36}$$

where $\gamma_{i,j} > 0$ is a design parameter. The time derivative of $V_{i,j}$ is

$$\begin{aligned}\dot{V}_{i,j} \le & -q_{i,1}\|e_i\|^2 + \frac{1}{2}\|P_i\|^2\|\varepsilon_i^*\|^2 + \frac{1}{\tau_i}\sum_{j=1}^{n_i}\tilde{\theta}_{i,j}^T\tilde{\theta}_{i,j} + \sum_{l=1}^{j-1}S_{i,l}\chi_{i,l+1} - \sum_{l=1}^{j-1}c_{i,l}S_{i,l}^2\\ & + \frac{1}{2}\varepsilon_{i,1}^{*2} + \sum_{l=1}^{j-1}\frac{\sigma_i}{\gamma_{i,l}}\tilde{\theta}_{i,l}^T\theta_{i,l} + S_{i,j-1}S_{i,j} - \sum_{l=1}^{j-1}\left(\frac{\chi_{i,l+1}^2}{\eta_{i,l+1}} - B_{i,l+1}(\cdot)\chi_{i,l+1}\right)\\ & + \sum_{l=1}^{j-2}\frac{1}{\tau_i}\tilde{\theta}_{i,l+1}^T\tilde{\theta}_{i,l+1} + S_{i,j}(\theta_{i,j}^T\varphi_{i,j}(\hat{\bar{x}}_{i,j}) + \tau_iS_{i,j} + S_{i,j+1} + \chi_{i,j+1} + \alpha_{i,j}\\ & - k_{i,j}\hat{x}_{i,1} + k_{i,j}x_{i,1} - \dot{z}_{i,j}) + \frac{1}{\gamma_{i,j}}\tilde{\theta}_{i,j}^T(\gamma_{i,j}\varphi_{i,j}(\hat{\bar{x}}_{i,j})S_{i,j} - \dot{\theta}_{i,j})\end{aligned} \tag{37}$$

Choose the intermediate control function $\alpha_{i,j}$ and the adaptation function $\theta_{i,j}$ as follows:

$$\alpha_{i,j} = k_{i,j}\hat{x}_{i,1} - S_{i,j-1} - k_{i,j}x_{i,1} - \theta_{i,j}^T\varphi_{i,j}(\hat{\bar{x}}_{i,j}) + \dot{z}_{i,j} - \tau_iS_{i,j} - c_{i,j}S_{i,j} \tag{38}$$

$$\dot{\theta}_{i,j} = \gamma_{i,j}S_{i,j}\varphi_{i,j}(\hat{\bar{x}}_{i,j}) - \sigma_i\theta_{i,j} \tag{39}$$

where $c_{i,j} > 0$ is a design constant and $\theta_{i,j}(0) = 0$.

Substituting (38) and (39) into (37), we can obtain

$$\begin{aligned}\dot{V}_{i,j} \leq & -q_{i,1}\|e_i\|^2 + \frac{1}{2}\|P_i\|^2\|\varepsilon_i^*\|^2 + \frac{1}{\tau_i}\sum_{j=1}^{n_i}\tilde{\theta}_{i,j}^T\tilde{\theta}_{i,j} + \sum_{l=1}^{j} S_{i,l}\chi_{i,l+1} \\ & - \sum_{l=1}^{j} c_{i,l}S_{i,l}^2 + \frac{1}{2}\varepsilon_{i,1}^{*2} + \sum_{l=1}^{j}\frac{\sigma_i}{\gamma_{i,l}}\tilde{\theta}_{i,l}^T\theta_{i,l} + S_{i,j}S_{i,j+1} \\ & - \sum_{l=1}^{j}\left(\frac{\chi_{i,l+1}^2}{\eta_{i,l+1}} - B_{i,l+1}(\cdot)\chi_{i,l+1}\right) + \sum_{l=1}^{j-1}\frac{1}{\tau_i}\tilde{\theta}_{i,l+1}^T\tilde{\theta}_{i,l+1}\end{aligned} \tag{40}$$

Step n_i: In the final design step, the actual control input u_i appears. From (12), we obtain

$$\dot{\hat{x}}_{i,n_i} = -k_{i,n_i}\hat{x}_{i,1} + \bar{\omega}_{i,n_i}^T\bar{u}_i + k_{i,n_i}x_{i,1} + \theta_{i,n_i}^T\varphi_{i,n_i}(\hat{X}) \tag{41}$$

Note that by (4), we have

$$\bar{\omega}_{i,n_i}^T\bar{u}_i = \sum_{j=1}^{l_i}\omega_{i,n_i,j}u_{i,j} = \sum_{j=j_1\ldots j_p}\omega_{i,n_i,j}\bar{u}_{i,j} + \sum_{j\neq j_1\ldots j_p}\phi_{i,j}(t)\omega_{i,n_i,j}b_{i,j}(\hat{\bar{x}}_{i,n_i})u_i \tag{42}$$

Substituting (42) into (41) results in

$$\dot{\hat{x}}_{i,n_i} = -k_{i,n_i}\hat{x}_{i,1} + \sum_{j=j_1\ldots j_p}\omega_{i,n_i,j}\bar{u}_{i,j} + \sum_{j\neq j_1\ldots j_p}\phi_{i,j}(t)\omega_{i,n_i,j}b_{i,j}(\hat{\bar{x}}_{i,n_i})u_i + k_{i,n_i}x_{i,1} + \theta_{i,n_i}^T\varphi_{i,n_i}(\hat{X}) \tag{43}$$

The time derivative of $S_{i,n_i} = \hat{x}_{i,n_i} - z_{i,n_i}$ is

$$\begin{aligned}\dot{S}_{i,n_i} = & -k_{i,n_i}\hat{x}_{i,1} + \sum_{j=j_1\ldots j_p}\omega_{i,n_i,j}\bar{u}_{i,j} + \sum_{j\neq j_1\ldots j_p}\phi_{i,j}(t)\omega_{i,n_i,j}b_{i,j}(\hat{\bar{x}}_{i,n_i})u_i \\ & + k_{i,n_i}x_{i,1} + \theta_{i,n_i}^T\varphi_{i,n_i}(\hat{X}) - \dot{z}_{i,n_i}\end{aligned} \tag{44}$$

Consider the Lyapunov function candidate

$$V_{i,n_i} = V_{i,n_i-1} + \frac{1}{2}S_{i,n_i}^2 + \frac{1}{2}\chi_{i,n_i}^2 + \frac{\tilde{\theta}_{i,n_i}^T\tilde{\theta}_{i,n_i}}{2\gamma_{i,n_i}} \tag{45}$$

where $\gamma_{i,n_i} > 0$ is a design parameter.

Choose the control u_i and the adaptation function $\theta_{i,j}$ as follows:

$$\begin{aligned}u_i = (g_i)^{-1}\Bigg[& k_{i,n_i}\hat{x}_{i,1} - S_{i,n_i-1} - k_{i,n_i}x_{i,1} - \theta_{i,n_i}^T\varphi_{i,n_i}(\hat{X}) \\ & + \dot{z}_{i,n_i} - \tau_i S_{i,n_i} - c_{i,n_i}S_{i,n_i} - \sum_{j=j_1\ldots j_p}\omega_{i,n_i,j}\bar{u}_{i,j}\Bigg]\end{aligned} \tag{46}$$

$$\dot{\theta}_{i,n_i} = \gamma_{i,n_i}S_{i,n_i}\varphi_{i,n_i}(\hat{X}) - \sigma_i\theta_{i,n_i} \tag{47}$$

where $g_i = \sum_{j\neq j_1\ldots j_p}\phi_{i,j}(t)\omega_{i,n_i,j}b_{i,j}(\hat{\bar{x}}_{i,n_i})$, $c_{i,n_i} > 0$ is a design constant and $\theta_{i,n_i}(0) = 0$.

$$\begin{aligned}\dot{V}_{i,n_i} \leq & -q_{i,1}\|e_i\|^2 + \frac{1}{2}\|P_i\|^2\|\varepsilon_i^*\|^2 + \frac{1}{\tau_i}\sum_{j=1}^{n_i}\tilde{\theta}_{i,j}^T\tilde{\theta}_{i,j} + \sum_{l=1}^{n_i-1}S_{i,l}\chi_{i,l+1} - \sum_{l=1}^{n_i}c_{i,l}S_{i,l}^2 \\ & + \frac{1}{2}\varepsilon_{i,1}^{*2} + \sum_{l=1}^{n_i}\frac{\sigma_i}{\gamma_{i,l}}\tilde{\theta}_{i,l}^T\theta_{i,l} - \sum_{l=1}^{n_i-1}\left(\frac{\chi_{i,l+1}^2}{\eta_{i,l+1}} - B_{i,l+1}(\cdot)\chi_{i,l+1}\right) + \sum_{l=1}^{n_i-1}\frac{1}{\tau_i}\tilde{\theta}_{i,l+1}^T\tilde{\theta}_{i,l+1}\end{aligned} \tag{48}$$

Finally, choose the Lyapunov function candidate for the whole system as

$$V = \sum_{l=1}^{m} V_{l,n_l} \tag{49}$$

The time derivative of V is

$$\begin{aligned}\dot{V} = \sum_{l=1}^{m} \dot{V}_{l,n_l} \leq \sum_{i=1}^{m} \Bigg\{ & - q_{i,1}\|e_i\|^2 + \frac{1}{2}\|P_i\|^2\|\varepsilon_i^*\|^2 + \frac{1}{\tau_i}\sum_{j=1}^{n_i}\tilde{\theta}_{i,j}^T\tilde{\theta}_{i,j} \\ & + \sum_{l=1}^{n_i-1} S_{i,l}\chi_{i,l+1} - \sum_{l=1}^{n_i} c_{i,l}S_{i,l}^2 + \frac{1}{2}\varepsilon_{i,1}^{*2} + \sum_{l=1}^{n_i}\frac{\sigma_i}{\gamma_{i,l}}\tilde{\theta}_{i,l}^T\theta_{i,l} \\ & - \sum_{l=1}^{n_i-1}\left(\frac{\chi_{i,l+1}^2}{\eta_{i,l+1}} - B_{i,l+1}(\cdot)\chi_{i,l+1}\right) + \sum_{l=1}^{n_i-1}\frac{1}{\tau_i}\tilde{\theta}_{i,l+1}^T\tilde{\theta}_{i,l+1}\Bigg\}\end{aligned} \tag{50}$$

Assumption 2: For all initial conditions, there exists positive constant $p_i > 0$, satisfying $V_{i,n_i}(0) \leq p_i$.

Theorem 4.1. *Under Assumptions 1 and 2 and suppose that $\theta_{i,1}$, $\theta_{i,j}$ and θ_{i,n_i} are adapted via adaptation laws (27), (39) and (47), respectively. Then for any initial conditions satisfying Assumption 2, there exist $c_{i,j}$, $\gamma_{i,j}$, $\eta_{i,j}$, σ_i, Q_i, π_i and $k_{i,j}$ ($i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n_i$) such that all signals of the closed-loop system consisting of (1) are semi-globally uniformly ultimately bounded. Moreover, the tracking error $S_{i,1}(t) = x_{i,1}(t) - y_{i,d}(t)$ can be made arbitrarily small by choosing appropriate design parameters.*

Proof: By using the Young's inequalities, we have

$$S_{i,l}\chi_{i,l+1} \leq \frac{1}{2}S_{i,l}^2 + \frac{1}{2}\chi_{i,l+1}^2 \tag{51}$$

Substituting (51) into (50) gives

$$\begin{aligned}\dot{V} = \sum_{l=1}^{m} \dot{V}_{l,n_l} \leq \sum_{i=1}^{m} \Bigg\{ & - q_{i,1}\|e_i\|^2 + \frac{1}{2}\|P_i\|^2\|\varepsilon_i^*\|^2 + \frac{2}{\tau_i}\sum_{j=1}^{n_i}\tilde{\theta}_{i,j}^T\tilde{\theta}_{i,j} \\ & - \sum_{l=1}^{n_i}\left(c_{i,l} - \frac{1}{2}\right)S_{i,l}^2 + \frac{1}{2}\varepsilon_{i,1}^{*2} + \sum_{l=1}^{n_i}\frac{\sigma_i}{\gamma_{i,l}}\tilde{\theta}_{i,l}^T\theta_{i,l} \\ & - \sum_{l=1}^{n_i-1}\left(\frac{\chi_{i,l+1}^2}{\eta_{i,l+1}} - \frac{\chi_{i,l+1}^2}{2} - B_{i,l+1}(\cdot)\chi_{i,l+1}\right)\Bigg\}\end{aligned} \tag{52}$$

By using the Young's inequality, we have

$$|B_{i,l+1}(\cdot)\chi_{i,l+1}| \leq \frac{\chi_{i,l+1}^2 B_{i,l+1}^2(\cdot)}{2\pi_i} + 2\pi_i \tag{53}$$

where π_i is a positive design constant. It is noted that

$$\tilde{\theta}_{i,l}^T\theta_{i,l} = \tilde{\theta}_{i,l}^T(\theta_{i,l}^* - \tilde{\theta}_{i,l}) \leq -\frac{1}{2}\tilde{\theta}_{i,l}^T\tilde{\theta}_{i,l} + \frac{1}{2}\theta_{i,l}^{*T}\theta_{i,l}^* \tag{54}$$

Substituting (53) and (54) into (52), we can obtain

$$\begin{aligned}\dot{V} = \sum_{l=1}^{m} \dot{V}_{l,n_l} \leq \sum_{i=1}^{m} \Bigg\{ & - q_{i,1}\|e_i\|^2 - \sum_{l=1}^{n_i} \left(c_{i,l} - \frac{1}{2}\right) S_{i,l}^2 \\ & - \sum_{l=1}^{n_i} \left(\frac{\sigma_i}{2\gamma_{i,l}} - \frac{2}{\tau_i}\right) \tilde{\theta}_{i,l}^T \tilde{\theta}_{i,l} - \sum_{l=1}^{n_i-1} \left(\frac{1}{\eta_{i,l+1}} - \frac{1}{2} - \frac{B_{i,l+1}^2(\cdot)}{2\pi_i}\right) \chi_{i,l+1}^2 \\ & + 2(n_i - 1)\pi_i + \frac{1}{2}\|P_i\|^2\|\varepsilon_i^*\|^2 + \frac{1}{2}\varepsilon_{i,1}^{*2} + \sum_{l=1}^{n_i} \frac{\sigma_i}{2\gamma_{i,l}} \theta_{i,l}^{*T} \theta_{i,l}^* \Bigg\}\end{aligned} \tag{55}$$

Let $A_{i,j} = \{\sum_{k=1}^{j} [S_{i,k}^2 + \frac{1}{\gamma_{i,k}} \tilde{\theta}_{i,k}^T \tilde{\theta}_{i,k}] + \sum_{k=1}^{j-1} \chi_{i,k+1}^2 + e_i^T P_i e_i \leq 2p_i\}$, $i = 1, \cdots, m$, $j = 1, 2, \cdots, n_i$. Since $A_{i,j}$ is a compact set and $B_{i,k+1}$ is a continuous function, there exists a positive constant $M_{i,k+1}$ such that $|B_{i,k+1}(\cdot)| \leq M_{i,k+1}$ on $A_{i,j}$, therefore, we have

$$\begin{aligned}\dot{V} = \sum_{l=1}^{m} \dot{V}_{l,n_l} \leq \sum_{i=1}^{m} \Bigg\{ & - q_{i,1}\|e_i\|^2 - \sum_{l=1}^{n_i} \left(c_{i,l} - \frac{1}{2}\right) S_{i,l}^2 \\ & - \sum_{l=1}^{n_i} \left(\frac{\sigma_i}{2\gamma_{i,l}} - \frac{2}{\tau_i}\right) \tilde{\theta}_{i,l}^T \tilde{\theta}_{i,l} - \sum_{l=1}^{n_i-1} \left(\frac{1}{\eta_{i,l+1}} - \frac{1}{2} - \frac{M_{i,l+1}^2}{2\pi_i}\right) \chi_{i,l+1}^2 \\ & + 2(n_i - 1)\pi_i + \frac{1}{2}\|P_i\|^2\|\varepsilon_i^*\|^2 + \frac{1}{2}\varepsilon_{i,1}^{*2} + \sum_{l=1}^{n_i} \frac{\sigma_i}{2\gamma_{i,l}} \theta_{i,l}^{*T} \theta_{i,l}^* \Bigg\}\end{aligned} \tag{56}$$

Choose the design parameters $\eta_{i,l+1}$, σ_i, $\gamma_{i,l}$, τ_i and $c_{j,k}$ such that $c_{i,l} - \frac{1}{2} > 0$, $\frac{\sigma_i}{2\gamma_{i,l}} - \frac{2}{\tau_i} > 0$ and $\frac{1}{\eta_{i,l+1}} - \frac{1}{2} - \frac{M_{i,l+1}^2}{2\pi_i} > 0$, respectively. Define $c = \min\{c_1, \cdots, c_m\}$ with $c_i = \left\{q_{i,1}/\lambda_{\max}(P_p), 2(c_{i,j} - 1/2), \sigma_i, 2\left(\frac{1}{\eta_{i,l+1}} - \frac{1}{2} - \frac{M_{i,l+1}^2}{2\pi_i}\right)\right\}$, $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, n_i$, $k = 1, 2, \cdots, j-1$.

From (56), we obtain

$$\dot{V} \leq -cV + \rho \tag{57}$$

where $\rho = \sum\limits_{i=1}^{m} \left\{2(n_i - 1)\pi_i + \frac{1}{2}\|P_i\|^2\|\varepsilon_i^*\|^2 + \frac{1}{2}\varepsilon_{i,1}^{*2} + \sum\limits_{l=1}^{n_i} \frac{\sigma_i}{2\gamma_{i,l}} \theta_{i,l}^{*T} \theta_{i,l}^*\right\}$.

From (57), and using the same arguments as in [23,24], it follows that all the signals in the closed-loop system are SUUB. Moreover, the tracking errors $S_{i,1}(t) = x_{i,1}(t) - y_{i,d}(t)$, $i = 1, \cdots, m$ can be made arbitrarily small by adjusting the design parameters appropriately.

5. **Simulation Studies.**

In this section, an example is given to show the effectiveness of the proposed fuzzy adaptive fault-tolerant control approach.

Example 5.1. [25]*: Consider two inverted pendulums connected by a spring with actuator failures. Denoting $x_{1,1} = \theta_1$ (angular position), $x_{1,2} = \dot{\theta}_1$ (angular rate), $x_{2,1} = \theta_2$ and $x_{2,2} = \dot{\theta}_2$; Angular rates $x_{1,2} = \dot{\theta}_1$ and $x_{2,2} = \dot{\theta}_2$ are not available for measurement. The dynamics equations of the inverted pendulums can be described as*

$$\begin{cases} \dot{x}_{1,1} = x_{1,2} \\ \dot{x}_{1,2} = f_{1,2}(x_{1,1}, x_{1,2}, x_{2,1}, x_{2,2}) + \frac{\bar{\omega}_{1,2}^T \bar{u}_1}{J_1} \\ y_1 = x_{1,1} \end{cases} \tag{58}$$

$$\begin{cases} \dot{x}_{2,1} = x_{2,2} \\ \dot{x}_{2,2} = f_{2,2}(x_{1,1}, x_{1,2}, x_{2,1}, x_{2,2}, \bar{u}_1) + \frac{\bar{\omega}_{2,2}^T \bar{u}_2}{J_2} \\ y_2 = x_{2,1} \end{cases} \tag{59}$$

where $f_{1,1}(\bar{x}_{1,1}) = 0$, $f_{2,1}(\bar{x}_{2,1}) = 0$, $f_{1,2}(\bar{x}_{1,2}) = \left(\frac{m_1 gr}{J_1} - \frac{kr^2}{4J_1}\right)\sin(x_{1,1}) + \frac{kr}{2J_1}(l-b) + \frac{kr^2}{4J_1}\sin(x_{2,1})$, $f_{2,2}(\bar{x}_{2,2}, \bar{u}_1) = \left(\frac{m_2 gr}{J_2} - \frac{kr^2}{4J_2}\right)\sin(x_{2,1}) + \frac{kr}{2J_2}(l-b) + \frac{kr^2}{4J_2}\sin(x_{1,2}) + u_{1,1} + u_{1,2}$, $\bar{\omega}_{1,2} = [6,6]^T$, $\bar{\omega}_{2,2} = [5,5]^T$, $\bar{u}_1 = [u_{1,1}, u_{1,2}]^T$, $\bar{u}_2 = [u_{2,1}, u_{2,2}]^T$.

The parameters in (58) and (59) are chosen as $m_1 = 2$kg, $m_2 = 2$kg, $J_1 = 1$kg, $J_2 = 1$kg, $k = 10$N/m, $r = 0.1$m, $l = 0.5$m, $g = 9.81$m/s^2 and $b = 0.4$m, where, in the example, $b < l$ so that the pendulums repel one another when both are in the upright position.

Choosing fuzzy membership functions as

$$\mu_{F_{i,j}^l}(\hat{x}_{i,j}) = \exp\left[-\frac{(\hat{x}_{i,j} + 3 - l)^2}{6}\right], \quad i = 1,2, \quad j = 2, \quad l = 1,2,3,4,5$$

Construct $\hat{f}_{1,2}(\hat{\bar{x}}_{1,2} \,|\theta_{1,2}) = \theta_{1,2}^{\mathrm{T}}\varphi_{1,2}(\hat{\bar{x}}_{1,2}, \hat{\bar{x}}_{2,2})$ and $\hat{f}_{2,2}(\hat{\bar{x}}_{1,2}, \hat{\bar{x}}_{2,2} \,|\theta_{2,2}) = \theta_{2,2}^{\mathrm{T}}\varphi_{2,2}(\hat{\bar{x}}_{1,2}, \hat{\bar{x}}_{2,2})$ according to [1]. Setting the parameters $k_{1,1} = 7$, $k_{1,2} = 30$, $k_{2,1} = 7$ and $k_{2,2} = 40$, then fuzzy state observer is

$$\begin{cases} \dot{\hat{x}}_{1,1} = \hat{x}_{1,2} + 7(x_{1,1} - \hat{x}_{1,1}) \\ \hat{x}_{1,2} = 6u_{1,1} + 6u_{1,2} + \hat{f}_{1,2}(\hat{\bar{x}}_{1,2}, \hat{\bar{x}}_{2,2} \,|\theta_{1,2}) + 30(x_{1,2} - \hat{x}_{1,2}) \end{cases} \tag{60}$$

$$\begin{cases} \dot{\hat{x}}_{2,1} = \hat{x}_{2,2} + 7(x_{2,1} - \hat{x}_{2,1}) \\ \hat{x}_{2,2} = 5u_{2,1} + 5u_{2,2} + \hat{f}_{2,2}(\hat{\bar{x}}_{2,2} \,|\theta_{2,2}) + 40(x_{2,2} - \hat{x}_{2,2}) \end{cases} \tag{61}$$

Setting $\phi_{1,1}(t) = \phi_{1,2}(t) = 0.8$, $\phi_{2,1}(t) = \phi_{2,2}(t) = 0.9$, $b_{1,1}(\hat{x}_{1,1}, \hat{x}_{1,2}) = 12$, $b_{1,2}(\hat{x}_{1,1}, \hat{x}_{1,2}) = 12$, $b_{2,1}(\hat{x}_{2,1}, \hat{x}_{2,2}) = 13$, $b_{2,2}(\hat{x}_{2,1}, \hat{x}_{2,2}) = 13$.

Choose the first intermediate control function $\alpha_{i,1}$ (26) and the control input u_i (46) and the parameters update laws (27) and (47). The design parameters are chosen as $\gamma_{1,1} = \gamma_{1,2} = 0.2$, $\gamma_{2,1} = \gamma_{2,2} = 0.1$, $c_{1,1} = c_{1,2} = c_{2,1} = c_{2,2} = 2$, $\tau_1 = \tau_2 = 4$, $\sigma_1 = \sigma_2 = 1$, $\eta_{1,2} = \eta_{2,2} = 0.01$. The actuator faults introduced for simulation are $u_{1,2} = 2$ when $t \geq 2$, and $u_{1,1} = 0.8\upsilon_{1,1}$ for $t \geq 3$; $u_{2,2} = 1$ when $t \geq 2$, and $u_{2,1} = 0.9\upsilon_{2,1}$ for $t \geq 3$. In the simulation, the reference signals are $y_{1,d} = \cos(t)$, $y_{2,d} = \cos(t)$, and the initial conditions are chosen as zeros. The simulation results are shown in Figures 1-5.

Remark 5.1. *It should be mentioned that the existing adaptive fuzzy tolerant control approaches in [22] cannot be applied to the inverted pendulum system systems (58) and (59), since the states in systems (58) and (59) are unmeasured. The existing adaptive fuzzy control approach in [11] can be applied to the inverted pendulum systems (58) and (59) provided that it is free of the actuator faults. However, the considered systems (58) and (59) in this paper have the actuator faults in the form of both loss of effectiveness and lock-in-place. Therefore, the mentioned adaptive fuzzy control approach in [11] cannot be applied to control the systems (58) and (59).*

6. **Conclusions.** This paper has developed an adaptive fuzzy fault tolerant control method for a class of unknown MIMO nonlinear systems with actuator faults in with unmeasured states. The considered faults are modeled as both loss of effectiveness and lock-in-place. With the help of fuzzy logic systems to approximate the unknown nonlinear functions, a fuzzy adaptive observer has been developed for estimating the unmeasured states. Combining the backstepping DSC technique with the nonlinear tolerant-fault control theory, a novel adaptive FTC approach has been constructed. It has been proved

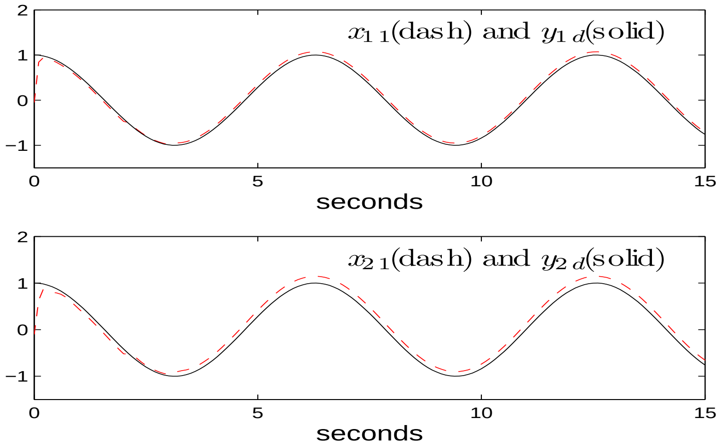

FIGURE 1. The curves of $x_{i,1}$ and $y_{i,d}$ $(i = 1, 2)$

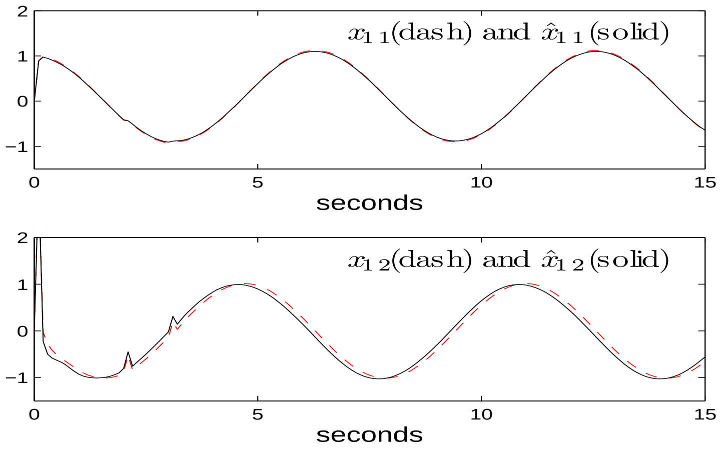

FIGURE 2. The curves of $x_{1,i}$ and $\hat{x}_{1,i}$ $(i = 1, 2)$

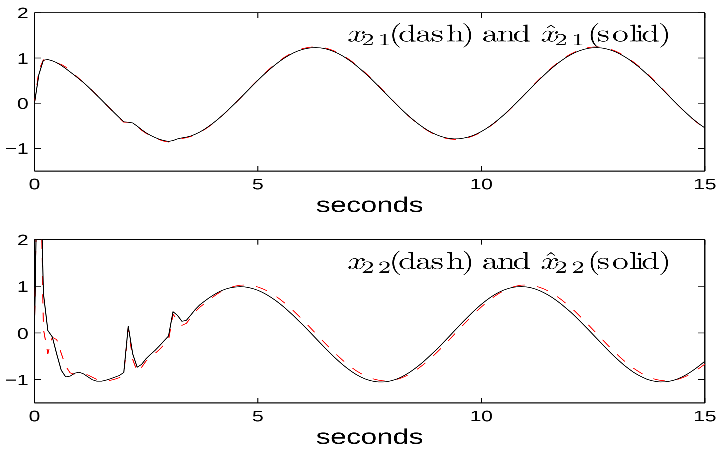

FIGURE 3. The curves of $x_{2,i}$ and $\hat{x}_{2,i}$ $(i = 1, 2)$

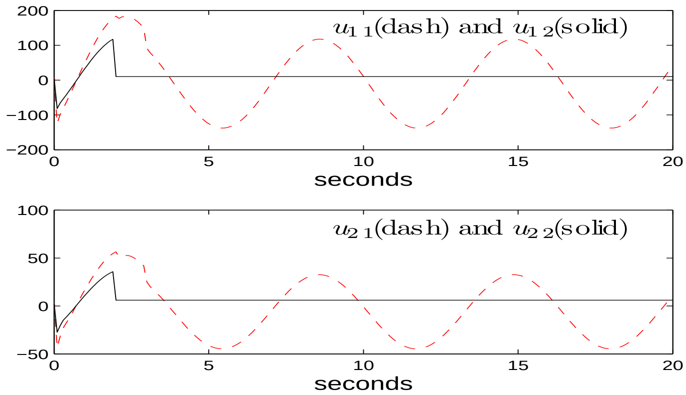

FIGURE 4. The curves of $u_{i,1}$ and $u_{i,2}$ $(i = 1, 2)$

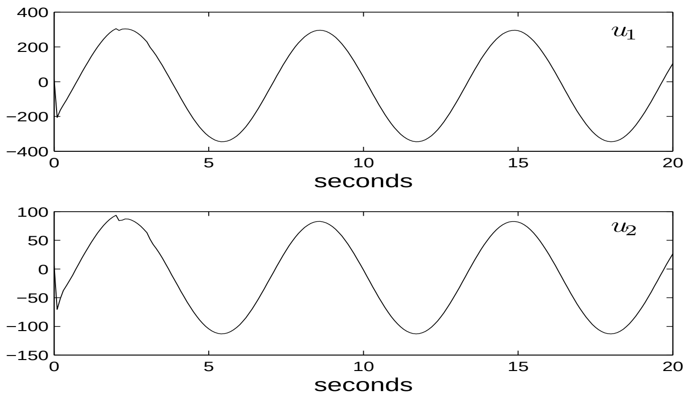

FIGURE 5. The curves of u_1 and u_2

that the proposed control approach can guarantee that all the signals of the resulting closed-loop system are bounded and the tracking error between the system output and the reference signal converges to a small neighborhood of zero by appropriate choice of the design parameters. Simulation results have been provided to show the effectiveness of the control approach. Future research will be concentrated on an adaptive fault-tolerant control design for discrete-time systems and related stability conditions that are based on the results of [26-28] and this paper.

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