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ROBUST QUANTIZED H_∞ CONTROL FOR NETWORK CONTROL SYSTEMS WITH MARKOVIAN JUMPS AND TIME DELAYS

Fangwen Li[1], Xingcheng Wang[1] and Peng Shi[2,3]

[1]College of Information Science and Technology
Dalian Maritime University
No. 1, Linghai Rd., Ganjingzi District, Dalian 116026, P. R. China
lifangwen2007@163.com; xc_wang@dl.cn

[2]School of Electrical and Electronic Engineering
The University of Adelaide
Adelaide, 5005 SA, Australia
peng.shi@vu.edu.au

[3]College of Engineering and Science
Victoria University
Melbourne VIC 8001, Australia

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Abstract. *This paper deals with the quantized H_∞ control problem for uncertain networked control system with Markovian jumps and time delays. In the study, network-induced delays and limited communication capacity due to signal quantization are both taken into consideration. The system contains time delays and Markovian jumps with partially known transition probabilities. By linear inequality approach, a sufficient condition is derived for the resulting closed-loop system to be stochastically stable with a prescribed H_∞ performance level. Finally, a numerical example is given to illustrate the effectiveness and efficiency of the proposed design method.*

Keywords: Network control system, Quantization, Markov jump linear system, Linear matrix inequality

1. **Introduction.** Networked control systems (NCSs) with Markovian jumps are typical complex stochastic dynamic systems, which can describe many real world systems, and more attention have been paid on stability analysis and control synthesis of this kind of complex stochastic dynamic systems, see for example [5-8] and the references therein. Network control systems become an important way to study complex systems due to their low cost, simple installation, maintenance and high reliability. Communication channels can reduce the cost of cables and power, simplify the installation and maintenance of the whole systems, and increase the reliability compared to the traditional point-to-point wiring system. NCSs have many applications such as remote surgery, unmanned aerial, vehicles and communication network. Now, more and more efforts have been devoted to both the stability and the control of the NCSs [1-4].

Due to the limited transmission capacity of the network and some devices in closed-loop systems, signals should be quantized before they sent to the next network node in practical. In order to get better performance of considered systems, more effects of quantization in NCSs should be took into consideration. The quantizer can be regarded as a coder which converts the continuous signal into piecewise continuous signal taking values in a finite set, which is usually employed when the observation and control signals are sent via limited communication channel. More attentions have been paid on the quantization problems in recent years, see for example, [11-14], and the references therein.

In NCSs, one of the important scheduling issues to treat is the effect of the network-induced delay on the system performance. For NCSs deal with different scheduling protocols, the network-induced delay may be constant, time-varying, or even random variable. There have been lots of works concerned with the analysis and synthesis problems for NCSs with network-induced delay, see for example, [9-12]. Among them, the time-delay NCSs modeled as Markov chains in NCSs have received much research attention. In the literature, there have been basically two approach in describing time-delay in Markov systems. Time-delay in Markovian jump systems independent and dependent on systems mode are reported in [15,17-20]. In [5-8], due to their practicality and simplicity in describing network-induced delays [5-8], stabilization and H_∞ problem are studied for employing Markovian systems to describe the network-included delay.

Looking into the existing results of NCSs, there are many works in studying time-delay modeled as Markov chains and quantization, respectively. However, in practice, network stochastic delay and quantization are quite often. However, there has been very limited work that has taken such type of multiple network-induced phenomenon into account. To the best of the authors' knowledge, up to know, little attention has been focus on NCSs with quantization and time-delay modeled as Markov jump system. On the other hand, NCSs lie at the intersection of control theory and communication theory. We need to consider the problem of robust stability and immeasurabilty of network together.

The goal of this paper is to study robust H_∞ control problem for uncertain NCSs with quantization and time-delays. Partially unknown transition probabilities of Markov chain with mode-dependent time-delays are used to model the system, and robust stochastically stable condition and quantized feedback controller are developed based on the quantization and delay-dependent with H_∞ performance. The sufficient conditions proposed are in linear matrix inequality (LMI) form. Finally, numerical examples are provided to illustrate the effectiveness of the proposed design approach.

2. **Problem Statement and Preliminaries.** Consider the following NCS:

$$\begin{aligned} x(k+1) = &A(r(k))x(k) + A_d(r(k))x(k-d(r(k))) \\ &+ B(r(k))u(k) + B_w(r(k))w(k) \\ z(k) = &C(r(k))x(k) + C_d(r(k))x(k-d(r(k))) \\ &+ D(r(k))u(k) + D_w(r(k))w(k) \end{aligned} \tag{1}$$

where for $k \in \mathcal{Z}$, $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}^m$ is the control input, $w(k) \in \mathbb{R}^p$ is the disturbance input which belongs to $\mathcal{L}_2[0, \infty)$, $z(k) \in \mathbb{R}^q$ is the output to be controlled. $d(r(k))$ is a constant, denoting the time-delay of the system when the system is in mode $r(k)$.

The parameter $r(k)$ represents a discrete-time homogeneous Markov chain taking values in a finite set $\mathcal{I} = \{1, 2, \cdots, N\}$ with the associated transition probability matrix $\Lambda \in \mathbb{R}^{N\times N}$ whose elements are given by $p_{ij} = \Pr\{r(k+1) = j | r(k) = i\}$, where $0 \le p_{ij} \le 1$, $\forall i, j \in \mathcal{I}$, and $\sum_{j=1}^{N} p_{ij} = 1$, $\forall i \in \mathcal{I}$.

In addition, the transition probabilities in Markov chain are considered to be partially accessed, that is, some elements in matrix Λ are unknown. For instance, system (1) with four modes, the transition probability matrix Λ may be in the form:

$$\Lambda = \begin{pmatrix} p_{11} & ? & p_{13} & p_{14} \\ p_{21} & ? & ? & ? \\ p_{31} & p_{32} & ? & p_{34} \\ ? & ? & p_{43} & p_{44} \end{pmatrix} \tag{2}$$

where "?" stands for the unknown element. For notation clarity, we denote that for any $i \in \mathcal{I}$

$$\mathcal{I}_k^i \triangleq \{j : p_{ij} \text{ is known}\}, \quad \mathcal{I}_{uk}^i \triangleq \{j : p_{ij} \text{ is unknown}\}. \tag{3}$$

To ease the presentation, in the following, we denote $A(r(k))$, $r(k) = i$ by A_i. The same notation will also be used for $A_d(r(k))$, $B(r(k))$, $B_w(r(k))$, $C(r(k))$, $C_d(r(k))$, $D(r(k))$, and $D_w(r(k))$.

Consider the uncertainties in system (1), we assume that

$$\begin{aligned} A(r(k)) &= \bar{A}(r(k)) + \Delta A(r(k)) \\ A_d(r(k)) &= \bar{A}_d(r(k)) + \Delta A_d(r(k)) \\ B(r(k)) &= \bar{B}(r(k)) + \Delta B(r(k)) \\ C(r(k)) &= \bar{C}(r(k)) + \Delta C(r(k)) \\ C_d(r(k)) &= \bar{C}_d(r(k)) + \Delta C_d(r(k)) \\ D(r(k)) &= \bar{D}(r(k)) + \Delta D(r(k)) \end{aligned}$$

where $\bar{A}(r(k))$, $\bar{A}_d(r(k))$, $\bar{B}(r(k))$, $\bar{C}(r(k))$, $\bar{C}_d(r(k))$, and $\bar{D}(r(k))$, for $r(k) = i$, $i \in \mathcal{I}$, are known real-valued constant matrices of appropriate dimensions that describe the nominal system. $\Delta A(r(k))$, $\Delta A_d(r(k))$, $\Delta B(r(k))$, $\Delta C(r(k))$, $\Delta C_d(r(k))$ and $\Delta D(r(k))$ are unknown matrices denoting the uncertainties in the system.

The admissible parameter uncertainties in this paper are assumed to be modeled as

$$\begin{pmatrix} \Delta A(r(k)) & \Delta A_d(r(k)) & \Delta B(r(k)) \\ \Delta C(r(k)) & \Delta C_d(r(k)) & \Delta D(r(k)) \end{pmatrix} = \begin{pmatrix} G_1(r(k)) \\ G_2(r(k)) \end{pmatrix} \Delta r(k) \begin{pmatrix} H_1(r(k)) & H_2(r(k)) & H_3(r(k)) \end{pmatrix}$$

with $\|\Delta r(k)\| \leq I$, $\forall k \in \mathcal{Z}$ and $\forall r(k) = i$, $i \in \mathcal{I}$.

Consider the quantization effect, it is assumed that the measurement signals will be quantized through the network before they are transmitted to the controller. The set of

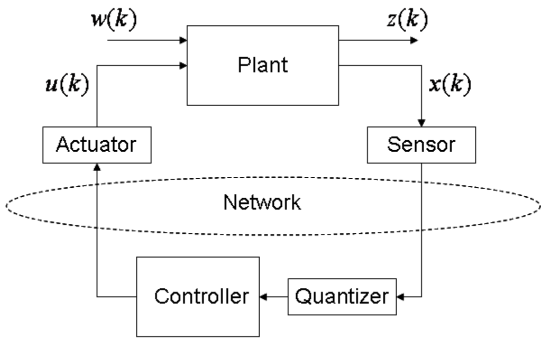

FIGURE 1. The structure of network control systems (1)

quantized levels is described as $\mathcal{U} = \{\pm u_i, u_i = \rho^i u_0, \pm 1, \pm 2, \ldots\} \cup \{0\}$, $0 < \rho < 1$, $u_0 > 0$, and the logarithmic quantizer $q(\cdot)$ as in [13] is applied

$$q(v) = \begin{cases} u_i, & \text{if } \frac{1}{1+\delta}\rho^i u_0 < v \leq \frac{1}{1-\delta}\rho^i u_0; \\ 0, & \text{if } v = 0; \\ -q(-v), & \text{if } v < 0 \end{cases} \tag{4}$$

where the parameter ρ is termed as quantization density, and $\delta = \frac{1-\rho}{1+\rho}$. From [13], we have

$$q(v) = (1 + \triangle_k)v \tag{5}$$

where $\triangle_k \in [-\delta, \delta]$, which is a suitable model for the logarithmic quantizer $q(v)$ with parameter δ.

Consider the quantizing effects are transformed into sector bounded uncertainties, associated to system (1), state feedback controller based on quantized state information is designed as

$$u(k) = K(r(k))q(x(k)) = K_i(I + \Delta_k)x(k), \ \ \|\Delta_k\| \leq \delta \tag{6}$$

where the matrix K_i are controller gains, and combining (1) and (6), the closed loop system is as follows:

$$\begin{aligned} x(k+1) &= \hat{A}_i x(k) + A_{di} x(k - d(r(k))) + B_{wi} w(k) \\ z(k) &= \hat{C}_i x(k) + C_{di} x(k - d(r(k))) D_{wi} w(k) \end{aligned} \tag{7}$$

where $\hat{A}_i = A_i + B_i K_i (I + \Delta_k)$, $\hat{C}_i = C_i + D_i K_i (I + \Delta_k)$.

2.1. **Several definitions and theorems.** In order to present the main results of this paper, we first introduce the following definitions and lemmas, which will be essential for the development of our main results. In order to present the main results of this paper, we first introduce the following definitions, which will be essential for our results.

Definition 2.1. *For system (1) is said to be stochastically stable, if for any initial $(x(0), r(0))$, the following holds*

$$E\left\{\sum_{k=0}^{\infty} \|x(k)\|^2 | x(0), r(0)\right\} < \infty \tag{8}$$

Definition 2.2. *Given the disturbance input $w(k) \in L_2$, a scalar $\gamma > 0$, system (1) is stochastically stable and with an H_∞ performance level γ if satisfies the following two requirements:*

1. When $w(k) = 0$, system (1) is stochastically stable in the sense of Definition 2.1.

2. When $w(k) \neq 0$, under zero initial conditions, the following inequality holds

$$E\left\{\sum_{k=0}^{\infty} \|z(k)\|^2\right\} < \gamma^2 \|w(k)\|^2 \tag{9}$$

Hence, the aim of this paper is to design state feedback controller K_i such that the system (7) with partially unknown transition probability Markovian chain is stochastically stable with an H_∞ performance level γ.

Lemma 2.1. *For any vectors $x, y \in \mathbb{R}^n$, matrices D, E and F with appropriate dimensions, and any scalar $\varepsilon > 0$, if $F^T F \leq I$, then*

$$DFE + E^T F^T D^T \leq \varepsilon D D^T \varepsilon + \varepsilon^{-1} E^T E \tag{10}$$

3. **Main Results.** Based on the previous results, our main purpose in this section is to develop the robust stochastically stable condition and design the feedback controller with an H_∞ performance for the NCS with time-varying delay, quantization and partially unknown transition probabilities Markovian chain. Firstly, sufficient conditions are given to ensure that the condition (8) holds.

Theorem 3.1. *Consider the system (7), when $w(k) = 0$, the system (7) with partially unknown transition probabilities Markovian chain is stochastically stable, if there exist matrices $P_i > 0$, $K_i > 0$, $i \in \mathcal{I}$ and $R > 0$ satisfying:*

$$\Phi_i = \begin{bmatrix} -P_i + (d_m - d_n + 1)R & * & * \\ 0 & -R & * \\ \hat{A}_i & A_{di} & -\bar{P}_i^{-1} \end{bmatrix} < 0 \tag{11}$$

where $\bar{P}_i = \sum_{j=1}^{N} p_{ij} P_j$, $d_m = \max\{d_i, i \in \mathcal{I}\}$, $d_n = \min\{d_i, i \in \mathcal{I}\}$.

Proof: Construct the following Lyapunov functional candidate for system (7) as

$$\begin{aligned} V(x(k), r(k)) &= V_1(x(k), r(k)) + V_2(x(k), r(k)) + V_3(x(k), r(k)) \\ V_1(x(k), r(k)) &= x^T(k) P(r(k)) x(k) \\ V_2(x(k), r(k)) &= \sum_{s=k-d(k)}^{k-1} x^T(s) R x(s) \\ V_3(x(k), r(k)) &= \sum_{j=k-d_2}^{k-d_1} \sum_{s=j}^{k-1} x^T(s) R x(s) \end{aligned} \tag{12}$$

Let $E\{\cdot\}$ stand for the mathematics statistical expectation of the stochastic process, one has from (7) for $r(k) = i$ and $r(k+1) = j$ is given by

$$E\{\Delta V(k)\} \triangleq E\{V(x(k+1), r(k+1)) | x(k), r(k)\} - V(x(k), r(k))$$

then for each $r(k) = i$, $i \in \mathcal{I}$, we obtain

$$\begin{aligned} E\{\Delta V_1(k)\} =& (\hat{A}_i x(k) + A_{di} x(k - d(r(k))))^T \bar{P}_i (\hat{A}_i x(k) + A_{di} x(k - d(r(k)))) \\ E\{\Delta V_2(k)\} =& x^T(k) R x(k) - x^T(k - d(k)) R x(k - d(k)) \\ &+ \sum_{s=k+1-d(k+1)}^{k-1} x^T(s) R x(s) - \sum_{s=k+1-d(k)}^{k-1} x^T(s) R x(s) \\ \le& x^T(k) R x(k) - x^T(k - d(k)) R x(k - d(k)) + \sum_{s=k-d_2+1}^{k-d_1} x^T(s) R x(s) \qquad (13) \\ E\{\Delta V_3(k)\} =& \sum_{j=k-d_2}^{k-d_1} \sum_{s=j}^{k} x^T(s) R x(s) - \sum_{j=k-d_2}^{k-d_1} \sum_{s=j}^{k-1} x^T(s) R x(s) \\ =& (d_2 - d_1) x^T(k) R x(k) - \sum_{s=k-d_2+1}^{k-d_1} x^T(s) R x(s) \end{aligned}$$

A combination of (13) leads to

$$E(\Delta V(k)) \le \eta^T(k) \bar{\Phi}_i \eta(k)$$

where

$$\eta^T(k) = \left[\ x^T(k) \quad x^T(k-d(k))\ \right]$$
$$\bar{\Phi}_i = diag\{-P_i + (d_m - d_n + 1)R, -R\} + [\hat{A}_i \quad A_{di}]^T \bar{P}_i [\hat{A}_i \quad A_{di}]$$

by Schur Lemma, we can get $\bar{\Phi}_i \leq \Phi_i$. From Theorem 3.1, we can obtain $\bar{\Phi}_i < 0$, that means

$$E\{\Delta V(k)\} \leq 0$$

thus

$$\begin{aligned} &E\{V(x(k+1), r(k+1))|x(k), r(k)\} - V(x(k), r(k)) \\ &\leq -\lambda_{\min}(-\Phi) \leq -\beta x(k)^T x(k) \end{aligned} \tag{14}$$

where $\lambda_{\min}(-\Phi)$ denotes the minimal eigenvalue of $-\Phi$ and $\beta = \inf\{\lambda_{\min}(-\Phi)\}$, from (14) we can obtain that for any $T \geq 1$

$$E\{V(x(T+1), r(T+1))\} - E\{V(x(0), r(0))\} \leq -\beta \sum_{k=0}^{T} E\{x(k)^T x(k)\}$$

Thus, the following holds for any $T \geq 1$

$$\begin{aligned} \sum_{k=0}^{T} E\{x(k)^T x(k)\} &\leq \frac{1}{\beta}(E\{V(x(0), r(0))\} - E\{V(x(T+1), r(T+1))\}) \\ &\leq \frac{1}{\beta} E\{V(x(0), r(0))\} \end{aligned}$$

Implying

$$\sum_{k=0}^{T} E\{x(k)^T x(k)\} \leq \frac{1}{\beta} E\{V(x(0), r(0))\} < \infty$$

Therefore, by Definition 2.1, it can be verified that the system (7) is stochastically stable. This complete the proof.

Theorem 3.2. *Consider system (7), when $w(k) \neq 0$, for given quantization density $\gamma > 0$, the system (7) with partially unknown transition probabilities Markovian chain is robust stochastically stable with an H_∞ performance level γ under zero initial condition, if there exist matrices $X_i > 0$, K_i, scalars $\varepsilon_{1i} > 0$, $i \in \mathcal{I}$ and $R > 0$ satisfying:*

$$\Omega_i = \begin{bmatrix} -X_i^{-1} + (d_m - d_n + 1)R & * & * & * & * & * \\ 0 & -R & * & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * \\ W_i^T \tilde{A}_i & W_i^T \bar{A}_{di} & W_i^T B_{wi} & \phi_{44} & * & * \\ \tilde{C}_i & \bar{C}_{di} & D_{wi} & \varepsilon_{1i} W_i^T G_{1i} G_{2i}^T & \phi_{55} & * \\ \tilde{H}_{1i} & H_{2i} & 0 & 0 & 0 & -\varepsilon_{1i} I \end{bmatrix} < 0 \tag{15}$$

where

$$\begin{aligned} \phi_{44} &= -\mathcal{X} + \varepsilon_{1i} W_i^T G_{1i} G_{1i}^T W_i \\ \phi_{55} &= -I + \varepsilon_{1i} G_{2i} G_{2i}^T \\ \tilde{H}_{1i} &= H_{1i} + H_{3i} K_i \\ \tilde{A}_i &= \bar{A}_i + \bar{B}_i K_i (I + \Delta_k) \end{aligned}$$

$$\tilde{C}_i = \bar{C}_i + \bar{D}_i K_i (I + \Delta_k)$$
$$\mathcal{X} = diag\{X_1, \cdots, X_N\}$$

Proof: First, by Theorem 3.1, the system (7) with $w(k) = 0$ is stochastically stable, so next, we proceed to prove the system with disturbance (7) has H_∞ performance level γ. For the next, the same Lyapunov functional as (12) and same techniques in the proof of Theorem 3.1 will be adopted to complete the proof of Theorem 3.2.

Define $\delta^T(k) = \begin{bmatrix} x^T(k) & x^T(k-d(k)) & w(k) \end{bmatrix}$, apply the inequality (15), we can obtain

$$E(\Delta V(k)) \leq \delta^T(k)\bar{\Omega}_i\delta(k) - (E(z^T(k)z(k)) - \gamma^2 w^T(k)w(k))$$

where

$$\bar{\Omega}_i = \begin{bmatrix} -P_i + (d_m - d_n + 1)R & * & * & * & * \\ 0 & -R & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * \\ \hat{A}_i & A_{di} & B_{wi} & -\bar{P}_i^{-1} & * \\ \hat{C}_i & C_{di} & D_{wi} & 0 & I \end{bmatrix} \tag{16}$$

For system (7) with partially unknown transition probabilities, it is easy to get

$$\begin{aligned} \bar{P}_i &= \sum_{j=1}^{N} p_{ij}P_j = \sum_{j\in I_k^i}^{N} p_{ij}P_j + \sum_{j\in I_{uk}^i}^{N} p_{ij}P_j \\ &\leq \sum_{j\in I_k^i}^{N} p_{ij}P_j + \left(1 - \sum_{j\in I_k^i}^{N} p_{ij}\right)\sum_{j\in I_{uk}^i}^{N} P_j = \tilde{P}_i. \end{aligned} \tag{17}$$

Noting that

$$\tilde{P}_i = W_i \mathcal{P} W_i^T$$

where

$$W_i = \left(\sqrt{p_{i1}}, \cdots, \sqrt{p_{ij}}, \sqrt{1 - \sum\nolimits_{j\in I_k^i}^{N} p_{ij}}, \cdots, \sqrt{1 - \sum\nolimits_{j\in I_k^i}^{N} p_{ij}}, \cdots, \sqrt{p_{iN}}\right)$$
$$\mathcal{P} = diag\{P_1, \cdots, P_N\}$$

and p_{ij} are known transition probabilities, and $1 - \sum\nolimits_{j\in I_k^i}^{N} p_{ij}$ stands for the unknown transition probabilities of elements.

According to Schur Lemma, we obtain $\bar{\Omega} < 0$ if and only if there exist

$$\tilde{\Omega}_i = \begin{bmatrix} -P_i + (d_m - d_n + 1)R & * & * & * & * \\ 0 & -R & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * \\ W_i^T\hat{A}_i & W_i^T A_{di} & W_i^T B_{wi} & -\mathcal{P}^{-1} * & \\ \hat{C}_i & C_{di} & D_{wi} & 0 & I \end{bmatrix} < 0 \tag{18}$$

Note that all admissible uncertainties of system (7), (18) can be written as

$$\tilde{\Omega}_i = \begin{bmatrix} -P_i + (d_m - d_n + 1)R & * & * & * & * \\ 0 & -R & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * \\ W_i^T\tilde{A}_i & W_i^T \bar{A}_{di} & W_i^T B_{wi} & -\mathcal{P}^{-1} * & \\ \tilde{C}_i & \bar{C}_{di} & D_{wi} & 0 & I \end{bmatrix}$$

$$+\begin{bmatrix} 0 & * & * & * & * \\ 0 & 0 & * & * & * \\ 0 & 0 & 0 & * & * \\ W_i^T G_{1i}\Delta_i \tilde{H}_{1i} & W_i^T G_{1i}\Delta_i H_{2i} & 0 & 0 & * \\ G_{2i}\Delta_i \tilde{H}_{1i} & G_{2i}\Delta_i H_{2i} & 0 & 0 & 0 \end{bmatrix} < 0 \tag{19}$$

In view of Lemma 2.1, we obtain that (19) holds, if and only if there exist scalars $\varepsilon_{1i} > 0$, such that

$$\begin{aligned} \hat{\Omega}_i = &\begin{bmatrix} -P_i + (d_m - d_n + 1)R & * & * & * & * \\ 0 & -R & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * \\ W_i^T \tilde{A}_i & W_i^T \bar{A}_{di} & W_i^T B_{wi} & -\mathcal{P}^{-1} & * \\ \tilde{C}_i & \bar{C}_{di} & D_{wi} & 0 & I \end{bmatrix} \\ &+ \varepsilon_{1i} \begin{bmatrix} 0 \\ 0 \\ 0 \\ W_i^T G_{1i} \\ G_{2i} \end{bmatrix} \Delta_i \begin{bmatrix} G_{1i}^T W_i & G_{2i}^T & 0 & 0 & 0 \end{bmatrix} \\ &+ \varepsilon_{1i}^{-1} \begin{bmatrix} \tilde{H}_{1i}^T \\ H_{2i}^T \\ 0 \\ 0 \\ 0 \end{bmatrix} \Delta_i \begin{bmatrix} \tilde{H}_{1i} & H_{2i} & 0 & 0 & 0 \end{bmatrix} < 0 \end{aligned} \tag{20}$$

By Schur Lemma, let $X_i = P_i^{-1}$. If matrix inequality (20) holds, from Theorem 3.2. $\Omega_i < 0$ is equal to $\hat{\Omega}_i < 0$. Then

$$E(\Delta V(k)) + E(z^T(k)z(k)) - \gamma^2 w^T(k)w(k) \le \delta^T(k)\bar{\Omega}_i\delta(k) < 0 \tag{21}$$

taking the sum of both sides of (21) from $k = 0$ to ∞, and recalling that $x(0) = 0$, the following inequality holds

$$E\left\{\sum_{k=0}^{\infty} z^T(k)z(k)\right\} \le \gamma^2 \sum_{k=0}^{\infty} w^T(k)w(k)$$

Therefore, by Definition 2.2, system (7) with partially unknown transition probability Markovian chain is stochastically stable with an H_∞ performance level γ.

4. **Control Design.**

In this section, we will consider the quantizer, and a robust quantized controller will be designed such that system (7) with time-delay and partially known transition probabilities Markovian chain is robustly stochastically stable and has a robust H_∞ performance level γ for all admissible parameter uncertainties.

Theorem 4.1. *Consider system (7), for given quantization density $\gamma > 0$, system (7) with partially unknown transition probabilities Markovian chain is robust stochastically stable with an H_∞ performance level γ under zero initial condition, if there exist matrices*

$X_i > 0$, $\tilde{R}_i > 0$, Y_i, and scalars $\varepsilon_{1i} > 0$ and $\varepsilon_{2i} > 0$, $i \in \mathcal{I}$ satisfying:

$$\Pi_i = \begin{bmatrix} \phi_{11} & * & * & * & * & * & * & * \\ 0 & -\tilde{R}_i & * & * & * & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * & * & * \\ \phi_{41} & W_i^T \bar{A}_{di} X_i & W_i^T B_{wi} & \phi_{44} & * & * & * & * \\ \phi_{51} & \bar{C}_{di} X_i & D_{wi} & \varepsilon_{1i} W_i^T G_{1i} G_{2i}^T & \phi_{55} & * & * & * \\ \phi_{61} & H_{2i} X_i & 0 & 0 & 0 & -\varepsilon_{1i} I & * & * \\ 0 & 0 & 0 & \bar{B}_i^T W_i & \bar{D}_i^T & 0 & -\varepsilon_{2i} I & * \\ Y_i & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-\varepsilon_{2i}}{\delta^2} I \end{bmatrix} < 0 \quad (22)$$

and the suitable controller (6) is $K_i = Y_i X_i^{-1}$ where

$$\begin{aligned} &\phi_{11} = -X_i + (d_m - d_n + 1)\tilde{R}_i && \phi_{41} = W_i^T(\bar{A}_i X_i + \bar{B}_i Y_i) \\ &\phi_{44} = -\mathcal{X} + \varepsilon_{1i} W_i^T G_{1i} G_{1i}^T W_i && \phi_{51} = \bar{C}_i X_i + \bar{D}_i Y_i \\ &\phi_{55} = -I + \varepsilon_{1i} G_{2i} G_{2i}^T && \phi_{61} = H_{1i} X_i + H_{3i} Y_i \end{aligned}$$

Proof: Consider the quantizer effect, (15) can be rewritten as

$$\Pi_i = \begin{bmatrix} -X_i^{-1} + (d_m - d_n + 1)R & * & * & * & * & * \\ 0 & -R & * & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * \\ W_i^T(\bar{A}_i + \bar{B}_i K_i) & W_i^T \bar{A}_{di} & W_i^T B_{wi} & \phi_{44} & * & * \\ \bar{C}_i + \bar{D}_i K_i & \bar{C}_{di} & D_{wi} & \varepsilon_{1i} W_i^T G_{1i} G_{2i}^T & \phi_{55} & * \\ \tilde{H}_{1i} & H_{2i} & 0 & 0 & 0 & -\varepsilon_{1i} I \end{bmatrix} + \begin{bmatrix} 0 & * & * & * & * & * \\ 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & * & * & * \\ W_i^T \bar{B}_i K_i \Delta_k & 0 & 0 & 0 & * & * \\ \bar{D}_i K_i \Delta_k & 0 & 0 & 0 & 0 & * \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0 \quad (23)$$

In view of Lemma 2.1, we obtain that (22) holds, if and only if there exist scalars $\varepsilon_{2i} > 0$, such that

$$\Pi_i = \begin{bmatrix} -X_i^{-1} + (d_m - d_n + 1)R & * & * & * & * & * \\ 0 & -R & * & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * \\ W_i^T(\bar{A}_i + \bar{B}_i K_i) & W_i^T \bar{A}_{di} & W_i^T B_{wi} & \phi_{44} & * & * \\ \bar{C}_i + \bar{D}_i K_i & \bar{C}_{di} & D_{wi} & \varepsilon_{1i} W_i^T G_{1i} G_{2i}^T & \phi_{55} & * \\ \tilde{H}_{1i} & H_{2i} & 0 & 0 & 0 & -\varepsilon_{1i} I \end{bmatrix} + \varepsilon_{2i}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ W_i^T \bar{B}_i \\ \bar{D}_i \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & \bar{B}_i^T W_i & \bar{D}_i^T & 0 \end{bmatrix}$$

$$+\varepsilon_{2i}\begin{bmatrix} K_i^T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\Delta_k^2\begin{bmatrix} K_i & 0 & 0 & 0 & 0 & 0 \end{bmatrix} < 0$$

due to

$$\Delta_k^2 \le \delta^2$$

By Schur Lemma, we can get

$$\Pi_i = \begin{bmatrix} -X_i^{-1}+(d_m-d_n+1)R & * & * & * & * & * & * & * \\ 0 & -R & * & * & * & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * & * & * \\ W_i^T(\bar{A}_i+\bar{B}_iK_i) & W_i^T\bar{A}_{di} & W_i^TB_{wi} & \phi_{44} & * & * & * & * \\ \bar{C}_i+\bar{D}_iK_i & D_{wi} & \varepsilon_{1i}W_i^TG_{1i}G_{2i}^T & \phi_{55} & * & * & * & \\ \tilde{H}_{1i} & H_{2i} & 0 & 0 & 0 & -\varepsilon_{1i}I & * & * \\ 0 & 0 & 0 & \bar{B}_i^TW_i & \bar{D}_i^T & 0 & -\varepsilon_{2i}I & * \\ K_i & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-\varepsilon_{2i}}{\delta^2}I \end{bmatrix} < 0 \tag{24}$$

Letting $K_i = Y_iX_i^{-1}$, multiplying the both sides of (23) by $diag\{X_i,\ X_i,\ I,\ I,\ I,\ I,\ I,\ I\}$, defining new matrix $\tilde{R}_i = X_iRX_i$, we can get condition (17). This completes the proof.

5. **Numerical Example.** In this section, an economic system [27] is considered to show the usefulness of the results above.

Consider system (7) with the following parameters. The system has three modes, $\mathcal{I} = \{1, 2, 3\}$. $d_1 = 1$, $d_2 = 2$, $d_3 = 3$, $\rho = 0.2$. The initial condition is selected as $x(0) = [0 \quad 0]$. The disturbance is a Gauss white noise, and the mode switching governed by partially unknown transition probabilities is supposed to be

$$\begin{pmatrix} 0.1 & ? & ? \\ 0.4 & 0.2 & 0.4 \\ ? & ? & ? \end{pmatrix}$$

the other parameters are set as follows:

$$\bar{A}_1 = \begin{bmatrix} 0 & 1 \\ -2.5 & 3.2 \end{bmatrix} \qquad \bar{A}_2 = \begin{bmatrix} 0 & 1 \\ -43.7 & 45.4 \end{bmatrix}$$

$$\bar{A}_3 = \begin{bmatrix} 0 & 1 \\ 5.3 & -5.2 \end{bmatrix} \qquad \bar{A}_{d1} = \begin{bmatrix} 0.3 & 0.1 \\ 0 & 0.2 \end{bmatrix}$$

$$\bar{A}_{d2} = \begin{bmatrix} 0.2 & 0.1 \\ 0 & 0.1 \end{bmatrix} \qquad \bar{A}_{d3} = \begin{bmatrix} 0 & 0.5 \\ 0 & 0.4 \end{bmatrix}$$

$$\bar{B}_1 = \bar{B}_2 = \bar{B}_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad B_{w1} = B_{w2} = B_{w3}\begin{bmatrix} 0 & 0.1 \\ 0 & 0.1 \end{bmatrix}$$

$$\bar{C}_1 = \bar{C}_2 = \bar{C}_3\begin{bmatrix} 0 & 0.1 \\ 0 & -0.1 \end{bmatrix} \qquad \bar{C}_{d1} = \bar{C}_{d2} = \bar{C}_{d3}\begin{bmatrix} 0.1 & 0 \\ 0.1 & 0 \end{bmatrix}$$

$$\bar{D}_1 = \bar{D}_2 = \bar{D}_3\begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad D_{w1} = D_{w2} = D_{w3} = \begin{bmatrix} 0 & 0.4 \\ 0 & 0.4 \end{bmatrix}$$

$$H_{11} = H_{12} = H_{13} \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0.1 \end{bmatrix} \qquad H_{21} = H_{22} = H_{23} \begin{bmatrix} 0.4 & 0.1 \\ -0.1 & 0.1 \end{bmatrix}$$

$$H_{31} = H_{32} = H_{33} \begin{bmatrix} 0 \\ -0.1 \end{bmatrix} \qquad G_{11} = G_{12} = G_{13} \begin{bmatrix} 0 & 0 \\ 0 & 0.1 \end{bmatrix}$$

$$G_{21} = G_{22} = G_{23} \begin{bmatrix} 0.1 & 0 \\ 0 & 0 \end{bmatrix}$$

Solving (17), the optimal value for H_∞ performance $\gamma = 0.9875$, and the quantized feedback controller gains matrices can be designed as

$$K_1 = \begin{bmatrix} 2.5008 & -3.0367 \end{bmatrix} \quad K_2 = \begin{bmatrix} 2.5000 & -3.0365 \end{bmatrix} \quad K_3 = \begin{bmatrix} 2.5010 & -3.0368 \end{bmatrix}$$

The simulation result of Markov chain is shown in Figure 2. There are three modes in the results, which are stochastic with partially unknown probabilities. The state response of system with disturbance is shown in Figure 3. It should be pointed out that the results in [22], time-delay, quantization and uncertain parameter are considered in system, but they are unavoidable in an economic system. we can also demonstrate that our system with these practical multiple network-induced phenomenons is still with better stochastically stability than those in [21,22]. Due to the complexity of the network disturbance, we enhance the disturbance of system, the results we obtain is less conservative than in [5,9,14]. At the same time it can show that our approach have better results in deal with multiple network-induced phenomenons. The state response of system with double disturbance is shown in Figure 4. It is easily observed that the proposed method has a better robust appearance.

6. **Conclusions.** In this paper, we have present a new approach on H_∞ control problem for uncertain network control system with time-varying delay and quantization. The system is modeled as Markovian jump linear system with mode-dependent time-delay, and

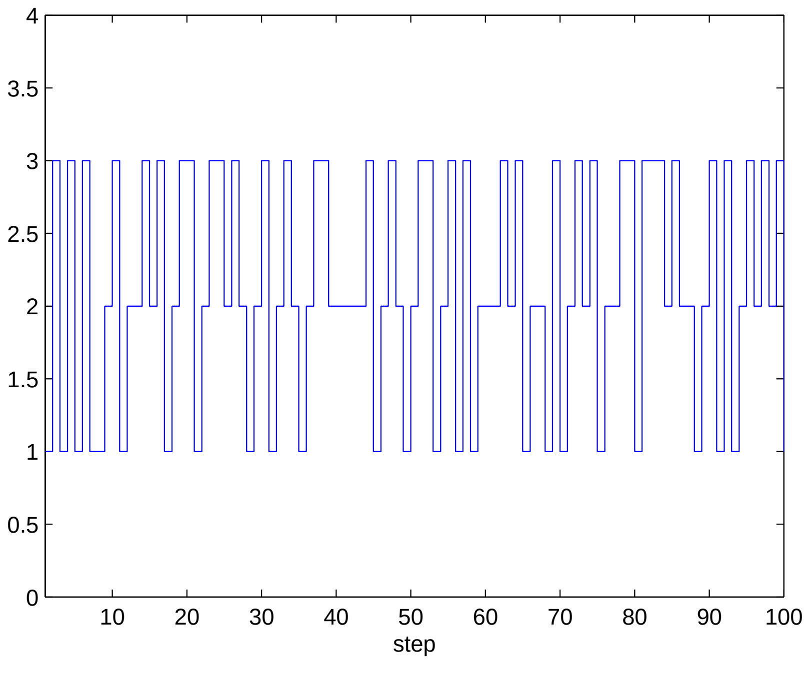

FIGURE 2. Parameters change of $r(k)$

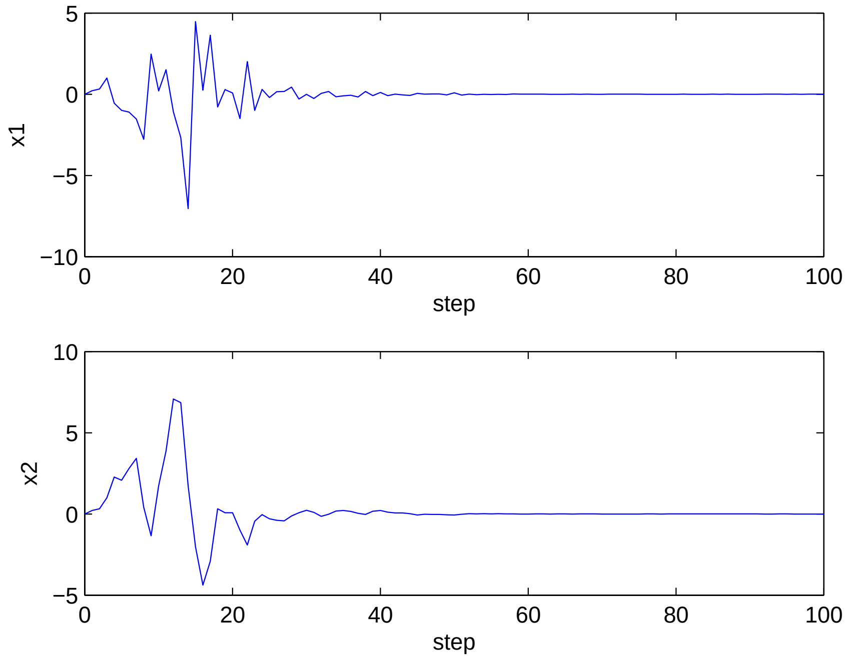

FIGURE 3. The state response of system with disturbance

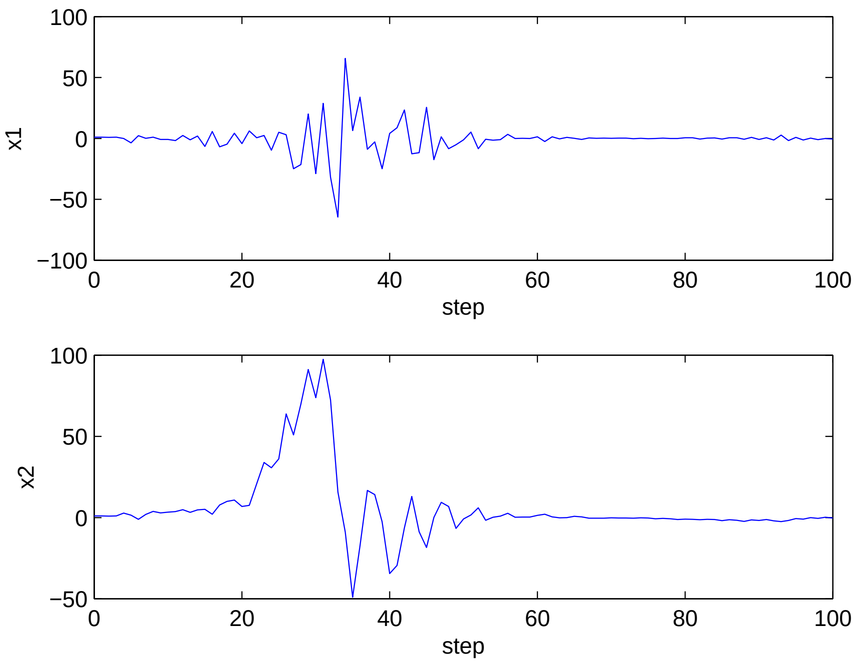

FIGURE 4. The state response of system with double disturbance

the transition probabilities are partially unknown. Based on the new model, sufficient conditions are developed for the robust stochastically stable of the system, and the quantized feedback controller gains are given in LMI form. A numerical example shows the effectiveness of the obtained approach. Further research work will focus on developing the approach to NCSs with nonlinear plant, using fuzzy logic theory to solve problem.

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