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ANALYSIS OF PRODUCTION PROCESSES USING A LEAD-TIME FUNCTION

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Abstract. *Manufacturers can minimize production costs by implementing a dynamic production model that follows a log-normal probability distribution of actual rate of return (RoR) data. Production costs are included in the model that includes internal and external factors introduced by external supplier companies (hereafter called suppliers). In this study, we calculated an expected loss value using a lead-time function and a loss function. We analyzed the changes in lead time in this simulation using a continuous expected loss-value calculation. Observing a production business receiving input from external suppliers, we dynamically modeled the specific production equipment procured from the supplier. To demonstrate the loss function, we present actual data from the production of a control device. Furthermore, we present the actual throughput data for a production flow process with high productivity (using a synchronous method) and in the absence of a production flow process (using an asynchronous method). The production efficiency of the synchronous process becomes clear from the actual data. For further verification, we confirmed the benefit of using the synchronization process to attempt to perform dynamic simulation.*

Keywords: Lead-time function, Production process, Log-normal distribution, Continuous expected loss value

1. Introduction.

For the improvement of the production processes, a conventional Operations Research approach (OR approach) was almost in previous research. To improve a production lead time, there are several studies to shorten production throughput [1, 2]. Moreover, various theories have been applied to improving and reforming production processes and increasing productivity. In [3], an analysis that uses the queuing model and applies a log-normal distribution to modeling a system in the steel industry is described.

Our several previous studies have proposed financial approaches to evaluate a production business including supplier [5, 6, 7].

To evaluate a production process, the lead-time of production system in the production stage by using a stochastic differential equation of the log-normal type, which is derived from its dynamic behavior, is modeled [5]. The use of a mathematical model that focuses on the selection process and adaptation mechanism of the production lead time is used [5]. Using this model and risk-neutral integral, the evaluation equation for the compatibility condition of the production lead time is defined and then calculated. Furthermore, it is clarified that the throughput of the production process was reduced [5, 6].

With respect to determining a throughput rate, an expected value and volatility of throughput of the whole process period is estimated by utilizing Kalman filter theory having been used for a state estimation problem in the control theory [6].

With respect to a physical approach, a state in which the production density of each process corresponded to the physical propagation of heat was introduced in our previous study [4].

Using this approach, the diffusion equation, which dominates the production process was shown. Moreover, we clarified that the production process was dominated by a diffusion equation [4].

To improve a production lead time, there are several studies to shorten production throughput (lead times) [1, 2]. From the time of product ordering, the lead time depends on the work required to make ready for production. Our several research results which were mathematical modelings and the evaluation method of the production processes have been reported.

The synchronization method is superior for improving throughput in production processes, which is used by a production flow process [8]. The production flow process is utilized for production of high-mix low-volume equipments, which are produced through several stages in the production process. This method is good for producing specific control equipment such as semiconductor manufacturing equipment in our experience. Then, we have reported that the production flow process has nonlinear characteristics in our previous study [10].

Moreover, a working-time delay is propagated through the stages in the production process. Its delays are due to volatility in the model. Indeed, the actual data indicated that in the production flow process, the delays were propagated to the successive stages [4].

In this study, we simulate a small-to-midsize firm without sufficient working capital to continue operations. Therefore, we need to raise working capital from financial institutions. Here, we call this cash flow. In essence, the rate of return (RoR) is at least proportional to the production lead time. In other words, if RoR forms a log-normal distribution, it is realistic to assume that the cash flow will also have the same log-normal distribution [5, 11].

We implemented a lead-time function and a loss function to calculate the expected loss value. In other words, it can be assumed that if the cash flow is critically required for lead time, it can be obtained before the production process. Furthermore, it is possible to identify lead times in advance as suitable targets, which is a very innovative approach.

To evaluate the total production of a business, we utilize the actual throughput data of a firm with high productivity and implement a dynamic simulation for evaluation to confirm effectiveness of the synchronous and asynchronous processes.

2. **Production Systems in the Manufacturing Equipment Industry.** The production methods used in manufacturing equipment are briefly covered in this paper. More information is provided in our report [9]. This system is considered to be a "Make-to-order system with version control", which enables manufacturing after orders are received from clients, resulting in "volatility" according to its delivery date and lead time. In addition, there is volatility in the lead time, depending on the content of the make-to-order products (production equipment).

From data for observed monthly rate of return (RoR), we calculate a probability density function (Figure 5) [9]. Results indicate that it conforms to a log-normal distribution (Figure 5, Theoretical). Our previous study provides further information.

$$f(x) = \frac{1}{\sqrt{2\pi}(x - k_p)\sigma_p} \exp\left\{ -\frac{1}{2} \left(\frac{(\ln x - k_p) - m}{\sigma_p} \right)^2 \right\} \tag{1}$$

where k_p is a displacement of x, σ_p is a volatility and m is an average.

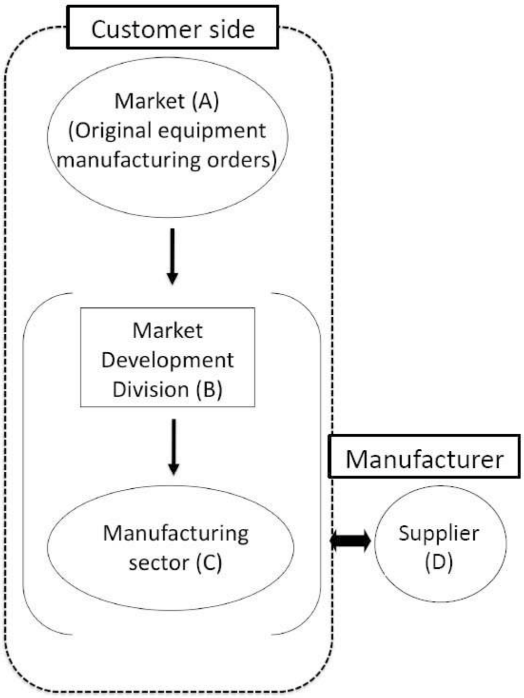

FIGURE 1. Business structure of company of research target

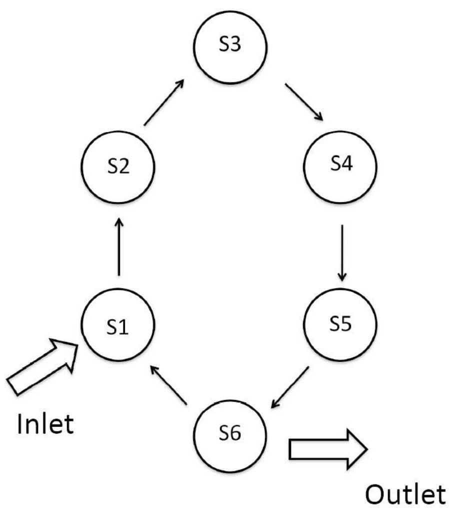

FIGURE 2. Production flow process

Production flow process. A manufacturing process that is termed as a production flow process is shown in Figure 2. The production flow process, which manufactures low volumes of a wide variety of products, is produced through several stages in the production process. In Figure 2, the process consists of six stages. In each step S1-S6 of the manufacturing process, materials are being produced.

The direction of the arrows represents the direction of the production flow. Production materials are supplied through the inlet and the end-product is shipped from the outlet [8].

3. **Lead Time Analysis Using a Lead-Time Function.** The lead-time function $f(y)$ is assumed as a log-normal probability density function so that we can calculate the lead time using a continuous expected value calculation as shown in Figure 4.

Assumption 3.1. *Lead time function of a probability density function with log-normal type.*

$$f(y) \equiv \frac{1}{\sqrt{2\pi}\sigma(y/y_0)} \exp\left\{ -\frac{(\ln(y/y_0) - \mu)^2}{2\sigma^2} \right\} \tag{2}$$

where, μ is an average value, σ is a volatility and y_0 is an initial lead time.

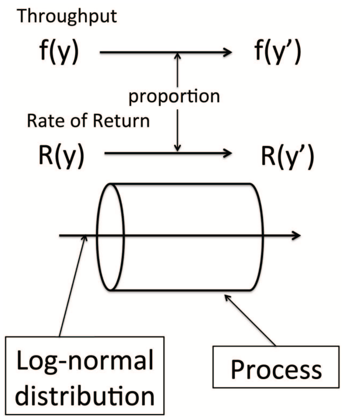

FIGURE 3. Throughput fluctuation in a process distribution amount

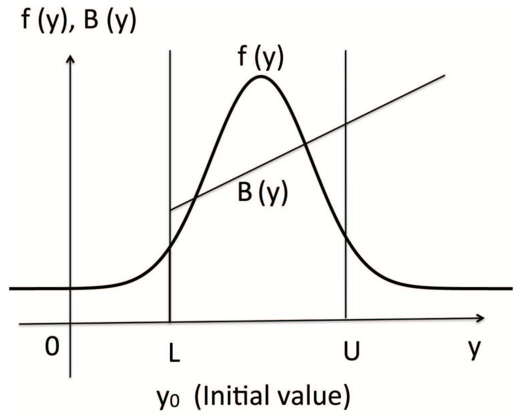

FIGURE 4. Lead time function $f(y)$ and loss function $B(y)$

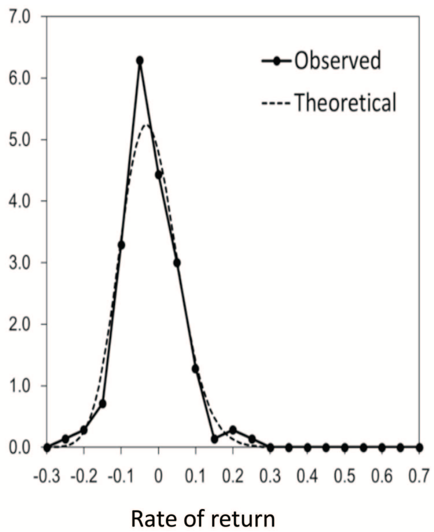

FIGURE 5. Probability density function of rate-of-return deviation: actual data (solid line) and data based on theoretical formula (dotted line)

Now, let F as a cash-in flow and let C_0 as a fixed cost, and we calculate a continuous expected loss value F.

$$\begin{aligned} F &= \int_{-\infty}^{\infty} f(y)B(y)dy + C_0 \\ &= \int_{-\infty}^{L} B(y)f(y)dy + \int_{L}^{U} B(y)f(y)dy + \int_{U}^{\infty} B(y)f(y)dy + C_0 \end{aligned} \tag{3}$$

where,

$$B(y) = py + q, \quad p \geq 0 \tag{4}$$

where p and q are constant parameters. L is a minimal lead time. U is a maximum lead time.

When $y < L$, production activities are not running. When $y > U$, the quantity ordered exceeds the physical limits of the production. Therefore, we must reduce the demand,

and the problem becomes an analysis of $L \le y \le U$.

$$F = \int_L^U (py+q)f(y)dy + C_0 \tag{5}$$

In general, the higher the lead time for a given product is, the lower the throughput is.
Therefore, the second term of Equation (5) is

$$\begin{aligned}(\text{The second term}) &= \int_L^U (py+q)f(y)dy + C_0 \\ &= \int_L^\infty py \cdot f(y)dy - \int_U^\infty q \cdot f(y)dy + C_0 \end{aligned} \tag{6}$$

From Equation (6), the first term of Equation (6) is

$$(\text{The first term}) = p\int_L^\infty f(y)\cdot ydy = \int_L^\infty py \cdot f(y)dy + \int_L^\infty q(U)\cdot f(y)dy \tag{7}$$

$$(\text{The first term}) = \int_L^\infty py \cdot f(y)dy = p\int_L^\infty \frac{1}{\sqrt{2\pi}\sigma(y/y_0)} \exp\left[-\frac{(\ln y - \ln y_0 - \mu)^2}{2\sigma^2}\right] dy \tag{8}$$

In Equation (8), let $\ln y = x$, $y = e^x$ and then $dy = e^x dx$.

$$(\text{The first term}) = p\int_L^\infty y \cdot f(y)dy = p\int_L^\infty \frac{y_0}{\sqrt{2\pi}\sigma} \exp\left[-\frac{(x - \ln y_0 - \mu)^2}{2\sigma^2}\right] e^x dx \tag{9}$$

Further, let $z = (x - \ln y_0 - \mu)/\sigma$ and then $dx = \sigma dz$.
The first term of Equation (9) is

$$\begin{aligned}&(\text{The first term}) \\ &= py_0 \int_{\ln L}^\infty \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}z^2\right) \exp(\sigma z + \ln y_0 + \mu)\cdot \sigma dz \\ &= py_0 \int_{\frac{\ln L - \ln y_0 - \mu}{\sigma}}^\infty \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}z^2\right) \exp(\sigma z + \ln y_0 + \mu) dz \\ &= py_0 \int_{\frac{\ln L - \ln y_0 - \mu}{\sigma}}^\infty \frac{1}{\sqrt{2\pi}} \cdot e^{\sigma z} \cdot e^{\ln y_0 + \mu} dz = py_0^2 \int_{\frac{\ln L - \ln y_0 - \mu}{\sigma}}^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left(z^2 - 2\sigma z + \sigma^2\right) + \frac{1}{2}\sigma^2} \cdot e^{\mu} dz \\ &= py_0^2 \int_{\frac{\ln L - \ln y_0 - \mu}{\sigma}}^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(z-\sigma)^2} \cdot e^{\mu + \frac{1}{2}\sigma^2} dz = py_0^2 e^{\left(\mu + \frac{1}{2}\sigma^2\right)} \Phi\left(\frac{\ln(L/y_0) - (\mu + \sigma^2)}{\sigma}\right) \end{aligned} \tag{10}$$

Applying the same method to the first term of Equation (6), the second term of Equation (6) becomes

$$\begin{aligned}(\text{The second term}) &= q\int_U^\infty f(y)dy \\ &= q\int_U^\infty \frac{1}{\sqrt{2\pi}\sigma(y/y_0)} \exp\left\{-\frac{(\ln y - \ln y_0 - \mu)^2}{2\sigma^2}\right\} dy \end{aligned} \tag{11}$$

In Equation (11), let $\ln y = x$, $y = e^x$ and then $dy = e^x dx$.

$$\begin{aligned}(\text{The second term}) &= qy_0 \int_{\ln U}^\infty \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(x - \ln y_0 - \mu)^2}{2\sigma^2}\right\} dx \\ &= q\int_{\frac{\ln U - \ln y_0 - \mu}{\sigma}}^\infty e^{-\frac{1}{2}z^2} dz = q\Phi\left(\frac{\ln(U/y_0) - \mu}{\sigma}\right) \end{aligned} \tag{12}$$

$$F = +py_0^2 e^{\left(\mu+\frac{1}{2}\sigma^2\right)}\Phi(d_1) - q\Phi(d_2) + C_0 \tag{13}$$

where,

$$d_1 = \frac{\ln(L/y_0) - (\mu+\sigma^2)}{\sigma}, \quad d_2 = \frac{\ln(U/y_0) - \mu}{\sigma}$$

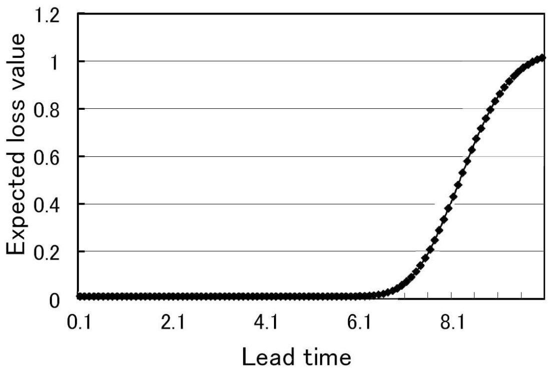

FIGURE 6. The expected loss function value for lead time distribution

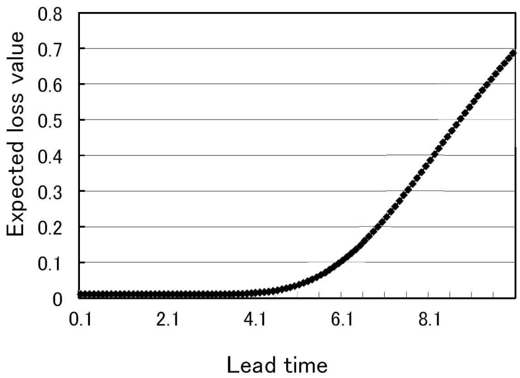

FIGURE 7. The expected loss function for lead time distribution

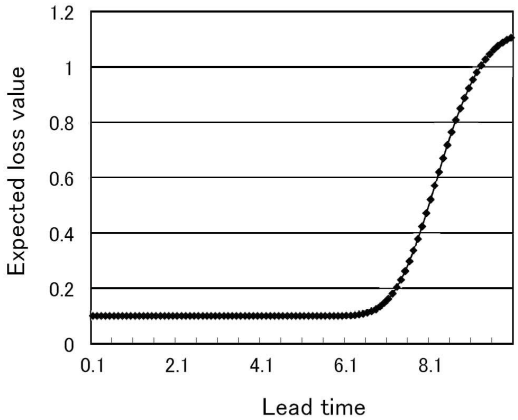

FIGURE 8. The expected loss function value for lead time distribution

TABLE 1. Parameter settings in Figures 6-8

Figure	Average μ	Volatility σ	C_0	y_0^2	p	q
Figure 6	0.9	0.1	0.01	1	0.5	0.2
Figure 7	0.9	0.3	0.01	1	0.5	0.2
Figure 8	0.9	0.1	0.1	1	0.5	0.2

4. **Actual Data Analysis of Test Run 1 through Test Run 3.** Each of the group, A, B, and C, in Tables 5, 7, and 9 indicates the work order in the group. The average (AVE) and standard deviation (STD) of each of the groups are as follows. These data are obtained by calculating the data in Tables 5-10.

In Figure 9, a throughput is proportional to an average value, but is not proportional to STD of Test runs 1, 2, and 3 in Figure 10. The STD is low, which indicates that the production process has high productivity and also has been synchronized. Therefore, the loss function $B(y)$ is practically reasonable as obtained from the actual data.

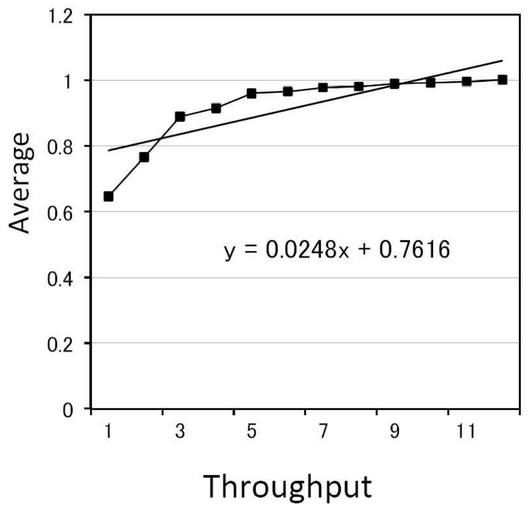

FIGURE 9. Average data of Test runs 1, 2 and 3

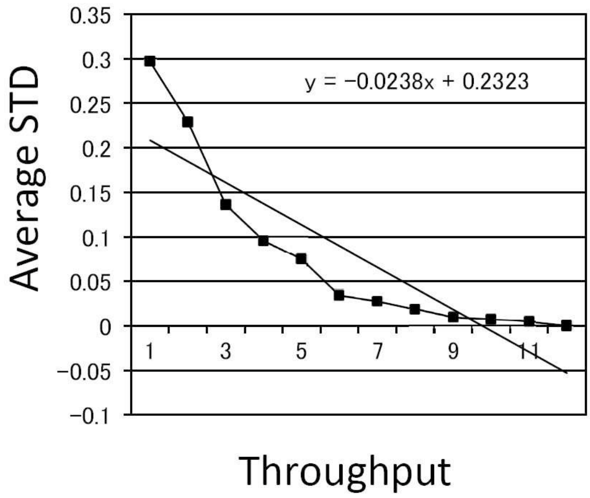

FIGURE 10. STD data of Test runs 1, 2 and 3

TABLE 2. AVE and STD data

	Test run 1		Test run 2		Test run 3	
	AVE	STD	AVE	STD	AVE	STD
A	0.766	0.229	0.916	0.075	0.989	0.009
B	0.980	0.136	0.994	0.005	0.96	0.034
C	0.647	0.297	1	0	0.992	0.007
Average	0.798	0.22	0.97	0.026	0.98	0.017

5. **Numerical Example.** Table 1 presents the parameter settings used to generate Figure 6, Figure 7, and Figure 8, again obtained by numerically calculating Equation (13). As found above, Figure 6 does not yield a larger expected loss value than the lead time in Figure 7. The expected loss value is a useful technique for planning the manufacturing process. If the fixed cost is larger, the expected loss is generated. Thus, volatility and fixed costs should be suppressed as much as possible.

The production throughput is evaluated using the number of equipment pieces in comparison with the target number of equipment pieces (production ranking) and simulating asynchronous and synchronous production (see Appendix). The asynchronous method is prone to worker fluctuations imposed by various delays, whereas worker fluctuations in the synchronous method are small. In terms of the production lead times results presented in the Appendix, the productivity ranking tests indicate that Test run 3 > Test run 2 > Test run 1, where Test run 1 is asynchronous and Test runs 2 and 3 are synchronous.

Here, the throughput values calculated from the throughput probability in Test run 1-Test run 3, are as follows.

- Test run 1: 4.4 (pieces of equipment)/6 (pieces of equipment) = 0.73
- Test run 2: 5.5 (pieces of equipment)/6 (pieces of equipment) = 0.92
- Test run 3: 5.7 (pieces of equipment)/6 (pieces of equipment) = 0.95

6. **Dynamic Simulation of Production Processes.** We attempted to perform a dynamic simulation of the production process by utilizing the simulation system that NTT DATA Mathematical Systems Inc. (www.msi.co.jp) has developed. With respect to the meaning of the individual parts in Figure 11, we conducted a simulation of the following procedure.

- When the simulation began, it generated one of the products on a "generate" parts go to "finish".
- In each process, including the six workers in parallel, the slowest worker waited till the work was completed.
- When the work of each process was completed, it moved to the next process.
- Simultaneously as each process was completed, it recorded the working time of each process.

With respect to Table 3 and Table 4,

- Process No. indicates each process (1-6).
- Average indicates the average time.
- STD indicates the standard deviation of process time (sec).
- Worker efficiency (W.E) indicates the efficiency of six workers.

"record" calculates the worker's operating time, which is obtained by multiplying the specified W.E data for the log-normally distributed random numbers in Table 3.

Figure 12 shows the operating time of processes 1-6 (record1-record6). As the working time of the synchronous process is less volatile, the work efficiency became higher than

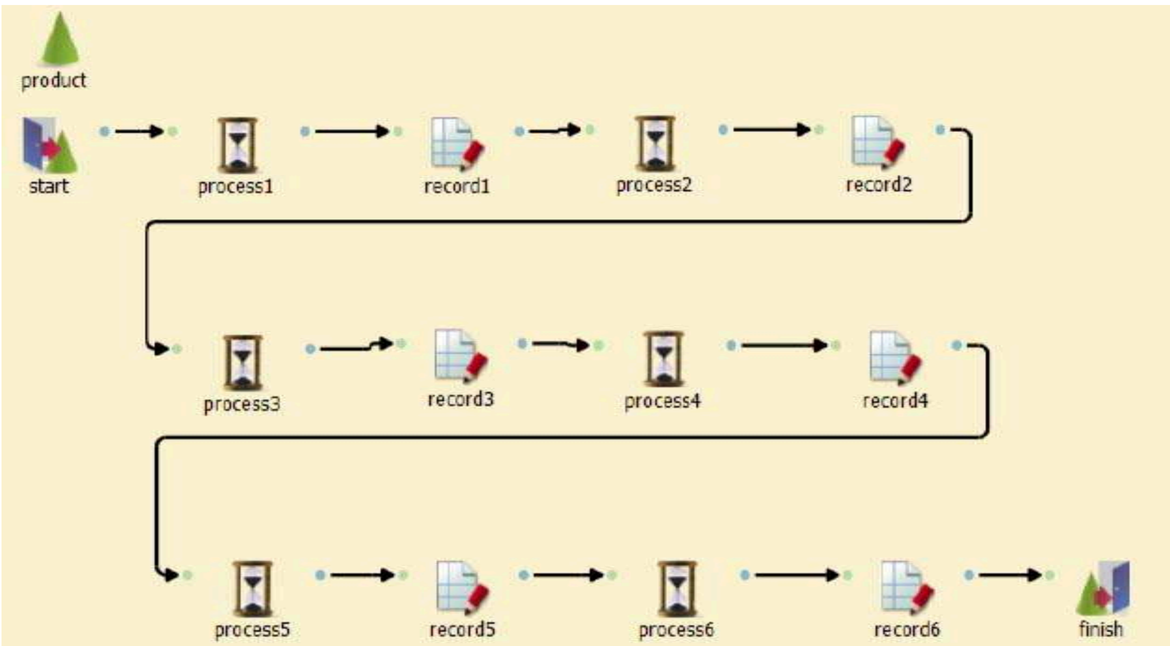

FIGURE 11. Simulation model of production flow system

TABLE 3. Working data for six production asynchronous processes

Process No.	No.1	No.2	No.3	No.4	No.5	No.6
Average	20	22	25	22	25	21
STD	2.1	2.5	1.6	1.9	2.0	1.9
W.E 1	0.83	1.0	0.66	0.76	0.88	0.91
W.E 2	1.27	1.26	1.21	1.31	1.17	1.20
W.E 3	0.96	1.11	1.01	1.12	0.88	0.89
W.E 4	0.92	0.96	1.06	0.98	0.91	0.9
W.E 5	1.2	1.03	1.07	0.89	1.03	1.1
W.E 6	1.09	1.1	1.2	0.98	1.13	0.89

TABLE 4. Working data for six production synchronous processes

Process No.	No.1	No.2	No.3	No.4	No.5	No.6
Average	20	20	20	20	20	20
STD	1.1	1.5	1.2	1.4	1.0	1.4
W.E 1	1.0	1.0	1.0	1.0	1.0	1.0
W.E 2	1.0	1.0	1.2	1.3	1.1	1.2
W.E 3	1.7	1.1	1.0	1.1	1.0	1.0
W.E 4	1.0	1.0	1.0	1.0	1.0	1.0
W.E 5	1.0	1.0	1.0	1.0	1.0	1.0
W.E 6	1.0	1.3	1.2	1.0	1.1	1.0

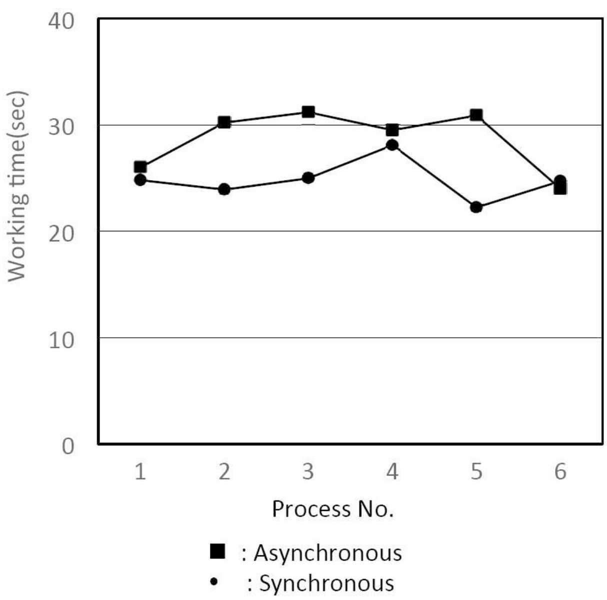

FIGURE 12. Working time for process number one through six

the asynchronous process. In Figure 12, the total working time of asynchronous and synchronous processes is 1241.7 (sec) and 586.4 (sec) respectively. The synchronous process shows more better production efficiency than the asynchronous process.

7. **Conclusions.** The value of a method that can calculate the expected loss value from a lead-time function and a loss function was verified with actual data. Moreover, it was

clarified through theoretical analysis that for improved productivity, it is important to minimize volatility and fixed costs. A comparison between synchronous and asynchronous methods revealed approximately 10% reduction in the results when using synchronous throughput. Therefore, synchronization (using the preprocess method; see Appendix) is recommended.

Using this method, a production company can also design a plan for the production process, and milestones can be added to advance the lead times closer to the target. For further verification, we determined the benefit of using the synchronization process to attempt dynamic simulation.

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Appendix A. Analysis of Actual Data in the Production Flow System. Figure 2 represents a manufacturing process called a flow production system, which is a manufacturing method employed in the production of control equipment. The flow production system, which in this case has six stages, is commercialized by the production of material in steps S1-S6 of the manufacturing process.

The direction of the arrow represents the direction of the production flow. In this system, production materials are supplied from the inlet and the end product will be shipped from the outlet.

Assumption A.1. *The production structure is nonlinear.*

Assumption A.2. *The production structure is a closed structure; that is, the production is driven by a cyclic system (production flow system).*

Assumption A.1 indicates that the determination of the production structure is considered a major factor, which includes the generation value of production or the throughput generation structure in a stochastic manufacturing process (hereafter called the manufacturing field). Because such a structure is at least dependent on the demand, it is considered to have a nonlinear structure.

Because the value of such a product depends on the throughput, its production structure is nonlinear. Therefore, Assumption A.1 reflects the realistic production structure and is somewhat valid. Assumption A.2 is completed in each step and flows from the next step until stage S6 is completed. Assumption A.2 is reasonable because new production starts from S1.

Based on the control equipment, the product can be manufactured in one cycle. The production throughput required to maintain 6 pieces of equipment/day is as follows:

$$\frac{(60 \times 8 - 28)}{3} \times \frac{1}{6} \simeq 25 \text{ (min)} \tag{14}$$

where the throughput of the previous process is set as 20 (min). In Equation (14), "28" represents the throughput of the previous process plus the idle time for synchronization. "8" is the number of processes and the total number of all processes is "8" plus the previous process. "60" is given by 20 (min) $\times$ 3 (cycles).

One process throughput (20 min) in full synchronization is

$$T_s = 3 \times 120 + 40 = 400 \text{ (min)} \tag{15}$$

Therefore, a throughput reduction of about 10% can be achieved. However, the time between processes involves some asynchronous idle time.

As a result, the above test-run is as follows.

- (Test run 1): Each throughput in every process (S1-S6) is asynchronous, and its process throughput is asynchronous. Table 5 represents the manufacturing time (min) in each process. Table 6 represents the variance in each process performed by workers. Table 5 represents the target time, and the theoretical throughput is given by $3 \times 199 + 2 \times 15 = 627$ (min).

  In addition, the total working time in stage S3 is 199 (min), which causes a bottleneck. Figure 13 is a graph illustrating the measurement data in Table 5, and it represents the total working time for each worker (K1-K9). The graph in Figure 14 represents the variance data for each working time in Table 5.
- (Test run 2): Set to synchronously process the throughput.

  The target time in Table 7 is 500 (min), and the theoretical throughput (not including the synchronized idle time) is 400 (min). Table 8 represents the variance data of each working process (S1-S6) for each worker (K1-K9).
- (Test run 3): The process throughput is performed synchronously with the reclassification of the process. The theoretical throughput (not including the synchronized idle time) is 400 (min) in Table 9.

TABLE 5. Total manufacturing time at each stage for each worker

	WS	S1	S2	S3	S4	S5	S6
K1	15	20	20	25	20	20	20
K2	20	22	21	22	21	19	20
K3	10	20	26	25	22	22	26
K4	20	17	15	19	18	16	18
K5	15	15	20	18	16	15	15
K6	15	15	15	15	15	15	15
K7	15	20	20	30	20	21	20
K8	20	29	33	30	29	32	33
K9	15	14	14	15	14	14	14
Total	145	172	184	199	175	174	181

TABLE 6. Volatility of Table 5

K1	1.67	1.67	3.33	1.67	1.67	1.67
K2	2.33	2	2.33	2	1.33	1.67
K3	1.67	3.67	3.33	2.33	2.33	3.67
K4	0.67	0	1.33	1	0.33	1
K5	0	1.67	1	0.33	0	0
K6	0	0	0	0	0	0
K7	1.67	1.67	5	1.67	2	1.67
K8	4.67	6	5	4.67	5.67	6
K9	0.33	0.33	0	0.33	0.33	0.33

TABLE 7. Total manufacturing time at each stage for each worker

	WS	S1	S2	S3	S4	S5	S6
K1	20	20	24	20	20	20	20
K2	20	20	20	20	20	22	20
K3	20	20	20	20	20	20	20
K4	20	25	25	20	20	20	20
K5	20	20	20	20	20	20	20
K6	20	20	20	20	20	20	20
K7	20	20	20	20	20	20	20
K8	20	27	27	22	23	20	20
K9	20	20	20	20	20	20	20
Total	180	192	196	182	183	182	180

Table 10 represents the variance data of Table 9. "WS" in the measurement tables represents the standard working time. This is an empirical value obtained from long-term experiments.

Table 8. Volatility of Table 7

K1	0	1.33	0	0	0	0
K2	0	0	0	0	0.67	0
K3	0	0	0	0	0	0
K4	1.67	1.67	0	0	0	0
K5	0	0	0	0	0	0
K6	0	0	0	0	0	0
K7	0	0	0	0	0	0
K8	2.33	2.33	0.67	1	0	0
K9	0	0	0	0	0	0

Table 9. Total manufacturing time at each stage for each worker

	WS	S1	S2	S3	S4	S5	S6
K1	20	18	19	18	20	20	20
K2	20	18	18	18	20	20	20
K3	20	21	21	21	20	20	20
K4	20	13	11	11	20	20	20
K5	20	16	16	17	20	20	20
K6	20	18	18	18	20	20	20
K7	20	14	14	13	20	20	20
K8	20	22	22	20	20	20	20
K9	20	25	25	25	20	20	20
Total	180	165	164	161	180	180	180

Table 10. Variance of Table 9

K1	0.67	0.33	0.67	0	0	0
K2	0.67	0.67	0.67	0	0	0
K3	0.33	0.33	0.33	0	0	0
K4	2.33	3	3	0	0	0
K5	1.33	1.33	1	0	0	0
K6	0.67	0.67	0.67	0	0	0
K7	2	2	2.33	0	0	0
K8	0.67	0.67	0	0	0	0
K9	1.67	1.67	1.67	0	0	0

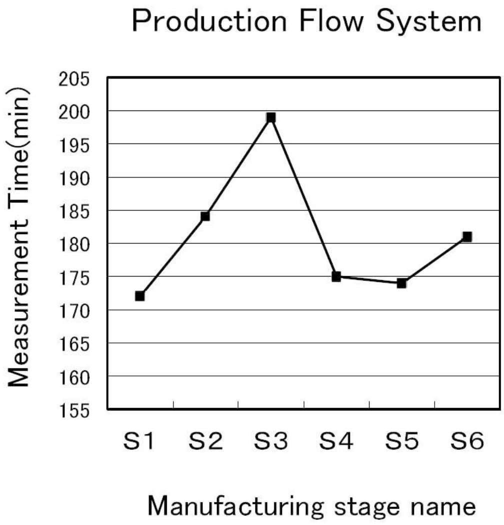

FIGURE 13. Total work time for each stage (S1-S6) in Table 5

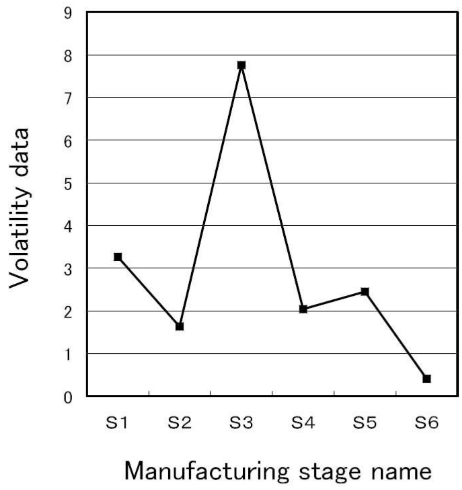

FIGURE 14. Volatility data for each stage (S1-S6) in Table 5